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► **To cite this version:**

Jose Ordonez-Miranda, Yunhui Wu, Masahiro Nomura, Sebastian Volz. Far-field thermal radiation driven by the temperature oscillations of macroscopic bodies. *Physical Review Applied*, 2024, 22 (5), pp.054053. 10.1103/PhysRevApplied.22.054053 . hal-04795387

**HAL Id: hal-04795387**

**<https://hal.science/hal-04795387v1>**

Submitted on 21 Nov 2024

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# Far-Field Thermal Radiation Driven by the Temperature Oscillations of Macroscopic Bodies

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(Dated: October 31, 2024)

We demonstrate that the time-averaged far-field thermal radiation between macroscopic bodies can be different from Planck's limit when their temperature difference is periodically modulated. This difference arises from the nonlinear dependence of the radiative heat flux on temperature and persists even for temperature-independent emissivities. By contrast, when the material emissivity depends on temperature, the modulated thermal radiation is driven by the second temperature derivative of emissivity and increases with the square of the temperature oscillations' amplitude. These temperature fluctuations generate thermal radiation even without a mean temperature difference. Our findings thus reveal a novel mechanism to control and modulate far-field radiation with potential applications in thermal management and energy harvesting.

## I. INTRODUCTION

Manipulation of the radiative heat currents is of primary interest to develop various technologies in thermophotovoltaic energy conversion, radiative cooling, thermal computing, and satellite management. The classical radiative heat transfer is described by Planck's law [1], which establishes an upper limit for the radiative heat flux between two macroscopic bodies separated by a distance  $d$  much longer than the dominant radiation wavelength  $\lambda_{th}$  ( $\approx 10 \mu\text{m}$  at room temperature). This Planck's limit can, however, be overcome by many orders of magnitude in the near- ( $d < \lambda_{th}$ ) and far-field ( $d \gg \lambda_{th}$ ) regimes characterized by the transmission of evanescent electromagnetic waves [2–8] and bodies with dimensions smaller than  $\lambda_{th}$  [9–11], respectively. Further, Tachikawa et al. [12] recently demonstrated that the Planck's limit can also be exceeded in the far-field radiation between two silicon microplates coated with silica nanolayers. All these previous works were conducted for stationary temperatures and clearly show that the super Planckian thermal radiation can be obtained when at least one of the dimensions of the radiating bodies and/or their separation distance is comparable to or smaller than  $\lambda_{th}$ .

The thermal radiation driven by temperature fluctuations periodically modulated in time has been explored less, but it is attracting an increasing interest due to its implications for radiative heat currents. By studying the thermal radiation between two bodies with an oscillating temperature difference, Latella et al. [13] demonstrated the existence of a radiative heat shuttling, a supplementary flux able to enhance or reduce the one produced by the mean temperature difference. These temperature oscillations were also exploited to amplify and control heat currents via a far-field thermal transistor [14] and thermal memristor [15], respectively. More recently, Yu and

Fan [16] showed that the time modulation of permittivity can also enhance, suppress, and reverse the near-field radiation between two bodies at fixed temperatures. Considering the nonlinear dependence of the radiative heat flux on both the temperature and permittivity, the periodic modulation of these parameters can be used to modulate heat currents. In addition, the periodic temperature fluctuations of bodies with a temperature-dependent emissivity generate a net radiative heat flux even in the absence of a mean temperature gradient [13, 17]. The first experimental observation of this latter shuttling effect was recently reported for the far-field radiation between two composites based on VO<sub>2</sub> [18]. These theoretical and experimental results thus show that the temperature fluctuations represent a promising mechanism to amplify heat exchanges, cool down solids, and insulate them from each other. This radiative amplification could lead to overcoming the Planck's limit of far-field radiation, however, it has not been explored yet.

In this work, we demonstrate that the classical Planckian regime of far-field thermal radiation can become effective super Planckian in presence of a temperature difference periodically modulated in time. This radiation enhancement increases with the amplitude of the temperature oscillations and appears for macroscopic materials of arbitrary geometry, even when their emissivity is independent of temperature. Our analytical results thus uncover a general thermal radiation mechanism driven by temperature fluctuations that could be useful to enhance the radiative heat flux of systems out of equilibrium.

## II. THEORETICAL MODEL

Let us consider two macroscopic bodies of arbitrary geometry exchanging heat by far-field thermal radiation due to their temperature difference periodically modulated in time  $t$ , as shown in Fig. 1(a). The emissivity  $\varepsilon_m$  of body  $m = 1$  and 2 depends on its tempera-

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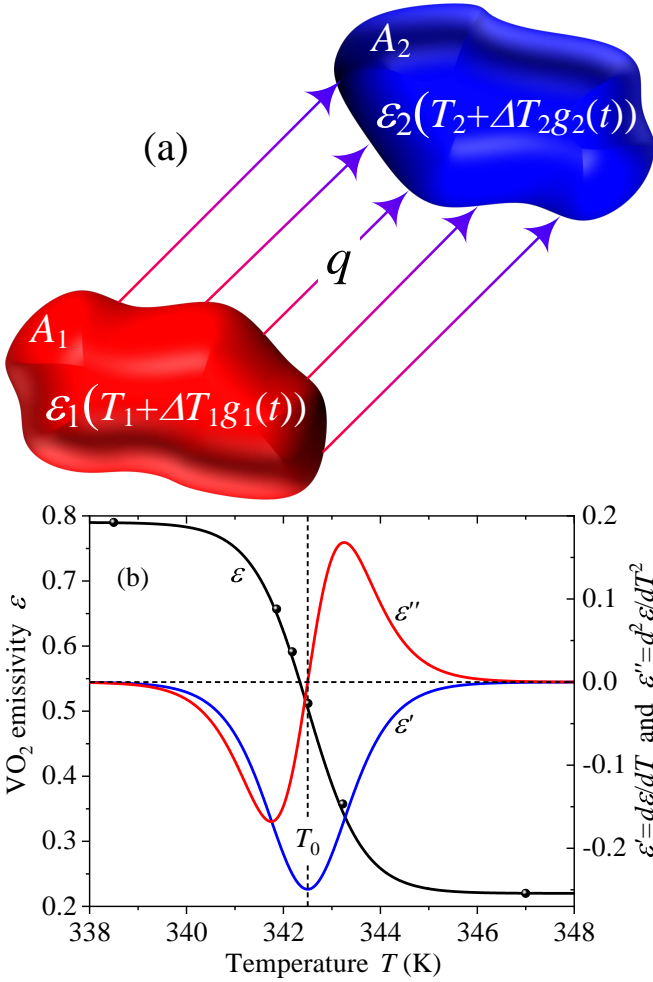


FIG. 1: Scheme of two bodies exchanging heat through a vacuum gap due to their temperature difference periodically modulated in time  $t$ . (b)  $\text{VO}_2$  emissivity and its temperature derivatives as functions of temperature. The points represent experimental data [19, 20], while the black line stands for their best fitting via Eq. (11).

ture  $T_m + \Delta T_m g_m(t)$ , where  $\Delta T_m > 0$  and  $g_m(t)$  is a periodic function of time with period  $\tau$  and magnitude  $|g_m(t)| \leq 1$ . This temperature dependence of emissivity is exhibited by solid-state phase-change materials, such as  $\text{VO}_2$  (see Fig. 1(b)), and a broad variety of materials in a wide enough temperature range. To ensure the thermalization of the bodies, we consider that  $\tau$  is much longer than their thermalization time (time required to reach the thermal equilibrium between its electrons and phonons), which typically ranges from a few to 100  $\mu\text{s}$ , for the case of  $\text{VO}_2$  [21] used in this work. This condition is good enough to guarantee that the heating frequency ( $\tau^{-1} \lesssim 1$  MHz) is much smaller than the spectral frequencies of the photons emitted by the two bodies, which are in the order of THz for temperatures around room temperature [22]. Under this quasi-steady state

and the assumption that  $T_1 > T_2$ , the far-field heat flux  $q$  (thermal power per unit area  $A_1$ ) exchanged by the two bodies is given by the Stefan-Boltzmann law, as follows [22]

$$q = \sigma \varepsilon \left[ (T_1 + \Delta T_1 g_1(t))^4 - (T_2 + \Delta T_2 g_2(t))^4 \right], \quad (1a)$$

$$\varepsilon^{-1} = \varepsilon_1^{-1} - 1 + \frac{A_1}{A_2} (\varepsilon_2^{-1} - 1) + F_{12}^{-1}, \quad (1b)$$

where  $\sigma$  is the Stefan-Boltzmann constant and  $F_{12}$  is the view (geometrical) factor determining the fraction of radiation that leaves surface  $A_1$  and strikes on surface  $A_2$ . For the sake of simplicity and clarity, we consider that body 2 is a blackbody ( $\varepsilon_2 = 1$ ) with dimensions much larger than its separation distance from body 1, such that  $F_{12} \approx 1$ . Under these conditions,  $\varepsilon = \varepsilon_1$  and Eq. (1a) reduces to

$$\frac{q}{\sigma} = \varepsilon_1 (T_1 + \delta T_1(t)) \left[ (T_1 + \delta T_1(t))^4 - (T_2 + \delta T_2(t))^4 \right], \quad (2)$$

where  $\delta T_m(t) = \Delta T_m g_m(t)$ . Considering that the temporal temperature fluctuations are much smaller than their corresponding steady-state counterparts ( $\Delta T_m \ll T_m$ ), the left-hand side of Eq. (2) can be expanded in a Taylor series. For an approximation up to  $(\Delta T_n/T_n)^2$ , this expansion yields

$$\frac{q}{\sigma} = \varepsilon_1 (T_1) p_0(t) + \varepsilon_1' (T_1) p_1(t) + \frac{\varepsilon_1'' (T_1)}{2} p_2(t), \quad (3)$$

where the modulated coefficients  $p_n(t)$  are defined by

$$p_0 = T_1^4 - T_2^4 + 4 [T_1^3 \delta T_1(t) - T_2^3 \delta T_2(t)] + 6 [T_1^2 \delta T_1^2(t) - T_2^2 \delta T_2^2(t)], \quad (4a)$$

$$p_1 = (T_1^4 - T_2^4) \delta T_1(t) + 4 [T_1^3 \delta T_1^2(t) - T_2^3 \delta T_1(t) \delta T_2(t)], \quad (4b)$$

$$p_2 = (T_1^4 - T_2^4) \delta T_1^2(t). \quad (4c)$$

The temporal fluctuations of the heat flux  $q$  are thus driven by its nonlinear dependence on the temperature oscillations  $\delta T_1(t)$  and  $\delta T_2(t)$ . As  $g_m(t) = \delta T_m(t)/\Delta T_m$  is a period function, it can be expressed as a sum of trigonometric functions via the following Fourier's series

$$g_m(t) = \sum_{n=1}^{\infty} \left[ a_{mn} \cos\left(\frac{2\pi n t}{\tau}\right) + b_{mn} \sin\left(\frac{2\pi n t}{\tau}\right) \right], \quad (5)$$

where  $a_{mn}$  and  $b_{mn}$  are the Fourier's coefficients of  $g_m(t)$  with common period  $\tau$  for  $m = 1$  and 2. The combination of Eqs. (4) and (5) predicts the following average coefficients  $\bar{p}_n = \tau^{-1} \int_0^\tau p_n(t) dt$  over one period

$$\bar{p}_2 = \frac{c_1}{2} (T_1^4 - T_2^4) \Delta T_1^2, \quad (6a)$$

$$\bar{p}_1 = 2\Delta T_1 (c_1 T_1^3 \Delta T_1 - c_2 T_2^3 \Delta T_2), \quad (6b)$$

$$\bar{p}_0 = T_1^4 - T_2^4 + 3 (c_1 T_1^2 \Delta T_1^2 - c_2 T_2^2 \Delta T_2^2). \quad (6c)$$

where the parameters  $c_m = \sum_{n=1}^{\infty} (a_{mn}^2 + b_{mn}^2)$  and  $c = \sum_{n=1}^{\infty} (a_{1n}a_{2n} + b_{1n}b_{2n})$  results from the quadratic dependence on  $\delta T_m(t)$ , while the contribution of the linear one vanishes. According to Eqs. (3) and (6), the net heat flux  $\bar{q}$  is hence generated by its nonlinear dependence on the amplitudes of the temperature fluctuations, as follows

$$\begin{aligned} \frac{\bar{q}}{\sigma} = & \varepsilon_{eff} (T_1^4 - T_2^4) + 2\varepsilon_1' \Delta T_1 (c_1 T_1^3 \Delta T_1 - c T_2^3 \Delta T_2) \\ & + 3\varepsilon_1 (c_1 T_1^2 \Delta T_1^2 - c_2 T_2^2 \Delta T_2^2). \end{aligned} \quad (7)$$

where  $\varepsilon_{eff} = \varepsilon_1(T_1) + \varepsilon_1''(T_1)c_1(\Delta T_1/2)^2$  is the effective emissivity of body 1, when its temperature oscillates with an amplitude  $\Delta T_1$ . Taking into account that  $c_1 > 0$ , these oscillations can be used as an emissivity enhancer of materials with  $\varepsilon_1'' > 0$ . In particular, according to the leading first term in Eq. (7), this condition  $\varepsilon_1'' > 0$  can generate effective super Planckian far-field radiation ( $\varepsilon_{eff} > 1$ ) for a high enough amplitude  $\Delta T_1$ . This key feature of macroscopic time-dependent systems is expected to appear for any material with an emissivity  $\varepsilon_1 > \varepsilon_{min}$ , where  $(\Delta T_1/2)^2 \varepsilon_{min}''(T_1) + \varepsilon_{min}(T_1) = 1$ . The minimum material emissivity supporting the effective super Planckian radiation is thus given by  $\varepsilon_{min} = 1 + A \cos(2T_1/\Delta T_1) + B \sin(2T_1/\Delta T_1)$ , with  $A$  and  $B$  being non-zero arbitrary constants. The far-field thermal radiation driven by a temperature difference periodically modulated in time thus becomes effective super Planckian with an effective emissivity  $\varepsilon_{eff}$  that increases with the square of the amplitude of the temperature oscillations and the second temperature derivative of the material emissivity, as defined in Eq. (7). Note that this definition of effective super Planckian regime is only valid for the time-averaged heat flux, as its unaveraged counterpart is still given by the Planck's law at each instant of time, as established by Eq. (1a). Besides  $\Delta T_1$ , the amplitude  $\Delta T_2$  of the blackbody temperature oscillations represents another degree of freedom to modulate  $q$ . This latter modulation can be seen clearer for the sinusoidal temperature oscillations  $g_m(t) = \cos(2\pi t/\tau + \alpha_m) = \cos(\alpha_m) \cos(2\pi t/\tau) - \sin(\alpha_m) \sin(2\pi t/\tau)$ , which indicates that  $a_{mn} = \cos(\alpha_m)\delta_{n1}$  and  $b_{mn} = -\sin(\alpha_m)\delta_{n1}$ , with  $\delta_{n1} = 1$  for  $n = 1$ , and  $\delta_{n1} = 0$  for  $n = 2, 3, \dots$ . These Fourier's coefficients yield  $c_1 = c_2 = 1$  and  $c = \cos(\alpha_1 - \alpha_2)$ , for which Eq. (7) takes the form

$$\begin{aligned} \frac{\bar{q}}{\sigma} = & \varepsilon_{eff} (T_1^4 - T_2^4) + 3\varepsilon_1 (T_1^2 \Delta T_1^2 - T_2^2 \Delta T_2^2) + \\ & 2\varepsilon_1' \Delta T_1 [T_1^3 \Delta T_1 - T_2^3 \Delta T_2 \cos(\alpha_1 - \alpha_2)]. \end{aligned} \quad (8)$$

The in-phase modulation ( $\alpha_1 = \alpha_2$ ) of the bodies' temperatures thus enhance the impact of  $\Delta T_2$  on  $\bar{q}$ , while the out-of-phase modulation ( $\alpha_1 - \alpha_2 = \pm\pi/2$ ) weakens it. The magnitude of  $\bar{q}$  can thus be tuned through the phase delay  $\alpha_1 - \alpha_2$  of the oscillating temperatures, provided that  $\Delta T_2 > 0$ . For  $\Delta T_2 = 0$ , the phase delay effect

disappears and Eq. (8) simplifies to

$$\frac{\bar{q}}{\sigma} = \varepsilon_{eff} (T_1^4 - T_2^4) + (3\varepsilon_1 + 2T_1\varepsilon_1') T_1^2 \Delta T_1^2. \quad (9)$$

The temperature modulation of only body 1 thus generates a net heat flux that increases with  $\Delta T_1^2$ , even if its emissivity does not change with temperature ( $\varepsilon_1' = 0 = \varepsilon_1''$ ). In this latter case, Eq. (9) becomes  $\bar{q} = \sigma\varepsilon_1 (T_1^4 - T_2^4 + 3T_1^2 \Delta T_1^2)$ , which, applied to our planet, explicitly shows that the daily temperature fluctuations on Earth around  $T_1$  enhance its thermal radiation with outer space. This enhancement keeps up even when both bodies are at the same mean temperature ( $T_1 = T_2 = T$ ). In this latter instance, the effect of  $\varepsilon_{eff}$  disappears and Eq. (8) reduces to

$$\begin{aligned} \frac{\bar{q}}{\sigma} = & 3\varepsilon_1 T^2 (\Delta T_1^2 - \Delta T_2^2) + \\ & 2\varepsilon_1' T^3 \Delta T_1 (\Delta T_1 - \Delta T_2 \cos(\alpha_1 - \alpha_2)). \end{aligned} \quad (10)$$

In the absence of a mean temperature difference ( $T_1 = T_2 = T$ ) and for a temperature independent emissivity ( $\varepsilon' = 0$ ), the net heat flux increases with the difference of the modulation amplitudes to the second power ( $\bar{q} \propto T^2 (\Delta T_1^2 - \Delta T_2^2)$ ). This proportionality shows that the temperature oscillations of common materials with an emissivity nearly independent of temperature, give rise to a thermal radiation regime different to the classical one driven by the fourth power of temperatures. The temperature dependence of emissivity ( $\varepsilon_1' \neq 0$ ), on the other hand, represents another degree of freedom to modulate the radiative heat flux, as established by Eq. (10). This modulation exists even if  $\Delta T_1 = \Delta T_2 = \Delta T$ , for which  $\bar{q} = 2\varepsilon_1' T^3 \Delta T^2 [1 - \cos(\alpha_1 - \alpha_2)]$  is strongly driven by the phase delay  $\alpha_1 - \alpha_2$ . The in-phase temperature modulation ( $\alpha_1 = \alpha_2$ ) thus eliminates  $\bar{q}$ , while the out-of-phase oscillations ( $\alpha_1 - \alpha_2 = \pm\pi$ ) maximize it. For an arbitrary phase delay, the direction of the net heat flux is no longer determined by the temperature difference, but rather by the sign of the permittivity slope  $\varepsilon_1'(T)$ . For  $\varepsilon_1'(T) > 0$ ,  $\bar{q} > 0$  and hence flows from body 1 to body 2, while it reverses its direction for  $\varepsilon_1'(T) < 0$ . In any case, the fact that  $\bar{q} \neq 0$ , for  $T_1 = T_2 = T$ , indicates that the temperature fluctuations could also enhance the far-field thermal radiation beyond the blackbody limit, for a relatively small temperature difference  $T_1 - T_2$ . Equation (7) and its particular cases in Eqs. (8)-(10) thus reveals the great potential of oscillating temperatures to enhance, suppress, and reverse the net heat flux of far-field radiation.

### III. RESULTS AND DISCUSSION

We now illustrate the predictions of our theory for the time-modulated far-field thermal radiation between a blackbody ( $\varepsilon_2 = 1$ ) and VO<sub>2</sub>, a phase-change material undergoing a metal-insulation transition around the temperature  $T_0 = 342.5$  K [23, 24]. This transition triggers

a steep emissivity variation in a narrow temperature interval, which could be exploited to enhance the radiative heat transfer. The measured VO<sub>2</sub> emissivity is shown in Fig. 1(a) and is well described by

$$\varepsilon_1(T) = \varepsilon_i + \frac{\varepsilon_m - \varepsilon_i}{1 + e^{-\beta(T-T_0)}}, \quad (11)$$

where  $\varepsilon_i = 0.79$  and  $\varepsilon_m = 0.22$  are the emissivities of VO<sub>2</sub> in its insulating and metallic phases, respectively, and  $\beta = 1.57 \text{ K}^{-1}$ . Equation (11) predicts that the VO<sub>2</sub> emissivity is bound between its values for the metallic and insulating phases ( $\varepsilon_m \leq \varepsilon_1 \leq \varepsilon_i$ ), whose ratio is  $\varepsilon_i/\varepsilon_m = 3.59$ . According to Fig. 1(a), this significant emissivity contrast appears in a relatively small temperature interval ( $\sim 345-340 = 5 \text{ K}$ ) and allows to modulate the radiative heat flux for temperatures across the VO<sub>2</sub> phase transition, in which  $\varepsilon'_1$  and  $\varepsilon''_1$  are not zero.

Figure 2 shows the heat flux  $q$ , temperature difference  $\delta T_1 = \Delta T_1 \cos(2\pi t/\tau)$ , and VO<sub>2</sub> emissivity  $\varepsilon_1$  as functions of time, when both the VO<sub>2</sub> and the blackbody are at the same average temperature  $T_1 = T_2 = T$ . For  $\delta T_1 > 0$  (green zone in  $0 < t/\tau < 0.25$ ), VO<sub>2</sub> is hotter than the blackbody and mainly keeps in its low-emissivity metallic phase that generate a relatively small heat flux  $q > 0$ . For  $\delta T_1 < 0$  (yellow zone in  $0.25 < t/\tau < 0.75$ ), the VO<sub>2</sub> temperature decreases and triggers its high-emissivity insulating phase, for which the heat flux magnitude increases and changes its direction to flow from the blackbody to VO<sub>2</sub>. The subsequent variations of  $\delta T_1 > 0$  (light blue zone in  $0.75 < t/\tau < 1$ ) increase again the VO<sub>2</sub> temperature, which induces its insulator-to-metal transition characterized by a low emissivity and hence low heat flux  $q > 0$ . This reversal in the heat flux direction occurs because the VO<sub>2</sub> emissivity decreases more rapidly than the increase in its temperature, resulting in a net reduction of radiated power from VO<sub>2</sub> despite its higher temperature. The combined time oscillations of  $\delta T_1$  and  $\varepsilon_1$  thus generate a heat flux with a strong nonlinear dependence on the temperature difference, despite the fact that  $10 \text{ K} = \Delta T_1 \ll T_0 = 342.5 \text{ K}$ . The phase transition of VO<sub>2</sub> excited with relatively small and smooth temperature fluctuations thus triggers sizable variations of the heat flux to generate a mean (net) heat flux  $\bar{q}$ , even when the average temperature of both bodies is the same ( $T_1 = T_2 = T$ ), as shown in Fig. 2(b). For temperatures inside the interval  $T_0 - \Delta T_1 \lesssim T \lesssim T_0 + \Delta T_1$  spanning over the VO<sub>2</sub> phase transition, the net heat flux flows from the blackbody to VO<sub>2</sub> ( $\bar{q} < 0$ ) due of the negative temperature derivative of the VO<sub>2</sub> emissivity ( $\varepsilon'(T) < 0$ ), as predicted by Eqs. (9) and (11). The minimum of the heat flux ( $\bar{q}_{\min} = -\sigma T_0^2 \Delta T_1^2 [\beta T_0 (\varepsilon_i - \varepsilon_m) - 3(\varepsilon_i + \varepsilon_m)]/2$ ) occurs at the temperature  $T \approx T_0$ , for which  $\varepsilon'(T)$  takes its negative minimum value  $-(\varepsilon_i - \varepsilon_m)\beta/4$ . This expression for  $\bar{q}_{\min}$  can be used to optimize the extraction of radiative heat flux from the blackbody to VO<sub>2</sub>. Outside of the temperature range  $T_0 - \Delta T \lesssim T \lesssim T_0 + \Delta T$ ,  $\bar{q}$  becomes positive, independent of  $T$ , and proportional

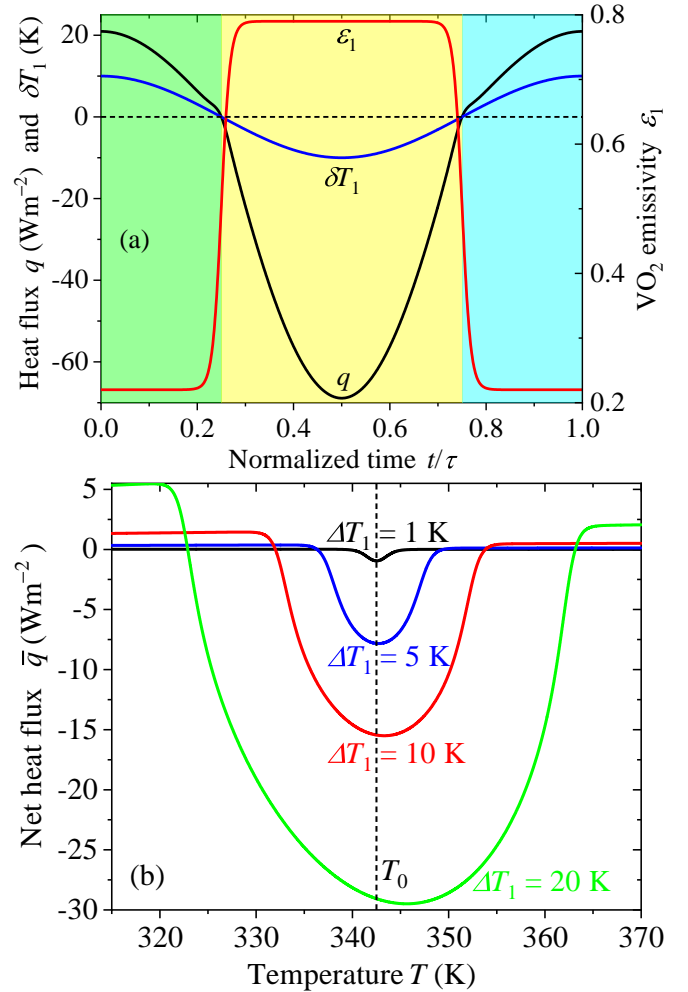


FIG. 2: (a) Time evolution of the heat flux, temperature difference  $\delta T_1 = \Delta T_1 \cos(2\pi t/\tau)$ , and VO<sub>2</sub> emissivity driving the far-field thermal radiation between VO<sub>2</sub> and a blackbody. (b) Net heat flux as a function of temperature  $T_1 = T_2 = T$  for four temperature amplitudes  $\Delta T_1$ . The green and blue zones stand for  $\delta T_1 > 0$ , while the yellow zone represents  $\delta T_1 < 0$ . Calculations were done with Eq. (2) for  $\Delta T_2 = 0$  and  $(T, \Delta T_1) = (T_0, 10 \text{ K})$  in (a).

to  $\Delta T_1^2$  (see Eq. (9)). At high temperature (metallic phase),  $\bar{q}$  is lower than at low temperature (insulating phase), as dictated by the VO<sub>2</sub> emissivity ( $\varepsilon_m < \varepsilon_i$ ). Further, the fact that  $\bar{q} = 0$  at  $T \approx T_0 \pm \Delta T$ , indicates that the oscillating temperature difference of the bodies can also be used to thermally insulate them from each other. The dynamical modulation of the radiating bodies' temperatures thus provides a new channel of thermal radiation that exists even in the absence of a stationary temperature difference.

Figure 3 shows the net heat flux  $\bar{q}$  as a function of the mean temperature  $T_1$  of VO<sub>2</sub> exchanging heat with a blackbody at temperature  $T_2 = T_0$ . Note that  $\bar{q}$  takes negative values for temperatures around  $T_1 = T_2$  in which

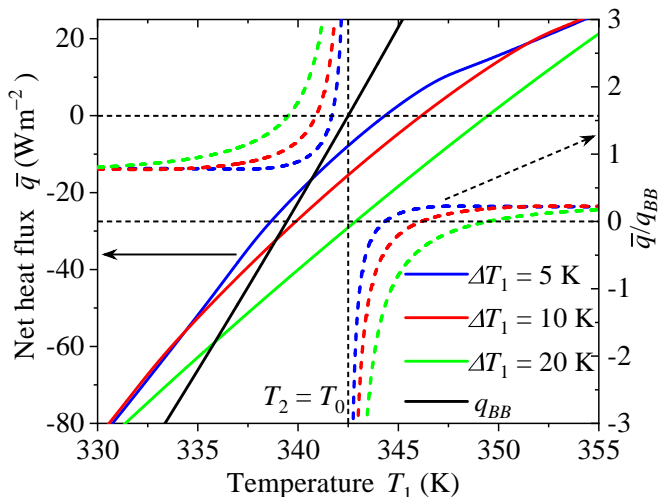


FIG. 3: Net heat flux  $\bar{q}$  as a function of temperature  $T_1 \neq T_2 = T_0$ , for three temperature amplitudes  $\Delta T_1$ . The black line stands for the blackbody limit  $q_{BB} = \sigma (T_1^4 - T_2^4)$ , while the dashed lines represent the ratio  $\bar{q}/q_{BB}$ . Calculations were done for  $\Delta T_2 = 0$ .

the VO<sub>2</sub> permittivity slope  $\varepsilon'(T_1) < 0$  (see Fig. 1(b)), as predicted by Eq. (9). The magnitude of  $\bar{q}$  can be greater than the blackbody limit  $q_{BB}$  (black solid line), as revealed by their ratio  $\bar{q}/q_{BB}$  increasing with the temperature amplitude  $\Delta T_1$ . This effective super Planckian radiation ( $|\bar{q}/q_{BB}| > 1$ ) appears in the vicinity of  $T_1 = T_2$ , in which  $\bar{q}$  can be greater than twice  $q_{BB}$ . The temperature fluctuations are thus expected to significantly enhance the far-field thermal radiation beyond the blackbody limit for a relatively small mean temperature dif-

ference  $T_1 - T_2$  of the radiating bodies.

#### IV. CONCLUSIONS

We have demonstrated that the far-field thermal radiation between two macroscopic bodies can become effective super Planckian when their temperature difference is periodically modulated in time. These temperature oscillations allow to control the magnitude and direction of the net heat flux, which exists even in the absence of a mean temperature difference. Unlike far-field thermal radiation driven by stationary temperatures, that excited by an oscillating temperature difference is determined by the temperature derivatives of the radiating bodies' emissivity and the oscillations' amplitude. For a VO<sub>2</sub> body exchanging heat with a blackbody, we have found a higher effective super Planckian heat flux for smaller temperature differences and larger amplitudes of the temperature fluctuations. Our findings thus uncover the potential of temperature oscillations to enhance the far-field thermal radiation beyond the Planck's limit, without going to the regime of near-field radiation.

#### ACKNOWLEDGMENTS

This work was supported by the CREST JST (Grant N° JPMJCR19I1) and KAKENHI JSPS (Grant N° 21H04635 and JP20J13729) projects.

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