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Private Wealth over the Life-Cycle: A Meeting between Microsimulation and Structural Approaches

Lino Galiana* Lionel Wilner[†]

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Abstract

This paper embeds a structural model of private wealth accumulation over the life-cycle within a dynamic microsimulation model designed for long-run projections of pensions. In such an environment, the optimal savings path results from consumption smoothing and bequests motives, on top of the mortality risk. Preferences are estimated based on a longitudinal wealth survey through a method of simulated moments. Simulations issued from these estimations replicate quite well a private wealth that is more concentrated than labor income. They enable us to compute “augmented” standards of living including capital income, hence to quantify both the countervailing role played by private wealth to earnings dropout after retirement and the impact of the mortality risk in this regard.

JEL Codes: C63; C88; D15.

Keywords: Microsimulation; Intertemporal Consumer Choice; Life-cycle; Inequality.

*Insee.

[†]Crest, 5 avenue Henry le Chatelier, 91120 Palaiseau, France. Phone: (+33)170266947.
Please address correspondence to lionel.wilner@ensae.fr.

1 Introduction

Drawing a comprehensive picture of inter-generational equity when analyzing pensions' redistributive effects requires to include capital income in the computation of standards of living. Up to now, dynamic microsimulation models have been widely used to provide long-run projections of, say, the share of pensions' expenditures in the GDP (in PAYGO systems), but they only include labor income as opposed to other sources of earnings. Though already useful to get a sense of pension reforms' financial and redistributive effects, this approach ignores private wealth, which we propose to consider in the current study.

The optimal design of pension schemes is a complex equation that involves many parameters. Among them, the differential between the productivity growth rate and the rate of return of financial assets, which has been epitomized by the $r > g$ debate, determines partly the frontier between funded and unfunded regimes (Samuelson, 1958; Diamond, 1965). Of particular importance too, the question of the existence and intensity of eviction between public pensions and private savings has been widely discussed (see, e.g., Feldstein, 1974; Barro and MacDonald, 1979; Attanasio and Rohwedder, 2003; Attanasio and Brugiavini, 2003). Yet even within pay-as-you-go (PAYGO) systems, when the share of capital income in total income is not negligible, the previous differential matters when comparing the standard of living of retirees with the one of working individuals. The latter comparison receives in fact much attention by policy makers, social insurers and institutions in charge of consulting them on these issues like the *Conseil d'Orientation des Retraites* (COR) in France -as far as achieving inter-generational equity is a concern. It must be recognized that indexation rules also play a key role in this respect. When contributions are a fixed fraction of labor earnings, hence following the productivity growth rate, and when pensions are indexed on the inflation rate, as is the case in some defined contributions (DC) systems, the relative standard of living of retirees depends directly on the relative position of the productivity growth rate *vis-à-vis* the inflation rate. This mechanism is detrimental to retirees in low-inflation regimes, as has been the case in most OECD countries since the '90s and up to the post-Covid era. However, a countervailing effect has to do with retirees disposing of more capital income due to their location in the life-cycle: they have been accumulating private wealth over their whole career and the share of capital income based on asset returns among their total income is far from negligible. In France, about 1/6 of retirees' earnings comes from capital income (Insee, 2018).

This paper develops a structural model of private wealth accumulation plugged into a dynamic microsimulation model at the individual level, which allows for fur-

ther developments related to inequality concerns. The structural model considers an agent’s intertemporal trade-off between consumption and savings, taking the income process as exogenous. It departs from the stylized life-cycle model in that it takes inheritance into account. The agent chooses her optimal consumption path, which results in wealth accumulation over time, given consumption smoothing and bequest motives, on the one hand, and a sequence of income realizations, on the other hand. For the sake of simplicity, the rate of asset returns is assumed to be homogeneous. To embed this model in the microsimulation, we assume that each simulated individual behaves as that agent; the comparative advantage of this method is to rely on the microsimulation to allow for fine heterogeneity as regards in particular the age at death, the sequence of labor earnings and the family status. By construction, both the timing of death and the income process are predetermined by the microsimulation framework; we assume that only the latter is known by the agent, i.e. that she faces a mortality risk but no income risk. Admittedly, the tractability of the structural model is required in order to alleviate the computational burden induced by the dynamic microsimulation framework and to be able to estimate its parameters: in particular, relying on some analytic, closed-form and interior solution avoids any issue with binding constraints, which induce some non-linearity in simulated consumption and savings paths, and which are not easy to handle with in the iterative procedure. For that reason, we assume away any uncertainty on earnings, we neglect borrowing constraints, and we posit some *ad hoc* bequest motive: though costly in terms of realism,¹ making those assumptions yield to a tractable model, the parameters of which can then be estimated through indirect inference (see below). The trade-off between consumption and savings is basically governed by the time preference rate, the curvature of the utility function and bequest motives. We estimate preferences from the observed distribution of private wealth, given simulated income realizations; we rely on a method of simulated moments to match actual and simulated statistics of that distribution. Moreover, we do so either in presence or in absence of any mortality risk, which enables us to quantify the effect of the latter on the wealth accumulation process. In particular, the mortality risk being higher at old ages leads individuals to save less (and to consume more) when young, which, in turn, refrains them from consuming at old ages -hence a hump-shaped pattern of consumption over the life-cycle. We resort to two waves (2015 and 2018) of the French longitudinal wealth survey, namely the *Enquête Patrimoine*. Finally, we simulate the path of private wealth for every individual in the microsimulation

¹At the same time, the microsimulation provides us with much heterogeneity in the revenue process, compared with usual approaches based on calibrations in the literature.

model; in particular, we assume the matrimonial property regime is a community of acquests, whereby each member of a couple detains 50% in all property acquired during the marriage, but disposes of her own initial wealth (before marriage) and possibly of her own inheritances. In details, we follow current legislative rules as regards capital's sharing rules within unions. The empirical approach developed here replicates not only observed patterns of wealth over the life-cycle as well as individual wealth's growth rates, but also the higher concentration for private wealth than for labor income, for instance. We show further that even a very parsimonious specification combined with the microsimulation yields a good fit of the share of net worth detained by the top 1%. We conclude by quantifying how much the inclusion of returns to capital in the definition of income affects the relative standard of living of retirees -a crucial target parameter that intervenes in the design of the pension system. In particular, we empirically assess the role played by the mortality risk as regards wealth accumulation, capital returns and the augmented standard of living of retirees.

Inheritance deserves special attention. From a theoretical viewpoint, there could be many reasons why there is money left on the table at the time of death; in particular, it is unclear whether agents have bequest motives related to dynastic behavior, or whether they are uncertain about the timing of their death. From an empirical perspective, we observe the distribution of bequests in the *Enquête Patrimoine* wealth survey. We do not model the profound motives that govern inheritance and that ultimately generate this empirical distribution in a structural fashion: rather, we adopt a reduced-form specification.² This constraint affects his savings decisions and refrains him from consuming everything. We allow further a fraction of individuals not to transmit anything at their death, consistently with what is observed in the data. From the heirs' point of view, inheritance occurs at the death of the last surviving parent as an income shock, after applying (rather complex, and hence simplified) legal inheritance rules which depend mainly on the marital status and on the number of children of the deceased. We estimate a reduced-form equation of bequests that depends on observed characteristics present both in the wealth survey and in the microsimulation model. Inverting the mapping between transmitted and inherited wealth, i.e., the inheritance tax function, yields estimates of the exogenous bequest motive. While building upon a structural approach, we do not use an overlapping generation model which

²The reason why we make this choice lies in that estimation through indirect inference would be infeasible: in the dynamic microsimulation, preferences' parameters would govern not only an ancestor's wealth accumulation path, but also the one of her descent. To avoid such "loops" of high-orders in the structural parameters, we assume instead that (for exogenous reason) every individual has some amount of capital that he wants to transmit to his descent.

would question tractability. Importantly, modelling inheritance enables us to depart from the life-cycle hypothesis and hence to yield a better fit.

The rest of the paper is organized as follows. A literature review is proposed in Section 2. Section 3 presents our model composed of both the “dynamic microsimulation” part and the “structural” part, i.e., the model of consumption and savings over the life-cycle. Section 4 is devoted to the empirical modelling of bequests. Section 5 is devoted to implementation, estimation and simulation. Our results are shown in Section 6. Section 7 concludes.

2 Literature

This paper intersects two strands of literature: (i) microsimulation, and (ii) intertemporal consumption and savings models.

2.1 Microsimulation

Microsimulation allows researchers to address the complexity of pensions systems, which more aggregate projection models cannot do. Its role in the evaluation of public policies is often undermined in empirical economics; however, it is one of the three main methods with reduced-form and structural approaches. The first dynamic microsimulation model of pensions in France was created by the French national institute in charge of economic studies and statistics, Insee (see [Division-Redistribution-Politiques-Sociales, 1999](#)), as *Destinie 1*, an acronym for *modèle Démographique, Economique et Social des Trajectoires INDIVIDUELLES sImulÉEs* (demographic, social and economic model for simulated individual trajectories). This microsimulation model has been widely used for projections and for either *ex ante* or *ex post* evaluations of pension reforms. *Destinie 1* incorporated already some reduced-form attempt to model wealth accumulation that allowed to compute individual standards of living including capital income on top of labor income. However, this module has been abandoned in the current version of the model, *Destinie 2*, which has prevailed from 2010 onwards.

Starting from an Italian dynamic microsimulation model for pensions, [Moriciano et al. \(2013\)](#) propose a supplementary module that projects the evolution of wealth over time. They adopt a semi-structural, approximate life-cycle approach in which reduced-form estimations are plugged into a structural model. By contrast, here we embed a structural intertemporal consumption model, the parameters of which are estimated on actual data, into the dynamic microsimulation model.

2.2 Consumption and savings over the life-cycle

Why do people save and accumulate wealth? A huge literature has been devoted to providing with some theoretical or empirical answers to that question. A first explanation deals with retirement, implies consumption smoothing over the life-cycle, and can be addressed thanks to intertemporal consumption models, see, e.g., the seminal contributions by [Ramsey \(1928\)](#), [Modigliani and Brumberg \(1954\)](#) and [Friedman \(1957\)](#). In certain environments, this motive explains why individuals transfer wealth across periods of time: they do so in order to equalize their marginal utility of consumption. From that viewpoint, the intertemporal elasticity of substitution (IES) which determines the curvature of the utility function is a key parameter that governs the way an agent actually smooths her consumption over time through the Euler equation. The estimation of this parameter has been the object of much attention since [Hall \(1978\)](#); structural estimations include at least [Blundell, Browning, and Meghir \(1994\)](#) and [Attanasio and Weber \(1995\)](#). On top of the IES, preferences for the future of both financial markets (encompassed in the interest rate) and private agents (the time preference rate, i.e., the discount factor) may also be recovered. In that vein, [Hurd \(1989\)](#) and [Cagetti \(2003\)](#) resort to indirect inference of the structural parameters of a life-cycle model with bequests based on wealth moments; we improve upon this methodology by relying on a dynamic microsimulation model in which we observe directly earnings, which avoids, e.g., some parametric approximation of the labor income path based on random walks combined with ARMA processes; moreover, our setting permits a variety of demographic trajectories. Some papers isolate the role of each parameter by resorting to different calibration values, as is done for instance by [Low \(2005\)](#). Another road consists in assuming financial markets and agents share the same discount rate, as [Blanchet et al. \(2016\)](#) do.

On top of retirement, other savings motives have been put forward, including precaution, namely the ability of facing possible future economic downturns. In uncertain environments, individuals face different risks as regards health, mortality, labor income or asset returns. In other words, individuals' life span is uncertain; they are exposed to earnings shocks, and they may detain risky assets. In such a context, precautionary savings may arise as an insurance against bad shocks, like the loss of autonomy at later ages for instance, or against the loss of revenues. This precautionary motive stems directly from agents' preferences, namely prudence behavior ([Kimball et al., 1990](#); [Eeckhoudt and Schlesinger, 2006](#)). In that vein, a strand of papers solve numerically and simulate life-cycle models under uncertainty in order to estimate its structural parameters, including [Deaton \(1991\)](#), [Carroll, Hall, and Zeldes \(1992\)](#) and [Gourinchas and Parker \(2002\)](#). These

empirical papers, also including [Dynan \(1993\)](#), have concluded to rather mixed evidence as regards the magnitude of this precaution motive: in France, it would typically account for less than 10% of wealth ([Arrondel and Calvo Pardo, 2008](#)). [Cagetti \(2003\)](#) estimates nevertheless that the amount of wealth at retirement implied by a model with precaution is twice as high as that implied by a pure life-cycle model. Our approach belongs to this strand of literature in the sense that we simulate a life-cycle model, but we stick to a certainty equivalent model; we depart from the standard framework in that we include a bequest motive and rely on heterogeneous income processes as provided by the microsimulation model. Similarly to these papers as well as to [Calvet et al. \(2021\)](#), we resort to a method of simulated moments when recovering preferences.

Bequest motives may also be invoked to explain why agents accumulate wealth. Albeit seldom interpreted as altruistic behavior ([Arrondel and Calvo Pardo, 2008](#)), bequest motives result more likely from dynastic behavior according which agents seek to maximize their intertemporal utility, taking their descent’s one into account. However, it has been shown that agents may form poor expectations about the timing of their death, which is a concurring explanation to why there is money left on the table at death. Determining which motive prevails empirically is beyond the scope of this paper.³ To sum up, it is uncertain whether non-zero wealth at death results from uncertainty (on the timing of death) or from bequest motives. As far as inheritance is concerned, a structural modelling is therefore more tricky⁴. Instead of specifying agents’ preferences on transmission as [Gan et al. \(2015\)](#) do, we adopt a reduced-form approach here that aims at replicating the observed distribution of bequests.

3 Model

3.1 Microsimulation

We rely on `Destinie 2`, the dynamic microsimulation model for pensions developed at Insee.⁵ Its setting is appropriate to bring a life-cycle model to the data since it is a dynamic projection of both demographic characteristics and labor market trajectories based on wealth survey data (*Patrimoine 2009-2010*, the French

³The flows of inherited wealth may be viewed as evidence of excess saving, see also the retirement-consumption puzzle above; this may lead to low real interest rates and *in fine* secular stagnation.

⁴See [Piketty and Saez \(2013\)](#) for a structural stand on that topic.

⁵The model is implemented in R with C++ for efficiency reasons. The source code is available at <https://github.com/InseeFr/Destinie-2>.

module of the Household Finance and Consumption Survey, HFCS). *Destinie 2* is composed of two parts: (i) a demographic module that simulates births, unions, divorces, migrations and deaths; (ii) a labor market module that simulates transitions across several states on the job market (private sector, public sector, unemployment, inactivity, etc.) as well as earnings dynamics. It is based on a representative sample of the *Patrimoine* survey which provides with retrospective information on past labor market trajectories of individuals as well as additional characteristics related to demography and wealth. The original sample of the survey is composed of nearly 37,000 individuals, but *Destinie 2* further draws with replacement as many individuals as necessary to obtain a working sample that is representative of French population at rate 1/1000, i.e., about 62,000 individuals. The simulation of individual events starts from the base year, 2009, i.e., the corresponding wave of *Patrimoine*, and stops in 2070. The model has three important features. First, it incorporates stochastic volatility due to randomness at the individual level as far as personal or professional events are concerned. However, at the aggregate level, macro outputs are statistically aligned on demographic and labor force projections provided by Insee. The second step of the microsimulation consists in computing pensions from previous individual trajectories generated before. Second, the statistical unit is the individual, but this individual belongs to a household, hence it allows (i) the computation of standards of living, defined as the ratio of available income over the number of consumption units (computed according to the modified OECD scale: 1 for the first adult, .5 for all other individuals aged 14 or more and .3 for all other individuals aged less than 14); (ii) the simulation of survivors benefits since family ties are available. The last feature yields projections that include derived, or indirect, pension rights, on top of direct rights that are usually computed by other microsimulation models based on individuals only. Third, this model simulates the three major regimes (private sector, public sector, self-employment), which goes beyond regime-specific models. Further details on the model are provided by [Blanchet, Le Minez, and Marino \(2017\)](#). In what follows, we proceed to a once-and-for-all, unique simulation.

Assumptions have to be made as regards wealth accumulation within households, and more precisely as regards formal unions. Three marital statuses prevail in France: (i) marriages, (ii) formal partnerships or civil unions called PACS (*Pacte Civil de Solidarité*) created in 1999, and (iii) informal unions. However, as emphasized for instance by [Frémeaux and Leturcq \(2019\)](#), as soon as capital is concerned, what matters is the marital property *regime*, as opposed to the marital status. The three main regimes are: (i) universal (or absolute) community property, (ii) community of acquests, or (iii) separate property. In an absolute

community regime, all financial assets are joint assets whatever their status (inherited, acquired before or after the marriage). Income received by either spouse is also considered to be a joint asset; so are returns to capital. However, this regime concerns only 2% of married couples in 2004 according to [Frémeaux and Leturcq \(2018\)](#). In a community of acquests, each spouse owns an undivided half-interest in all property acquired during the marriage, except for (i) property acquired by gift or inheritance during the marriage, which is separate property; (ii) separate property acquired before the marriage, which remains separate property; or (iii) property that is acquired during a period when the couple is permanently living separately and apart (e.g., legal separation, actual or *de facto*), which is also separate property. This regime has been the default matrimonial property regime since the corresponding reform in 1965. Finally, the separate property regime concerns about 10% of married couples in 2010 and involves a complete individualization of wealth within the household.⁶ Informal unions and PACS being absent from *Destinie*, we assume that married couples are ruled by the community of acquests that concerns the vast majority of unions;⁷ an extension could consist in introducing a share of separate property, depending on observable characteristics for instance. By construction, the community of acquests assumes an equal (50-50) share on the wealth acquired by the couple, while separate property is equivalent to full individualization of wealth; in the latter case, the information on the marital status is irrelevant. Another extension might consist in enabling couples to dispose of the household's wealth according to some prenuptial agreement. Put differently, some flexibility could be obtained by considering partial pooling of resources within the household, the weight of each spouse being equal to her/his share in total income, for instance. In France, [Morin \(2014\)](#) documents that the average income share of married women amounted to 36% in 2011, which could serve as a benchmark.

In a pure life-cycle model without uncertainty on the timing of death, inheritance does not exist because agents die without any wealth (net of debt). However, to replicate the actual distribution of inheritance, we introduce inheritance

⁶The share of individualized wealth exhibits an increasing trend between 1998 and 2010, driven especially by the top of the distribution of capital, which results in upward-sloping within-household inequality, hence in gender inequality. See, e.g., [Frémeaux and Leturcq \(2013\)](#) on that issue.

⁷Since 2007, separate property has been the default matrimonial regime of PACS. Between 1999 and 2006, the default regime resembled to a community of acquests. However, even though PACS constitute an increasing share of newly formed couples (the annual inflows of new contracted PACS is about 180,000, while the number of marriages is about 220,000), they are still a minority, about 1.4 millions in 2011, as far as the stock of unions is concerned (married individuals being at least 23 millions).

to account for its weight in households’ wealth.⁸ To our needs, we implement a simplified version of French inheritance rules. Basically, the inherited amount depends on the marital status and on the number of children, which yields four distinct situations. In the absence of children and of marriage, the heirs are the ascendants –the parents, to simplify. When the deceased was married, the inheritance is split between the spouse and the parents, the former receiving one half of the inheritance when the two parents are alive and 3/4 of it when only one parent survives his deceased child. When the deceased had children without being married, her descendants (her children, to simplify) receive the whole inheritance. When the deceased was married, the inheritance is split between the spouse, on the one hand, and equally among the children, on the other hand; the spouse is allowed to choose between (i) the usufruct of the whole inheritance, and (ii) 1/4 of the whole inheritance, which is the solution that we will implement because allocating returns to private wealth this way makes more sense, as opposed to what might prevail for real estate, for instance –and also for computational reasons. Complications and numerous exceptions to these general rules arise: (i) when there is a will; (ii) when there are children from former unions; (iii) when individuals refuse to become heirs, etc. The idea is that freedom in inheritance is limited by the rule of “rightful heirs”: legitimate heirs like children cannot be spoiled even when a will is available.

3.2 Intertemporal consumption over the life-cycle

Assumptions We consider a stylized,⁹ structural model of intertemporal consumption *à la* Yaari (1965) where the environment is uncertain as regards mortality risk but where the labor supply is fixed. In particular, there is no uncertainty on earnings, since within the microsimulation framework there is no labor income risk.

For now, we consider a tractable structural model, which we estimate from microsimulated individual trajectories. We believe that a homogeneous behavior combined with heterogeneous trajectories on both family and labor markets will lead to reasonable, realistic dispersion in outcomes, which we will be able to verify from our simulations below. Besides, we assume perfect financial markets, and

⁸For a summary of inheritance flows estimates and an extended discussion on the Kotlikoff–Summers–Modigliani controversy regarding the share of current wealth that comes from inheritance, see Piketty and Zucman (2015).

⁹Though stylized, the model embeds Modigliani and Brumberg (1954) and Friedman (1957) as special cases.

rule out any borrowing constraint. We consider a unique financial asset¹⁰ with rate of return equal to the real interest rate r which, by assumption, is net of depreciation. We further assume time separability of intertemporal preferences, the agent's discount factor being denoted by $\beta = \frac{1}{1+\delta}$ and such that $0 \leq \delta \leq r$. Instantaneous preferences are represented by some Constant Relative Risk Aversion (CRRA) utility function: $u(C) = \frac{C^{1-\gamma}}{1-\gamma}$, $\gamma > 0$. In this certain environment, $\gamma = 1/\sigma$ is rather the inverse of the intertemporal elasticity of substitution (IES) than the coefficient of relative risk aversion. When $\gamma = 1$, the utility function tends to the logarithm case: $u(C) = \log(C)$. We consider a finite horizon model in which time is discrete, more precisely annual as is the case in the dynamic microsimulation model. Let T denote the maximal living age of the agent who starts her active life at time $t = 0$ and who retires at some exogenous time $0 < t_r < T$ that does not result from any optimization.¹¹ During her lifetime, this individual is alive at time t with probability π_t , and this information is not revised over the life-cycle. Her labor earnings process $\{Y_t^r\}$ is exogenous and comes from the dynamic microsimulation model. We consider inheritance as a temporary positive income shock. The income process $\{Y_t\}$ may therefore integrate such exogenous shocks; in our model, though, the only additional exogenous shocks are received bequests from deceased parents. In other words, $Y_t = Y_t^r + H_t^r$ where H_t^r is either positive or equal to zero.

By assumption, the individual is endowed at the beginning of her life-cycle an initial stock of capital K_0 . One can think K_0 as an initial sum to start active life. Besides, we impose that the agent has some bequest motive and seeks to give some financial amount H^g to her descent. As a counterpart of this transmission, heirs incur a temporary income shock H^r when they receive their inheritance, a fraction of H^g , typically at the death of the last surviving parent. The difference between H^g and H^r stems from the possible presence of multiple heirs and from the inheritance tax schedule. This “targeted bequest” is a reduced-form, convenient way of sticking to a tractable model without resorting to an overlapping generations model, and without invoking dynastic motives that would complicate further the analysis, on top of making the estimation possibly infeasible.

¹⁰this asset can be viewed as a portfolio composed itself of heterogeneous assets, the weighted average of their returns being equal to the real interest rate.

¹¹In France, most people retire at 62, which corresponds to one of the legal retirement dates, but also to the moment from which an individual may ask for his pensions rights without experiencing any penalty. Other papers depart from this assumption and have tempted to approximate the decision-making that rules the timing of retirement: see, among others, [Stock and Wise \(1990\)](#) as well as [Mahieu and Blanchet \(2004\)](#).

The problem and its analytical solution The consumer maximizes the discounted sum of her instantaneous utilities that depend only on the consumption path:¹²

$$\max_{\{C_t\}} \sum_{t=0}^{T-1} \beta^t \pi_t u(C_t) \quad (1)$$

subject to the law of private wealth accumulation, denoting by $R = 1 + r$:

$$K_{t+1} = R(K_t + Y_t - C_t) \quad (2)$$

where K_t accounts for the stock of private wealth that is available at the beginning of time t . In other words, we assume that earnings and consumption decisions are made at the beginning of the period. Recursion yields:

$$K_t = R^t K_0 + \sum_{\tau=1}^t R^\tau (Y_{t-\tau} - C_{t-\tau}). \quad (3)$$

The no-Ponzi game constraint, i.e., the condition forbidding debt at death $K_T - H^g \geq 0$, leads to the following intertemporal budget constraint:

$$\sum_{t=0}^{T-1} R^{-t} C_t \leq K_0 + \sum_{t=0}^{T-1} R^{-t} Y_t - R^{-T} H^g \quad (4)$$

This constraint binds at the optimum of the consumer's program.¹³ As a result, the agent ends her life with no capital left on the table, once the transmission has been accounted for. This result would not hold in the presence of uncertainty. Alternatively, this model can be thought of as some limit case of a model with pure joy of giving, or warm-glow, with a utility of bequest that is associated with a degree of altruism equal to zero. Empirically, a null degree of altruism, consistent among others with accidental bequest and uncertain lifetime, cannot be excluded: see, for instance, [Cagetti \(2003\)](#) on that topic.

The first-order condition of the program that consists in maximizing (1) un-

¹²Consumption should be interpreted as a marshallian demand for a composite good, which may well result from a trade-off between leisure and consumption, possibly including preference for inactivity, among others. In any case, consumption allows us to embed such programs in a single-dimension index, which helps justify the fact that the optimal pattern may be non-decreasing over time. This specification encompasses less parsimonious models including explicit preferences for non-work income, for instance.

¹³If it did not, its Lagrange multiplier would be equal to zero. Hence the marginal utility of consumption would be null, which is equivalent to consumption tending to infinity, but violates the intertemporal budget constraint (4).

der (4) yields an Euler equation:

$$\pi_t u'(C_t) = \beta R \pi_{t+1} u'(C_{t+1}) \iff \log \frac{C_{t+1}/\pi_{t+1}^\sigma}{C_t/\pi_t^\sigma} = \sigma \log(\beta R) \quad (5)$$

Denoting by $C'_t = C_t/\pi_t^\sigma$ and combining equations (4) and (5) yields:

$$\begin{aligned} C'_t &= C'_0 (\beta R)^{\sigma t} \\ C'_0 &= \frac{K_0 + \sum_{t=0}^{T-1} R^{-t} Y_t - R^{-T} H^g}{\sum_{t=0}^{T-1} \alpha^t \pi_t^\sigma} \end{aligned} \quad (6)$$

where $\alpha = \beta^\sigma R^{\sigma-1}$. Let $S_t = R^t \frac{\sum_{\tau=0}^{t-1} \alpha^\tau \pi_\tau^\sigma}{\sum_{\tau=0}^{T-1} \alpha^\tau \pi_\tau^\sigma}$. The mortality risk generates a hump-shaped profile for consumption as documented empirically in [Attanasio and Weber \(1995\)](#) and theoretically in [Drouhin \(2015\)](#), for instance. More precisely, this pattern is due to survival probabilities π_t decreasing sharply at old ages, which causes consumption to fall at the end of the lifetime. In contrast, at younger ages, the mortality risk can be neglected, hence consumption increases provided that agents are patient enough, i.e. such that $\beta R > 1$).

The current stock of wealth is given by:

$$K_t = [R^t - S_t] K_0 + \left[\sum_{\tau=1}^t R^\tau Y_{t-\tau} - S_t \sum_{\tau=0}^{T-1} R^{-\tau} Y_\tau \right] + S_t R^{-T} H^g \quad (7)$$

Given the simplifying assumptions adopted above, the model is tractable and admits a closed-form solution, which will be useful for simulation-based inference. Indeed, tractability is a requirement to bring a structural consumption model to a 0.1% representative population simulated over more than 60 years. Given the maximal living age T , the survival sequence $(\pi_t)_{0 \leq t \leq T-1}$ and the income sequence $(Y_t)_{0 \leq t \leq T-1}$, individual consumption and wealth dynamics are identified by the vector of parameters (β, R, σ) , namely: preferences for the future of both financial markets and the agent as well as the intertemporal elasticity of substitution.

3.3 The life-cycle model within the microsimulation framework

Bringing the previous theory to simulated data is a novel and ambitious task which requires to adapt slightly the analysis. It is now necessary to allow for heterogeneity as regards individual trajectories, including the timing of death; in the microsimulation model, this terminal moment T_i actually differs across individuals. We assume therefore that each individual endowed with characteris-

tics $X_i = (\{Y_{it}^r\}_{t=0}^{T_i-1}, T_i, \{H_{it}\}_{t=0}^{T_i-1})$, where $H_{it} = (H_i^g, H_{it}^r)$, behaves as the previous agent, given her preferences encompassed by the vector of structural parameters.¹⁴ As noted earlier, in a world with perfect financial markets, inheritance is akin to a transitory income shock, from children’s viewpoint. In the following, we denote $Y_{it} = Y_{it}^r + H_{it}^r$. We also assume that individuals make their choices by considering real flows instead of nominal flows.

From the previous model, the current amount of capital K_{it} is a deterministic function of (i) the simulated sequence of labor earnings $\{Y_{it}\}_{t=0}^{T_i-1}$, including received bequests H_i^r , (ii) the time of death T_i , (iii) the targeted bequest H_i^g , and (iv) structural parameters. While the labor earnings process, net of bequests, is given by the dynamic microsimulation model, inherited wealth H_i^r has to be estimated from the *Enquête Patrimoine* wealth survey and simulated as a function of sociodemographics that are common to both *Enquête Patrimoine* and *Destinie 2*, see next section on that issue. By contrast, transmitted bequests H_i^g must be recovered from inherited wealth after inverting the latter given the inheritance tax schedule (the Appendix provides with more details on this tax scheme) and taking the possible presence of multiple heirs into account.

Besides, the *Enquête Patrimoine* wealth survey provides with information on the amount of private wealth $K_{i,2009}$ available for every individual i surveyed in 2009, and thus in the microsimulation model. Since the initial amount of capital is:

$$K_{i0} = \frac{K_{it} - S_t R^{-T} H_i^g - \left[\sum_{\tau=1}^t R^\tau Y_{i,t-\tau} - S_t \sum_{\tau=0}^{T_i-1} R^{-\tau} Y_{i,\tau} \right]}{R^t - S_t}, \quad (8)$$

we are able to recover the initial wealth for any given value of the vector of structural parameters, replacing t by the time elapsed since school-leaving age for each individual i present in the survey. We use the observed value in the survey to determine K_{it} , where t accounts for the time that corresponds to 2009 in individual i ’s life-cycle dynamics. This procedure is made possible because each individual in *Destinie 2* has an identifier that can be matched to *Enquête Patrimoine*. The knowledge of K_{i0} allows us to compute then the optimal consumption path for each individual of our simulated data using (6), and hence to simulate the entire wealth path from (7). As a result, observed and simulated wealth coincide in 2009, by construction. Finally, for simulated individuals who are not observed in the survey (for instance because they were not born at that time), we draw in

¹⁴This vector of structural parameters is assumed to be homogeneous here; extensions could make it vary depending on X_i . For instance, the rate of return of financial assets is likely to be higher for richer individuals -who may also be more patient. Such extensions might be required to replicate the extreme concentration of capital at the top of the distribution.

the joint distribution of age and observed wealth, and compute the corresponding initial amount of capital, i.e., the initial wealth that fits equation (8) at these values.

As regards the real interest rate, ideally that rate of return to private wealth assets would be derived at the equilibrium of the capital market. We choose not to model both sides of this market here, which we feel is beyond the scope of this paper. Moreover, even a complete model of private wealth could not be enough since housing prices are also likely to impact this rate, see DSGE models for instance. We take this rate as exogenous and resort to a calibration approach, starting with $r = 3\%$. This assumption is all the more plausible that in practice, the discount window is determined by central banks.¹⁵ The interest rate depends nevertheless on the asset considered: it is higher for stocks than for bonds, and for life insurance than for savings. It tends also to increase with wealth itself due to composition effects: richer individuals tend to own more diversified portfolios, more risky assets, and hence to benefit from higher rates of return. Extensions should therefore consider heterogeneous rates of return, especially as far as replicating the high concentration of private wealth at the top of the distribution is a concern.

Last, in order to compute the empirical counterparts of survival probabilities, we consider the values provided by the Human Mortality Database (HMD). Those values vary with age, hence over time in the spirit of our life-cycle model. They are also heterogeneous across two supplementary dimensions: sex and year of birth.

4 Inheritance

It is not the purpose of this paper to address the empirical plausibility of different bequest motives (accidental, altruism, exchange, warm glow, retrospective) surveyed for instance by [Arrondel and Masson \(2006\)](#). Instead, recognizing that there is a substantial heterogeneity in these motives, we propose simply to reproduce the observed distribution of inherited wealth in order to simulate corresponding bequest vector $H_i = (H_i^g, H_i^r)$ at the individual level in the microsimulation.

This exercise constitutes a module of the microsimulation model *per se*. First, we consider a 2-step reduced-form estimation of received bequests H_i^r à la [Laibson et al. \(1998\)](#) and [Cagetti \(2003\)](#) as a function of relevant socio-demographics. More

¹⁵An alternative justification could lie in that our working sample is made up of individuals who are not numerous enough (0.1% on the whole) to influence that rate, and hence who behave as price takers.

precisely, the *Enquête Patrimoine* 2014-2015 wealth survey yields information on a sub-sample of individuals about their inherited wealth. The latter information is only available in brackets: the corresponding variable in the data is ordered and polytomous. We know only that the inherited amount H_i^r , when positive, is (1) lower than €3,000, comprised between (2) €3,000 and €8,000, (3) €8,000 and €15,000, (4) €15,000 and €30,000, (5) €30,000 and €60,000, (6) €60,000 and €100,000, (7) €100,000 and €150,000, (8) €150,000 and €200,000, (9) €200,000 and €250,000, or (10) higher than €250,000. In a first step, we estimate a Probit model on a dummy equal to one when the individual receives some bequest (extensive margin). In a second step, conditionally on receiving some positive bequest, we resort to an ordered Probit model with known thresholds to explain the amount inherited (intensive margin). Such a model assumes a relationship between the observed variable $Y_i = \log(H_i^r)$ and some unobserved latent variable Y_i^* such that $\forall k \in \llbracket 1, 10 \rrbracket$,

$$Y_i = k \iff Y_i^* \in [s_k; s_{k+1}[, \quad (9)$$

or equivalently

$$Y_i = \sum_{k=1}^{10} k \mathbf{1}\{s_k \leq Y_i^* < s_{k+1}\}. \quad (10)$$

We posit a linear model on the latent, unobserved variable Y_i^* :

$$Y_i^* = D_i' \mu + \varepsilon_i, \quad (11)$$

where D_i are socio-demographic characteristics and ε_i follows a normal distribution with mean 0 and variance σ_ε^2 . We use the information on thresholds $\{s_k\}_{k=1}^{11}$ which are known both by survey respondents and by the researcher ($s_1 = -\infty$, $s_2 = \log(3,000)$, \dots , $s_{11} = +\infty$).

It will be crucial for the simulation exercise that follows to restrict our attention to covariates that are common to both the survey and the dynamic microsimulation model. Hence in our empirical specification, sociodemographics D_i are composed of the logarithm of income,¹⁶ age, school-leaving age as a proxy for education, and a female dummy. We adopt an unrestricted form for school-leaving age and we include 5-year age dummies in the list of covariates.

The model is estimated under the assumption of D_i being exogenous. Since the thresholds are known, both the location -the constant μ_0 - and the scale $-\sigma_\varepsilon$ - of the model are identified. Under these assumptions, the vector $(\mu, \sigma_\varepsilon)$ can be estimated easily: the maximum likelihood estimator (MLE) is consistent and asymptotically

¹⁶Income is the sum of labor income, pensions and unemployment benefits.

normal (CAN) as the number of individuals grows large.

Table 1 displays our estimates from a sample composed of 10,344 individuals surveyed in *Enquête Patrimoine* 2014-2015 and answering to the inheritance question. Among them, 8,012 individuals declare both positive income and positive bequests. Bequests are found to be positively correlated with both income and education. They also vary significantly over the life-cycle, highest inherited amounts being mostly found after 45.¹⁷

From previous estimates, we are then able to simulate a temporary amount of inherited wealth $H_i^r(D_i; \hat{\mu}, \hat{\sigma}_\varepsilon)$ for each individual present in the dynamic microsimulation model. For individual i , inheritance is triggered in the year of the parent's death. Inheritance can be seen as a temporary income shock for the child perspective.¹⁸ For example, if a child i were to receive $H_{it}^r = \text{€}170,000$ at time t , given her characteristics at the time when her parent dies, her deceased parent j has to bequeath him a little more due to the inheritance tax (see the Appendix for corresponding tax schedule). This amount, denoted by H_j^g in (8), is around €170,500 to take the first tax bracket into account.

As Table 2 shows, the median simulated bequest is about €27,000 while the average amounts to nearly €178,000. The simulated distribution is positively skewed and exhibits a huge excess kurtosis, which is consistent with the observed distribution of bequests. The comparison between observed and predicted bequests is provided by Figure 1. The fit of the model sounds more than correct at the intensive margin: the differential between actual and simulated shares of individuals in each bracket of inherited amount is very small. As expected, the extensive margin is slightly more difficult to fit; this is a concern for the first bracket, i.e., for the 2,332 individuals with no bequest (22,5% of the estimation sample). It turns out that the model under-predicts the number of individuals in that situation by 800; at the same time, the next two brackets are slightly over-predicted, which suggests that the model attributes small amounts of bequests to these individuals.

Second, the estimation of transmitted wealth H_i^g consists in inverting the inheritance tax schedule and recover the money that has virtually been transmitted by the deceased to each heir separately. We compute therefore a weighted sum of such pecuniary amounts within a family according to the inheritance rules,

¹⁷After controlling for age, income and education, gender is not significant.

¹⁸This way of understanding inheritance as a temporary income shock means that the child knows the date of death of the parent and does not communicate it, since the parent does not know it. This simplifying assumption, which implies considering, from the child's point of view, inheritance as an exogenous phenomenon, makes it possible to have a tractable multigeneration model

which yields the corresponding bequest. Finally, applying the fiscal legislation to the latter gives the predicted bequest for each of the heirs. More details on the inheritance tax scheme are provided in Appendix.

5 Estimation

This section presents how we estimate the parameters governing agents’ preferences of the structural model combined with microsimulated data, given previous reduced-form estimates of bequests.

5.1 Method of simulated moments

A first approach could consist in calibrating these parameters to some plausible values reported in the literature for instance. It is tempting to invoke estimates of both IES and time discount factor provided, among others, by [Stock and Wise \(1990\)](#), [Mahieu and Blanchet \(2004\)](#) or [Low \(2005\)](#); see, e.g., [Pemberton \(1997\)](#) for a summary of other references and some calibration values.

However, we believe that estimating the vector of preferences is a more satisfactory approach than the calibration, the main difference being that our estimator will be based on some statistical criterion. In this validation step, we impose that our model matches moments, or rather auxiliary statistics, of the distribution of private wealth observed in the data. Our empirical strategy is therefore a method of simulated moments ([Pakes, 1986](#); [McFadden, 1989](#); [Pakes and Pollard, 1989](#)), a special case of indirect inference ([Gouriéroux, Monfort, and Renault, 1993](#)) or simulation-based inference ([Gouriéroux and Monfort, 1993](#)).

We aim at fitting empirical features of the distribution of wealth which are not necessarily moments, but some statistics of this distribution. Namely, we consider two sets of auxiliary statistics s_1 and s_2 of the data: (i) s_1 accounts for an inverse hyperbolic sine (IHS) transform¹⁹ of private wealth held by each 5-year cohort between 20 and 85, which yields 13 “cross-sectional moments” (relying on *between* or individual variability), and (ii) s_2 refers to the median growth rate of private wealth between 2015 and 2018 for each 5-year cohort between 30 and 70, which yields 8 “longitudinal moments” (relying on *within* or longitudinal variability). Note that the latter auxiliary statistics resemble to the change in individual wealth-to-income ratio used by [Calvet et al. \(2021\)](#) in order to identify the time preference

¹⁹The IHS transform $\log(x + \sqrt{1 + x^2})$ is well-known to accommodate very dispersed distributions like net worth with full support over the real line: see, e.g. [Burbidge, Magee, and Robb \(1988\)](#).

rate, up to the normalization by earnings. *In fine*, we dispose of 21 moments to overidentify β (see, e.g., [Einav, Finkelstein, and Mahoney \(2018\)](#) for a similar methodology).

Given the model above and these auxiliary statistics, it could be tempting to achieve overidentification of both parameters β , the discount factor, and γ (or equivalently σ , the IES), given that the real interest rate has been calibrated. Yet a simple identification argument imposes to normalize further, say σ , to some usual value found in the literature. Remember from the model that the observation of capital-based auxiliary statistics will only enable us to recover an estimate of the parameter $\alpha = \beta^\sigma R^{\sigma-1}$. But then it follows that $\beta = \hat{\alpha}^{\frac{1}{\sigma}} R^{\frac{1-\sigma}{\sigma}}$: hence, provided that an estimate $\hat{\alpha}$ of α is available from the empirical counterpart of equation (7), the model adds up some nonlinear constraint on these parameters. It is then impossible to truly separate them in this setting; empirically, we pick up a value of the IES close to one as in [Blundell, Browning, and Meghir \(1994\)](#),²⁰ and estimate β only. As a result, the model cannot be more parsimonious in the dimension of agents' structural preferences; recall however that it conveys much heterogeneity in individual trajectories thanks to the microsimulation.

Our estimation procedure is classical minimum distance. Our estimate of β consists in finding $\hat{\beta}$ that minimizes the distance between theoretical (or predicted) statistics from our simulated model and its empirical counterpart. To account for the fact that the number of individuals differs across 5-year age groups, we resort first to a diagonal weighting matrix that assigns more weight to empirical moments formed by more numerous individuals. By construction, the trace of this first-step weight matrix is 2 (the sum of weights being 1 for the first “cross-sectional” set of moments and 1 for the second “longitudinal” set of moments). Formally, we denote by $g(\beta) = \omega(\hat{s} - s(\beta))$ where $s(\beta)$ is the simulated vector of statistics predicted by our model, \hat{s} is its empirical counterpart, namely the sample statistics observed in the survey data, and ω designates the previous weight (equal to the share of the considered age group in the population). Our estimation procedure chooses the parameter value that results in simulated statistics matching observed statistics as closely as possible:

$$\hat{\beta} = \arg \min_{\beta} g'(\beta) W g(\beta), \quad (12)$$

²⁰In a different vein, using data on labor supply behavior, [Chetty \(2006\)](#) estimates a coefficient of relative risk aversion that is close to one, and that 2 is a tight upper bound for this parameter shaping the curve of the utility function. In a meta-analysis, he also stresses that the central estimate is near 1, quoting, e.g., the point estimate obtained by [Blundell, Duncan, and Meghir \(1998\)](#), 0.93. Based on a structural life-cycle model with mortality risk and bequest motives, [Hurd \(1989\)](#) finds a value of 1.12 for γ ; he resorts to a nonlinear least squares method in order to match simulated and actual wealth moments as we do.

In other words, we aim at minimizing an objective function usually called the “goodness-of-fit” (GoF). W is a weighting matrix; since the optimal matrix that yields an efficient estimator is unknown to us, we resort to feasible 2-step GMM in order to get an estimate for $\widehat{\beta}$ (Hansen, 1982).

After minimizing the weighted difference between empirical and theoretical sets of moments, we end up with a first-stage estimate $\widehat{\beta}^{(1)}$ which we use to compute an estimated residual covariance matrix, the inverse of that matrix being our optimal weighting matrix. The asymptotic distribution of the resulting estimate $\widehat{\beta}^{(2)}$ minimizing the GoF function associated with that matrix is a normal distribution whose variance is given by formula (C.15) in Appendix C of Cagetti (2003). A correcting term $1 + \tau$ enters multiplicatively the usual sandwich formula of that variance-covariance matrix, which accounts for the ratio of the number of observations to the number of simulated points -more profoundly, for the variance of the estimator that stems from the simulation. Empirically, as in Cagetti (2003), this term is extremely low given our simulated sample size, and can be innocuously approximated to 1.

The practical implementation consists in resorting to Nelder and Mead (1965)’s algorithm based on the simplex method in order to find the solution of the minimization problem (12).

To build previous auxiliary statistics of the distribution of wealth, we rely on three waves of the *Patrimoine* survey, a wealth survey conducted jointly by the Insee and the French central bank. This wealth survey is the French part of the Household Finance and Consumption Survey (HFCS), a harmonized system of wealth surveys supervised by the European Central Bank. Table 3 sums up the way we used the different waves of the survey. These waves of the same longitudinal survey provide with information on the individual trajectories of private wealth accumulation between 2015 and 2018. This information was not available before 2015: merging previous *Patrimoine* surveys yields only a pseudo-panel, while from 2015 onwards, about 4,700 individuals are followed over time. Remember also that the 2009 wave corresponds to the wave that has been used to generate the microsimulated population (every individual belonging to this first wave can be matched to an individual present in Destinie 2), and that we use this information to ensure that the simulated life-cycle wealth dynamics is consistent with the observed value in 2009 thanks to (8). A methodological contribution of this paper is therefore to capitalize on the panelization of that French wealth survey in order to estimate structural parameters governing preferences over the future.

5.2 Implementation

Even with a tractable model derived from (8), bringing a life-cycle savings model to 60 years of microsimulated data for a sample of 62,000 individuals is computationally burdensome, especially with the supplementary ambition to estimate structural parameters from indirect inference. Hence program efficiency is a major concern. Our implementation is `Rcpp` based (Eddelbuettel et al., 2011) to integrate `C++` efficiency within `R` statistical framework.²¹ The `R` implementation is itself optimized by us resorting to the `data.table` framework (Dowle et al., 2019). To simulate 60 years of capital accumulation over this microsimulated sample representative of French population at rate 1/1000, this optimization yields a performing model both in terms of time (a microbenchmark test yields a median time of 3.5 seconds over 50 replications²²) and memory (less than 900Mb of RAM). To reduce the risk of unexpected program behavior, many unit tests checking the consistency with model hypotheses (e.g. $K_T - H^g = 0$ for all individuals) are included in the public `R` package.

As previously mentioned, adapting the model to a microsimulated framework requires special attention on matrimonial dynamics. Remember that we assume that the unique matrimonial regime is the community of acquests, associated with equal income splitting within couples. Since we are able to match every microsimulated agent with a true individual observed in 2009, we compute the initial wealth in (8) at the individual level. As a result, t is individual-specific: somebody who started to work in 1999 will have $t = 10$ while another agent closer from retirement might have a t equal to, say, 40. K_{i0} is then determined from the structural parameters, the date of death T_i as well as the income sequence $(\{Y_{it}\}_{0 \leq t \leq T_i-1})$. Given K_{i0} , the remaining dynamics of wealth accumulation is derived thanks to equation (2).

5.3 Monte-Carlo analysis

To check that the above estimation method makes sense and is feasible, we perform Monte-Carlo experiments in which we pick up some values of parameters for which we simulate the model. In this exercise we neutralize the mortality risk by setting $\pi_t = 1, \forall t < T_i$. We then estimate structural parameters²³ of agents'

²¹The source code is available on Gitlab at <https://gitlab.insee.fr/patrimoine-destinie/capitulation> and takes the form of several `R` packages.

²²Tests have been performed on a 64-bit Windows Machine with an Intel Core i5-6300U 2.4GHz processor.

²³In such an exercise we consider indifferently β or γ since from a statistical viewpoint there is no reason to prefer one parameter to another.

preferences based on the indirect inference estimation procedure presented previously, and we compare these values with the ones fixed by us. Reassuringly, the auxiliary statistics chosen seem informative enough about true preferences, which gives therefore some credit to the estimation approach adopted here. In Table 4 we check that different true values of parameters (0.96 and 0.98 as regards β , 0.5 and 2 as regards γ) can actually be recovered by our estimation procedure. Changing the initial point does not seem to perturb much the estimation -neither does the restriction to moments based on the population aged between 40 and 60.

6 Results

6.1 Parameter estimation

We now present the results from the estimation issued from the overidentification of the parameter β based on previous statistics of the empirical distribution of private wealth. Those estimates are obtained under the assumption that $r = 0.03$. Though it is possible to pick up any arbitrary value of γ as explained above, we focus on the case where $\gamma = 1$; the latter assumption means that, since the effect of the interest rate on the consumption growth is given approximately by

$$\frac{d \log(C_{t+1}/C_t)}{dr} \approx \frac{d \log(C_{t+1}/C_t)}{d \log(1+r)} = \sigma,$$

an increase in the real interest rate by 100bp increases consumption growth by about $\sigma = 1\%$. Table 5 reports the estimates obtained for β given different values of γ , neutralizing the mortality risk (i.e. imposing that $\pi_t = 1$ for any t before T_i) or not. First, all those estimates are rather high and close to 1, mostly comprised between .99 and 1.01, which suggests that much patience is required to accommodate both observed life-cycle profiles of wealth and corresponding growth rates. Yet it should be kept in mind that a lower degree of patience would be obtained in the case of a higher EIS, i.e. a lower γ , which reassuringly tempers that caveat. Second, as expected, a higher degree of patience is needed to rationalize the accumulation of wealth in the presence of a mortality risk, especially large amounts of capital observed at older ages when the survival probably begins to fall substantially. From a structural life-cycle model with mortality risk and bequest motives, [Gan et al. \(2015\)](#) estimate nevertheless very similar time discount factors, based on indirect inference from wealth moments, too. When $r = 0.03$, their coefficient of relative risk aversion is about 0.81 (0.18) and their time discount factor cannot be distinguished from 1. Our coefficients are therefore in line with

these estimates and previous estimates found in the literature, including [Cagetti \(2003\)](#). Figure 2 examines now the quality of the prediction as regards the 13 cross-sectional moments. The model performs well at middle ages from 40 to 65; it over-predicts the wealth detained by individuals aged less than 30, and under-predicts the wealth detained by those aged 70 or more. Hopefully, the vast majority of observations in the survey lie in the 40-65 range (corresponding weights in the estimation are depicted in the same Figure). More generally, the relatively poor fit at extreme ages should not be surprising: as [Calvet et al. \(2021\)](#) recall, simulated life-cycle models do not match well empirical patterns of wealth before 40 for at least four reasons: (i) housing purchases, (ii) transfers from relatives, (iii) investments in education, and (iv) changes in family size. The fit is also expected to be poor after retirement due to health shocks.

The model behaves quite properly in the longitudinal dimension, when it comes about replicating the accumulation of wealth as observed in the panelized sample of wealth between 2015 and 2018. Figure 3 reports the fit of the model in this dimension: old age (after 65) and the 45-50 put aside, the model yields quite satisfactory predictions in this respect. It shall be remembered that the structural part of the pure life-cycle model, i.e., leaving bequests aside, is extremely parsimonious.

Figure 4 evaluates how the model fits the cross-sectional distribution of net wealth observed in the *Enquête Patrimoine* 2017-2018, i.e., after nine annual iterations.²⁴ Observed and simulated distributions of net wealth look very close in IHS scale. From this viewpoint, all simulated quantiles of the distribution of net wealth resemble the ones observed. The only exception concerns the bottom of the distribution, namely below the first decile. It can be explained by the absence of borrowing constraint in the model which tends mechanically to predict more negative net wealth. The fact that the IHS compresses the distribution of wealth at the extremes but expands it around zero also helps to explain the mismatch at the bottom of the distribution.

Finally, Figure 5 emphasizes the importance of introducing the mortality risk in the model in order to replicate a hump-shaped profile of consumption over the life-cycle. In the absence of any mortality risk, i.e. when $\pi_t = 1$ for any t before T , and since $\hat{\beta}R > 1$, predicted consumption would keep on increasing over time at that power rate. By contrast, in the presence of the mortality risk (and in the absence of any income risk, i.e. of any precaution motive for savings), that consumption pattern exhibits some inverted-U shaped relationship; the corresponding acme being reached at old ages due to observed survival probabilities decreasing

²⁴Though observed wealth in 2015 is used as external validation, the model starts in 2009.

substantially, i.e. faster than previous power rate, after 75.²⁵

6.2 Simulation of net wealth patterns over the life-cycle

Given previous estimations, we now simulate life-cycle patterns of net wealth. Such simulations are performed from 2009 to 2070, i.e., to the end of the horizon of *Destinie 2*, but are only presented up to 2040. The results are displayed first by Figure 6 which depicts the accumulation of private wealth over the life-cycle as a series of annual cross-sections. By construction, the initial simulation coincides with the observed pattern of net wealth in *Patrimoine 2009-2010*, which corresponds to the curve in brightest red. It is striking to see how each simulation reflects the structure imposed by the model: these wealth profiles become mechanically more hump-shaped as simulated time goes by. They are consistent with empirical facts based on the 2015 wealth survey, for instance Ferrante and Solotareff (2018). These results suggest that despite its extreme parsimony, the model achieves reasonable projection properties in terms of median private wealth and its variation all over the life-cycle.

Figure 7 depicts then how the share of capital income among total income (defined as the sum of labor and financial capital income) evolves over the life-cycle. The empirical evidence is more scarce: to the best of our knowledge, the only stylized fact on French data is provided by Insee (2018). This share would be equal to 10.2% on average, but would amount to 16.6 % among retirees. An important motivation for the empirical approach proposed here is precisely related to the computation of adjusted standards of living that would take capital income into account. It is therefore interesting to observe that the model is able to replicate this share, and its hump-shaped pattern over the life-cycle. At old ages, this share is roughly higher, though, than it was at younger ages, which again has implications for inter-generational equity concerns, and for the funding of pensions schemes.

6.3 Simulated inequality in labor and capital income

To estimate the impact of taking capital income into account when computing inequality indices, we thus resort to our simulations and confront them with observed data. We consider as before that the rate of return to private wealth, namely the real interest rate, is equal to 3%, and emphasize that our results are mechanically sensitive to that assumption.

²⁵Introducing a precautionary motive in the model would make that acme occur earlier in the life-cycle.

Table 6 shows that the top 10% would perceive slightly less than 30% of total income; this figure is actually below the estimate from the 2015 wealth survey, 39.5%, partly because the microsimulation model *Destinie 2* alone, i.e., without the structural model of wealth accumulation developed here, already underestimates the concentration of labor income. This is due to the fact that the intensive margin of labor at the individual level is absent from the microsimulation model: individuals either work full time, or don't work at all. As a result, the top 10% (resp. 1%) perceives 32% (resp. 9.3%) of labor income in the data while the model predicts 26.7% (6.1%) only.

As far as the concentration of net wealth is concerned, the model predicts a higher concentration for wealth than for labor income: see, e.g., the top shares displayed in Table 6.²⁶ Yet it fails to replicate accurately the share detained by the top 10%, namely 66.5%: it under-predicts that share (only 48.8%). The picture looks far less dramatic as regards the top 1% where the differential between observed (22.7%) and simulated (21.3%) shares is considerably reduced. There are at least two reasons why the concentration at the top of the distribution is still reproduced inaccurately: (i) the estimation procedure implemented here aims at fitting median wealth (and their growth rates), and not higher quantiles of the distribution for instance; (ii) both observed and unobserved heterogeneity in structural parameters are missing. In particular, high income individuals likely benefit from highest rates of return. Besides, replicating the very high concentration of net worth at the top of the distribution is an active area of research (see, e.g., [Blanchet, Fournier, and Piketty, 2021](#)), which would probably require more reduced-form, statistical approaches than the structural method adopted here.

Last, the role of the mortality risk on the unequal detention of wealth can be assessed from Table 6 by comparing bottom and middle panels.²⁷ Neutralizing the mortality risk by setting $\pi_t = 1$, for any t before T , leads to a slight decrease in inequality: this risk, which depends on time, date of birth, and sex, amplifies slightly differences in wealth accumulation across individuals, not by much according to our simulations. In 2015, that mortality risk alone would be responsible of a higher concentration at the top, about 1.5pp (resp. 2.9pp) for the top 10% (resp. top 1%) -to be compared with the baseline 47.3% (resp. 18.6%).

To go beyond those simulations, the microsimulation combined with the struc-

²⁶The Gini coefficient amounts to about 0.65 for wealth and to nearly 0.4 for income.

²⁷Note that the mortality risk has both a direct effect, through agents' behavior (holding preferences unchanged), and an indirect effect, through the estimation of impatience: a higher degree of patience is required to rationalize the data in presence of mortality risk. We do not disentangle the former from the latter in the current analysis.

tural model can be considered as a useful projection tool for policy analysis such as the one on retirement issues. Figure 8 shows the projected change in labor income inequality as well as in wealth inequality, based on projected top shares. As expected in the absence of any shock in the microsimulation environment, and especially in the absence of any mortality risk, the model predicts that this indicator remains relatively stable over time. However, the introduction of uncertainty, due to mortality risk, tends to amplify inequality as simulated time goes by.

6.4 Computing “augmented” standards of living

Last, we construct “augmented” standards of living defined as the ratio of total income, including both labor and capital income, over the number of consumption units in the household, according to the definition provided by the modified OECD scale. The projected evolution of the relative standard of living of retirees, defined as the ratio between the median standard of living of retirees and the one that prevails in the whole population, is then depicted by Figure 9. As simulated time goes by, the relative standard of living (based on a definition that excludes capital income) decreases: this is because pensions have been indexed on the inflation rate only since 1987 while for individuals in the labor force, earnings grow at a higher pace, namely the long-term productivity rate.²⁸ Including capital income drives this relative standard of living upwards by slightly less than 2pp at the beginning of the projection (2010-2020), but at about +8pp twenty years after. This differential is related to retirees being older and disposing of more wealth. Note also that this gap widens as simulated time goes by because the rate of return of capital income exceeds the long-term productivity growth rate.

Figure 10 isolates further the mere role of the mortality risk in this regard: by keeping the time preference rate unchanged, namely at its estimated value in the absence of any mortality risk (about 0.994). Two curves are plotted, corresponding to projected augmented relative standards of living respectively in the absence of any mortality risk, i.e. when $\pi_t = 1, \forall t < T$ (in blue) and with some mortality risk, i.e. when $\pi_t < 1, \forall t < T$ (in red). Due to the mortality risk, aggregate wealth is lower, which hurts relatively more older individuals disposing of more wealth, those individuals being at the end of the accumulation process. Remember nevertheless that introducing the mortality risk leads to a higher estimated time preference rate in order to rationalize the data, which tempers the previous diagnosis, acts thus as a countervailing mechanism and explains why the blue dashed curve of that Figure lies, in fact, in-between the red and blue solid curves of Figure 9.

²⁸Calibrated to 1.5% in our simulations.

7 Conclusion

This paper has embedded a structural model of wealth accumulation with bequests within a microsimulation framework that conveys much heterogeneity at the individual level, especially as regards career and family history. In this environment, parameters governing agents' preferences are recovered through indirect inference. Simulations based on this estimation enable us to replicate observed wealth patterns over the life-cycle as well as the dynamics of individual accumulation. It has then been possible to quantify implications on the “augmented” standard of living which includes capital income, private wealth acting as a countervailing mechanism to the relative dropout after retirement due to indexation of pensions on inflation. Last, we isolate and quantify the role of the sole mortality risk.

Our simulations are helpful to empirically quantify the importance of considering total income (capital income on top of labor income) instead of the sole labor earnings. From that viewpoint, they are of certain interest as far as the optimal design of pensions is concerned. To illustrate, let us consider two workers with identical life-time working paths and similar family status; yet A has accumulated more private wealth than B, either from inheritance or by saving more. Should A then be entitled to lower pension benefits? The answer depends on social preferences as regards the redistributive nature of pensions, and of social insurance, more generally. It is yet remarkable that in Germany, the eligibility condition for unemployment benefits,²⁹ i.e. another component of social insurance, namely unemployment insurance, explicitly includes private wealth on top of income, which hence shows that it is possible to condition social benefits on wealth. Disposing of appropriate methodological tools in the projection of pensions is a corresponding prerequisite; we see our work as a contribution in that direction.

We last mention some extensions that are left for further research. Including heterogeneity in asset returns, among other structural parameters, is a challenging task so as to replicate the extreme concentration of wealth at the top of the distribution.³⁰ The introduction of borrowing constraints is also expected to help improve the fit at the bottom of the distribution. Another extension would consist in matching not only the median but also other (higher) quantiles of the distribution of wealth or/and its growth rate.

²⁹*Bürgergeld* have replaced former ALG II since 2023.

³⁰A recent literature emphasizes that wealthy individuals benefit from much higher asset returns. [Lusardi, Michaud, and Mitchell \(2017\)](#) stress the role played by financial knowledge in this respect; [Fagereng et al. \(2019\)](#) find that the wealthiest do not save more, but that they do face higher capital gains; [Bach, Calvet, and Sodini \(2020\)](#) show that such heterogeneity explains most of the increase in top shares.

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We are indebted to Théo Guichaoua for outstanding research assistance. We are grateful to Mathias André, Luc Arrondel, Carole Bonnet, Nicolas Drouhin, François Le Grand, André Masson, Benoît Rapoport, Sébastien Roux, and Maxime Tô for valuable comments as well as to the attendees of the 37th Journées de Microéconomie Appliquée (Annecy 2021, on-line) conference, of the "Pensions and ageing" workshop (CDC Paris, 2019) and of the Insee-D2E (Paris 2019) seminar.

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Figures

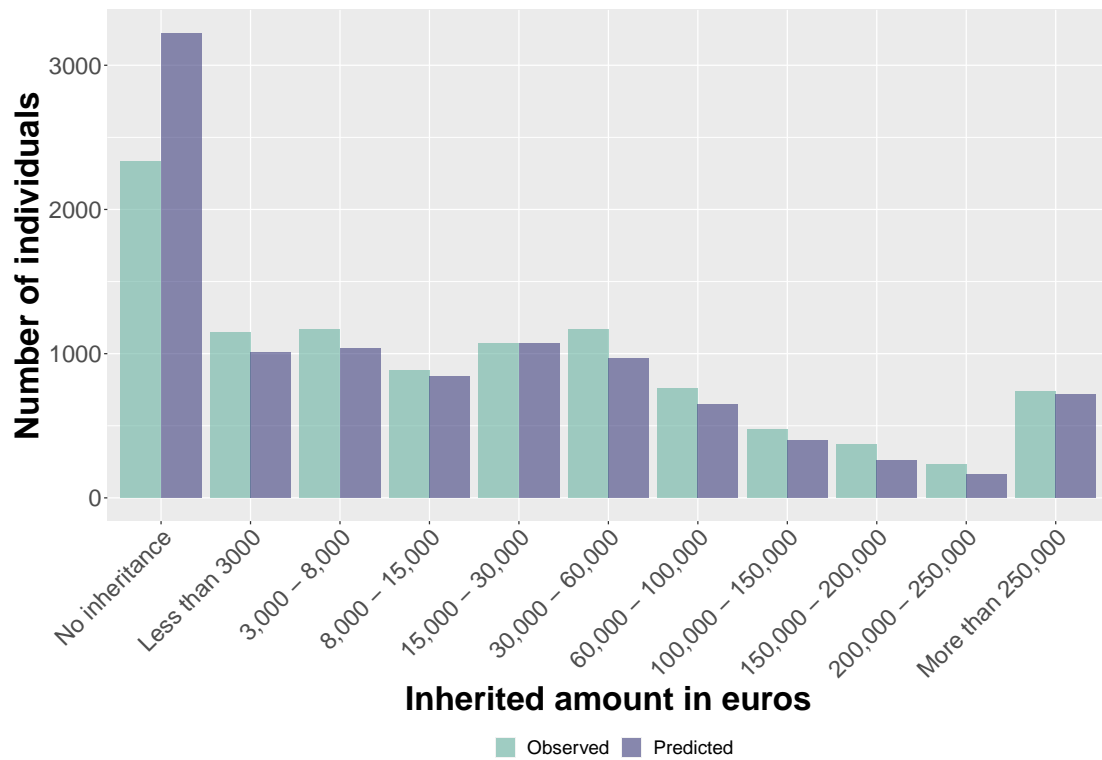


Figure 1: Fitting the empirical distribution of bequests

Source. Enquête Patrimoine 2014-2015.

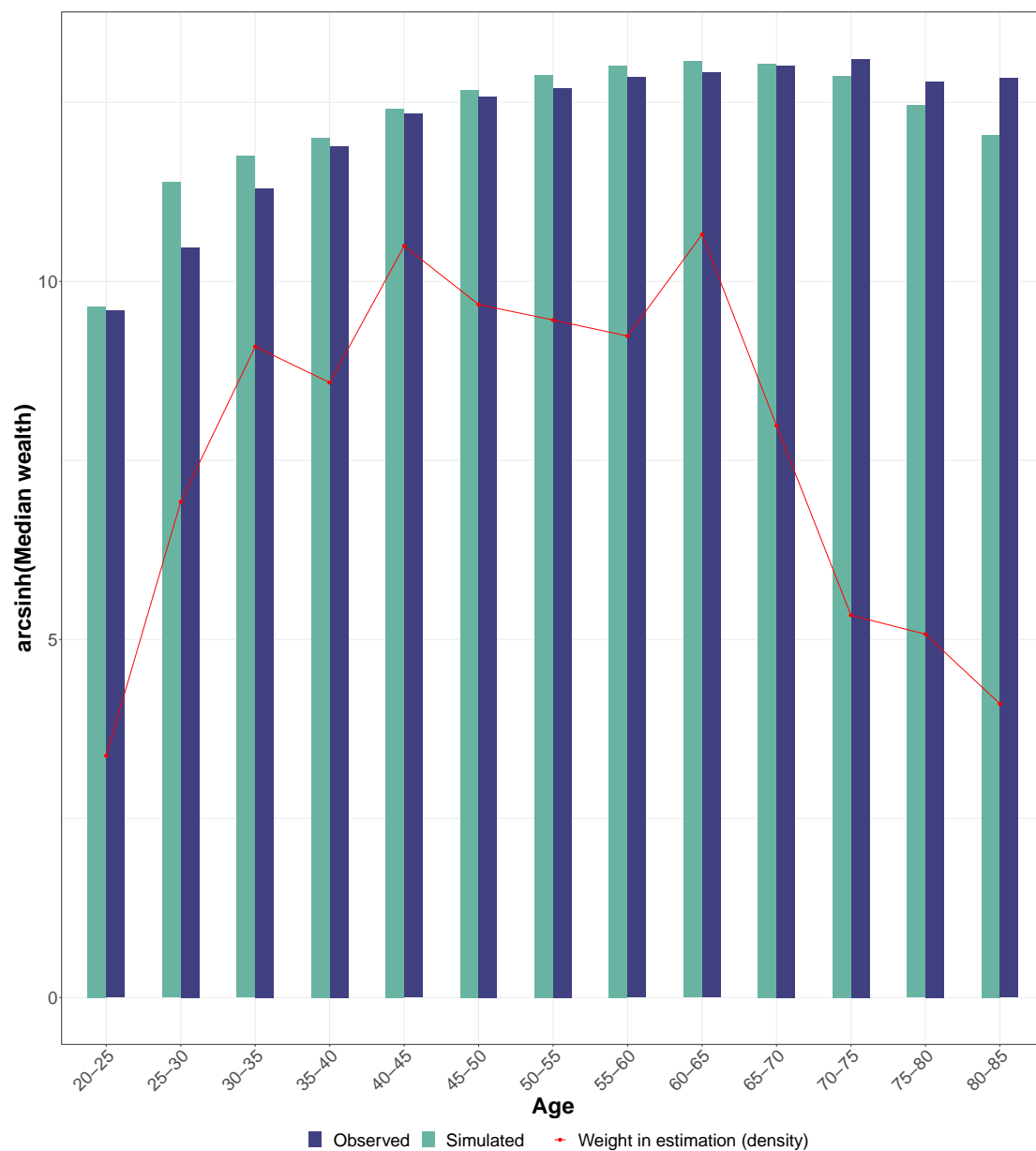
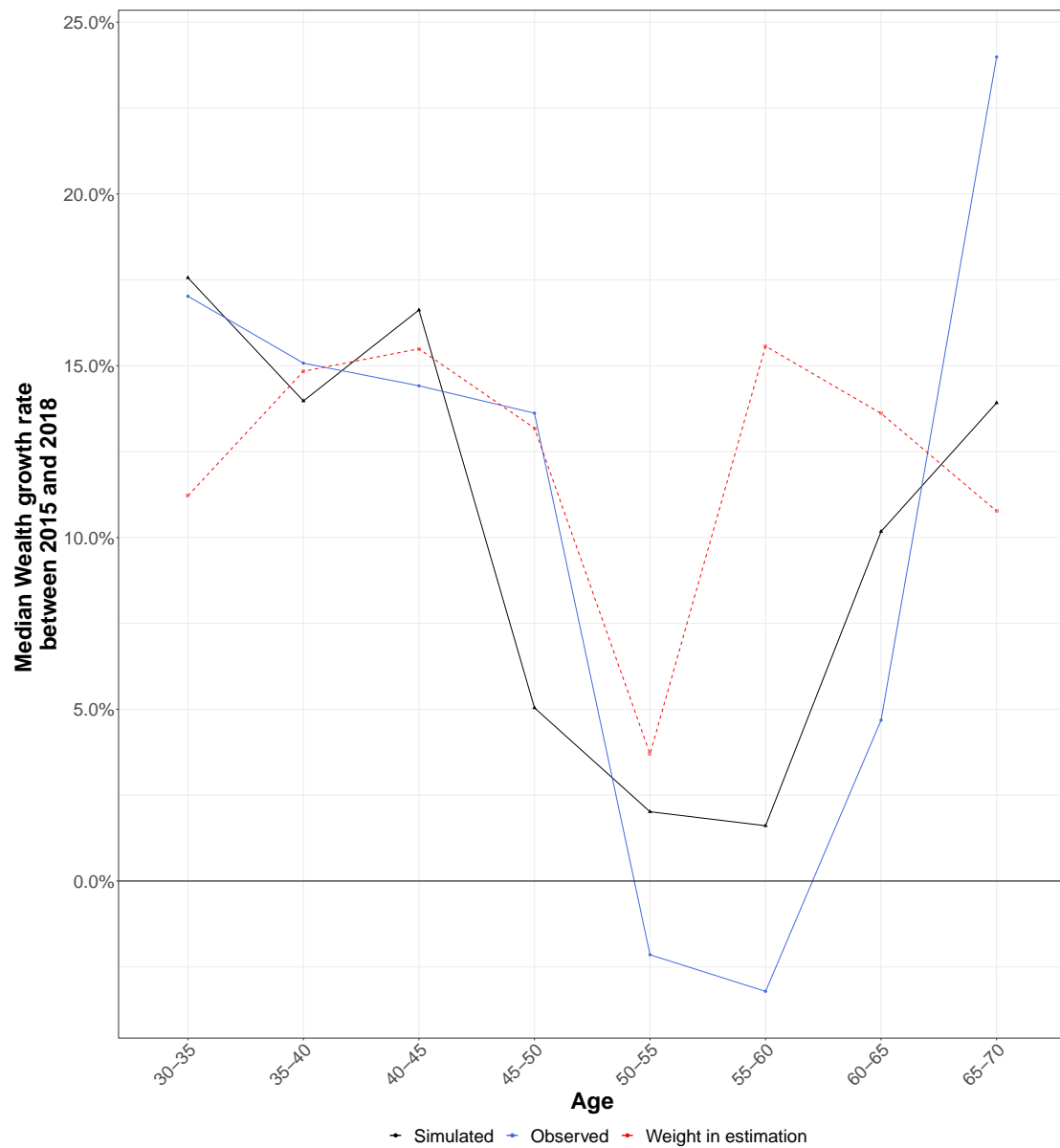


Figure 2: Quality of the fit - first set of moments (median wealth in 2015, by age)

Sources: *Enquête Patrimoine* 2014-2015 (observed); microsimulation model *Destinie 2* associated with the structural model (simulated).

Lecture. In 2015, the simulated median wealth of individuals aged between 40 and 45 is about 12.2 in IHS scale (around 99,000 €); the observed median wealth is equal to 12.3 in that same scale (about 110,000 €). Sample weights for each moment in the estimation procedure are in red.

Figure 3: Quality of the fit - second set of moments (median growth rate of wealth between 2015 and 2018, by age)



Sources: *Enquêtes Patrimoine* 2014-2015 and 2017-2018 (observed); microsimulation model *Destinie 2* associated with the structural model (simulated).

Lecture. Between 2015 and 2018, the observed median wealth growth rate between 2015 and 2018 was 14.4% for people aged 40 to 45; the corresponding simulated rate is 12.8%. Sample weights for each moment in the estimation procedure are in red.

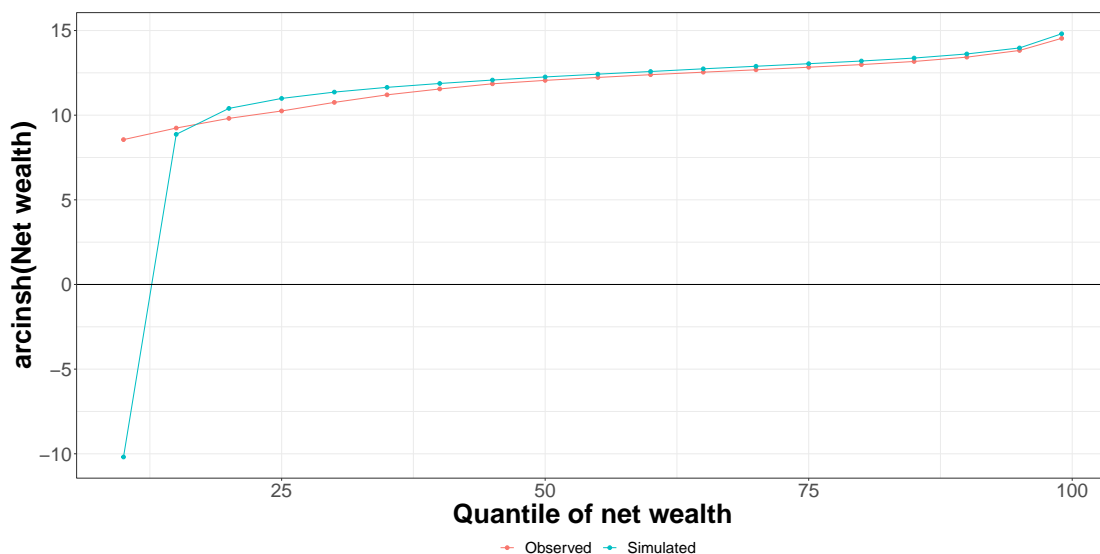


Figure 4: Fitting the empirical distribution of wealth in 2018

Sources: *Enquête Patrimoine 2017-2018* (observed); microsimulation model *Destinie 2* associated with the structural model (simulated).

Lecture. In 2018, the simulated median wealth is equal to 12.22 on a IHS scale (around 100,000 €) while the observed median wealth is equal to 12.05 (around 86,000 €).

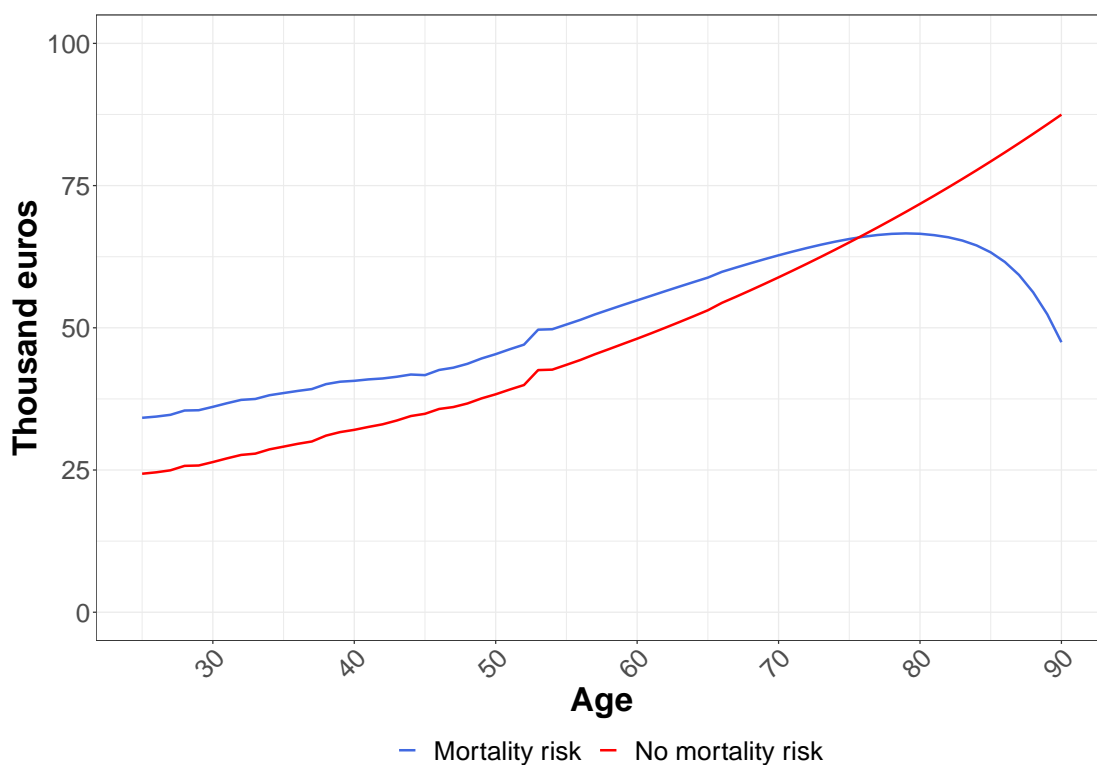


Figure 5: Consumption pattern over the life-cycle: the role of the mortality risk

Calibration: $\beta = 0.99, \gamma = 1, r = .03$

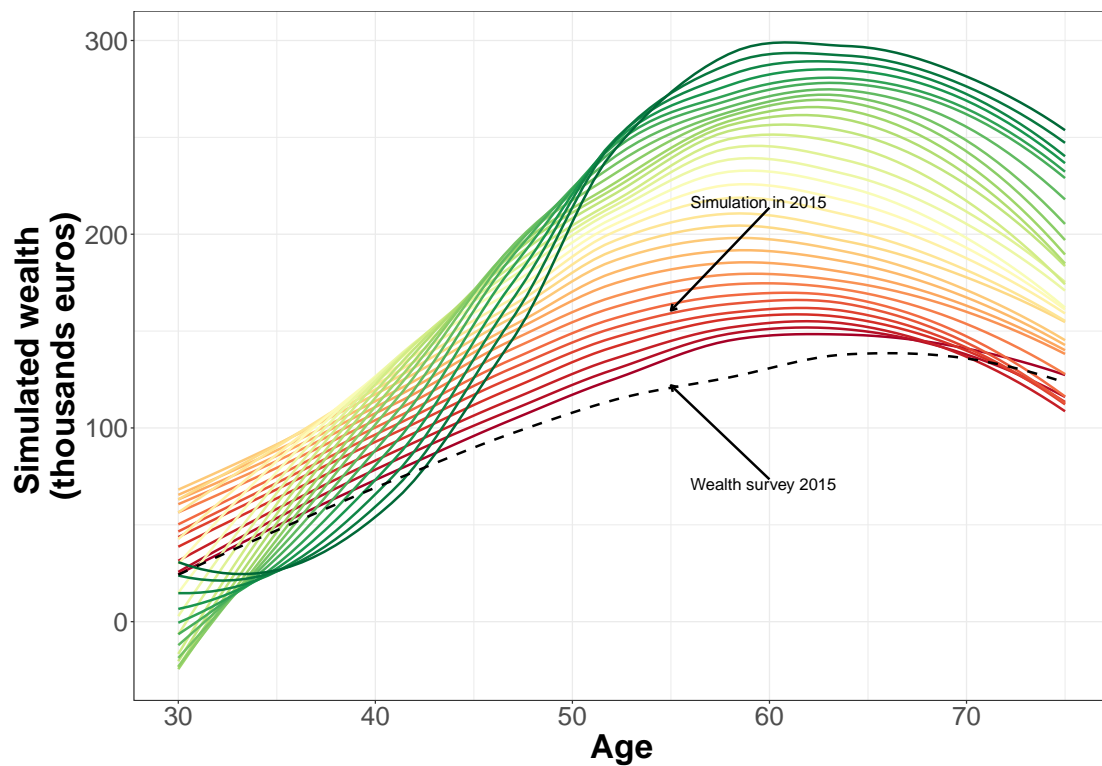


Figure 6: Life-cycle patterns of wealth (simulations from 2009 to 2040)

Sources: *Enquête Patrimoine* 2014-2015 (observed); microsimulation model *Destinie 2* associated with the structural model (simulated).

Lecture. Life-cycle profile of wealth observed (dash) in *Enquête Patrimoine* 2014-2015. Simulations from 2009 (red) to 2040 (green) using the microsimulation model *Destinie 2* associated with the structural model. By construction, observed and simulated curves coincide in 2009 (brightest red).

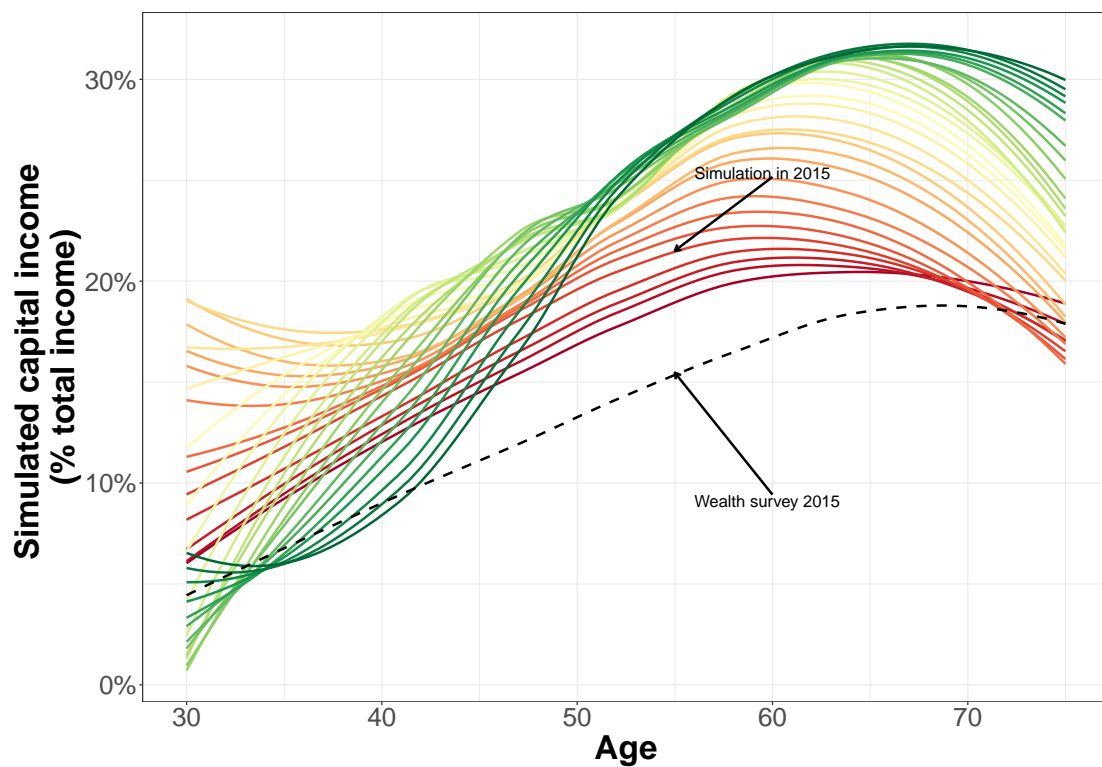
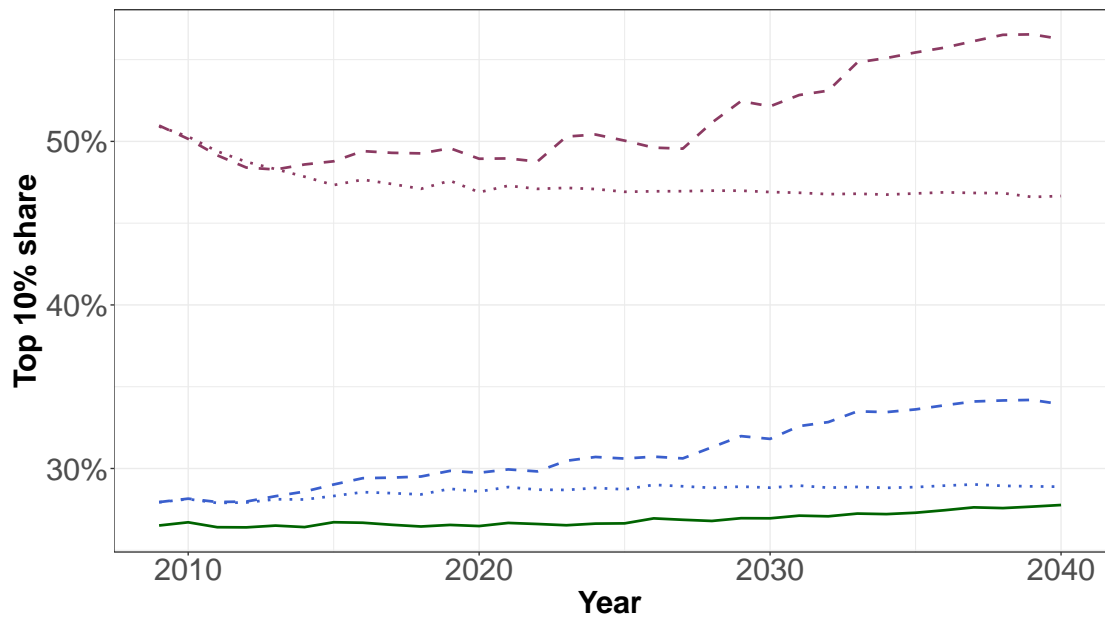


Figure 7: Capital income share over the life-cycle (simulations from 2009 to 2040)

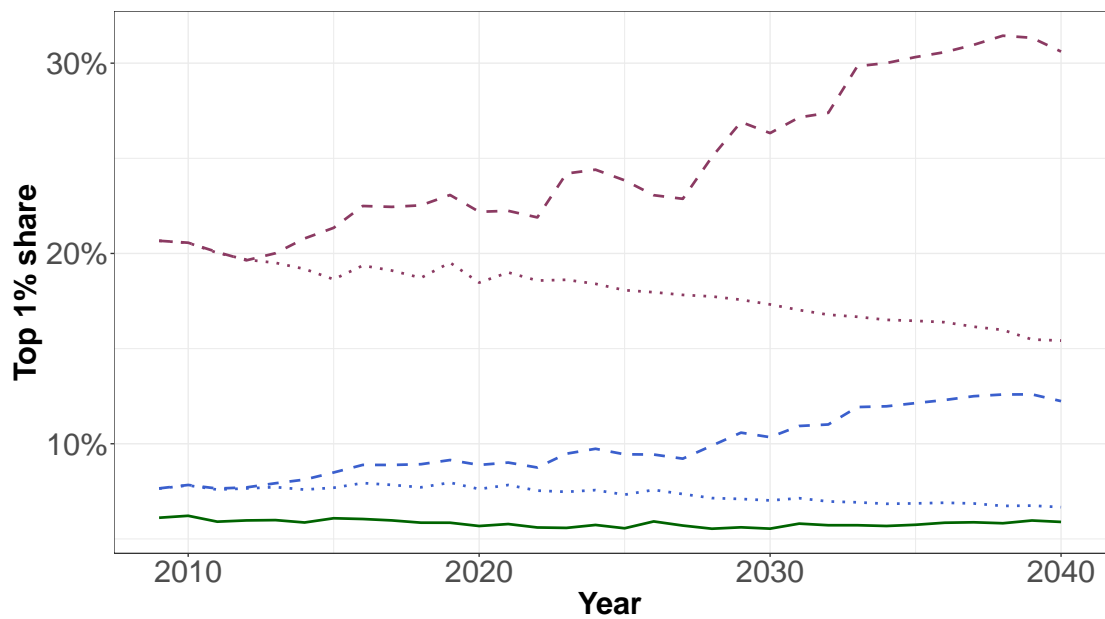
Sources: *Enquête Patrimoine* 2014-2015 (observed); microsimulation model *Destinie 2* associated with the structural model (simulated).

Lecture. Life-cycle profile of the share of capital income in total income: observed (dash) in *Enquête Patrimoine* 2014-2015; simulated from 2009 (red) to 2040 (green) using the microsimulation model *Destinie 2* associated with the structural model.



Model – Common component – Mortality risk ·· No mortality risk
Variable – Labor income – Total income – Wealth

(a) Top 10%



Model – Common component – Mortality risk ·· No mortality risk
Variable – Labor income – Total income – Wealth

(b) Top 1%

Figure 8: Simulated long-run wealth inequality

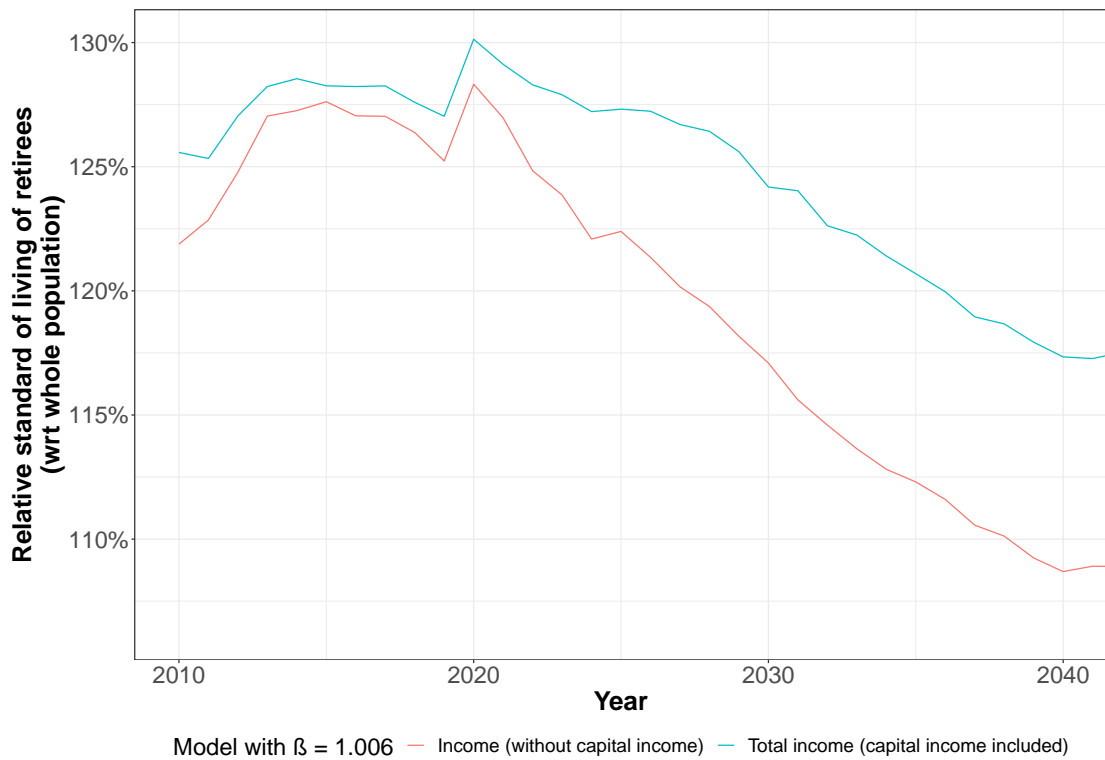


Figure 9: Relative standard of living of retired individuals

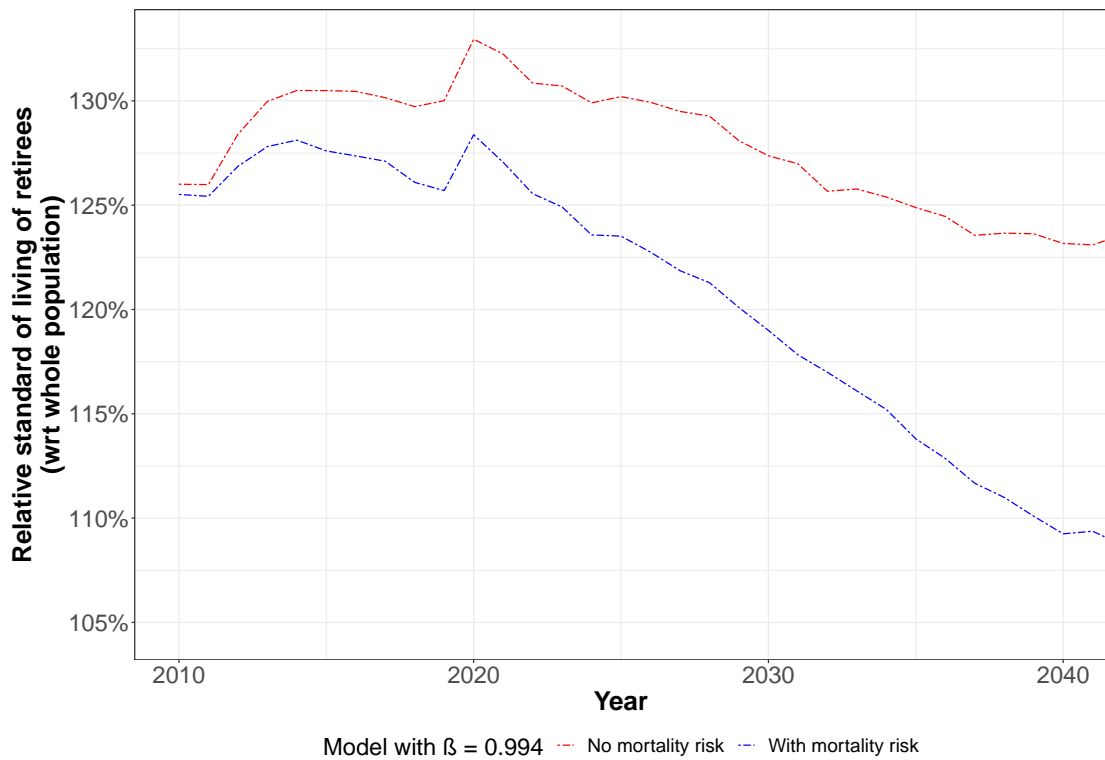


Figure 10: Relative standard of living of retired individuals: Illustrating the role of the mortality risk

Tables

Table 1: Model of bequests (2-step estimation)

	<i>Dependent variable:</i>	
	Prob. bequest (0/1) (SELECTION)	Log(bequest) (OUTCOME)
Constant	1.293*** (0.298)	8.021*** (0.351)
Female		0.070* (0.043)
Log(income)		0.084*** (0.023)
log(σ)		0.557*** (0.010)
Age dummies	Yes	Yes
School-leaving age dummies	Yes	Yes
Observations	10,344	8,012
Log likelihood (by obs.)	-0.478	-2.141
Bayesian information criterion	10,084	34,520
Model	Probit	Interval regression

Note: *p<0.1; **p<0.05; ***p<0.01

Sample of individuals whom both parents are deceased.

The probability of receiving a bequest once both parents have died is estimated by a Probit model from *Enquête Patrimoine 2014-2015*. The intensity of bequest is estimated from an ordered Probit with known thresholds (a.k.a interval regression) using declared positive received bequests in the same survey.

Table 2: Simulated distribution of inherited wealth

Q1	Median	Mean	Q3	P90	# of obs.
7,430	26,987	180,122	93,731	330,497	19,615

Table 3: Estimation procedure: external validation

Use	Information	Wealth survey year	Sample	Restriction to sample
Fit K_{2009}	Individual matching with microsimulation model	2009	All	
Moment s_1	Median wealth held by 5-years cohort (arcsinh scaled)	2015	All	People aged from 20 to 85 in 2015 (13 moments)
Moment s_2	Median growth rate between 2015 and 2018 $K_{i,2018}/K_{i,2015}$ (5 years cohorts)	2015 ; 2018	Panelized	People aged from 30 to 70 in 2015 with non-negative wealth (8 moments)

Note. Moments are weighted by the corresponding sample size in wealth surveys *Enquête Patrimoine* 2009-2010, 2014-2015 and 2017-2018.

Table 4: Monte-Carlo experiments

<i>Scenario:</i>	Variation on β			Variation on γ	
	(1) $\beta^{(0)} = 0.75$	(2) $\beta^{(0)} = 1.2$	(3) $\beta^{(0)} = 0.75$	(1) $\gamma^{(0)} = 1.2$	(3) $\gamma^{(0)} = 3$
<i>All moments:</i>					
Main specification	0.980	0.980	0.960	2.000	0.500
Restrict to population aged from 40 to 60	0.980	0.980	0.960	2.000	0.500
<i>First set of moments only:</i>					
Main specification	0.980	0.980	0.960	2.000	0.500
Restrict to population aged from 40 to 60	0.980	0.980	0.960	2.000	0.500
<i>Second set of moments only:</i>					
Main specification	0.980	0.980	0.960	2.000	0.500
Restrict to population aged from 40 to 60	0.980	0.980	0.960	2.104	0.469

Scenario: No mortality risk ($\pi_t = 1, \forall t$)

(1): ($r = 3\%, \beta = 0.98, \gamma = 2$)

(2): ($r = 8\%, \beta = 0.96, \gamma = 2$)

(3): ($r = 3\%, \beta = 0.98, \gamma = 0.5$)

Table 5: Estimated parameters

	$\gamma=0.8$	$\gamma = 1$	$\gamma = 1.2$
Discount factor β (model with mortality risk)	1.000*** (2e-05)	1.006*** (7e-05)	1.011*** (2e-04)
Discount factor β (model without mortality risk)	0.989*** (4e-04)	0.994*** (4e-04)	0.998*** (0.001)
Number of moments	21		

Note.

*p<0.1; **p<0.05; ***p<0.01

Model estimated by minimum distance between sample moments from Table 3 and simulated moments computed in the microsimulation model.

Table 6: Top income and wealth simulated shares (in %)

GROUP	LABOR INCOME	TOTAL INCOME	NET WEALTH
WEALTH SURVEY			
Top 10 %	32	39.5	66.5
Top 1 %	9.3	12.6	22.7
SIMULATED DATA (MORTALITY RISK)			
Top 10 %	26.7	29	48.8
Top 1 %	6.1	8.5	21.3
SIMULATED DATA (NO MORTALITY RISK)			
Top 10 %	26.7	28.3	47.3
Top 1 %	6.1	7.7	18.6

Total income is the sum of labor income (labor earnings, unemployment benefits, and retirement pensions) and of capital income. Negative wealth is bottom-coded to zero.

Appendix

Inheritance in France: legal rules

Donation rights depend on the nature of the family ties between the deceased and the successor. The descendants of the deceased have an inalienable right to the deceased's wealth. Regardless of the will of the deceased, the children or their successors inherit 3/4 of the deceased's wealth. A spouse either chooses to inherit 1/4 of the deceased's wealth or retains the usufruct of all the deceased's wealth. For the sake of simplicity, we consider only the first situation in our inheritance model.

Two steps are required to determine the amount of the inheritance tax:

1. Subtract a deduction from the bequest to get the taxable amount
2. Apply a progressive tax schedule to get the inherited amount

Deduction and tax rates are applied to each individuals. Tax rates differ depending on whether the heir is in direct line or not. For now, we consider only legal rates for children (see Table 7).

As explained in Section 4, since the model is estimated on children's inherited wealth, we need to invert the taxation scheme above in order to recover the amount that has been bequeathed by the deceased.

Table 7: Inheritance taxation rules

Taxable share after deduction	Marginal tax rate
Less than 8 072 €	5 %
Between 8 072 and 12 109 €	10 %
Between 12 109 and 15 932 €	15 %
Between 15 932 and 552 324 €	20 %
Between 552 324 and 902 838 €	30 %
Between 902 838 and 1 805 677 €	40 %
More than 1 805 677 €	45 %

France applies an automatic deduction (*abattement*) to inheritance taxation. Hence, to derive taxable inheritance amount, 159 325 € are automatically deducted from the amount given.