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# A Generic Instrumental Variable Approach for Industrial Robots Identification

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***Abstract***— This paper deals with the important topic of industrial robots identification. The usual identification method is based on the inverse dynamic identification model and least squares technique. This method has been successfully applied on several industrial robots. Good results are obtained provided a well-tuned derivative bandpass filtering of joint positions to calculate joint velocities and accelerations is used. However, we can doubt whether the bandpass filtering is well tuned or not. An alternative is instrumental variable techniques that are robust to data filtering and statistically optimal. In this paper, we propose a generic instrumental variable approach suitable for robots identification. Instruments set is the inverse dynamic model built from simulated data calculated from simulation of the direct dynamic model. The simulation is based on previous estimates and assumes the same reference trajectories and the same control law structure for both actual and simulated robots. At last, gains of simulated controller are updated according to instrumental variable estimates to obtain a valid instruments set at each step of the algorithm. The proposed approach validates the inverse and direct dynamic models at the same time, is not sensitive to initial conditions and has a fast convergence. Experimental results obtained on a six degrees of freedom industrial robot show the effectiveness of our approach: 60 dynamic parameters are identified in 3 iterations.

***Index Terms***—Closed loop identification, Instrumental variable method, Rigid robot dynamics.

## I. INTRODUCTION

THE usual robots identification method is based on the inverse dynamic identification model and least squares technique. This method, called Inverse Dynamic Identification Model – Least Squares method (IDIM – LS), has been successfully applied to identify inertial parameters of several prototypes and industrial robots [1][2][3][4][5] among others. Good results are obtained provided a well-tuned derivative bandpass filtering of joint positions to calculate joint velocities and accelerations is used. However, even with guidelines for bandpass filtering tuning given in [5], we can doubt whether IDIM – LS estimates are unbiased or not.

This leads us to try other identification methods: the Extended Kalman Filter (EKF) [6], the Set Membership Uncertainty [7], an algorithm based on Linear Matrix Inequality (LMI) tools [8] and a maximum likelihood (ML) approach [9][10]. However, these techniques do not really improve IDIM – LS method and they were not validated on 6 degrees of freedom (DOF) industrial robots.

Another approach is the Instrumental Variable technique (IV) introduced by Reiersøl in 1941 [12]. In [13][14][20][21][24], IV methods are studied for linear systems. However, these works are mostly theoretically oriented and validated on low dimensional linear systems. This may explain why there are few real world applications, especially in robotics [25][26]. This shows that a gap must be bridged between theory and control engineering practices.

In this paper, we propose a generic IV approach relevant for identification of any rigid robot. Instruments set is the inverse dynamic model built from simulated data calculated from simulation of the direct dynamic model. The simulation assumes the same reference trajectories and the same control law structure for both actual and simulated robots and is based on previous IV estimates. This defines an iterative algorithm stopping when convergence is achieved. Finally, gains of simulated controller are updated according to IV estimates to obtain a valid instruments set at each step of the algorithm. This algorithm, called IDIM – IV, validates the inverse and direct dynamic models of robot at the same time, improves the noise immunity of estimates with respect to corrupted data in the observation matrix resulting from noisy measurements and/or bad tuning of joint positions bandpass filtering, is not sensitive to initialization and has a fast convergence.

A condensed version of this work has been presented in [15] and [22]. This paper contains detailed proofs to enlighten the theoretical understanding of IDIM – IV method, gives additional experimental results and deals with experimental validation of statistical assumptions.

The paper is organized as follows: Section II reviews the usual identification technique IDIM – LS. Section III presents IDIM – IV identification method. Tests that check statistical assumptions are introduced in section IV. The modeling and experimental identification of TX40 robot are presented in section V.

Finally, section VI is the conclusion.

## II. IDIM: INVERSE DYNAMIC IDENTIFICATION MODEL TECHNIQUE

In this part, we recall the main steps of IDIM – LS identification method. Full details can be found in [23].

### A. Inverse dynamic model of robots

The inverse dynamic model (IDM) of  $n$  moving links robot calculates the  $(n \times 1)$  joint torques vector  $\boldsymbol{\tau}$  as a function of generalized coordinates and their derivatives [16]:

$$\boldsymbol{\tau} = \mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{N}(\mathbf{q}, \dot{\mathbf{q}}) \quad (1)$$

Where  $\mathbf{q}$ ,  $\dot{\mathbf{q}}$  and  $\ddot{\mathbf{q}}$  are respectively the  $(n \times 1)$  vectors of generalized joint positions, velocities and accelerations;  $\mathbf{M}(\mathbf{q})$  is the  $(n \times n)$  matrix of robot inertia;  $\mathbf{N}(\mathbf{q}, \dot{\mathbf{q}})$  is the  $(n \times 1)$  vector of centrifugal, Coriolis, gravitational and friction torques.

The modified Denavit and Hartenberg (DHM) notation allows obtaining a IDM linear in relation to a set of base dynamic parameters  $\boldsymbol{\beta}$ :

$$\boldsymbol{\tau} = \mathbf{IDM}(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}})\boldsymbol{\beta} \quad (2)$$

Where  $\mathbf{IDM}(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}})$  is the  $(n \times b)$  matrix of basis functions of bodies dynamics and  $\boldsymbol{\beta}$  is the  $(b \times 1)$  vector of base parameters.

Equation (2) represents the Inverse Dynamic Identification Model (IDIM). Base parameters are the minimum number of dynamic parameters from which the IDM can be calculated. They are obtained from standard dynamic parameters by regrouping some of them with linear relations [17][18]. Standard parameters of a link  $j$  are:  $XX_j$ ,  $XY_j$ ,  $XZ_j$ ,  $YY_j$ ,  $YZ_j$  and  $ZZ_j$  the six components of the inertia matrix of link  $j$  at the origin of frame  $j$ ;  $MX_j$ ,  $MY_j$  and  $MZ_j$  the components of the first moment of link  $j$ ;  $M_j$  the mass of link  $j$ ;  $Ia_j$  a total inertia moment for rotor and gears of actuator  $j$ ;  $Fv_j$  and  $Fc_j$  the viscous and Coulomb friction parameters of joint  $j$ .

### B. Data acquisition

Usually, data available from robots controllers are the following: measurements of  $\mathbf{q}$  and measurements of the  $(n \times 1)$  control signals vector  $\mathbf{v}_\tau$  calculated according to control law. Robots are mostly position – controlled. Control laws widely used in robotics are PD, PID, computed torque (flatness control) and passive controls [16]. When identifying base parameters, PD control is preferred to the others because it is

easy to tune and excellent tracking is not necessary [23]. Motors actuating moving links are PI current – controlled. Current closed loop has a bandwidth greater than 500(Hz). Then, in the frequency range of dynamics (less than 10(Hz)), its transfer function is modeled as a static gain [23]. Control signal of motor  $j$ ,  $v_{\tau_j}$ , is connected with the motor  $j$  current reference.  $\tau$  is given by:

$$\tau = \mathbf{G}_\tau \mathbf{v}_\tau \quad (3)$$

Where  $\mathbf{G}_\tau$  is the  $(n \times n)$  diagonal matrix of drive gains. Diagonal components of  $\mathbf{G}_\tau$  have *a priori* values given by manufacturers that can be checked with special tests.

### C. Data filtering

In (2),  $\mathbf{q}$  is estimated with  $\hat{\mathbf{q}}$  obtained by filtering measurements of  $\mathbf{q}$  through a lowpass Butterworth filter in both the forward and reverse directions using *filtfilt* Matlab function.  $(\hat{\dot{\mathbf{q}}}, \hat{\ddot{\mathbf{q}}})$  are calculated with a central differentiation algorithm of  $\hat{\mathbf{q}}$ . In doing so, we avoid distortion when calculating  $\mathbf{IDM}(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}})$  coefficients. This point is discussed in [5].  $\mathbf{IDIM}$  given by (2) is sampled at a measurement frequency  $f_m$  while robot is tracking some reference trajectories  $(\mathbf{q}_r, \dot{\mathbf{q}}_r, \ddot{\mathbf{q}}_r)$ .

$\tau$  being perturbed by high frequency disturbances and since there is no high frequency information because of lowpass filtered data  $(\hat{\mathbf{q}}, \hat{\dot{\mathbf{q}}}, \hat{\ddot{\mathbf{q}}})$ , a parallel decimation procedure is used to eliminate torque ripples and information free high frequency samples. The parallel decimation is carried out with *decimate* Matlab function. This point is discussed in [5] too.

### D. IDIM – LS estimates

After data acquisition, sampling and parallel decimation, we obtain the following over-determined linear system:

$$\mathbf{Y}(\tau) = \mathbf{W}(\hat{\mathbf{q}}, \hat{\dot{\mathbf{q}}}, \hat{\ddot{\mathbf{q}}}) \boldsymbol{\beta} + \boldsymbol{\rho} \quad (4)$$

Where  $\mathbf{Y}(\tau)$  is the  $(r \times 1)$  measurements vector built from actual torques  $\tau$ ;  $\mathbf{W}(\hat{\mathbf{q}}, \hat{\dot{\mathbf{q}}}, \hat{\ddot{\mathbf{q}}})$  is the  $(r \times b)$  observation matrix built from  $\mathbf{IDM}(\hat{\mathbf{q}}, \hat{\dot{\mathbf{q}}}, \hat{\ddot{\mathbf{q}}})$ ;  $\boldsymbol{\rho}$  is the  $(r \times 1)$  vector of error terms;  $r$  is the number of rows in (4).

In (4)  $\mathbf{Y}$  and  $\mathbf{W}$ , equations of each joint  $j$  are regrouped together. Thus,  $\mathbf{Y}$  and  $\mathbf{W}$  are partitioned so that:

$$\mathbf{Y}(\tau) = \begin{bmatrix} \mathbf{Y}^1 \\ \vdots \\ \mathbf{Y}^n \end{bmatrix}, \quad \mathbf{W}(\hat{\mathbf{q}}, \hat{\dot{\mathbf{q}}}, \hat{\ddot{\mathbf{q}}}) = \begin{bmatrix} \mathbf{W}^1 \\ \vdots \\ \mathbf{W}^n \end{bmatrix}, \quad \text{with } \mathbf{Y}^j = \begin{bmatrix} \tau_j(1) \\ \vdots \\ \tau_j(n_e) \end{bmatrix}, \quad \mathbf{W}^j = \begin{bmatrix} \mathbf{IDM}^j(\hat{\mathbf{q}}(1), \hat{\dot{\mathbf{q}}}(1), \hat{\ddot{\mathbf{q}}}(1)) \\ \vdots \\ \mathbf{IDM}^j(\hat{\mathbf{q}}(n_e), \hat{\dot{\mathbf{q}}}(n_e), \hat{\ddot{\mathbf{q}}}(n_e)) \end{bmatrix}, \quad \mathbf{IDM}^j(\hat{\mathbf{q}}(\cdot), \hat{\dot{\mathbf{q}}}(\cdot), \hat{\ddot{\mathbf{q}}}(\cdot)) \text{ is}$$

the  $j^{\text{th}}$  row of the  $(n \times b)$  matrix of the basis functions  $\text{IDM}(\hat{\mathbf{q}}(\cdot), \hat{\mathbf{q}}(\cdot), \hat{\mathbf{q}}(\cdot))$  given by (2).  $\mathbf{Y}^j$  and  $\mathbf{W}^j$  represent the  $n_e$  equations of a subsystem  $j$ ,  $n_e$  being the number of rows in  $\mathbf{Y}^j$  and  $\mathbf{W}^j$ .

Using base parameters and “exciting” reference trajectories [1][19], we get a well conditioned matrix  $\mathbf{W}$ . A good conditioning number of  $\mathbf{W}$  means that base parameters are well excited and they can be well identified.

### E. Statistical analysis

$\boldsymbol{\rho}$  is assumed to have zero mean, to be serially uncorrelated and to be heteroskedastic e.g. to have a diagonal covariance matrix  $\boldsymbol{\Omega}$  partitioned so that:

$$\boldsymbol{\Omega} = \text{diag}(\sigma_1^2 \mathbf{I}_{n_e} \quad \cdots \quad \sigma_j^2 \mathbf{I}_{n_e} \quad \cdots \quad \sigma_n^2 \mathbf{I}_{n_e})$$

Where  $\mathbf{I}_{n_e}$  is the  $(n_e \times n_e)$  identity matrix. Heteroskedasticity assumption is based on the fact that robots are non linear MIMO systems.

$\sigma_j^2$  is the error variance calculated from subsystem  $j$  OLS solution e.g. :

$$\mathbf{Y}^j = \mathbf{W}^j(\hat{\mathbf{q}}, \hat{\mathbf{q}}, \hat{\mathbf{q}})\boldsymbol{\beta} + \boldsymbol{\rho}^j \quad (5)$$

Thus, the Weighted LS (WLS) estimator is used to estimate  $\boldsymbol{\beta}$ . The WLS solution of (4) is given by:

$$\hat{\boldsymbol{\beta}}_{LS} = (\mathbf{W}^T \boldsymbol{\Omega}^{-1} \mathbf{W})^{-1} \mathbf{W}^T \boldsymbol{\Omega}^{-1} \mathbf{Y} \quad (6)$$

Usually, such weighting operations normalize error terms in (4). Indeed, with:

$$\bar{\boldsymbol{\rho}} = \boldsymbol{\Omega}^{-1/2} \boldsymbol{\rho} \quad (7)$$

One obtains  $\boldsymbol{\Sigma}_{\bar{\boldsymbol{\rho}}\bar{\boldsymbol{\rho}}} = E(\bar{\boldsymbol{\rho}}\bar{\boldsymbol{\rho}}^T) = \boldsymbol{\Omega}^{-1/2} E(\boldsymbol{\rho}\boldsymbol{\rho}^T) \boldsymbol{\Omega}^{-1/2} = \mathbf{I}_r$ .

The estimated covariance matrix of the WLS estimator is:

$$\boldsymbol{\Sigma}_{LS} = (\mathbf{W}^T \boldsymbol{\Omega}^{-1} \mathbf{W})^{-1} \quad (8)$$

$\hat{\sigma}_{\hat{\boldsymbol{\beta}}_{LS}(i)}^2 = \boldsymbol{\Sigma}_{LS}(i, i)$  is the  $i^{\text{th}}$  diagonal coefficient of  $\boldsymbol{\Sigma}_{LS}$ . The relative standard deviation  $\% \hat{\sigma}_{\hat{\boldsymbol{\beta}}_{LS}(i)}$  of  $\hat{\boldsymbol{\beta}}_{LS}(i)$ , the  $i^{\text{th}}$  component of  $\hat{\boldsymbol{\beta}}_{LS}$ , is then given by:

$$\% \hat{\sigma}_{\hat{\boldsymbol{\beta}}_{LS}(i)} = \frac{100 * \hat{\sigma}_{\hat{\boldsymbol{\beta}}_{LS}(i)}}{|\hat{\boldsymbol{\beta}}_{LS}(i)|} \text{ for } |\hat{\boldsymbol{\beta}}_{LS}(i)| \neq 0 \quad (9)$$

IDIM – LS identification method was successfully applied on several prototypes and industrial robots

(see the references given in introduction). This identification method is illustrated in Fig. 1.

*F. Limitations of DIDIM – LS method*

However, to provide unbiased results, measurements of  $\mathbf{q}$  and  $\mathbf{v}_\tau$  must be accurate enough at high sampling rate. Furthermore, the bandpass filtering described in this part must be well tuned. At last, the direct dynamic model (DDM) given by (10) is validated *a posteriori*.

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} = \boldsymbol{\tau} - \mathbf{N}(\mathbf{q}, \dot{\mathbf{q}}) \tag{10}$$

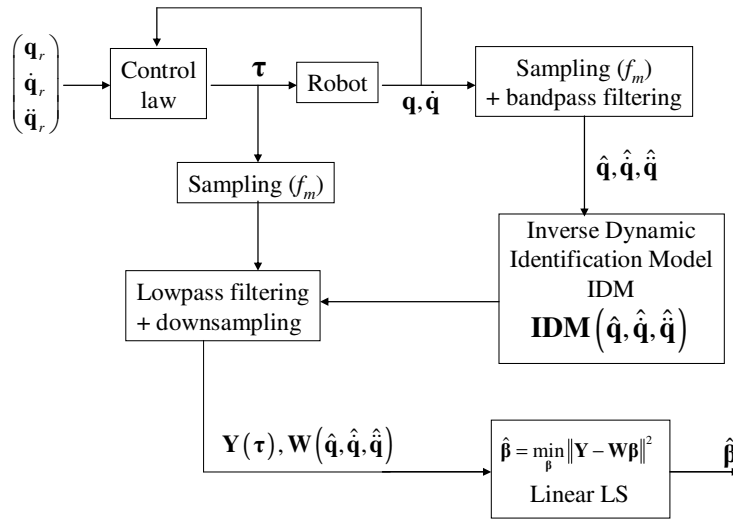


Fig. 1. IDIM LS Identification scheme

An alternative for eliminating bias of IDIM – LS estimates is the Instrumental Variable (IV) method that deals with the problem of noisy observation matrix and can be statistically optimal.

*G. Brief theoretical background of IV method*

It is well known that LS estimates are unbiased if the following assumption holds [11], chapter 7:

$$E(\mathbf{W}^T \boldsymbol{\rho}) = 0 \tag{11}$$

Where  $E(\cdot)$  is the expectation operator.

In this case,  $\mathbf{W}$  is not correlated with  $\boldsymbol{\rho}$ . A violation of assumption (11) leads to biased LS estimates [11].  $\mathbf{W}(\hat{\mathbf{q}}, \hat{\mathbf{q}}_d, \hat{\mathbf{q}}_{dd})$  being built from noisy measured data, we can doubt whether  $\mathbf{W}(\hat{\mathbf{q}}, \hat{\mathbf{q}}_d, \hat{\mathbf{q}}_{dd})$  is correlated with  $\boldsymbol{\rho}$  or not, even if the bandpass filtering data described in the previous section is used. That is the reason why it is interesting to use the IV method introduced by Reiersøl in 1941 [12]. This method consists in introducing

an  $(r \times b)$  instrumental matrix denoted  $\mathbf{Z}$  such that (4) becomes:

$$\mathbf{Z}^T \mathbf{Y} = \mathbf{Z}^T \mathbf{W} \boldsymbol{\beta} + \mathbf{Z}^T \boldsymbol{\rho}$$

We make the following assumptions:

$$E(\mathbf{Z}^T \mathbf{W}) \text{ exists, is finite and of full rank } b \quad (12)$$

$$E(\mathbf{Z}^T \boldsymbol{\rho}) = 0 \quad (13)$$

Then, the simple IV estimator provides unbiased estimates given by:

$$\hat{\boldsymbol{\beta}}_{SIV} = (\mathbf{Z}^T \mathbf{W})^{-1} \mathbf{Z}^T \mathbf{Y}$$

IV methods were widely studied and applied to linear systems, Box – Jenkins model especially, [13][14][20][21][24] among others. According to these works, a good manner consists in building  $\mathbf{Z}$  from only simulated data. These simulated data are outputs of an auxiliary model which is the noise – free mathematical model of the system to be identified. Instruments can be constructed on previous IV estimates denoted as  $\hat{\boldsymbol{\beta}}_{IV}^{k-1}$ . This defines an iterative process stopping when convergence is reached.

However, these works are mostly theoretically oriented and validated on low dimensional linear systems. Furthermore, in many real world applications, these methods cannot be used as is. This may explain why IV methods are rarely employed in robotics, see [25][26].

In the following section, we aim at bridging the gap between theory and control engineering practices by proposing a generic IV approach relevant for rigid robots identification.

### III. INSTRUMENTAL VARIABLE APPROACH FOR ROBOTS IDENTIFICATION

#### A. Choice of a valid instrumental variable matrix

Because we have  $\text{rank}(\mathbf{Z}) = \text{rank}(\mathbf{W}) = b$ , the system is called “just identified”. In this case, the true model is assumed to be:

$$\mathbf{Y} = \mathbf{W}(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}) \boldsymbol{\beta} + \mathbf{e}_y$$

Where  $\mathbf{e}_y$  is the  $(r \times 1)$  vector of measurement noise,  $\mathbf{W}(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}})$  is the noise – free observation matrix denoted as  $\mathbf{W}_{nf}$  from now. We make the following assumption:  $E(\mathbf{W}_{nf}^T \mathbf{e}_y) = 0$ . We have the following relation:

$$\mathbf{W} = \mathbf{W}_{nf} + \mathbf{V}$$

Where  $\mathbf{V}$  is a  $(r \times b)$  matrix of error terms uncorrelated with  $\mathbf{W}_{nf}$  and  $\mathbf{e}_y$ , that is  $E(\mathbf{V}^T \mathbf{W}_{nf}) = 0$  and

$E(\mathbf{V}^T \mathbf{e}_y) = 0$ . Since we have  $\boldsymbol{\rho} = \mathbf{e}_y - \mathbf{V}\boldsymbol{\beta}$ ,  $\mathbf{W}$  is correlated with  $\boldsymbol{\rho}$  leading to biased LS estimates. A  $(r \times b)$  valid instrumental matrix is:

$$\mathbf{Z} = \mathbf{W}_{nf} = \mathbf{W}(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}) \quad (14)$$

To show that, the IDM is assumed to well specified. Hence, we obtain:

$$\mathbf{W} = \mathbf{Z} + \mathbf{V} = \mathbf{W}_{nf} + \mathbf{V}$$

Thus, it follows:

$$E(\mathbf{Z}^T \mathbf{W}) = E(\mathbf{W}_{nf}^T \mathbf{W}_{nf}) + E(\mathbf{W}_{nf}^T \mathbf{V}) = E(\mathbf{W}_{nf}^T \mathbf{W}_{nf})$$

Finally, the following relations hold:

$$\text{rank}(E(\mathbf{Z}^T \mathbf{W})) = \text{rank}(E(\mathbf{W}_{nf}^T \mathbf{W}_{nf})) = b \quad (15)$$

$$E(\mathbf{Z}^T \boldsymbol{\rho}) = E(\mathbf{W}_{nf}^T \boldsymbol{\rho}) = 0 \quad (16)$$

$$\text{Indeed: } E(\mathbf{W}_{nf}^T \boldsymbol{\rho}) = E(\mathbf{W}_{nf}^T \mathbf{e}_y) - E(\mathbf{W}_{nf}^T \mathbf{V})\boldsymbol{\beta} = 0.$$

Hence, with  $\mathbf{Z} = \mathbf{W}_{nf}$ , assumptions (12) and (13) hold. Now, we must choose and simulate a valid auxiliary model to build an instrumental matrix  $\hat{\mathbf{Z}}$  that is as close as possible to  $\mathbf{Z}$  given by (14).

### B. Choice and simulation of a valid auxiliary model

For robots, the auxiliary model is the DDM (direct dynamic model) given by (10). Simulation of the DDM is performed assuming same reference trajectories and the same control law structure for both actual and simulated robots. In addition, simulation of DDM is based on previous IV estimates. Hence, at step  $k$ , where  $k$  is the  $k^{\text{th}}$  IV estimates, simulated joint accelerations are given by:

$$\mathbf{M}(\mathbf{q}_s, \hat{\boldsymbol{\beta}}_{IV}^{k-1}) \ddot{\mathbf{q}}_s = (\boldsymbol{\tau}_s - \mathbf{N}(\mathbf{q}_s, \dot{\mathbf{q}}_s, \hat{\boldsymbol{\beta}}_{IV}^{k-1})) \quad (17)$$

By integrating (17), we get the  $(n \times 1)$  vectors of simulated joint velocities  $\dot{\mathbf{q}}_s$  and positions  $\mathbf{q}_s$ . The  $(n \times 1)$  vector of simulated torque  $\boldsymbol{\tau}_s$  is given by  $\boldsymbol{\tau}_s = \mathbf{G}_\tau \mathbf{v}_{\tau,s}$ , where  $\mathbf{v}_{\tau,s}$  is the  $(n \times 1)$  vector of simulated control signals calculated according to the control law.

Like measurements, simulated data are sampled at a measurement frequency  $f_m$ . The  $(n_m \times b)$  instrumental variable matrix is then  $\hat{\mathbf{Z}}_{f_m} = \mathbf{W}_{\delta f_m}(\mathbf{q}_s, \dot{\mathbf{q}}_s, \ddot{\mathbf{q}}_s, \hat{\boldsymbol{\beta}}_{IV}^{k-1})$ . Where  $\mathbf{W}_{\delta f_m}(\mathbf{q}_s, \dot{\mathbf{q}}_s, \ddot{\mathbf{q}}_s, \hat{\boldsymbol{\beta}}_{IV}^{k-1})$  is the  $(n_m \times b)$  sampled matrix of  $\text{IDM}(\mathbf{q}_s, \dot{\mathbf{q}}_s, \ddot{\mathbf{q}}_s, \hat{\boldsymbol{\beta}}_{IV}^{k-1})$ .

Each column of  $\hat{\mathbf{Z}}_{fm}$  is resampled at a lower rate (parallel decimation). Then we have:

$$\hat{\mathbf{Z}} = \mathbf{W}_\delta(\mathbf{q}_s, \dot{\mathbf{q}}_s, \ddot{\mathbf{q}}_s, \hat{\boldsymbol{\beta}}_{IV}^{k-1})$$

Compared with IDIM – LS and the other methods cited in introduction, IDIM – IV uses IDM and DDM. Thus, both DDM and IDM are validated at the same time. This is the first contribution of our approach.

Unfortunately, a simple simulation of the DDM to get  $\mathbf{q}_s$ ,  $\dot{\mathbf{q}}_s$  and  $\ddot{\mathbf{q}}_s$  is not enough to build  $\mathbf{Z}$  defined by (14). Indeed, simulation of the DDM is based on previous IV estimates  $\hat{\boldsymbol{\beta}}_{IV}^{k-1}$  and we may obtain  $\hat{\mathbf{Z}} \neq \mathbf{Z} = \mathbf{W}_{nf}$ . So, the choice of initial values  $\hat{\boldsymbol{\beta}}_{IV}^0$  is crucial even if IV algorithms are known to be quite robust to initialization [21]. In fact, a bad choice of  $\hat{\boldsymbol{\beta}}_{IV}^0$  may lead to algorithm divergence or invalid IV estimates because of violation of relations (12) and (13).

Thus, we propose an insensitive to initial conditions IV algorithm. This assumes that the following condition:

$$\left( \mathbf{q}_s(\hat{\boldsymbol{\beta}}_{IV}^k), \dot{\mathbf{q}}_s(\hat{\boldsymbol{\beta}}_{IV}^k), \ddot{\mathbf{q}}_s(\hat{\boldsymbol{\beta}}_{IV}^k) \right) \approx (\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}) \nabla \hat{\boldsymbol{\beta}}_{IV}^k \quad (18)$$

is satisfied at any iteration  $k$ , starting with  $k = 0$ .

This is possible by tacking the same control law structure for both actual and simulated robots with the same performances given by bandwidth, stability margin or closed loop poles. Because parameters of the simulated robot, e.g.  $\hat{\boldsymbol{\beta}}_{IV}^k$ , change at each iteration  $k$ , gains of the simulated controller must be updated according to  $\hat{\boldsymbol{\beta}}_{IV}^k$ .

For example, let us consider a PD control law for each joint  $j$ . The inverse dynamic model (IDM) (1) for the joint  $j$ , can be written as a decoupled double integrator perturbed by a coupling torque, such that:

$$\boldsymbol{\tau}_j = \mathbf{M}_{j,j}(\mathbf{q})\ddot{\mathbf{q}}_j - \mathbf{p}_j$$

$\mathbf{p}_j$  is considered as a perturbation given by:

$$\mathbf{p}_j = -\sum_{i \neq j}^n \mathbf{M}_{j,i}(\mathbf{q})\ddot{\mathbf{q}}_i - \mathbf{N}_j(\mathbf{q}, \dot{\mathbf{q}})$$

$\mathbf{M}_{j,i}(\mathbf{q})$  is approximated by a constant inertia  $J_j$ , given by:

$$J_j = ZZ_j + Ia_j + \max_q \left( \mathbf{M}_{j,i}(\mathbf{q}) - ZZ_j - Ia_j \right)$$

$J_j$  is the maximum value of inertia moment with respect to  $\mathbf{q}$ . This gives the smallest stability margin of the closed loop second order transfer function while  $\mathbf{q}$  varies. It must be taken at least as  $ZZ_j + Ia_j$  which can be calculated from *a priori* CAD values. The DDM of a joint  $j$  is approximated by a double integrator as following:

$$\ddot{\mathbf{q}}_j = \frac{(\boldsymbol{\tau}_j + \mathbf{p}_j)}{\mathbf{M}_{j,j}(\mathbf{q})} \approx \frac{(\boldsymbol{\tau}_j + \mathbf{p}_j)}{J_j}$$

We understand it makes sense to use linear techniques to tune closed – loop performances of each joint  $j$  closed – loop.

Let us consider the joint  $j$  PD control of the actual robot illustrated in Fig. 2.

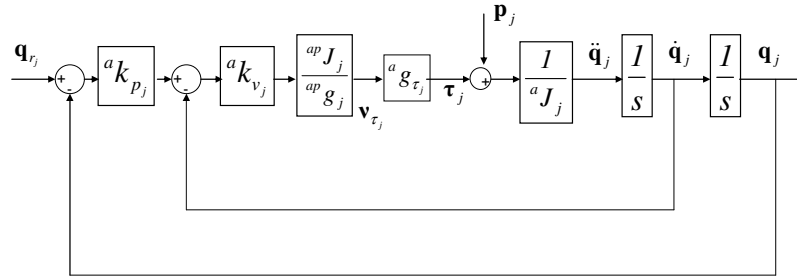


Fig. 2. Joint PD control of the actual robot

The control input is given by:

$$\mathbf{v}_{\tau_j} = \left( {}^a k_{p_j} {}^a k_{v_j} (\mathbf{q}_{r_j} - \mathbf{q}_j) - {}^a k_{v_j} \dot{\mathbf{q}}_j \right) \frac{{}^{ap} J_j}{{}^{ap} g_{\tau_j}} \quad (19)$$

$\tau_j$  is given by:

$$\tau_j = {}^a g_{\tau_j} \mathbf{v}_{\tau_j} \quad (20)$$

Where  ${}^a g_{\tau_j}$  is the actual drive gain,  ${}^a J_j$  is the actual value of  $J_j$ ,  ${}^{ap} J_j$  and  ${}^{ap} g_{\tau_j}$  are *a priori* values of the actual unknown values  ${}^a J_j$  and  ${}^a g_{\tau_j}$  respectively.

If *a priori* values are equal to actual ones, then  ${}^a k_{p_j}$  and  ${}^a k_{v_j}$  are the PD control gains of the normalized double integrator system  $1/s^2$ . Closed – loop performances are chosen with the desired 2 poles of the second order closed – loop transfer function characterized by  ${}^d \omega_{n_j}$  and  ${}^d \zeta_j$ .  ${}^d \omega_{n_j}$  is the desired natural frequency which characterizes the closed – loop bandwidth and  ${}^d \zeta_j$  is the desired damping which



$\forall \hat{\boldsymbol{\beta}}_{IV}^k$ . This algorithm not sensitive to initial values  $\hat{\boldsymbol{\beta}}_{IV}^0$  is the second contribution of our approach.

In the following sections, it is reasonable to make the following approximation  $\hat{\mathbf{Z}} \approx \mathbf{Z}$ .

### C. Algorithm initialization

In [23], it is proposed to take a regular inertia matrix  $\mathbf{M}(\mathbf{q}_s, \hat{\boldsymbol{\beta}}_{IV}^0)$  to have a good initialization for numerical integration of the DDM. It is obtained with:

$$\hat{\boldsymbol{\beta}}_{IV}^0 = 0, \text{ except for, } Ia_j^0 = 1, \text{ for } j = 1, n \quad (24)$$

### D. Calculation of IDIM – IV estimates

After data acquisition, data filtering and parallel decimation, we obtain:

$$\mathbf{Z}^T \mathbf{Y}(\tau) = \mathbf{Z}^T \mathbf{W}(\hat{\mathbf{q}}, \hat{\dot{\mathbf{q}}}, \hat{\ddot{\mathbf{q}}}) \boldsymbol{\beta} + \mathbf{Z}^T \boldsymbol{\rho}$$

Where  $\mathbf{Z}$  is our  $(r \times b)$  instrumental variable matrix given by (23).  $\mathbf{Y}$  and  $\mathbf{W}$  are defined by (4).

In  $\mathbf{Y}$ ,  $\mathbf{W}$  and  $\mathbf{Z}$ , equations of each joint  $j$  are regrouped together. Thus, like  $\mathbf{Y}$  and  $\mathbf{W}$ ,  $\mathbf{Z}$  is partitioned so that:

$$\mathbf{Z} = \begin{bmatrix} \mathbf{Z}^1 \\ \vdots \\ \mathbf{Z}^n \end{bmatrix} \text{ with } \mathbf{Z}^j = \begin{bmatrix} \mathbf{IDM}^j(\mathbf{q}_s(1), \dot{\mathbf{q}}_s(1), \ddot{\mathbf{q}}_s(1)) \\ \vdots \\ \mathbf{IDM}^j(\mathbf{q}_s(n_e), \dot{\mathbf{q}}_s(n_e), \ddot{\mathbf{q}}_s(n_e)) \end{bmatrix},$$

$\mathbf{IDM}^j(\mathbf{q}_s(\cdot), \dot{\mathbf{q}}_s(\cdot), \ddot{\mathbf{q}}_s(\cdot))$  is the  $j^{\text{th}}$  row of the  $(n \times b)$  matrix of the basis functions  $\mathbf{IDM}(\mathbf{q}_s(\cdot), \dot{\mathbf{q}}_s(\cdot), \ddot{\mathbf{q}}_s(\cdot))$  given by (2). Partitions of  $\mathbf{Y}$  and  $\mathbf{W}$  are given in (4).

$\mathbf{Y}^j$ ,  $\mathbf{W}^j$  and  $\mathbf{Z}^j$  represent the  $n_e$  equations of a subsystem  $j$ .

Because  $\boldsymbol{\rho}$  is assumed to be heteroskedastic (see II), IV estimates are given by:

$$\hat{\boldsymbol{\beta}}_{IV}^k = (\mathbf{Z}^T \boldsymbol{\Omega}^{-1} \mathbf{W})^{-1} \mathbf{Z}^T \boldsymbol{\Omega}^{-1} \mathbf{Y} \quad (25)$$

This solution is called Weighted IV estimates (WIV).

Like with LS techniques, such weighting operations normalize error terms. However, when using IDIM – IV method,  $\sigma_j^2$  is the error variance calculated from subsystem  $j$  IV solution e.g. :

$$(\mathbf{Z}^j)^T \mathbf{Y}^j = (\mathbf{Z}^j)^T \mathbf{W}^j(\hat{\mathbf{q}}, \hat{\dot{\mathbf{q}}}, \hat{\ddot{\mathbf{q}}}) \boldsymbol{\beta} + (\mathbf{Z}^j)^T \boldsymbol{\rho}^j$$

The covariance matrix of IV estimates is given by:

$$\boldsymbol{\Sigma}_{IV} = (\mathbf{Z}^T \boldsymbol{\Omega}^{-1} \mathbf{Z})^{-1}$$

$\hat{\sigma}_{\hat{\beta}_{IV}^k(i)}^2 = \Sigma_{IV}(i, i)$  is the  $i^{\text{th}}$  diagonal coefficient of  $\Sigma_{IV}$ . Relative standard deviation  $\% \hat{\sigma}_{\hat{\beta}_{IV}^k(i)}$  is given by:

$$\% \hat{\sigma}_{\hat{\beta}_{IV}^k(i)} = \frac{100 * \hat{\sigma}_{\hat{\beta}_{IV}^k(i)}}{|\hat{\beta}_{IV}^k(i)|} \text{ for } |\hat{\beta}_{IV}^k(i)| \neq 0 \quad (26)$$

Where  $\hat{\beta}_{IV}^k(i)$  is an IDIM – IV estimation of  $\beta(i)$  at step  $k$ .

#### E. Convergence criterion

This process is iterated until its convergence:

$$\frac{\|\mathbf{p}_k\| - \|\mathbf{p}_{k-1}\|}{\|\mathbf{p}_{k-1}\|} \leq tol_1 \text{ and } \max_{i=1, \dots, b} \frac{|\hat{\beta}_{IV}^k(i) - \hat{\beta}_{IV}^{k-1}(i)|}{|\hat{\beta}_{IV}^{k-1}(i)|} \leq tol_2 \quad (27)$$

Where  $\|\mathbf{p}_k\|$  is the 2-norm of  $\mathbf{p}$  at step  $k$ .

The parameters  $tol_1$  and  $tol_2$  are values ideally chosen to be small to get fast convergence with good accuracy. A good compromise consists in choosing  $tol_1$  and  $tol_2$  between 2.5% and 5.0%.

#### F. Algorithm of the IDIM – IV identification method

The scheme of IDIM – IV method is illustrated in Fig. 4.

IDIM – IV identification algorithm is summarized as follows:

*Compute the inverse and direct dynamic models with SYMORO+ software;*

*Compute  $\mathbf{W}$  and  $\mathbf{Y}$  according to (4);*

*Step 0: initialize IDIM – IV with the regular initialization given by (24);*

*While*  $\left( \frac{\|\mathbf{p}_k\| - \|\mathbf{p}_{k-1}\|}{\|\mathbf{p}_{k-1}\|} \geq tol_1 \text{ \& \& } \max_{i=1, \dots, b} \frac{|\hat{\beta}_{IV}^k(i) - \hat{\beta}_{IV}^{k-1}(i)|}{|\hat{\beta}_{IV}^{k-1}(i)|} \geq tol_2 \right)$  *do:*

*Simulate the DDM by updating gains of the simulated controller with (22);*

*Compute  $\hat{\mathbf{Z}} \approx \mathbf{Z} = \mathbf{W}_{nf}$  as described in section B;*

*Compute IV solution with (25);*

*End of while.*

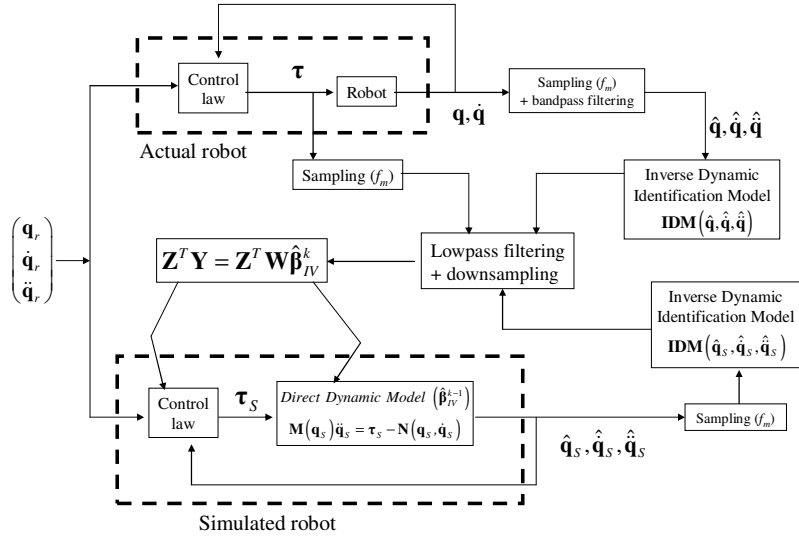


Fig. 4. Scheme of the IDIM – IV identification method

Since DDM and IDM can be calculated with SYMORO+ software, IDIM – IV is a “fully – automated” identification method. This is the third contribution of our approach.

#### IV. VALIDATION OF STATISTICAL ASSUMPTIONS AND MODEL REDUCTION

##### A. Introduction

In many papers, statistical assumptions are not verified while estimates quality depends on them. This is particularly true when one faces real world systems identification.

In Statistics, statistical tests are carried out with homoskedastic errors [11]. That is the reason why the tests described in this sections use  $\bar{\rho}$  and not  $\rho$ . In addition, according to (7), if statistical assumptions made on  $\bar{\rho}$  hold, those made on  $\rho$  hold too.

##### B. Statistical assumptions made on $\bar{\rho}$

###### 1) Normality of $\bar{\rho}$

Normality assumption is crucial to get unbiased estimates. If this assumption is violated, then both LS and IV theories do not work. The Kolmogorov – Smirnov test (KS – test) allows doing that [11].

KS – test is a nonparametric test for equality of continuous one dimensional probability distribution that can be used to compare a sample with a reference probability distribution. The KS – test quantifies a distance between the empirical distribution function (EDF) of the sample and the cumulative distribution function (CDF) of the reference distribution.

In our case,  $\bar{\rho}$  resulting from a normalization of  $\rho$ , the reference distribution is  $N(0,1)$ . The null

hypothesis is then  $H_0 : \bar{\rho} \sim N(0,1)$ . The EDF of  $\bar{\rho}$  is compared with the CDF of the reference distribution via a KS – test with a 0.05 level of significance. KS – test is carried out with *kstest* MATALB function.

If the KS – test rejects  $H_0$ , something goes wrong with measurements, bandpass filtering is badly scaled or IDM is misspecified. For misspecified IDM, some effects such as non linear frictions, stiffness and so forth, are missing and they must be included (see [30] for a good overview).

## 2) Independent samples test

It is important to check if samples are independent or not. Indeed, if  $\bar{\rho}$  is serially correlated, then relative deviations given by (9) or (26) are no longer consistent because they are underestimated.

A simple way to find correlation between samples consists in using linear regressions. For the  $i^{\text{th}}$  sample of  $\bar{\rho}$ , one writes :

$$\bar{\rho}(i) = a_1 \bar{\rho}(i-1) + a_2 \bar{\rho}(i-2) + \dots + a_p \bar{\rho}(i-p)$$

Where each  $a_i$  is a coefficient and  $p$  is an dependence order chosen by the user with  $p \ll n_e$ .

We obtain the following over-determined linear system:

$$\mathbf{y}_\rho = \mathbf{\Phi} \mathbf{a} + \mathbf{u} \quad (28)$$

Where  $\mathbf{y}_\rho = \begin{bmatrix} \bar{\rho}(p+1) \\ \vdots \\ \bar{\rho}(n_e) \end{bmatrix}$ ,  $\mathbf{\Phi} = \begin{bmatrix} \bar{\rho}(p) & \dots & \bar{\rho}(1) \\ \vdots & & \vdots \\ \bar{\rho}(n_e-1) & \dots & \bar{\rho}(n_e-p) \end{bmatrix}$ ,  $\mathbf{a} = \begin{bmatrix} a_1 \\ \vdots \\ a_p \end{bmatrix}$  and  $\mathbf{u}$  is the error assumed to be serially

uncorrelated and to have zero mean.

Estimates of  $\mathbf{a}$ ,  $\hat{\mathbf{a}}$ , are LS solution of (28):

$$\hat{\mathbf{a}} = (\mathbf{\Phi}^T \mathbf{\Phi})^{-1} \mathbf{\Phi}^T \mathbf{y}_\rho \quad (29)$$

The readjusted  $R_{\bar{\rho}}^2$  is given by [11]:

$$R_{\bar{\rho}}^2 = 1 - \frac{\|\mathbf{y}_\rho - \mathbf{\Phi} \hat{\mathbf{a}}\|^2}{\|\mathbf{y}_\rho\|^2} \quad (30)$$

$\bar{\rho}$  is serially uncorrelated if each  $\hat{a}_i$  is close to zero with large deviation and if  $R_{\bar{\rho}}^2$  is close to zero (typically less than 0.1). Roughly speaking, in this case, columns of  $\mathbf{\Phi}$  do not explains variations observed on  $\mathbf{y}_\rho$ .

If  $\bar{\rho}$  is serially correlated, then information free useless samples are present and they must be removed. The parallel filter cutoff frequency must be therefore rescaled according to the order  $p$ .

### C. Model reduction

Some dynamic parameters remain poorly identifiable because they have a poor contribution on dynamics. They can be cancelled to simplify inverse and direct models.

In papers written by Gautier and Khalil, it is suggested that parameters such that  $\% \hat{\sigma}_{\hat{\beta}_{ls}}$  is greater than a bound between 20% and 30% are cancelled to keep a set of essential parameters of a simplified dynamic model without loss of accuracy (see [3] for instance). However, there is neither formal proof nor test that validates or rejects such statement.

In statistics, the F – statistic is widely used to validate/invalidate model reduction [11]. It is assumed that  $H_0 : \bar{\mathbf{p}} \sim N(0,1)$  holds. From  $b$  base parameters,  $bc$  parameters may constitute the set of essential parameters. The F – statistic is used as follows:

1. First, one runs IDIM – IV method with the  $b$  base parameters and one computes  $\|\bar{\mathbf{p}}\|$ ;
2. Second, one runs IDIM – IV method with the  $bc$  essential parameters and one computes  $\|\bar{\mathbf{p}}_c\|$ , the error norm obtained with the reduced model;

3. Third, one calculates  $\hat{F} = \frac{(\|\bar{\mathbf{p}}_c\|^2 - \|\bar{\mathbf{p}}\|^2)}{\|\bar{\mathbf{p}}\|^2} \left( \frac{n_e - b}{b - bc} \right)$ .

If  $\hat{F}$  is less than or compatible with  $F_{(1-\alpha), (b-bc), (n_e-b)}$ , then the F – statistic accepts the model reduction. Otherwise, the model reduction is rejected. The F – statistic is carried out with *vartest2* MATLAB function.

Parameters that show the largest relative deviations are eliminated first and this process is executed in a decreasing way ( $\% \hat{\sigma}_{\hat{\beta}_{vw}} = 60\%$ , ...,  $\% \hat{\sigma}_{\hat{\beta}_{vw}} = 30\%$ ) until the F – statistic fails.

It is important to note that IDIM – IV method is used instead of IDIM – LS because IDIM – LS estimates may be biased. It is also suggested to perform the KS – test to check the normality of  $\bar{\mathbf{p}}_c$ . If the KS – test fails, it does not make sense to run the F – statistic.

## V. EXPERIMENTAL IDENTIFICATION RESULTS

### A. Modeling of TX40 Robot

Stäubli TX40 robot has a serial structure with six rotational joints. Robot kinematics is defined using the modified Denavit and Hartenberg notation (see Fig. 5).

Geometric parameters defining TX40 frames are given in Table 1:  $\sigma_j = 0$  means that joint  $j$  is rotational;  $\alpha_j$  and  $d_j$  give respectively the angle and distance between  $z_{j-1}$  and  $z_j$  along  $x_{j-1}$ ;  $\theta_j$  and  $r_j$  give respectively the angle and the distance between  $x_{j-1}$  and  $x_j$  along  $z_j$ . Because all joints are rotational,  $\theta_j$  is

the position variable of joint  $j$  e.g.  $\mathbf{q}_j = \theta_j$ .

TX40 robot is characterized by a coupling between joints 5 and 6. This coupling effect adds two additional parameters:  $f_{v_{m6}}$  the motor 6 viscous friction coefficient and  $f_{c_{m6}}$  the motor 6 dry friction coefficient. Full details about TX40 modeling are given in [27]. TX40 has 60 base dynamic parameters. The columns of  $\mathbf{IDM}(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}})$  in (2) are obtained using the Newton – Euler recursive algorithm. SYMORO+ software is used to automatically calculate the customized symbolic expressions of models [16].

TABLE 1  
GEOMETRIC PARAMETERS OF TX40 ROBOT

j	$\sigma_j$	$\alpha_j$	$d_j$	$\theta_j$	$r_j$
1	0	0	0	$\theta_1$	0
2	0	$-\pi/2$	0	$\theta_2$	0
3	0	0	$d_3 = 0.225\text{m}$	$\theta_3$	$r_3 = 0.035\text{m}$
4	0	$\pi/2$	0	$\theta_4$	$r_4 = 0.225\text{m}$
5	0	$-\pi/2$	0	$\theta_5$	0
6	0	$\pi/2$	0	$\theta_6$	0

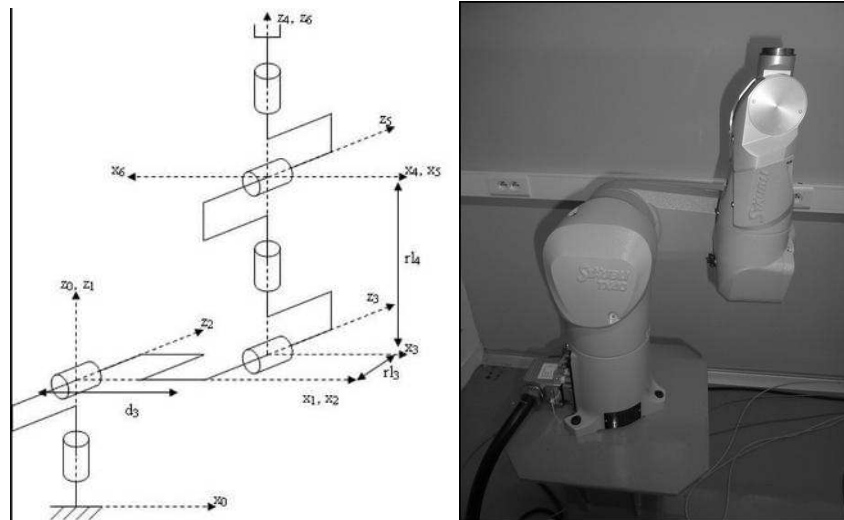


Fig. 5. Link frames of TX40 Stäubli robot

Joint positions and control signals are stored with a sampling frequency measurement  $f_m = 5\text{KHz}$ .

Robot simulation is carried out with the same reference trajectories and with the same PD control law structure as actual TX40 robot. In addition, gains of the simulated controller are updated with (22), at each

step of IDIM – IV. IDIM – IV identification method is initialized with all base parameters equal to 0 except  $Ia_j = 1$  for  $j \neq 5$  and  $Ia_5 = 2$  because of the coupling effect. At last, we choose  $tol_1 = tol_2 = 2.5\%$ .

We use a *C MEX S-Function of SIMULINK* on a 2011 laptop PC with INTEL i7 CPU to run the DDM simulation. For a 8s trajectory duration, one step of IDIM – IV takes 3.5s.

Reference trajectories  $(\mathbf{q}_r, \dot{\mathbf{q}}_r, \ddot{\mathbf{q}}_r)$  are fifth order polynomials. Since we have  $cond(\mathbf{W}) = 200$ , reference trajectories excite well all base parameters [19].

### B. IDIM-LS and IDIM – IV methods with well tuned bandpass filtering

IDIM – LS and IDIM – IV methods are carried out with a filtered position  $\hat{\mathbf{q}}$ , calculated with a 50(Hz) cut-off frequency forward and reverse fourth order Butterworth filter and with velocities  $\dot{\hat{\mathbf{q}}}$  and accelerations  $\ddot{\hat{\mathbf{q}}}$ , calculated with a central difference algorithm of  $\hat{\mathbf{q}}$ . Butterworth filter is tuned according to rules given in [5][23] and recalled in section II. The maximum bandwidth for joint 6 is  $\omega_{dyn} = 10(\text{Hz})$  leading to choose  $\omega_{fq} > 5 * \omega_{dyn}$ ,  $\omega_{fq} > 50(\text{Hz})$ ,  $\omega_{fq}$  being the filter cutoff frequency. Then we choose a 50(Hz) cut-off frequency.

Parallel decimation is carried out with a lowpass Tchebyshef filter with a cutoff frequency  $\omega_{fp} > 2 * \omega_{dyn}$ ,  $\omega_{fp} > 20(\text{Hz})$ ,  $\omega_{fp}$  being the parallel filter cutoff frequency. Then we choose a 10(Hz) cut-off frequency. According to the relation  $\omega_{fp} = 2 * \pi * 0.8 * f_m / (2 * n_d)$ , the sample rate  $f_m$  is divided by  $n_d = 100$ .

The normality assumption of  $\bar{\boldsymbol{\rho}}$  is validated because the KS – test accepts  $H_0 : \bar{\boldsymbol{\rho}} \sim N(0,1)$ . The histogram of  $\bar{\boldsymbol{\rho}}$  obtained with IDIM – IV method is plotted in Fig. 7. It matches a Gaussian distribution and we have  $\hat{\sigma}_{\rho,LS} = \hat{\sigma}_{\rho,IV} = 1.03 \approx 1.0$ . So, errors terms in  $\boldsymbol{\rho}$  are normalized and heteroskedasticity is well taken into account. Furthermore, there is no missing effect such as non linear friction or stiffness. IDM is well specified. The test of independency described in section IV was run. We have  $R_{\bar{\boldsymbol{\rho}}}^2 = 0.05 < 0.1$  and coefficients  $a_i$  are small with large relative deviations. So, samples of  $\bar{\boldsymbol{\rho}}$  can be considered independent. Finally, all statistical assumptions made on  $\boldsymbol{\rho}$  hold in practice.

IDIM – LS and IDIM – IV estimates are given in Table 2 and Table 3 respectively. IDIM – IV method needs only 3 steps to converge (see Table 4). F – statistic accepts to cancel parameters such that  $\% \hat{\sigma}_{\hat{\beta}_{LS}(i)}$  or  $\% \hat{\sigma}_{\hat{\beta}_{IV}(i)}$  is greater than 30%. Indeed, we have:  $\|\bar{\boldsymbol{\rho}}\| = 48.5$ ,  $\|\bar{\boldsymbol{\rho}}_c\| = 49$ ,  $n_e = 2160$ ,  $b = 60$  and  $b_c = 28$  leading to  $\left( \frac{49^2 - 48.5^2}{48.5^2} \right) \left( \frac{2160 - 60}{60 - 28} \right) \approx 1.4$ . This value is smaller than  $F_{0.95,32,2100} \approx 1.6$ . Finally, from 60 base parameters, only 28 are well identified with good relative standard deviation. These parameters define a set of essential

dynamic parameters.

IDIM – LS estimates match those provided by IDIM – IV method. According to the Hausman's theory [28], IDIM – LS estimates are unbiased. Like the other identification methods cited in introduction, IDIM – IV does not improve IDIM – LS coupled with good bandpass filtering data. In fact, in such case, we can write  $\mathbf{W}(\hat{\mathbf{q}}, \hat{\mathbf{q}}, \hat{\mathbf{q}}) \approx \mathbf{W}(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}) = \mathbf{W}_{nf}$  and this explains why IDIM – LS estimates are unbiased.

Direct comparisons have been performed (see Fig. 6). Estimated torques constructed with IDIM – LS and IDIM – IV estimates fit measured torques. Since we have  $\|\mathbf{Y} - \mathbf{W}\hat{\boldsymbol{\beta}}_{LS}\|/\|\mathbf{Y}\| = 5\%$  and  $\|\mathbf{Y} - \mathbf{Z}\hat{\boldsymbol{\beta}}_{IV}\|/\|\mathbf{Y}\| = 6\%$ , identification results are of good quality.

Norm of error relative to filtered joint position calculated at each step  $k$  and for each axis  $j$  is given in Table 5. Since these relative errors are very small, less than 0.2%, relation (18) is always satisfied. This result emphasizes effectiveness of gains updating of simulated controller given by (22).

TABLE 2:  
IDIM – LS ESTIMATES

	$\hat{\boldsymbol{\beta}}_{LS}$	$\% \hat{\sigma}_{\hat{\boldsymbol{\beta}}_{LS}}$		$\hat{\boldsymbol{\beta}}_{LS}$	$\% \hat{\sigma}_{\hat{\boldsymbol{\beta}}_{LS}}$
ZZ <sub>1R</sub>	1.25	1.1	Fc <sub>3</sub>	6.10	1.8
FV <sub>1</sub>	8.18	0.6	MX <sub>4</sub>	-0.02	16.0
Fc <sub>1</sub>	6.57	2.2	Ia <sub>4</sub>	0.03	8.8
XX <sub>2R</sub>	-0.48	2.6	FV <sub>4</sub>	1.14	1.4
XZ <sub>2R</sub>	-0.16	4.3	Fc <sub>4</sub>	2.30	2.5
ZZ <sub>2R</sub>	1.08	1.0	MY <sub>5R</sub>	-0.03	13.0
MX <sub>2R</sub>	2.20	2.5	Ia <sub>5</sub>	0.04	8.8
FV <sub>2</sub>	5.67	1.0	FV <sub>5</sub>	1.88	1.8
Fc <sub>2</sub>	7.76	1.8	Fc <sub>5</sub>	2.90	2.9
XX <sub>3R</sub>	0.13	9.4	Ia <sub>6</sub>	0.01	9.4
ZZ <sub>3R</sub>	0.12	7.6	FV <sub>6</sub>	0.68	1.5
MY <sub>3R</sub>	-0.60	2.2	Fc <sub>6</sub>	2.10	2.5
Ia <sub>3</sub>	0.09	8.8	fv <sub>m6</sub>	0.63	1.6
FV <sub>3</sub>	2.02	1.6	fc <sub>m6</sub>	1.80	3.7

TABLE 3:

IDIM – IV ESTIMATES AFTER 3 STEPS

	$\hat{\beta}_{IV}^3$	$\% \hat{\sigma}_{\hat{\beta}_{IV}}$		$\hat{\beta}_{IV}^3$	$\% \hat{\sigma}_{\hat{\beta}_{IV}}$
ZZ <sub>1R</sub>	1.25	1.3	Fc <sub>3</sub>	6.0	1.9
Fv <sub>1</sub>	8.20	0.7	MX <sub>4</sub>	-0.02	20.0
Fc <sub>1</sub>	6.55	2.6	Ia <sub>4</sub>	0.03	9.4
XX <sub>2R</sub>	-0.48	2.9	Fv <sub>4</sub>	1.15	1.5
XZ <sub>2R</sub>	-0.16	4.8	Fc <sub>4</sub>	2.27	2.6
ZZ <sub>2R</sub>	1.09	1.2	MY <sub>5R</sub>	-0.03	14.0
MX <sub>2R</sub>	2.21	2.9	Ia <sub>5</sub>	0.04	11.0
Fv <sub>2</sub>	5.68	1.2	Fv <sub>5</sub>	1.90	2.0
Fc <sub>2</sub>	7.77	2.1	Fc <sub>5</sub>	2.80	3.5
XX <sub>3R</sub>	0.13	10.0	Ia <sub>6</sub>	0.01	10.9
ZZ <sub>3R</sub>	0.12	8.8	Fv <sub>6</sub>	0.69	1.6
MY <sub>3R</sub>	-0.60	2.3	Fc <sub>6</sub>	2.00	2.8
Ia <sub>3</sub>	0.10	9.2	fv <sub>m6</sub>	0.63	1.8
Fv <sub>3</sub>	2.03	1.8	fc <sub>m6</sub>	1.81	4.2

TABLE 4:

IDIM – IV ESTIMATES CONVERGENCE

	$\hat{\beta}_{IV}^0$	$\hat{\beta}_{IV}^1$	$\hat{\beta}_{IV}^2$	$\hat{\beta}_{IV}^3$
ZZ <sub>1R</sub>	1.0	1.24	1.25	1.25
Fv <sub>1</sub>	0.0	8.18	8.20	8.20
Fc <sub>1</sub>	0.0	6.54	6.54	6.54
XX <sub>2R</sub>	0.0	-0.47	-0.48	-0.48
XZ <sub>2R</sub>	0.0	-0.15	-0.16	-0.16
ZZ <sub>2R</sub>	1.0	1.08	1.09	1.09
MX <sub>2R</sub>	0.0	2.20	2.21	2.21
Fv <sub>2</sub>	0.0	5.62	5.68	5.68
Fc <sub>2</sub>	0.0	7.75	7.77	7.77
XX <sub>3R</sub>	0.0	0.125	0.13	0.13
ZZ <sub>3R</sub>	0.0	0.12	0.12	0.12

MY <sub>3R</sub>	0.0	-0.60	-0.60	-0.60
Ia <sub>3</sub>	1.0	0.09	0.10	0.10
Fv <sub>3</sub>	0.0	2.00	2.03	2.03
Fc <sub>3</sub>	0.0	6.00	6.0	6.0
MX <sub>4</sub>	0.0	-0.01	-0.02	-0.02
Ia <sub>4</sub>	1.0	0.03	0.03	0.03
Fv <sub>4</sub>	0.0	1.13	1.15	1.15
Fc <sub>4</sub>	0.0	2.26	2.27	2.27
MY <sub>5R</sub>	0.0	-0.025	-0.03	-0.03
Ia <sub>5</sub>	2.0	0.04	0.04	0.04
Fv <sub>5</sub>	0.0	1.90	1.90	1.90
Fc <sub>5</sub>	0.0	2.75	2.80	2.80
Ia <sub>6</sub>	1.0	0.009	0.01	0.01
Fv <sub>6</sub>	0.0	0.64	0.69	0.69
Fc <sub>6</sub>	0.0	1.95	2.00	2.00
f <sub>v</sub> <sub>m6</sub>	0.0	0.61	0.63	0.63
f <sub>c</sub> <sub>m6</sub>	0.0	1.78	1.81	1.81

TABLE 5:

NORM OF ERROR RELATIVE TO FILTERED JOINT POSITION

$\frac{\ \hat{\mathbf{q}}_j - \mathbf{q}_{s,j}\ }{\ \hat{\mathbf{q}}_j\ }$	$k = 0$	$k = 1$	$k = 2$	$k = 3$
Joint 1	0.080%	0.078%	0.078%	0.078%
Joint 2	0.050%	0.045%	0.045%	0.045%
Joint 3	0.050%	0.048%	0.048%	0.048%
Joint 4	0.051%	0.050%	0.050%	0.050%
Joint 5	0.100%	0.097%	0.097%	0.097%
Joint 6	0.120%	0.119%	0.119%	0.119%

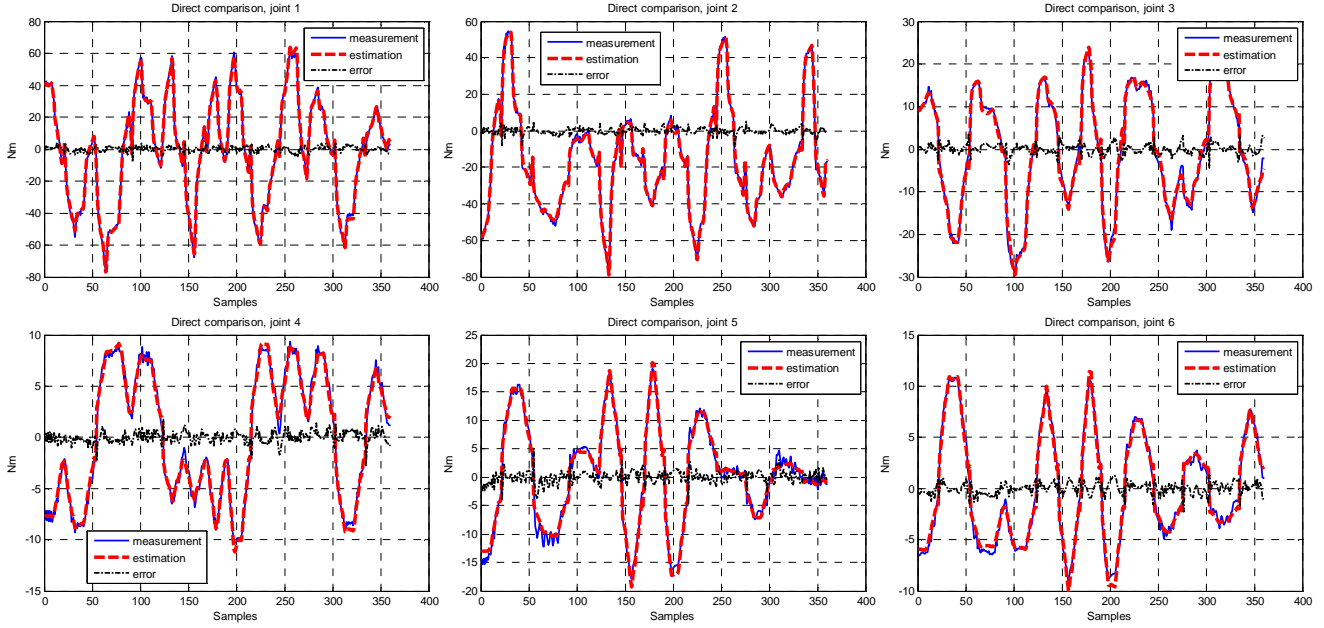


Fig. 6. Direct validations performed for joint 1, 2, 3, 4, 5 and 6. Blue: measurement; red: estimation; black: error.

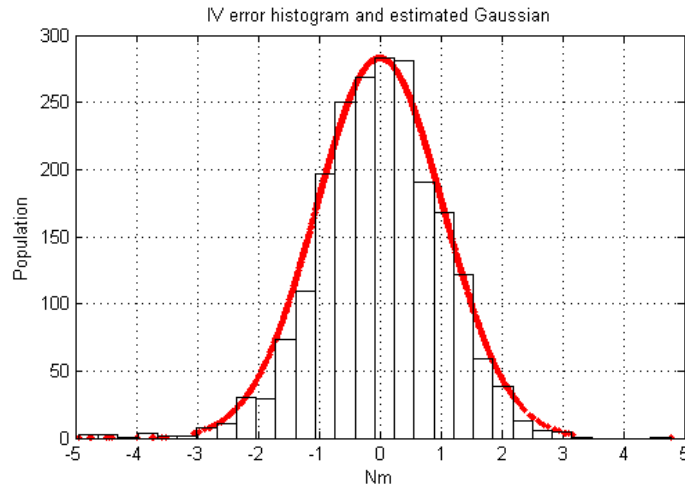


Fig. 7. Histogram of IV error and estimated Gaussian with data filtering

C. IDIM-LS and IDIM – IV methods without bandpass filtering

IDIM – LS and IDIM – IV methods are carried out with measurements of  $\mathbf{q}$  and with  $(\hat{\mathbf{q}}, \hat{\dot{\mathbf{q}}})$  calculated by a central difference algorithm of  $\mathbf{q}$  measurements without lowpass Butterworth filtering. There is no parallel decimation. IDIM – IV starts with the regular initialization. IDIM – LS and IDIM – IV estimates are given in Table 6 and Table 7. Once again, IDIM – IV method needs 3 steps to converge (see Table 4). We give the essential parameters because the model reduction is accepted by the F – statistic.

IDIM – LS estimates do not match those calculated with IDIM – IV method. Since IDIM – IV estimates

given in Table 7 stick to those given in Table 3, IDIM – LS estimates are biased. IDIM – LS fails because of the too large noise in the observation matrix  $\mathbf{W}(\mathbf{q}, \hat{\mathbf{q}}, \hat{\hat{\mathbf{q}}})$  coming from derivation of  $\mathbf{q}$  without lowpass filtering. In fact, we have  $E(\mathbf{W}^T \boldsymbol{\rho}) \neq 0$ .

IDIM – IV succeeds because the instrumental matrix  $\hat{\mathbf{Z}}_{fm} = \mathbf{W}_{\delta_{fm}}(\mathbf{q}_s, \hat{\mathbf{q}}_s, \hat{\hat{\mathbf{q}}}_s, \hat{\boldsymbol{\beta}}_{IV}^{k-1})$  is calculated with the simulated values  $(\mathbf{q}_s, \hat{\mathbf{q}}_s, \hat{\hat{\mathbf{q}}}_s)$  that are very close to the actual ones  $(\mathbf{q}, \hat{\mathbf{q}}, \hat{\hat{\mathbf{q}}})$  thanks to gains updating performed at each step of the algorithm.

This validation shows that IDIM – IV cancels the bias of IDIM – LS, coming from a noisy estimation of  $(\mathbf{q}, \hat{\mathbf{q}}, \hat{\hat{\mathbf{q}}})$  that gives a too noisy observation matrix  $\mathbf{W}(\mathbf{q}, \hat{\mathbf{q}}, \hat{\hat{\mathbf{q}}})$ . This result was expected because this a property of IV methods.

However, we can notice that IDIM – IV has lost its efficiency compared with IDIM – IV coupled with a parallel decimation. Indeed, deviations given in Table 7 are greater than those given in Table 3. This experimental result shows that parallel decimation can be related with “optimal prefilters” used in [21][24].

The normality assumption of  $\bar{\boldsymbol{\rho}}$  is validated because the KS – test accepts  $H_0 : \bar{\boldsymbol{\rho}} \sim N(0,1)$ . The histogram of  $\bar{\boldsymbol{\rho}}$  obtained with IDIM – IV method is plotted in Fig. 8. It matches a Gaussian distribution and we have  $\hat{\sigma}_{\rho, IV} = 1.02 \approx 1.0$ . So, errors terms in  $\boldsymbol{\rho}$  are normalized and heteroskedasticity is well taken into account. The test of independency described in section IV was run. We have  $R_{\bar{\boldsymbol{\rho}}}^2 = 0.07 < 0.1$  and coefficients  $a_i$  are small with large relative deviations. So, samples of  $\bar{\boldsymbol{\rho}}$  can be considered independent. Finally, all statistical assumptions made on  $\boldsymbol{\rho}$  hold in practice.

TABLE 6:

IDIM – LS ESTIMATES WITHOUT DATA FILTERING

	$\hat{\boldsymbol{\beta}}_{LS}$	$\% \hat{\sigma}_{\hat{\boldsymbol{\beta}}_{LS}}$		$\hat{\boldsymbol{\beta}}_{LS}$	$\% \hat{\sigma}_{\hat{\boldsymbol{\beta}}_{LS}}$
ZZ <sub>1R</sub>	0.06	5.5	Fc <sub>3</sub>	5.56	1.4
Fv <sub>1</sub>	8.10	0.4	MX <sub>4</sub>	0.06	2.8
Fc <sub>1</sub>	6.06	1.3	Ia <sub>4</sub>	0.01	11.5
XX <sub>2R</sub>	-0.08	4.1	Fv <sub>4</sub>	1.20	1.9
XZ <sub>2R</sub>	-0.02	6.7	Fc <sub>4</sub>	2.30	3.5
ZZ <sub>2R</sub>	0.05	3.2	MY <sub>5R</sub>	-0.02	8.1
MX <sub>2R</sub>	4.20	0.7	Ia <sub>5</sub>	0.01	6.8
Fv <sub>2</sub>	5.15	0.6	Fv <sub>5</sub>	1.84	1.9

Fc <sub>2</sub>	8.26	0.9	Fc <sub>5</sub>	2.85	1.5
XX <sub>3R</sub>	-0.01	20.0	Ia <sub>6</sub>	0.001	19.0
ZZ <sub>3R</sub>	-0.05	3.2	Fv <sub>6</sub>	0.68	2.2
MY <sub>3R</sub>	-0.30	1.8	Fc <sub>6</sub>	2.00	3.8
Ia <sub>3</sub>	0.05	2.2	fv <sub>m6</sub>	0.64	1.8
Fv <sub>3</sub>	2.21	1.05	fc <sub>m6</sub>	1.74	3.62

TABLE 7:

IDIM – IV ESTIMATES WITHOUT DATA FILTERING

	$\hat{\beta}_{IV}^3$	$\% \hat{\sigma}_{\beta_{IV}}$		$\hat{\beta}_{IV}^3$	$\% \hat{\sigma}_{\beta_{IV}}$
ZZ <sub>1R</sub>	1.25	2.6	Fc <sub>3</sub>	5.9	3.4
Fv <sub>1</sub>	8.25	1.7	MX <sub>4</sub>	-0.02	40.0
Fc <sub>1</sub>	6.50	6.6	Ia <sub>4</sub>	0.03	13.0
XX <sub>2R</sub>	-0.48	6.0	Fv <sub>4</sub>	1.16	1.9
XZ <sub>2R</sub>	-0.16	10.0	Fc <sub>4</sub>	2.20	3.8
ZZ <sub>2R</sub>	1.08	2.4	MY <sub>5R</sub>	-0.03	21.7
MX <sub>2R</sub>	2.20	5.8	Ia <sub>5</sub>	0.04	17.0
Fv <sub>2</sub>	5.68	2.3	Fv <sub>5</sub>	1.95	2.6
Fc <sub>2</sub>	7.73	4.1	Fc <sub>5</sub>	2.80	5.5
XX <sub>3R</sub>	0.13	20.0	Ia <sub>6</sub>	0.01	15.1
ZZ <sub>3R</sub>	0.11	19.0	Fv <sub>6</sub>	0.69	2.2
MY <sub>3R</sub>	-0.60	4.2	Fc <sub>6</sub>	2.00	4.0
Ia <sub>3</sub>	0.10	15.0	fv <sub>m6</sub>	0.64	2.4
Fv <sub>3</sub>	2.06	2.8	fc <sub>m6</sub>	1.79	5.8

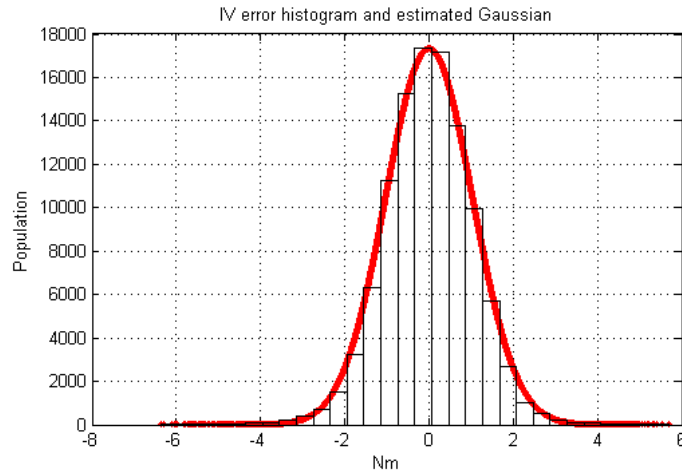


Fig. 8. Histogram of IV error and estimated Gaussian without data filtering

*D. IDIM – IV method compared with Total Least Squares (TLS) technique*

IDIM – IV method is now compared with TLS method because one can resort with TLS method when facing noisy observation matrix. Details about TLS method can be found in [29] and many papers of the same authors.

TLS method is carried with and without bandpass filtering. When coupled with a well tuned bandpass filtering, TLS estimates stick to IDIM – LS estimates given in Table 2. Like other approaches, TLS technique does not improve IDIM – LS method. Without bandpass filtering, TLS method provides estimates given in Table 8. In this case, TLS estimates do not match IDIM – IV estimates. It comes that the TLS estimator is biased. Unlike IDIM – IV method, TLS cannot cancel the bias resulting from a too noisy observation matrix  $\mathbf{W}(\mathbf{q}, \hat{\mathbf{q}}, \hat{\mathbf{q}})$ . This result shows that IDIM – IV is more effective than TLS technique.

TABLE 8:

TLS ESTIMATES WITHOUT BANDPASS FILTERING

	$\hat{\beta}_{LS}$	$\% \hat{\sigma}_{\beta_{LS}}$		$\hat{\beta}_{LS}$	$\% \hat{\sigma}_{\beta_{LS}}$
$ZZ_{1R}$	0.11	5.0	$Fc_3$	5.60	1.6
$Fv_1$	8.05	0.5	$MX_4$	0.01	2.5
$Fc_1$	6.00	1.4	$Ia_4$	0.01	13.0
$XX_{2R}$	-0.12	4.0	$Fv_4$	1.18	2.5
$XZ_{2R}$	-0.08	6.3	$Fc_4$	2.30	3.7
$ZZ_{2R}$	0.43	5.1	$MY_5$	-0.03	8.0

			R		
MX <sub>2R</sub>	3.21	0.9	Ia <sub>5</sub>	0.02	7.0
Fv <sub>2</sub>	5.22	0.6	Fv <sub>5</sub>	1.91	2.1
Fc <sub>2</sub>	8.20	1.0	Fc <sub>5</sub>	2.80	1.8
XX <sub>3R</sub>	-0.01	22.0	Ia <sub>6</sub>	0.005	20.0
ZZ <sub>3R</sub>	0.05	3.3	Fv <sub>6</sub>	0.68	2.0
MY <sub>3R</sub>	-0.40	2.1	Fc <sub>6</sub>	2.00	4.1
Ia <sub>3</sub>	0.07	2.2	f <sub>v</sub> m <sub>6</sub>	0.65	2.0
Fv <sub>3</sub>	2.10	1.4	f <sub>c</sub> m <sub>6</sub>	1.75	3.8

#### E. IDIM – IV method compared with classical Output Error Method (OE)

Because natural Output Error (OE) overcome the problem of noisy observation matrix, IDIM – IV method is compared with a classical OE technique (see [23], section III). OE identification methods minimize a quadratic error between an actual output and a simulated output of the system assuming both the actual and the simulated systems have the same input. For robots identification, it is more suitable to choose the closed-loop OE method (CLOE) [23]. Taking measured joint positions as outputs, the actual outputs vector is  $\mathbf{y}_q = \mathbf{q}$  and the simulated output vector is  $\mathbf{y}_s = \mathbf{q}_s$ .  $\mathbf{q}_s$  is obtained from integration of the DDM (10). The criterion to be minimized is therefore:

$$J(\boldsymbol{\beta}) = \|\mathbf{y}_q - \mathbf{y}_s\|^2 = (\mathbf{y}_q - \mathbf{y}_s)^T (\mathbf{y}_q - \mathbf{y}_s)$$

The minimization of  $J(\boldsymbol{\beta})$  is a nonlinear LS problem. Estimates can be computed using algorithms such as gradient method or Newton methods. These methods, based on a first or second order Taylor's expansion of  $J(\boldsymbol{\beta})$ , are available in *lsqnonlin* MATLAB function.

DDM simulation is performed without updating gains of the simulated controller and the Gauss – Newton (GN) algorithm is initialized with acceptable values. The *lsqnonlin* MATLAB function is used. GN algorithm converges after 1000 iterations and we retrieve IDIM – IV estimates given in Table 3. However, if GN algorithm is initialized with the regular initialization, it does not converge. As expected, classical OE methods are not suitable for 6 DOF robots identification: they converge slowly and they are sensitive to initialization whereas IDIM – IV converges after 3 iterations only and is not sensitive to initialization.

## VI. CONCLUSION

In this paper has presented a generic IV method suitable for robots identification, called IDIM – IV, was successfully applied on a 6 DOF industrial robot manufactured by STAUBLI.

This identification method combines the inverse and the direct dynamic models. These models are validated at the same time. Until now, the inverse and direct models were validated separately.

IDIM – IV algorithm improves noise immunity of estimates with respect to corrupted data in  $W$  coming from noisy measurements and/or bad tuning of bandpass filters of joint positions. A bandpass filtering is not needed to get unbiased estimates. However, if IDIM – IV method is used without parallel decimation, it may loss its efficiency.

Gains of simulated controller being updated at each step of IDIM – IV according to IDIM – IV estimates, the algorithm is not sensitive to initialization and has a fast convergence. Only 3 iterations are needed to identify 60 dynamic parameters. With classical IV methods, at last 5 iterations are needed to identify low dimensional systems.

IDIM – IV was also compared with TLS and OE methods. Experimental results show that IDIM – IV is more effective than these two approaches.

At last, statistical assumptions were experimentally validated with rigorous statistical tests. In many papers, statistical assumptions are rarely verified while they are crucial to obtain good estimates.

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