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# Kondo-Zeno crossover in the dynamics of a monitored quantum dot

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We study the dynamics of a quantum dot coupled to a metallic bath and subject to continuous monitoring of its total charge. The dynamics averaged over measurement noise is described by a dissipative Anderson impurity model with local dephasing, that we solve using an extension of the Non-Crossing Approximation. We show that the decay rate of an initially polarized spin, which is suddenly coupled to the bath and to the monitoring protocol, displays a crossover from Kondo screening, with a decay rate controlled by interactions, to Quantum Zeno effect, with a decay rate which decreases with bare dissipation as the dephasing or monitoring rate is increased. Using a Schrieffer-Wolff transformation on the Lindbladian we derive an effective model for the long-time dynamics which is described, at weak dissipation, by a non-Hermitian Kondo model with complex-valued spin exchange. As the dephasing is increased heating due to doublon production takes over and control the spin decay. Our results show that Kondo effect is robust to a weak charge monitoring, as we further confirm by computing the dot spectral function.

*Introduction* - A quantum spin-1/2 coupled to a metallic bath represents one of the simplest many-body problem. At low-temperatures the spin is collectively screened by the bath into a many-body singlet, a manifestation of the Kondo effect [1–5] which leaves behind unique equilibrium and dynamical signatures, including an exponentially-slow spin dynamics [6, 7]. Initially observed in metals with diluted magnetic impurities, the Kondo effect has been later realized in quantum dots and various mesoscopic setups [8–11]. A different dissipative mechanism to freeze or slow down the dynamics of a system is the Quantum Zeno Effect [12, 13], where projective measurements or strong continuous monitoring leads to a complete localization of the dynamics. The Zeno Effect has been observed in different quantum platforms, from cavity and circuit QED [14, 15] to ultracold atoms [16–18], and has recently raised new interest in many-body settings [19–21].

Recent experimental progress in quantum simulation platforms both in the solid state and with ultracold atoms have opened new windows to explore the physics of dissipative systems and offer now the possibility to explore the interplay between Kondo screening and Zeno effect. This is the case of quantum dots coupled to a quantum point contact acting as a monitoring device [22–28] or of ultracold alkaline-earth atoms where Kondo physics has been realized [29–31], which are naturally exposed to correlated dissipative processes, such as dephasing due to spontaneous emission [32, 33] or two-body losses due to inelastic scattering [16, 34, 35]. The physics of these dissipative quantum impurity models has started to be explored only recently, with a focus on non-interacting chains with localized single particle losses [20, 36–39] or pumps [40, 41] or local dephasing [42–46]. Non-Hermitian quantum impurity models, arising from a postselection over quantum trajectories, have also been studied [47–49]. Recently the effect of projective measurements on the Kondo effect in the steady-

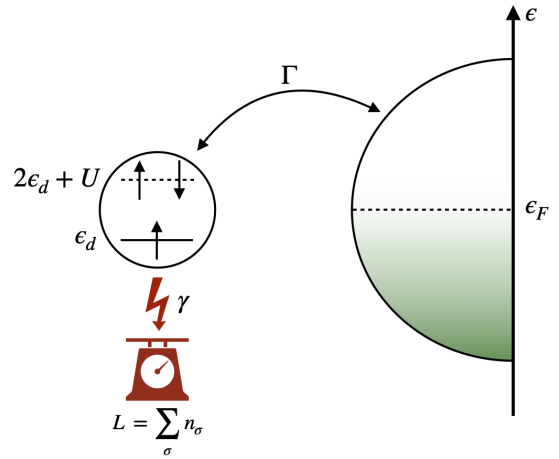


FIG. 1. Sketch of the setup for the Kondo-Zeno Crossover. A singly-occupied quantum dot is coupled to a large metallic bath via a hybridization  $\Gamma$  and to a measurement apparatus continuously monitoring, with rate  $\gamma$ , its total charge. Upon averaging over the monitoring noise the dynamics of the dot plus metallic bath is described by Lindblad master equation with a jump operator  $L = \sum_{\sigma} n_{\sigma}$ , describing charge dephasing.

state have been discussed [50]. In addition to their intrinsic interest, dissipative quantum impurity models also arise as effective description of open Markovian lattice models in the large connectivity limit, within Dynamical Mean-Field Theory [51].

In this Letter we investigate the effect of continuous monitoring on the dynamics of Kondo effect. Specifically we study the time evolution of an interacting quantum dot coupled to a metallic bath, in presence of additional dissipation (dephasing) due to monitoring of quantum dot's total charge (see Fig. 1). Since both Coulomb repulsion and charge monitoring compete to freeze charge fluctuations without affecting the spin degrees of freedom one can expect a non-trivial effect on

Kondo physics. We solve the resulting dissipative Anderson Impurity model by means of a recently developed self-consistent dynamical map based on the Non-Crossing Approximation (NCA) [51, 52]. This method treats non-perturbatively both electron-electron interactions and local dissipation. We show that the dynamics of an initially polarised spin displays a crossover from Kondo to Zeno screening, as dissipation is increased, as highlighted by a non-monotonous spin-relaxation rate as a function of dissipation. Using a Schrieffer-Wolff transformation we show that the effective model controlling this crossover takes the form of a non-Hermitian Kondo model, with complex (i.e. coherent and dissipative) Kondo coupling and in presence of additional dissipative terms describing doublon production. The former controls the physics at weak monitoring and leads to a robust Kondo effect even in presence of dissipation, as we further confirm by computing the spectral function and the impurity entanglement, while the latter dominate at strong dephasing and give rise to local heating.

*Anderson Impurity Model under Continuous Monitoring* - We consider a model for an interacting spinful quantum dot coupled to a large metallic bath and to a monitoring device which measures weakly but continuously its total charge (see Fig. 1), as realized in the homodyne detection or the quantum-state diffusion protocol [27, 53]. The dynamics of the system, upon averaging over the monitoring noise, is described by a dissipative Anderson impurity model with local charge dephasing for which the density matrix  $\rho_t$  of the system (dot plus bath) evolves according to the many-body Lindblad master equation [54]

$$\partial_t \rho_t = -i[H, \rho_t] + \gamma \left( L \rho_t L^\dagger - \frac{1}{2} \{L^\dagger L, \rho_t\} \right) \quad (1)$$

where  $H$  is the dot plus bath Hamiltonian given by the Anderson Impurity Model (AIM) [5]

$$H = \sum_{\mathbf{k}, \sigma} \varepsilon_{\mathbf{k}} c_{\mathbf{k}, \sigma}^\dagger c_{\mathbf{k}, \sigma} + \sum_{\mathbf{k}, \sigma} (V_{\mathbf{k}} d_\sigma^\dagger c_{\mathbf{k}, \sigma} + h.c.) + H_{\text{dot}}. \quad (2)$$

Here the first term describes the Hamiltonian of the metallic bath with fermionic operators  $c_{\mathbf{k}, \sigma}, c_{\mathbf{k}, \sigma}^\dagger$ , the second term describes the hybridization between dot and bath with coupling  $V_{\mathbf{k}}$  and the last term describes the dot Hamiltonian

$$H_{\text{dot}} = \varepsilon_d \sum_{\sigma} d_\sigma^\dagger d_\sigma + U n_\uparrow n_\downarrow \quad (3)$$

with gate voltage  $\varepsilon_d$  and Coulomb repulsion  $U$ . The action of the monitoring device is taken into account at the Markovian level by a jump operator proportional to the dot total charge, i.e.  $L = L^\dagger = \sum_{\sigma} n_\sigma \equiv n$ , where  $\gamma$  in Eq. (1) is the monitoring (or dephasing) rate [27]. We note that this type of dissipation preserves particle-hole symmetry, namely under the trans-

formation  $n \rightarrow 1 - n$  the dissipative part of the Lindblad master equation remains unaffected. As such, if we choose  $\varepsilon_d = -U/2$  such that the coherent evolution is also particle-hole symmetric we can conclude that the system remains half-filled under the dynamics generated by Eq. (1). In the following we consider a metallic bath at zero temperature with a semicircular density of states of bandwidth  $W$ , giving rise to an hybridization function  $\Gamma(\varepsilon) = 2\pi \sum_{\mathbf{k}} V_{\mathbf{k}}^2 \delta(\varepsilon - \varepsilon_{\mathbf{k}}) = \Gamma \sqrt{1 - (\varepsilon/W)^2}$ .

*Hybridization Expansion and Non-Crossing Dynamical Map* - To tackle the dissipative AIM we first reformulate the Lindblad many-body dynamics in Eq. (1) using the purification/superfermion representation [55–62]. This amounts to represent density matrices as pure states  $|\rho_t\rangle$  in an extended Hilbert space  $\mathcal{H} \otimes \tilde{\mathcal{H}}$  that contains a copy of our degrees of freedom. In this formalism the Lindblad master equation takes the form of a Schrodinger-like equation  $\partial_t |\rho_t\rangle = \mathcal{L} |\rho_t\rangle$  generated by a non-Hermitian operator  $\mathcal{L}$ , the Lindbladian, which reads [54]

$$\mathcal{L} = -i \left( H - \tilde{H} \right) + \gamma \left( n \tilde{n} - \frac{1}{2} n^2 - \frac{1}{2} \tilde{n}^2 \right) \quad (4)$$

where  $H$  is given in Eq. (2) and  $\tilde{H}$  takes the very same form in terms of fermions living in the  $\tilde{\mathcal{H}}$  Hilbert space,  $\tilde{c}_{\mathbf{k}, \sigma}, \tilde{c}_{\mathbf{k}, \sigma}^\dagger$  and  $\tilde{d}_\sigma, \tilde{d}_\sigma^\dagger$  for the bath and dot operators respectively, which satisfy the usual fermionic algebra [54]. We note the analogy between the doubling of the degrees of freedom due to  $\mathcal{H}$  and the Keldysh formalism [63, 64]: the first term in Eq. (4) can be seen as the coherent evolution along the two independent Keldysh contours while the dissipation term induces an explicit coupling between them and a back-action (non-Hermitian contribution).

We can then perform an exact hybridization expansion in the system-bath coupling, starting from the density matrix in the interaction picture [42, 52, 65], and obtain an exact equation for the dressed impurity time-evolution operator  $\mathcal{V}(t, 0) = \text{Tr}_{\text{bath}} [\exp(\mathcal{L}t)]$ , which reads [54]

$$\partial_t \mathcal{V}(t, 0) = \mathcal{L}_{\text{dot}} \mathcal{V}(t, 0) + \int_0^t d\tau \Sigma(t, \tau) \mathcal{V}(\tau, 0) \quad (5)$$

where  $\mathcal{L}_{\text{dot}} = -i(H_{\text{dot}} - \tilde{H}_{\text{dot}}) + \gamma(n\tilde{n} - \frac{1}{2}n^2 - \frac{1}{2}\tilde{n}^2)$  is the impurity Lindbladian, while the self-energy  $\Sigma(t, \tau)$  takes into account the effect of the metallic bath and it is a priori given by an infinite class of one-particle irreducible diagrams. To close the hierarchy we use the NCA dynamical map [52] corresponding to a self-consistent approximation on the series for  $\mathcal{V}$ , by keeping only the compact diagrams in which hybridization lines do not cross [6, 66–71]. This amounts to restrict the self-energy to the form  $\Sigma(t, \tau) \equiv \Sigma_{\text{NCA}}(t, \tau)$

$$\begin{aligned} \Sigma_{\text{NCA}}(\tau, \bar{\tau}) = & -i \sum_{\sigma, \alpha, \bar{\alpha}} \Psi_\sigma^\alpha \mathcal{V}(\tau, \bar{\tau}) \bar{\Psi}_\sigma^{\bar{\alpha}} \Delta_\sigma^{\alpha \bar{\alpha}}(\tau, \bar{\tau}) \\ & + i \sum_{\sigma, \alpha} \bar{\Psi}_\sigma^{\bar{\alpha}} \mathcal{V}(\tau, \bar{\tau}) \Psi_\sigma^\alpha \Delta_\sigma^{\alpha \bar{\alpha}}(\bar{\tau}, \tau) \end{aligned} \quad (6)$$

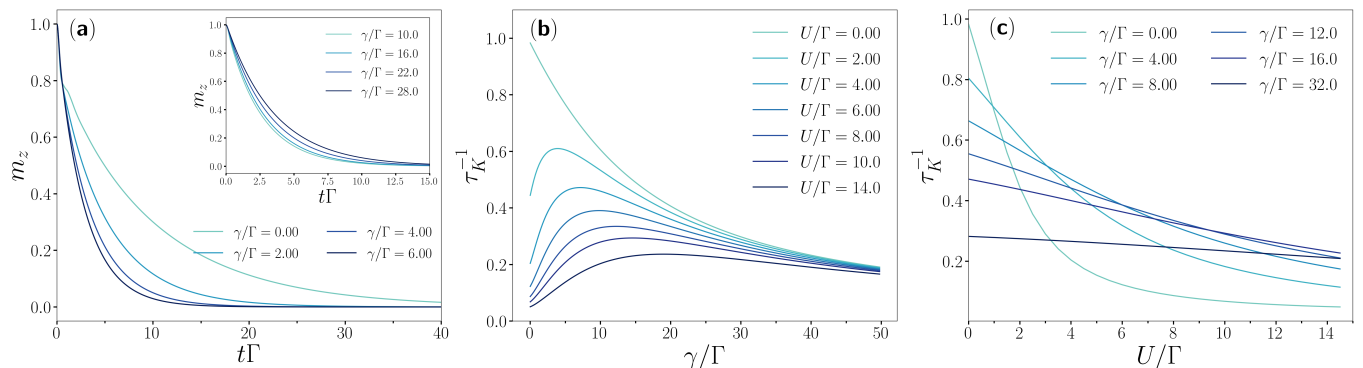


FIG. 2. Impurity Spin Dynamics and Kondo-Zeno Crossover - (a) Dynamics of the impurity magnetization starting from a polarised state  $\rho(0) = |\uparrow\rangle\langle\uparrow|$  and evolving under the dynamics of the half-filled dissipative Anderson Impurity Model, with  $U = 8\Gamma$ . The impurity spin decays to zero at long times with an exponential decay  $m_z \sim \exp(-t/\tau_K)$ . (b) The decay rate  $\tau_K^{-1}$  depends non-monotonously on the monitoring rate  $\gamma$ . For small dissipation the decay is faster, however upon increasing  $\gamma$  we see that the decay of the impurity slows down. The crossover between these two regimes, characteristic of the Quantum Zeno effect, is set by  $\gamma \sim U$ . (c) The crossover is also visible in the dependence of  $\tau_K^{-1}$  from the interaction  $U$ .

which is itself a function of  $\mathcal{V}(t, 0)$ , hence the self-consistent non-perturbative nature of this method.

In Eq. (6) we have collected the dot operators in  $\mathcal{H}$  and  $\tilde{\mathcal{H}}$  into a field [54]  $\tilde{\Psi}_\sigma = (d_\sigma^\dagger, \tilde{d}_\sigma)$  whose components are labelled by the index  $\alpha = 0, 1$  and introduced the real-time hybridization function  $\Delta_\sigma^{\alpha\bar{\alpha}}(\tau, \bar{\tau})$ , that keeps into account the full non-Markovian nature of the metallic bath (see Ref. 54 for definition.)

*Spin Dynamics and Kondo-Zeno Crossover* - We consider an initially polarised spin up and then suddenly switch-on the monitoring and the coupling to the metallic bath. In Fig. 2(a) we plot the dynamics of the dot magnetization  $m_z(t) = \text{Tr}\rho_t(n_\uparrow - n_\downarrow)$  at fixed interaction  $U = 8\Gamma$  and for different values of the monitoring rate  $\gamma$ . We see that the magnetization decays to zero at long times for all values of  $\gamma$ . Interestingly, the spin dynamics first accelerates upon adding a weak dissipation, then for large enough  $\gamma$  it slows down again, see inset of Fig. 2(a). The spin decay rate  $\tau_K^{-1}$ , that we extract from an exponential fit  $m_z \sim \exp(-t/\tau_K)$  depends non-monotonously on  $\gamma$ , as we see in Fig. 2(b): it first grows at small dissipation since the system is more dissipative, then reaches a maximum around  $\gamma \sim U$  and decreases at large  $\gamma$  as  $1/\gamma$ , indicating the freezing of the system dynamics due to the strong observation, a signature of the Quantum Zeno effect [16, 20, 51, 72, 73]. The weak monitoring regime  $\gamma < U$  is particularly interesting as it shows a strong dependence on the interaction  $U$ . Indeed we see in Fig. 2(b) that in the weakly correlated regime the spin-decay rate is linear at small  $\gamma$ , while for  $U \gg \Gamma$  a sub-linear dependence emerges, suggesting a robustness of Kondo physics to weak charge monitoring. In Fig. 2(c) we plot the spin decay rate as a function of interaction  $U$ , for different values of  $\gamma$ . In absence of dephasing,  $\gamma = 0$ , the decay of the magnetization is due to the hybridization with the metallic bath leading to a

Kondo singlet and occurs on time scales that depends strongly on the interaction, a signature of the Kondo effect [5, 7, 74, 75]. On the other hand in the strongly dissipative Zeno regime we see that the decay rate is almost independent of  $U$ , see Fig. 2(c). The crossover between Kondo decay due to singlet formation and Zeno decay due to dephasing is one of the main result of this work. We note in addition that the Kondo-Zeno crossover also controls the dynamics of the impurity entanglement entropy that shows a sharp maximum at short times for dissipation smaller than  $\gamma \sim U$  [54]. In the following we provide a physical understanding of the basic mechanism behind the crossover. Before that it is useful to discuss the dynamics in the charge sector.

*Doublon Dynamics* - We now consider the dynamics of the dot double occupation  $D(t) = \text{Tr}(\rho_t n_\uparrow n_\downarrow)$ , starting again from a singly occupied polarised spin up. In Fig. 3(a) we plot the dynamics of  $D(t)$  for different values of dissipation, at fixed  $U = 8\Gamma$ . While in the unitary case ( $\gamma = 0$ ) the doublon fraction displays a non-monotonous time-dependence approaching the steady state value with oscillations, as  $\gamma$  increases the dynamics becomes exponential,  $D(t) \sim D_{ss}(1 - \exp(-t/\tau_D))$  with a characteristic relaxation rate  $\tau_D^{-1}$  and a steady-state value  $D_{ss}$  that strongly depend on interaction and dephasing. In Fig. 3(b) we see that the doublon relaxation rate also displays a non-monotonic behavior with a sharp maximum and decreases at large  $\gamma$  as  $1/\gamma$ . Comparing Fig. 3(b) and Fig. 2(c) we see that the doublon relaxation rate is generally larger than the spin one, i.e. doublon dynamics is faster both at zero dissipation (as expected) and for finite  $\gamma$ . The steady-state value  $D_{ss}$  increases monotonously with  $\gamma$  (see Fig. 3(c)), and ultimately reaches the value  $D_{ss} \sim 1/4$  for  $\gamma \rightarrow \infty$ , indicating that locally the impurity is heating up towards infinite temperature. We emphasize however that the steady-state is reached on

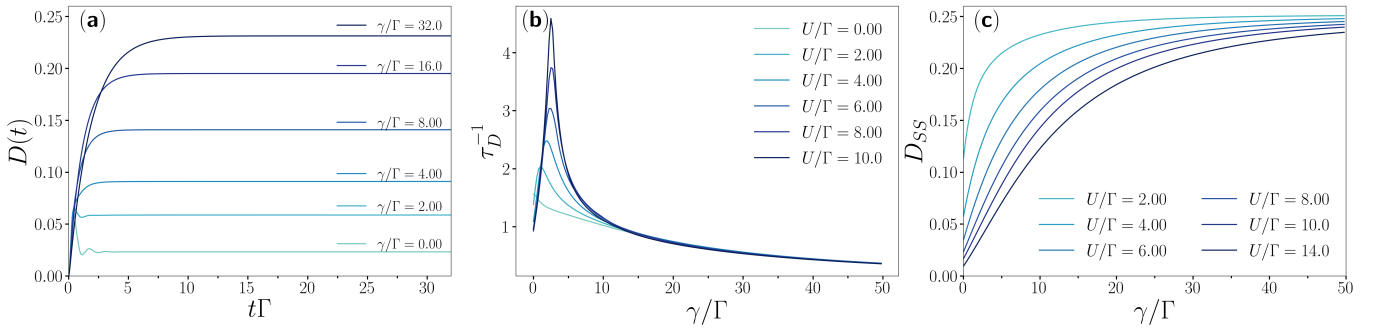


FIG. 3. Dynamics of the doublon fraction starting from a polarised state  $\rho(0) = |\uparrow\rangle\langle\uparrow|$  and evolving under the dynamics of the half-filled dissipative Anderson Impurity Model. (a) The double occupancy  $D(t)$  for  $U = 8\Gamma$  increases with time and saturates rapidly to a steady-state on a characteristic time scale  $\tau_D$ . (b) The doublon thermalisation rate  $\tau_D^{-1}$  as a function of  $\gamma$  shows again signature of the Zeno effect, namely it increases with  $\gamma$  through a maximum and then decreases, again as  $1/\gamma$ . (c) The steady state value of double occupancy  $D_{ss}$  increases with  $\gamma$ , yet we see that for large interaction doublons are suppressed at least for small  $\gamma$ .

time scales  $t \gg \tau_D$ , hence for  $\gamma = \infty$  the system remains frozen in the initial spin polarised state, as expected from the Quantum Zeno effect. Strong interactions and coupling to the cold metallic bath however suppress heating by keeping down the doublon fraction, at least for  $\gamma < U$ .

*Schrieffer-Wolff Transformation* – In order to understand the emergence of the Zeno-Kondo crossover we consider a dissipative Schrieffer-Wolff transformation [54, 76–78] on the Lindbladian of the dissipative AIM in Eq.(4). Specifically we consider the non-unitary (similarity) transformation  $\mathcal{L}_{\text{eff}} = e^S \mathcal{L} e^{-S} = \mathcal{L} + [S, \mathcal{L}] + \frac{1}{2}[S, [S, \mathcal{L}]] + \dots$  with a generator  $S$  that we fix so to decouple the diagonal and the off-diagonal (with respect to the hybridization) sectors of the Lindbladian. Specifically we take  $S$  of the form

$$S = \sum_{\mathbf{k}\sigma} X_{\mathbf{k}\sigma} \left( c_{\mathbf{k}\sigma}^\dagger d_\sigma + c_{\mathbf{k}\sigma} d_\sigma^\dagger \right) + \sum_{\mathbf{k}\sigma} \tilde{X}_{\mathbf{k}\sigma} \left( \tilde{c}_{\mathbf{k}\sigma}^\dagger \tilde{d}_\sigma + \tilde{c}_{\mathbf{k}\sigma} \tilde{d}_\sigma^\dagger \right) \quad (7)$$

where  $X_{\mathbf{k}\sigma}, \tilde{X}_{\mathbf{k}\sigma}$  are written in terms of local impurity operators  $d_\sigma, d_\sigma^\dagger$  and  $\tilde{d}_\sigma, \tilde{d}_\sigma^\dagger$  [54]. After projection onto the single occupied sector  $n = \tilde{n} = 1$  the effective Lindbladian to second order in  $V_{\mathbf{k}}$  takes the form [54]

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{bath}} - i \left( H_{\text{eff,K}} - \tilde{H}_{\text{eff,K}} \right) \quad (8)$$

where  $\mathcal{L}_{\text{bath}} = -i(H_{\text{bath}} - \tilde{H}_{\text{bath}})$  is the Lindbladian of the bath,  $H_{\text{eff,K}}$  describes a spin-exchange Kondo coupling between the impurity and the bath electrons,

$$H_{\text{eff,K}} = - \sum_{\mathbf{q}\mathbf{k}} J_{\mathbf{q}\mathbf{k}} \left( \Phi_{\mathbf{q}}^\dagger \frac{\vec{\sigma}}{2} \Phi_{\mathbf{k}} \right) \cdot \vec{S}_{\text{d}} \quad (9)$$

where  $\Phi_{\mathbf{k}}^\dagger = (c_{\mathbf{k}\uparrow}, c_{\mathbf{k}\downarrow})$  and  $\vec{S}_{\text{d}} = \sum_{\sigma\sigma'} d_\sigma^\dagger \vec{\tau}_{\sigma\sigma'} d_{\sigma'}$  is the dot spin with  $\vec{\tau}_{\sigma\sigma'}$  the Pauli matrices. Eq. (9) describes a

non-Hermitian Kondo model [47, 79], due to the complex-valued spin-exchange coupling  $J_{\mathbf{q}\mathbf{k}}$  renormalised by dissipation, which reads (by setting  $\epsilon_{\mathbf{k}} = \epsilon_{\mathbf{q}} = 0$  and dropping the momentum dependence of the hybridization  $V_{\mathbf{k}} \simeq V$ )

$$J_{\mathbf{q}\mathbf{k}} = J_R + iJ_I = - \frac{8V^2U}{U^2 + \gamma^2} - i \frac{8V^2\gamma}{U^2 + \gamma^2} \quad (10)$$

We stress that differently from previous works here the non-Hermitian Kondo model does not arise from post-selection and no-click limit [47]. Rather it emerges from the competition between Coulomb repulsion and charge monitoring: deep in the Kondo regime, when charge fluctuations are frozen due to strong Coulomb repulsion, the leftover local moment can only dissipate through a bath-mediated spin exchange, given in Eq. (10), while other dissipative processes are blocked at particle-hole symmetry [54].

The effective Lindbladian in Eq. (8) captures the competition between Kondo and Zeno screening. This is more clearly seen by considering first the weak dissipation regime  $V \ll \gamma \ll U$ . Here we have  $J_R \gg J_I$  and the system is in the Kondo regime, with few excited doublons that can be safely projected out, yet with a small imaginary Kondo coupling. Results on the non-Hermitian Kondo model suggests a transition from Kondo to a non-Kondo state above a critical value of  $J_I$  [47, 79, 80]. This picture is compatible with our NCA results both for the spin-decay rate, which at large  $U$  shows robustness against dissipation (See. Fig. 2(b)) and for the impurity spectral function, a key quantity to characterize the emergence of Kondo effect [5]. In absence of dissipation the spectrum shows a characteristic three peak structure, with coherent low-energy Kondo peak and incoherent Hubbard bands at high-frequency. As we show in Ref. [54] a weak charge dephasing,  $\gamma \ll U$ , strongly renormalises the latter, whose location drifts with  $\gamma$ , while leaves practically unaffected the Kondo peak. On the other hand, as we increase the dissipation above  $\gamma \sim U$

new physics emerge. First, we see from Eq. (10) that  $J_I$  becomes maximal and then decreases to zero as  $1/\gamma$ . This is consistent with the spin-decay rate  $\tau_K^{-1}$  which therefore in this regime appears to be controlled by dissipation rather than Kondo screening. Furthermore for  $\gamma \sim U$  other dissipative channels open up beyond the single-occupied manifold, such as doublon-holon production terms, which enter the effective Lindbladian and control the dynamics of the system [54]. As confirmed by the NCA results in this regime doublon production becomes substantial and the system locally heats up, leading also to a decay of the initially polarized spin.

*Conclusions* – In this work we have studied the dynamics of an interacting quantum dot under continuous monitoring of its charge. Focusing on the dynamics averaged over the measurement noise and described by a dissipative Anderson Impurity model, we have shown how the interplay of interaction and local dissipation strongly affects the dynamics of the system. In particular we have highlighted a crossover from Kondo to Quantum Zeno effect in the dynamics of an initially polarised spin. We have shown that this can be understood at weak dissipation in terms of a non-Hermitian Kondo model, obtained as effective model at long times through a Schrieffer-Wolff transformation on the Lindbladian. On the other hand, upon increasing the dephasing rate, heating due to doublon production dominates the spin decay. Our results highlights the robustness of Kondo effect to weak charge monitoring, before heating starts to kick in.

This work opens up several avenues for future research. First, the signatures of Kondo-Zeno crossover in the steady-state transport are worth exploring. Furthermore looking at the dynamics of the conditional state and associated non-linear probes, such as block-entanglement entropy or purity of quantum trajectories, could unveil measurement-induced transitions in this problem. Finally, the role of monitoring different observables is also an important question to explore.

While completing this manuscript, a related work appeared analyzing dissipative implementations of Kondo physics [81].

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[1] P. W. Anderson, Localized magnetic states in metals, *Phys. Rev.* **124**, 41 (1961).

- [2] J. Kondo, Resistance Minimum in Dilute Magnetic Alloys, *Progress of Theoretical Physics* **32**, 37 (1964), <https://academic.oup.com/ptp/article-pdf/32/1/37/5193092/32-1-37.pdf>.
- [3] P. Nozières, A “fermi-liquid” description of the kondo problem at low temperatures, *Journal of Low Temperature Physics* **17**, 31 (1974).
- [4] K. G. Wilson, The renormalization group: Critical phenomena and the kondo problem, *Rev. Mod. Phys.* **47**, 773 (1975).
- [5] A. C. Hewson, *Cambridge Studies in Magnetism* (Cambridge University Press, Cambridge, 1993).
- [6] P. Nordlander, M. Pustilnik, Y. Meir, N. S. Wingreen, and D. C. Langreth, How long does it take for the kondo effect to develop?, *Phys. Rev. Lett.* **83**, 808 (1999).
- [7] F. B. Anders and A. Schiller, Real-time dynamics in quantum-impurity systems: A time-dependent numerical renormalization-group approach, *Phys. Rev. Lett.* **95**, 196801 (2005).
- [8] D. Goldhaber-Gordon, H. Shtrikman, D. Mahalu, D. Abusch-Magder, U. Meirav, and M. A. Kastner, Kondo effect in a single-electron transistor, *Nature* **391**, 156 (1998).
- [9] L. Kouwenhoven and L. Glazman, Revival of the kondo effect, *Physics World* **14**, 33 (2001).
- [10] M. Pustilnik and L. Glazman, Kondo effect in quantum dots, *Journal of Physics: Condensed Matter* **16**, R513 (2004).
- [11] C. Latta, F. Haupt, M. Hanl, A. Weichselbaum, M. Claassen, W. Wuester, P. Fallahi, S. Faelt, L. Glazman, J. von Delft, H. E. Türeci, and A. Imamoglu, Quantum quench of kondo correlations in optical absorption, *Nature* **474**, 627 (2011).
- [12] B. Misra and E. C. G. Sudarshan, The Zeno’s paradox in quantum theory, *Journal of Mathematical Physics* **18**, 756 (1977).
- [13] P. Facchi and S. Pascazio, Quantum zeno subspaces, *Phys. Rev. Lett.* **89**, 080401 (2002).
- [14] A. Signoles, A. Facon, D. Grosso, I. Dotsenko, S. Haroche, J.-M. Raimond, M. Brune, and S. Gleyzes, Confined quantum zeno dynamics of a watched atomic arrow, *Nature Physics* **10**, 715 (2014).
- [15] L. Bretheau, P. Campagne-Ibarcq, E. Flurin, F. Mallet, and B. Huard, Quantum dynamics of an electromagnetic mode that cannot contain  $|i\rangle_n/|i\rangle_n$  photons, *Science* **348**, 776 (2015), <https://www.science.org/doi/pdf/10.1126/science.1259345>.
- [16] J. J. García-Ripoll, S. Dürr, N. Syassen, D. M. Bauer, M. Lettner, G. Rempe, and J. I. Cirac, Dissipation-induced hard-core boson gas in an optical lattice, *New Journal of Physics* **11**, 013053 (2009).
- [17] F. Schäfer, I. Herrera, S. Cherukattil, C. Lovecchio, F. S. Cataliotti, F. Caruso, and A. Smerzi, Experimental realization of quantum zeno dynamics, *Nature Communications* **5**, 3194 (2014).
- [18] Y. S. Patil, S. Chakram, and M. Vengalattore, Measurement-induced localization of an ultracold lattice gas, *Phys. Rev. Lett.* **115**, 140402 (2015).
- [19] Y. Li, X. Chen, and M. P. A. Fisher, Quantum zeno effect and the many-body entanglement transition, *Phys. Rev. B* **98**, 205136 (2018).
- [20] H. Fröml, A. Chiocchetta, C. Kollath, and S. Diehl, Fluctuation-Induced Quantum Zeno Effect, *Physical Review Letters* **122**, 040402 (2019).

- [21] A. Biella and M. Schiró, Many-Body Quantum Zeno Effect and Measurement-Induced Subradiance Transition, *Quantum* **5**, 528 (2021).
- [22] M. Avinun-Kalish, M. Heiblum, A. Silva, D. Mahalu, and V. Umansky, Controlled dephasing of a quantum dot in the kondo regime, *Phys. Rev. Lett.* **92**, 156801 (2004).
- [23] K. Kang and G. L. Khym, Entanglement, measurement, and conditional evolution of the kondo singlet interacting with a mesoscopic detector, *New Journal of Physics* **9**, 121 (2007).
- [24] T. Aono, Dephasing in a quantum dot coupled to a quantum point contact, *Phys. Rev. B* **77**, 081303 (2008).
- [25] E. V. Sukhorukov, A. N. Jordan, S. Gustavsson, R. Leturcq, T. Ihn, and K. Ensslin, Conditional statistics of electron transport in interacting nanoscale conductors, *Nature Physics* **3**, 243 (2007).
- [26] M. S. Ferguson, L. C. Camenzind, C. Müller, D. E. F. Biesinger, C. P. Scheller, B. Braunecker, D. M. Zumbühl, and O. Zilberberg, Measurement-induced population switching, *Phys. Rev. Res.* **5**, 023028 (2023).
- [27] H. M. Wiseman and G. J. Milburn, *Quantum Measurement and Control* (Cambridge University Press, Cambridge, England, 2009).
- [28] S. Sankar, C. Bertrand, A. Georges, E. Sela, and Y. Meir, Detector-tuned overlap catastrophe in quantum dots (2024), [arXiv:2402.04207](https://arxiv.org/abs/2402.04207) [[cond-mat.mes-hall](https://arxiv.org/abs/2402.04207)].
- [29] L. Riegger, N. Darkwah Oppong, M. Höfer, D. R. Fernandes, I. Bloch, and S. Fölling, Localized magnetic moments with tunable spin exchange in a gas of ultracold fermions, *Phys. Rev. Lett.* **120**, 143601 (2018).
- [30] R. Zhang, Y. Cheng, P. Zhang, and H. Zhai, Controlling the interaction of ultracold alkaline-earth atoms, *Nature Reviews Physics* **2**, 213 (2020).
- [31] M. Kanász-Nagy, Y. Ashida, T. Shi, C. u. u. u. P. m. c. Moca, T. N. Ikeda, S. Fölling, J. I. Cirac, G. Zaránd, and E. A. Demler, Exploring the anisotropic kondo model in and out of equilibrium with alkaline-earth atoms, *Phys. Rev. B* **97**, 155156 (2018).
- [32] F. Gerbier and Y. Castin, Heating rates for an atom in a far-detuned optical lattice, *Phys. Rev. A* **82**, 013615 (2010).
- [33] R. Bouganne, M. Bosch Aguilera, A. Ghermaoui, J. Beugnon, and F. Gerbier, Anomalous decay of coherence in a dissipative many-body system, *Nature Physics* **16**, 21 (2020).
- [34] T. Tomita, S. Nakajima, I. Danshita, Y. Takasu, and Y. Takahashi, Observation of the mott insulator to superfluid crossover of a driven-dissipative bose-hubbard system, *Science Advances* **3**, 10.1126/sciadv.1701513 (2017).
- [35] K. Honda, S. Taie, Y. Takasu, N. Nishizawa, M. Nakagawa, and Y. Takahashi, Observation of the sign reversal of the magnetic correlation in a driven-dissipative fermi gas in double wells, *Phys. Rev. Lett.* **130**, 063001 (2023).
- [36] F. m. c. Damant, E. Mascarenhas, D. Pekker, and A. J. Daley, Controlling quantum transport via dissipation engineering, *Phys. Rev. Lett.* **123**, 180402 (2019).
- [37] A.-M. Visuri, T. Giamarchi, and C. Kollath, Symmetry-protected transport through a lattice with a local particle loss, *Phys. Rev. Lett.* **129**, 056802 (2022).
- [38] A.-M. Visuri, T. Giamarchi, and C. Kollath, Nonlinear transport in the presence of a local dissipation, *Phys. Rev. Res.* **5**, 013195 (2023).
- [39] A.-M. Visuri, J. Mohan, S. Uchino, M.-Z. Huang, T. Esslinger, and T. Giamarchi, DC transport in a dissipative superconducting quantum point contact, [arXiv e-prints](https://arxiv.org/abs/2304.00928), [arXiv:2304.00928](https://arxiv.org/abs/2304.00928) (2023), [arXiv:2304.00928](https://arxiv.org/abs/2304.00928) [[cond-mat.quant-gas](https://arxiv.org/abs/2304.00928)].
- [40] P. L. Krapivsky, K. Mallick, and D. Sels, Free fermions with a localized source, *Journal of Statistical Mechanics: Theory and Experiment* **2019**, 113108 (2019).
- [41] P. L. Krapivsky, K. Mallick, and D. Sels, Free bosons with a localized source, *Journal of Statistical Mechanics: Theory and Experiment* **2020**, 063101 (2020).
- [42] M. Schiro and O. Scarlatella, Quantum impurity models coupled to Markovian and non-Markovian baths, *The Journal of Chemical Physics* **151**, 044102 (2019).
- [43] F. Tonielli, R. Fazio, S. Diehl, and J. Marino, Orthogonality catastrophe in dissipative quantum many-body systems, *Phys. Rev. Lett.* **122**, 040604 (2019).
- [44] P. E. Dolgirev, J. Marino, D. Sels, and E. Demler, Non-gaussian correlations imprinted by local dephasing in fermionic wires, *Phys. Rev. B* **102**, 100301 (2020).
- [45] A. P. Chaudhari, S. P. Kelly, R. J. Valencia-Tortora, and J. Marino, Zeno crossovers in the entanglement speed of spin chains with noisy impurities, *Journal of Statistical Mechanics: Theory and Experiment* **2022**, 103101 (2022).
- [46] J. a. Ferreira, T. Jin, J. Mannhart, T. Giamarchi, and M. Filippone, Transport and nonreciprocity in monitored quantum devices: An exact study, *Phys. Rev. Lett.* **132**, 136301 (2024).
- [47] M. Nakagawa, N. Kawakami, and M. Ueda, Non-Hermitian Kondo Effect in Ultracold Alkaline-Earth Atoms, *Physical Review Letters* **121**, 203001 (2018).
- [48] T. Yoshimura, K. Bidzhiev, and H. Saleur, Non-hermitian quantum impurity systems in and out of equilibrium: Noninteracting case, *Phys. Rev. B* **102**, 125124 (2020).
- [49] M. Stefanini and J. Marino, Orthogonality catastrophe beyond luttinger liquid from post-selection (2023), [arXiv:2310.00039](https://arxiv.org/abs/2310.00039) [[cond-mat.stat-mech](https://arxiv.org/abs/2310.00039)].
- [50] M. Hasegawa, M. Nakagawa, and K. Saito, Kondo effect in a quantum dot under continuous quantum measurement (2021), [arXiv:2111.07771](https://arxiv.org/abs/2111.07771) [[cond-mat.mes-hall](https://arxiv.org/abs/2111.07771)].
- [51] O. Scarlatella, A. A. Clerk, R. Fazio, and M. Schiró, Dynamical mean-field theory for markovian open quantum many-body systems, *Phys. Rev. X* **11**, 031018 (2021).
- [52] O. Scarlatella and M. Schiró, Self-consistent dynamical maps for open quantum systems, *SciPost Phys.* **16**, 026 (2024).
- [53] N. Gisin and I. C. Percival, The quantum-state diffusion model applied to open systems, *J. Phys. A: Math. Theor.* **25**, 5677 (1992).
- [54] Supplemental material, where we detail the non-crossing dynamical map, the schrieffer-wolff transformation, and we present additional numerical data (spectral function, entanglement entropy). it includes [51, 52, 56, 58, 65].
- [55] T. Prosen, Third quantization: a general method to solve master equations for quadratic open fermi systems, *New Journal of Physics* **10**, 043026 (2008).
- [56] A. A. Dzhioev and D. S. Kosov, Superfermion representation of quantum kinetic equations for the electron transport problem, *The Journal of Chemical Physics* **134**, 044121 (2011), [https://pubs.aip.org/aip/jcp/article-pdf/doi/10.1063/1.3548065/13788434/044121\\_1\\_online.pdf](https://pubs.aip.org/aip/jcp/article-pdf/doi/10.1063/1.3548065/13788434/044121_1_online.pdf).
- [57] U. Harbola and S. Mukamel, Superoperator nonequilibrium green's function theory of many-body systems; ap-

- lications to charge transfer and transport in open junctions, *Physics Reports* **465**, 191 (2008).
- [58] A. Dorda, M. Nuss, W. von der Linden, and E. Arrigoni, Auxiliary master equation approach to nonequilibrium correlated impurities, *Phys. Rev. B* **89**, 165105 (2014).
- [59] E. Arrigoni and A. Dorda, Master equations versus keldysh green's functions for correlated quantum systems out of equilibrium, in *Out-of-Equilibrium Physics of Correlated Electron Systems*, edited by R. Citro and F. Mancini (Springer International Publishing, Cham, 2018) pp. 121–188.
- [60] D. Werner, J. Lotze, and E. Arrigoni, Configuration interaction based nonequilibrium steady state impurity solver, *Phys. Rev. B* **107**, 075119 (2023).
- [61] Y. TAKAHASHI and H. UMEZAWA, Thermo field dynamics, *International Journal of Modern Physics B* **10**, 1755 (1996), <https://doi.org/10.1142/S0217979296000817>.
- [62] I. Ojima, Gauge fields at finite temperatures—“thermo field dynamics” and the kms condition and their extension to gauge theories, *Annals of Physics* **137**, 1 (1981).
- [63] M. Seclì, *Topology and Nonlinearity in Driven-Dissipative Photonic Lattices: Semiclassical and Quantum Approaches*, 1st ed. (SISSA, 2021).
- [64] A. McDonald and A. A. Clerk, Third quantization of open quantum systems: Dissipative symmetries and connections to phase-space and keldysh field-theory formulations, *Phys. Rev. Res.* **5**, 033107 (2023).
- [65] M. Vanhovecke and M. Schirò, Diagrammatic monte carlo for dissipative quantum impurity models, *Phys. Rev. B* **109**, 125125 (2024).
- [66] N. E. Bickers, Review of techniques in the large-  $N$  expansion for dilute magnetic alloys, *Rev. Mod. Phys.* **59**, 845 (1987).
- [67] E. Müller-Hartmann, Self-consistent perturbation theory of the anderson model: Ground state properties, *Z. Physik B - Condensed Matter* **57**, 281 (1984).
- [68] Y. Meir, N. S. Wingreen, and P. A. Lee, Low-temperature transport through a quantum dot: The anderson model out of equilibrium, *Phys. Rev. Lett.* **70**, 2601 (1993).
- [69] M. Eckstein and P. Werner, Nonequilibrium dynamical mean-field calculations based on the noncrossing approximation and its generalizations, *Phys. Rev. B* **82**, 115115 (2010).
- [70] R. Härtle, G. Cohen, D. R. Reichman, and A. J. Millis, Decoherence and lead-induced interdot coupling in nonequilibrium electron transport through interacting quantum dots: A hierarchical quantum master equation approach, *Phys. Rev. B* **88**, 235426 (2013).
- [71] A. Erpenbeck and G. Cohen, Resolving the nonequilibrium Kondo singlet in energy- and position-space using quantum measurements, *SciPost Phys.* **10**, 142 (2021).
- [72] D. Rossini, A. Ghermaoui, M. B. Aguilera, R. Vatrè, R. Bouganne, J. Beugnon, F. Gerbier, and L. Mazza, Strong correlations in lossy one-dimensional quantum gases: From the quantum Zeno effect to the generalized Gibbs ensemble, arXiv:2011.04318 [cond-mat] (2020).
- [73] M. Seclì, M. Capone, and M. Schirò, Steady-state quantum zeno effect of driven-dissipative bosons with dynamical mean-field theory, *Phys. Rev. A* **106**, 013707 (2022).
- [74] L. Kohn and G. E. Santoro, Quench dynamics of the anderson impurity model at finite temperature using matrix product states: entanglement and bath dynamics, *Journal of Statistical Mechanics: Theory and Experiment* **2022**, 063102 (2022).
- [75] M. M. Wauters, C.-M. Chung, L. Maffi, and M. Burdello, Simulations of the dynamics of quantum impurity problems with matrix product states, *Phys. Rev. B* **109**, 115101 (2024).
- [76] E. M. Kessler, Generalized schrieffer-wolff formalism for dissipative systems, *Phys. Rev. A* **86**, 012126 (2012).
- [77] L. Rosso, F. Iemini, M. Schirò, and L. Mazza, Dissipative flow equations, *SciPost Phys.* **9**, 091 (2020).
- [78] M. Nakagawa, N. Tsuji, N. Kawakami, and M. Ueda, Dynamical sign reversal of magnetic correlations in dissipative hubbard models, *Phys. Rev. Lett.* **124**, 147203 (2020).
- [79] P. Kattel, A. Zhakenov, P. R. Pasnoori, P. Azaria, and N. Andrei, Dissipation driven phase transition in the non-hermitian kondo model (2024), arXiv:2402.09510 [cond-mat.str-el].
- [80] J. A. S. Lourenço, R. L. Eneias, and R. G. Pereira, Kondo effect in a  $\mathcal{PT}$ -symmetric non-hermitian hamiltonian, *Phys. Rev. B* **98**, 085126 (2018).
- [81] M. Stefanini, Y.-F. Qu, T. Esslinger, S. Gopalakrishnan, E. Demler, and J. Marino, Dissipative realization of kondo models (2024), arXiv:2406.03527 [cond-mat.quant-gas].

## Supplemental Material to ‘Kondo-Zeno crossover in the dynamics of a monitored quantum dot’

In this Supplemental Material, we discuss:

1. The stochastic dynamics of a monitored quantum dot and the Lindblad evolution for the averaged state
2. The hybridization expansion and Non-Crossing Approximation for dissipative quantum impurity models
3. Additional Results: spectral function and impurity entanglement entropy
4. The Schrieffer Wolff transformation and the effective Kondo model

### LINDBLAD DYNAMICS FROM CONTINUOUS MONITORING

We consider a quantum dot coupled to a metallic lead and to a monitoring device which measures some hermitian operator  $O$ . We focus here on a homodyne detection scheme corresponding to a quantum-state diffusion protocol described by the following stochastic Schrodinger equation

$$d|\psi(\xi_t)\rangle = -idtH|\psi(\xi_t)\rangle + d\xi_t (O - \langle O \rangle) |\psi(\xi_t)\rangle - \frac{\gamma dt}{2} (O - \langle O \rangle)^2 |\psi(\xi_t)\rangle \quad (\text{S1})$$

Here we follow the standard terminology and define the state  $|\psi(\xi_t)\rangle$  as the ensemble of quantum trajectories, each labeled by a specific history of measurement outcomes  $\xi_t$ . The first term in Eq. (S1) describes the deterministic evolution, a coherent dynamics generated by the dot+lead Hamiltonian

$$H = \sum_{\mathbf{k},\sigma} \varepsilon_{\mathbf{k}} c_{\mathbf{k},\sigma}^\dagger c_{\mathbf{k},\sigma} + \sum_{\mathbf{k},\sigma} (V_{\mathbf{k}} d_\sigma^\dagger c_{\mathbf{k},\sigma} + h.c.) + \varepsilon_d \sum_{\sigma} d_\sigma^\dagger d_\sigma + U n_\uparrow n_\downarrow \quad (\text{S2})$$

The continuous monitoring of the operator  $O$  has two effects on the evolution of the state of the system, as seen in the last terms in Eq. (S1). It introduces a local stochastic term whose noise couples to the fluctuations  $O - \langle O \rangle$  as well as a back-action term proportional to the fluctuation squared,  $(O - \langle O \rangle)^2$ . Here, we defined the expectation value  $\langle \circ \rangle_\xi = \langle \psi(\xi_t) | \circ | \psi(\xi_t) \rangle$  and the stochastic real variable  $d\xi_t$ , satisfying the standard rules from Itô calculus  $\overline{d\xi_t} = 0$  and  $(d\xi_t)^2 = dt$ .  $d\xi_t$  can be interpreted as an infinitesimal fluctuating noise term.

From the stochastic Schrodinger equation in Eq. (S1) one can obtain the density matrix  $\rho(\xi_t) = |\psi(\xi_t)\rangle\langle\psi(\xi_t)|$  and write down an equation of motion for its evolution. Using the properties of the Itô noise this reads

$$d\rho(\xi_t) = -i[H, \rho(\xi_t)] + O\rho(\xi_t)O - \frac{1}{2} \{\rho(\xi_t), O^2\} + d\xi_t \{O - \langle O \rangle_\xi, \rho(\xi_t)\} \quad (\text{S3})$$

Due to the properties of the Itô noise which is uncorrelated at different times, the average of Eq. (S3) yields the following Lindblad equation for the averaged state  $\rho_t \equiv \overline{\rho(\xi_t)}$

$$\partial_t \rho_t = -i[H, \rho_t] + O\rho_t O - \frac{1}{2} \{O^2, \rho_t\} \quad (\text{S4})$$

which coincides with the one given in the main text for Hermitian jump operator.

### HYBRIDIZATION EXPANSION AND NON-CROSSING DIAGRAMS

#### Vectorization of the Lindbladian

We start by applying the superfermion or vectorization formalism [56, 58] to the case of the master equation for a dissipative quantum impurity model, i.e.

$$\partial_t \rho_t = -i[H, \rho_t] + L\rho_t L^\dagger - \frac{1}{2} \{L^\dagger L, \rho_t\} \quad (\text{S5})$$

where  $H = H_{\text{dot}} + H_{\text{bath}} + H_{\text{hyb}}$  is the dot plus bath Hamiltonian given in the main text while  $L$  is the impurity jump operator, corresponding in our case to the dot total charge  $L = \sum_{\sigma} n_{\sigma}$ . We introduce the Hilbert spaces  $\mathcal{H}$  for the impurity and the bath degrees of freedom and its doubled tilde-version  $\tilde{\mathcal{H}}$  and the associated identity operators

$$I = \sum_m |m\rangle\langle m| \quad (\text{S6})$$

$$\tilde{I} = \sum_m |\tilde{m}\rangle\langle \tilde{m}|. \quad (\text{S7})$$

written in terms of two orthonormal basis  $|m\rangle, |\tilde{m}\rangle$ . We can also duplicate all the degrees of freedom in the problem, namely the impurity and the bath fermions, and introduce the associated creation/annihilation operators  $d_{\sigma}, \tilde{d}_{\sigma}$  and  $c_{\mathbf{k},\sigma}, \tilde{c}_{\mathbf{k},\sigma}$  and their Hermitian conjugate. The key step is now to vectorize the identity operator, introducing the so called left vacuum [56, 58] (or vectorized identity)

$$|I\rangle = \sum_m (-i)^m |m\rangle \otimes |\tilde{m}\rangle \quad (\text{S8})$$

The vectorized identity is particularly useful as it allows to write any operator  $O$  as a vector

$$|O\rangle = O|I\rangle = O \otimes \tilde{I}|I\rangle \quad (\text{S9})$$

In particular, the vectorized density matrix reads:

$$|\rho\rangle = \rho|I\rangle \quad (\text{S10})$$

Using the left vacuum one can write down the trace of any operator over the density matrix as

$$\langle O(t) \rangle = \text{Tr}(\rho_t O) \equiv \langle I|O|\rho_t\rangle \quad (\text{S11})$$

Following these rules we can rewrite the Lindblad master equation as a non-unitary Schrodinger type of equation

$$\partial_t |\rho_t\rangle = \mathcal{L}|\rho_t\rangle \quad (\text{S12})$$

where  $\mathcal{L}$  is the vectorized Lindbladian. The advantage of the vectorization formalism is that the superoperator structure usually needed to treat Lindbladian problems and the associated hybridization expansion is now encoded by doubling the local Hilbert space and working with an additional quantum number, similar to an orbital degrees of freedom. The Lindbladian  $\mathcal{L}$  has now three contributions

$$\mathcal{L} = \mathcal{L}_{\text{dot}} + \mathcal{L}_{\text{bath}} + \mathcal{L}_{\text{hyb}} \quad (\text{S13})$$

The first one  $\mathcal{L}_{\text{dot}}$  is the free Lindbladian for the dissipative impurity. By using the super-fermions rules [58, 62] ( $d_{\sigma}|I\rangle = -i\tilde{d}_{\sigma}^{\dagger}|I\rangle$  and  $d_{\sigma}^{\dagger}|I\rangle = -i\tilde{d}_{\sigma}|I\rangle$ ) and since we consider only the dissipation on the impurity degrees of freedom, we can formally write the impurity Lindbladian  $\mathcal{L}_{\text{dot}}$  as:

$$\mathcal{L}_{\text{dot}} = -i \left( H_{\text{dot}} - \tilde{H}_{\text{dot}} \right) + \left( s_L L \tilde{L} - \frac{1}{2} L^{\dagger} L - \frac{1}{2} \tilde{L}^{\dagger} \tilde{L} \right) \quad (\text{S14})$$

where  $s_L$  is an extra sign depending on the fermionic ( $s_L = -i$ ) or bosonic ( $s_L = 1$ ) nature of the jumps operator. The second term in Eq. (S13) is the bath Lindbladian and reads  $\mathcal{L}_{\text{bath}} = -i \left( H_{\text{bath}} - \tilde{H}_{\text{bath}} \right)$ . Finally the impurity-bath Lindbladian  $\mathcal{L}_{\text{hyb}}$  can be written in compact form by introducing the following fields

$$\Phi_{\sigma} = \sum_{\mathbf{k}} V_{\mathbf{k}} \begin{pmatrix} c_{\mathbf{k},\sigma} \\ \tilde{c}_{\mathbf{k},\sigma}^{\dagger} \end{pmatrix} \quad \Psi_{\sigma} = \begin{pmatrix} d_{\sigma} \\ \tilde{d}_{\sigma}^{\dagger} \end{pmatrix} \quad (\text{S15})$$

which group together the operators living in the space  $\mathcal{H}$  and  $\tilde{\mathcal{H}}$ . Using these fields we can write the system-bath term in a more compact way:

$$\mathcal{L}_{\text{hyb}} = -i \left( H_{\text{hyb}} - \tilde{H}_{\text{hyb}} \right) = -i \sum_{\sigma\alpha} \left( \bar{\Phi}_{\sigma}^{\alpha} \Psi_{\sigma}^{\alpha} + \bar{\Psi}_{\sigma}^{\alpha} \Phi_{\sigma}^{\alpha} \right) \quad (\text{S16})$$

where we have introduced a label  $\alpha = 0, 1$  which denotes the Hilbert space  $\mathcal{H}$  or  $\tilde{\mathcal{H}}$  ( $d_\sigma = \Psi_\sigma^{\alpha=0}$  and  $\tilde{d}_\sigma^\dagger = \Psi_\sigma^{\alpha=1}$ ). At this point we can write the formal solution of the vectorized master equation (S12)

$$|\rho_t\rangle = \mathcal{T}_t \exp\left(\int_0^t \mathcal{L}(\tau) d\tau\right) |\rho_0\rangle \quad (\text{S17})$$

where we have introduced the time ordering operator  $\mathcal{T}_t$  in the Superfermions representation. Unlike the standard Keldysh time-ordering, here the time ordering is defined as:

$$t_\alpha > \bar{t}_\beta = \begin{cases} t > \bar{t} & \text{if } \alpha = \beta \in \mathcal{H}, \tilde{\mathcal{H}} \\ \alpha \in \mathcal{H} & \beta \in \tilde{\mathcal{H}} \end{cases} \quad (\text{S18})$$

This ordering allows to define a time-ordering operator  $\mathcal{T}_t$  such that two operators,  $\psi_1$  and  $\psi_2$ , being  $\psi$  a creation or annihilation fermionic operator living in the  $\mathcal{H}(\tilde{\mathcal{H}})$  Hilbert space, anticommute under time-ordering:

$$\mathcal{T}_t \psi_1(t_\alpha) \psi_2(t_\beta) = \begin{cases} \psi_1(t_\alpha) \psi_2(t_\beta) & \text{if } t_\alpha > t_\beta \\ -\psi_2(t_\beta) \psi_1(t_\alpha) & \text{otherwise} \end{cases} \quad (\text{S19})$$

Eq. (S17) represents the starting point to perform the hybridization expansion, namely an expansion order by order in the system-bath coupling  $\mathcal{L}_{\text{hyb}}$ , as we will discuss in the next section.

### Hybridization Expansion and NCA Dynamical Map

In this Section we derive the perturbation series (called Hybridization expansion) for the evolution superoperator in the powers of the impurity-bath coupling, up to all orders in this coupling. In the interaction picture we can write the time evolution operator as,

$$|\rho_t\rangle = \mathcal{V}_0(t) \mathcal{T}_t \exp\left(-i \int_0^t \mathcal{L}_{\text{hyb}}(s) ds\right) |\rho_0\rangle \quad (\text{S20})$$

where  $\mathcal{V}_0(t) = \exp(\mathcal{L}_0 t)$  is the time evolution operator in the Superfermions representation in the decoupled limit, i.e.  $\mathcal{L}_0 = \mathcal{L}_{\text{dot}} + \mathcal{L}_{\text{bath}}$ . In fact the time dependence of  $\mathcal{L}_{\text{hyb}}$  is given in function of the free evolution  $\mathcal{L}_{\text{hyb}}(\tau) = e^{-\mathcal{L}_0 \tau} \mathcal{L}_{\text{hyb}} e^{\mathcal{L}_0 \tau}$  by using a standard Taylor expansion for the bath-impurity terms, we can write all the order of  $|\rho_t\rangle$

$$|\rho_t\rangle = \sum_n \frac{(-i)^n}{n!} \int_0^t d\tau_1 \cdots d\tau_n \mathcal{V}_0(t) \mathcal{T}_t [\mathcal{L}_{\text{hyb}}(\tau_1) \mathcal{L}_{\text{hyb}}(\tau_2) \cdots \mathcal{L}_{\text{hyb}}(\tau_n)] |\rho_0\rangle \quad (\text{S21})$$

due to the fact that the coupling between the bath and the impurity degrees of freedom is linear, we only have to consider the even terms of the expansion

$$|\rho_t\rangle = \sum_n \frac{(-1)^n}{2n!} \int_0^t d\tau_1 \cdots d\tau_{2n} \mathcal{V}_0(t) \mathcal{T}_t [\mathcal{L}_{\text{hyb}}(\tau_1) \mathcal{L}_{\text{hyb}}(\tau_2) \cdots \mathcal{L}_{\text{hyb}}(\tau_{2n})] |\rho_0\rangle \quad (\text{S22})$$

in terms of the spinors  $\Psi$  and  $\Phi$ , the time evolution of the density matrix becomes

$$|\rho_t\rangle = \sum_n \sum_{\{\sigma, \bar{\sigma}\}} \frac{(-1)^n}{(n!)^2} \int_0^t d\tau_1 \cdots d\tau_n \int_0^t d\bar{\tau}_1 \cdots d\bar{\tau}_n \mathcal{V}_0 \mathcal{T}_t [\bar{\Psi}_\sigma(\tau_1) \Phi_\sigma(\tau_1) \cdots \bar{\Phi}_\sigma(\tau_n) \Psi_\sigma(\tau_n)] |\rho_0\rangle \quad (\text{S23})$$

We now take the average over the bath  $Tr_{\text{bath}}[\cdots] = \langle I_{\text{bath}} | \cdots | \rho_0 \rangle$ , using the fact that the initial state  $|\rho_0\rangle$  is factorized, initially we take the initial density matrix as  $\rho_0 = \rho_{\text{dot},0} \otimes \rho_{\text{bath},0}$ . Then since the bath is non-interacting, we can use the Wick theorem in order to trace out the bath degrees of freedom and obtain the hybridization expansion for the dressed impurity propagator  $\langle I_{\text{bath}} | \rho_t \rangle \equiv \mathcal{V}(t, 0) | \rho_{\text{dot},0} \rangle$  which finally reads [65]

$$\mathcal{V}(t, 0) | \rho_{\text{dot},0} \rangle = \sum_n \sum_{\{\sigma, \bar{\sigma}\}} \frac{(-i)^n}{(n!)^2} \int_0^t \prod_i d\tau_i d\bar{\tau}_i \exp(\mathcal{L}_{\text{dot}} t) \mathcal{T}_t [\bar{\Psi}_\sigma(\tau_1) \cdots \Psi_\sigma(\tau_n)] | \rho_{\text{dot},0} \rangle \text{Det}_\sigma [\{\bar{\Phi}_\sigma, \Phi_\sigma\}] \text{Det}_{\bar{\sigma}} [\{\bar{\Phi}_{\bar{\sigma}}, \Phi_{\bar{\sigma}}\}] \quad (\text{S24})$$

where  $\text{Det}_\sigma [\{\bar{\Phi}_\sigma, \Phi_\sigma\}]$  are determinants built out of the real-time hybridization functions of the bath defined as

$$\Delta_\sigma^{\alpha\bar{\alpha}}(\tau, \bar{\tau}) = -i \langle I_{\text{bath}} | \mathcal{T}_t [\bar{\Phi}_\sigma^\alpha(\tau) \Phi_\sigma^{\bar{\alpha}}(\bar{\tau})] | \rho_{\text{bath},0} \rangle \quad (\text{S25})$$

For a non-interacting bath, as the one we consider here, a simple calculation gives [65]

$$\Delta_\sigma^{01}(\tau, \bar{\tau}) = \int d\epsilon (1 - n_F(\epsilon)) \Gamma_\sigma(\epsilon) e^{-i\epsilon(\tau - \bar{\tau})} \quad (\text{S26})$$

$$\Delta_\sigma^{10}(\tau, \bar{\tau}) = - \int d\epsilon n_F(\epsilon) \Gamma_\sigma(\epsilon) e^{-i\epsilon(\tau - \bar{\tau})} \quad (\text{S27})$$

where  $n_F(\epsilon)$  is the Fermi distribution and  $\Gamma(\epsilon)$  the energy-dependent hybridization for the spin channel  $\sigma$ , defined in the main text. From this result the diagonal components of  $\Delta_\sigma^{\alpha\bar{\alpha}}(\tau, \bar{\tau})$  can be obtained, for example  $\Delta_\sigma^0(\tau, \bar{\tau}) = i\theta(\tau - \bar{\tau})\Delta_\sigma^{10}(\tau, \bar{\tau}) + i\theta(\bar{\tau} - \tau)\Delta_\sigma^{01}(\tau, \bar{\tau})$ .

The hybridization expansion we have derived in Eq. (S24) is a bare expansion in the dot-bath coupling. It can be reorganized in a self-consistent diagrammatic expansion by identifying one-particle irreducible diagrams and introducing a self-energy  $\Sigma(\tau, \tau')$  to obtain a Dyson equation for  $\mathcal{V}(t, 0)$  of the form [52]

$$\mathcal{V}(t, 0) = \mathcal{V}_{\text{dot}}(t, 0) + \int_0^t d\tau \int_0^\tau d\bar{\tau} \mathcal{V}_{\text{dot}}(t, \tau) \Sigma(\tau, \bar{\tau}) \mathcal{V}(\bar{\tau}, 0) \quad (\text{S28})$$

which can be rewritten as in the main text upon taking a time-derivative on both sides of the equation and using  $\partial_t \mathcal{V}_{\text{dot}}(t, 0) = \mathcal{L}_{\text{dot}} \mathcal{V}_{\text{dot}}(t, 0)$ . As in any interacting diagrammatic theory the self-energy is not known in closed form. It can however reorganised as an expansion in diagrams with increasing number of crossing hybridization lines [51, 52]. The lowest order self-consistent level of this hierarchy is the so called non-crossing approximation (NCA) which corresponds to keeping only the compact diagrams in which hybridization lines do not cross. The formal expression for the NCA self-energy is therefore the one given in the main text, which reads

$$\Sigma_{NCA}(\tau, \bar{\tau}) = -i \sum_\sigma \sum_{\alpha, \bar{\alpha}} [\Psi_\sigma^\alpha \mathcal{V}(\tau, \bar{\tau}) \bar{\Psi}_\sigma^{\bar{\alpha}} \Delta_\sigma^{\alpha\bar{\alpha}}(\tau, \bar{\tau}) - \bar{\Psi}_\sigma^{\bar{\alpha}} \mathcal{V}(\tau, \bar{\tau}) \Psi_\sigma^\alpha \Delta_\sigma^{\alpha\bar{\alpha}}(\bar{\tau}, \tau)] \quad (\text{S29})$$

which describes a rainbow diagram with a dot vertex  $\bar{\Psi}_\sigma^{\bar{\alpha}}$  at time  $\bar{\tau}$ , a dressed dot propagation from  $\bar{\tau}$  to  $\tau$  with propagator  $\mathcal{V}(\tau, \bar{\tau})$  and a hybridization line  $\Delta_\sigma^{\alpha\bar{\alpha}}(\tau, \bar{\tau})$  and finally another dot vertex operator  $\Psi_\sigma^\alpha$  at time  $\tau$ , with the second term in Eq. (S29) describing the diagram with  $\tau, \bar{\tau}$  exchanged.

#### Steady-state Condition

Within NCA we can obtain an equation directly for the steady-state density matrix of the impurity [52], which is useful for example when computing spectral functions. If we assume a unique steady state, the associate density matrix is defined as  $|\rho_{\text{dot,ss}}\rangle = \mathcal{V}(\infty, 0)|\rho_{\text{dot},0}\rangle$ . While in principle this would require to perform the full transient dynamics from an arbitrary initial condition, here we show how to obtain  $|\rho_{\text{ss}}\rangle$  directly from the stationary state propagator  $\mathcal{V}(t, t')$ .

By definition, the steady-state density matrix satisfy the condition,

$$\partial_t \mathcal{V}(t, 0) |\rho_{\text{dot},0}\rangle \xrightarrow{t \rightarrow \infty} 0 \quad (\text{S30})$$

where  $\rho_0$  is an arbitrary initial state. by using the previous Dyson equation, we get:

$$\lim_{t \rightarrow \infty} \partial_t \mathcal{V}(t, 0) |\rho_{\text{dot},0}\rangle = \lim_{t \rightarrow \infty} \left( \mathcal{L}_{\text{dot}} \mathcal{V}(t, 0) + \int_0^t d\tau \Sigma_{NCA}(t, \tau) \mathcal{V}(\tau, 0) \right) |\rho_{\text{dot},0}\rangle = 0 \quad (\text{S31})$$

by making a change of variable in the convolution integral, the condition reads

$$\lim_{t \rightarrow \infty} \left( \mathcal{L}_{\text{dot}} \mathcal{V}(t, 0) + \int_0^t d\tau \Sigma_{NCA}(t, t - \tau) \mathcal{V}(t - \tau, 0) \right) |\rho_{\text{dot},0}\rangle = 0 \quad (\text{S32})$$

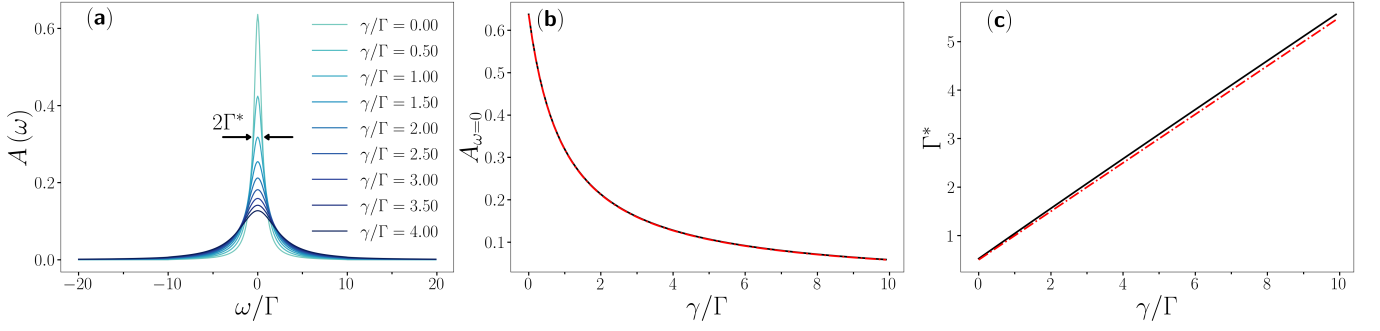


FIG. S1. Dissipative Resonant Level Model - (a) Impurity spectral function for the half-filled dissipative resonant level model, corresponding to  $U = -2\epsilon_d = 0\Gamma$  and increasing values of dephasing  $\gamma$ . (b) Spectral function value at zero frequency as a function of  $\gamma$ . (c) Width  $\Gamma^*$  of the zero frequency peak as a function of  $\gamma$ . In both cases the agreement between NCA (solid line, for  $W = 10\Gamma$ ) and the exact solution (dashed line, see main text) is excellent.

Now, we have to assume some hypothesis, the first one is that the system is supposed to loose memory of the initials conditions, this mean that the Self Energy must vanish when  $\tau \approx t \rightarrow \infty$ , under this hypothesis the convolution integral in the Dyson equation can be cutoff when  $t - \tau > t_{memory}$ . Then when the Self energy is non-zeros, the propagator  $\mathcal{V}(t - \tau, 0)$  is stationary and can be replace by  $\mathcal{V}(\infty, 0)$ . So under the previous hypothesis, the steady state density matrix satisfies:

$$\left( \mathcal{L}_{\text{dot}} + \int_0^\infty \Sigma_{NCA}(\tau) d\tau \right) |\rho_{\text{dot,ss}}\rangle = 0 \quad (\text{S33})$$

This condition depends only on the stationary state propagator  $\mathcal{V}(\tau)$  through the NCA self-energy  $\Sigma_{NCA}(\tau)$ .

## ADDITIONAL RESULTS

In this Section we present additional NCA results on the dissipative Anderson Impurity Model: (i) the impurity spectral function and (ii) the impurity entanglement entropy.

### Impurity Spectral Function

We start from the retarded Green's function of the dot which is defined as

$$G_\sigma^R(t, t') = -i\theta(t - t') \langle \{d_\sigma(t), d_\sigma^\dagger(t')\} \rangle \quad (\text{S34})$$

where  $\langle \dots \rangle$  implies the average over the steady state and  $\{A, B\} = AB + BA$  is the anti-commutator. The spectral function of the impurity,  $A_\sigma(\omega)$ , is defined by going in Fourier space and taking the imaginary part of the retarded Green's function, i.e.

$$A_\sigma(\omega) = -\frac{1}{\pi} \text{Im} [G_\sigma^R(\omega)] \quad (\text{S35})$$

We can obtain the impurity Green's function from the knowledge of the NCA dynamical map superoperator  $\mathcal{V}(t)$  and the impurity steady-state density matrix (S33), a result analog to the quantum regression theorem for Lindblad evolution [51]. In particular, using the superfermion representation one can show that

$$\begin{aligned} G_\sigma^R(t, 0) &= -i\theta(t) \langle \{d_\sigma(t), d_\sigma^\dagger\} \rangle \\ &= -i\theta(t) \langle I | d_\sigma \mathcal{V}(t) d_\sigma^\dagger | \rho_{\text{dot,ss}} \rangle - \theta(t) \langle I | d_\sigma \mathcal{V}(t) \tilde{d}_\sigma | \rho_{\text{dot,ss}} \rangle \end{aligned} \quad (\text{S36})$$

In practice we compute first the steady-state using the condition (S33) and then solve the NCA equations at long-times to obtain  $\mathcal{V}(t)$  and perform Fourier transform to obtain the spectral function.

We start discussing the non-interacting case,  $U = 0$ , corresponding to a dissipative Resonant Level model. We stress that due to the monitoring of the total charge the model is still interacting, i.e. non-gaussian. However the

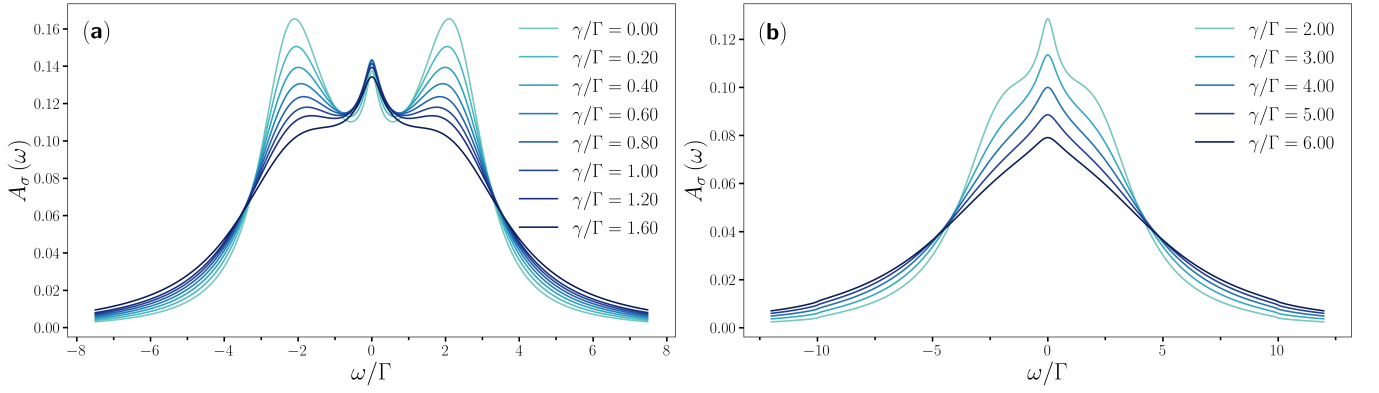


FIG. S2. Dissipative Anderson Impurity Model - (a) Impurity spectral function for the half-filled dissipative AIM and for  $U = -2\epsilon_d = 4\Gamma$  and increasing values of  $\gamma$ . We see that the low-frequency structure is robust to dephasing. (b) Upon increasing further  $\gamma$  the central peak eventually merges with the incoherent background.

specific nature of the dissipative interaction makes it possible to close exactly the equations of motion for the retarded Green's function in terms of a Dyson equation. We obtain therefore

$$G_\sigma^R(\omega) = \frac{1}{(G_{0\sigma}^R)^{-1} - \Sigma_\sigma^R(\omega)} \quad (\text{S37})$$

where the bare retarded Green's function of the impurity reads  $(G_{0\sigma}^R)^{-1} = \omega - \epsilon_d + i\eta$ , while the self-energy  $\Sigma_\sigma^R(\omega)$

$$\Sigma_\sigma^R(\omega) = -i\frac{\gamma}{2} + \sum_{\mathbf{k}} \frac{|V_{\mathbf{k}}|^2}{\omega - \epsilon_{\mathbf{k}} + i\eta} = \mathcal{P}\mathcal{P} \sum_{\mathbf{k}} \frac{|V_{\mathbf{k}}|^2}{\omega - \epsilon_{\mathbf{k}}} - \frac{i}{2} [\gamma + \Gamma(\omega)] \quad (\text{S38})$$

where  $\Gamma(\epsilon) = 2\pi \sum_{\mathbf{k}} |V_{\mathbf{k}}|^2 \delta(\epsilon - \epsilon_{\mathbf{k}})$  is the hybridization function. One can therefore obtain the spectral function in closed form in the wide-bandwidth limit

$$A_\sigma(\omega) = -\frac{1}{\pi} \text{Im} [G_\sigma^R(\omega)] = \frac{1}{2\pi} \frac{(\gamma + \Gamma)}{(\omega - \epsilon_d)^2 + (\gamma + \Gamma)^2/4} \quad (\text{S39})$$

From this expression we can immediately read out the renormalised width  $\Gamma^*$ ,

$$\Gamma^* = \frac{1}{2} (\gamma + \Gamma) \quad (\text{S40})$$

In Fig. (S1)(a) we plot the spectral function obtained with NCA in the  $U = 0$  case for increasing value of the dephasing rate. We see that the resonance at the Fermi level is broadened and at the same time the value at  $\omega = 0$  decreases with  $\gamma$ . A comparison with the exact result for the  $U = 0$  case is shown in Fig. (S1)(b-c), concerning the weight at  $\omega = 0$  and the width of the resonance, demonstrating the perfect agreement between NCA and the exact result. We note that from the exact expression in Eq. (S39) we conclude that the value of the spectral function at  $\omega = 0$  is affected by the dissipative interaction (monitoring rate/dephasing), even though the system remains half-filled. This is different than in the unitary case, where for  $U \neq 0$  the value of the zero frequency spectral function is not renormalised and points towards a breakdown of the Luttinger theorem for dissipative quantum impurity models.

We then move to the interacting case,  $U \neq 0$ , which is more relevant for the present work. In Fig. (S2)(a-b) we plot the impurity spectral function for  $U = 4\Gamma$  and increasing values of the dephasing  $\gamma$ . First, for  $\gamma = 0$ , we recognize the three peak structure of the Anderson impurity spectral function, with the Kondo peak at  $\omega = 0$  and the incoherent Hubbard bands centered around  $\omega = \pm U/2$ . Introducing a small dephasing rate  $\gamma < U$  has a strong effect on the Hubbard bands whose position shifts and whose amplitude shrinks down. On the other hand the Kondo peak appears robust to small dephasing, its width and value at zero frequency remaining essentially constant for small  $\gamma$ . This appears to be in good agreement with the result on the spin decay rate shown in the main text. Indeed we can say that for small dephasing the spectral function is only weakly renormalised with respect to the unitary case. On the other hand when  $\gamma$  is further increased we see two effects taking place, namely the value of the spectral function decreases and the width of the resonance increases, until for  $\gamma > U$  only a broad resonance is left in the spectral function.

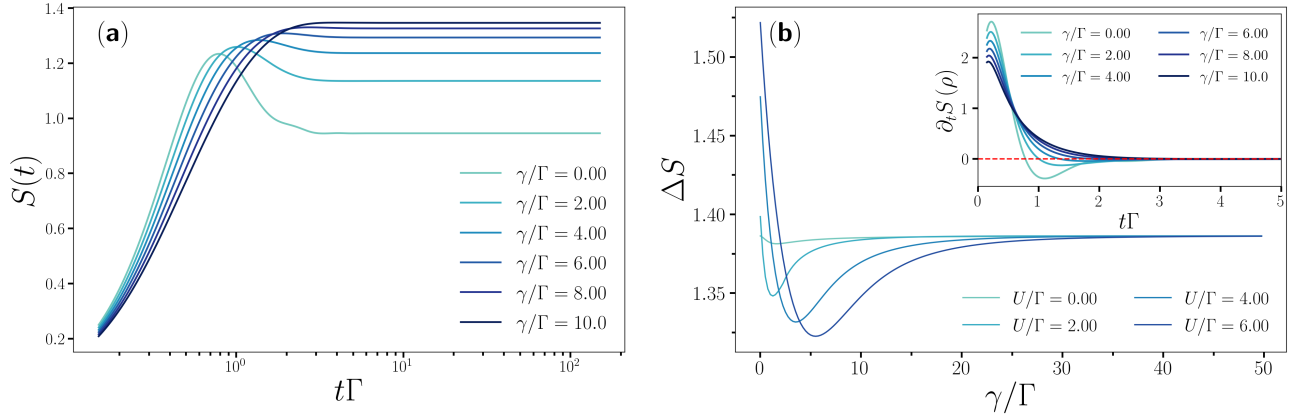


FIG. S3. Impurity Entanglement Entropy Dynamics - (a) Dynamics of the impurity entropy after tracing out the metallic bath. For small dissipation we see a clear maximum which is then suppressed for increasing dissipation, when the entanglement entropy is monotonously growing towards the steady-state. The crossover between the two regimes is again controlled by a scale of the Zeno type, as we show in the panel (b). We first show the total entanglement rate, which displays a zero of negative values for small  $\gamma$ . The total change in entropy between the maximum value and the steady-state has a minimum at a value of dissipation  $\gamma \sim U$ .  $\Delta S = 2 \max(S) - S_{SS}$

### Impurity Entanglement Entropy

We consider the dynamics of the impurity entanglement entropy, obtained by tracing out the metallic bath and computing the thermal entropy of the reduced density matrix of the system, i.e.

$$S(t) = -\text{Tr}(\rho_t \log \rho_t) \quad (\text{S41})$$

where  $\rho_t$  is the reduced density matrix of the system after tracing out the metallic bath. This can be directly obtained from the NCA approach which works directly with the evolution operator of the reduced system, i.e.  $\rho_t = \mathcal{V}(t, 0)\rho_0$ . We emphasize therefore that the state of the system is mixed to begin with, due to the dephasing, therefore the entropy of entanglement also takes contribution from the thermal entropy. In Fig. S3 we plot the dynamics of the entropy as a function of time for different values of the dissipation rate  $\gamma$ . In the unitary case,  $\gamma = 0$ , we observe a maximum at short times followed by an approach to a steady-state value. As we include dissipation the maximum remains well visible for small and moderate values of  $\gamma$ , while for large dissipation the dynamics of the entanglement entropy is monotonous in time towards the steady-state. Intriguing the crossover between the two regimes of entanglement dynamics is also controlled by the Zeno scale. To appreciate this point we plot the quantity  $\Delta S = 2\max(S) - S_{SS}$ , which for a monotonously growing entropy dynamics reduces to the steady-state value, while in the general case quantify the level of non-monotonicity in  $S(t)$ , and it is related to the time-integral of absolute value of the entanglement entropy rate  $\partial_t S$ . We see that  $\Delta S$  has a clear minimum as a function of dissipation, for  $\gamma_* \sim U$  as expected from the Zeno effect.

### SCHRIEFFER-WOLFF TRANSFORMATION

In this section, we provide a detailed derivation of the non-Hermitian Kondo model by using the generalized Schrieffer-Wolff (SW) transformation. We start from the Lindbladian in the superfermion representation that we write as

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{\text{hyb}} \quad (\text{S42})$$

where  $\mathcal{L}_0$  is the diagonal part (with respect to the hybridization) of the Lindbladian that includes the decoupled impurity and bath terms as well as the local charge dephasing  $\mathcal{L}_0 = \mathcal{L}_{\text{dot}} + \mathcal{L}_{\text{bath}}$ , while  $\mathcal{L}_{\text{hyb}} = -i(H_{\text{hyb}} - \tilde{H}_{\text{hyb}})$  is the off-diagonal term. As in the equilibrium case, the SW transformation allows us to integrate out the bath-impurity coupling terms by introducing a non-unitary (similarity) transformation  $S$  such that the new Lindbladian

$$\mathcal{L}_{\text{eff}} = e^S \mathcal{L} e^{-S} = \mathcal{L} + [S, \mathcal{L}] + \frac{1}{2} [S, [S, \mathcal{L}]] + \dots \quad (\text{S43})$$

is diagonal order by order in a perturbative expansion. In particular this can be achieved by choosing the generator  $S$  to be fully off-diagonal with respect to the hybridization and to satisfy the condition

$$\mathcal{L}_{\text{hyb}} + [S, \mathcal{L}_0] = 0 \quad (\text{S44})$$

which then gives an expression for the effective Lindbladian which reads

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_0 + \frac{1}{2}[S, \mathcal{L}_{\text{hyb}}] \quad (\text{S45})$$

In the following we first derive the form of the generator  $S$  of the Schrieffer-Wolff transformation and then write down the resulting effective Lindbladian operator.

### The Generator

The condition for the generator suggests that  $S$  should have a structure both diagonal and off-diagonal in the two Hilbert spaces. We therefore parametrize it as following

$$S = \sum_{\mathbf{k}\sigma} X_{\mathbf{k}\sigma} \left( c_{\mathbf{k}\sigma}^\dagger d_\sigma - hc \right) + \sum_{\mathbf{k}\sigma} \tilde{X}_{\mathbf{k}\sigma} \left( \tilde{c}_{\mathbf{k}\sigma}^\dagger \tilde{d}_\sigma - hc \right) \quad (\text{S46})$$

where the operators  $X_{\mathbf{k}\sigma}$  read

$$X_{\mathbf{k}\sigma} = s_{\mathbf{k}} + t_{\mathbf{k}} n_{\bar{\sigma}} + x_{\mathbf{k}} \tilde{n} + w_{\mathbf{k}} n_{\bar{\sigma}} \tilde{n} + h_{\mathbf{k}} \tilde{n}_\uparrow \tilde{n}_\downarrow + g_{\mathbf{k}} n_{\bar{\sigma}} \tilde{n}_\uparrow \tilde{n}_\downarrow \quad (\text{S47})$$

$$= (s_{\mathbf{k}} + t_{\mathbf{k}} n_{\bar{\sigma}}) + (x_{\mathbf{k}} + w_{\mathbf{k}} n_{\bar{\sigma}}) \tilde{n} + (h_{\mathbf{k}} + g_{\mathbf{k}} n_{\bar{\sigma}}) \tilde{n}_\uparrow \tilde{n}_\downarrow \quad (\text{S48})$$

and similarly for  $\tilde{X}_{\mathbf{k}}$  upon switching tilde and not-tilde, i.e.

$$\tilde{X}_{\mathbf{k}\sigma} = \tilde{s}_{\mathbf{k}} + \tilde{t}_{\mathbf{k}} \tilde{n}_{\bar{\sigma}} + \tilde{x}_{\mathbf{k}} n + \tilde{w}_{\mathbf{k}} \tilde{n}_{\bar{\sigma}} n + \tilde{h}_{\mathbf{k}} n_\uparrow n_\downarrow + \tilde{g}_{\mathbf{k}} \tilde{n}_{\bar{\sigma}} n_\uparrow n_\downarrow \quad (\text{S49})$$

$$= (\tilde{s}_{\mathbf{k}} + \tilde{t}_{\mathbf{k}} \tilde{n}_{\bar{\sigma}}) + (\tilde{x}_{\mathbf{k}} + \tilde{w}_{\mathbf{k}} \tilde{n}_{\bar{\sigma}}) n + (\tilde{h}_{\mathbf{k}} + \tilde{g}_{\mathbf{k}} \tilde{n}_{\bar{\sigma}}) n_\uparrow n_\downarrow \quad (\text{S50})$$

While the first two terms in  $X_{\mathbf{k}\sigma}, \tilde{X}_{\mathbf{k}\sigma}$  are diagonal in the Hilbert spaces, and represent straightforward generalisation of the generator for the standard AIM, the last terms represent the new addition due to dissipation. These terms couple the two sectors  $\mathcal{H}$  and  $\tilde{\mathcal{H}}$  and they are needed to satisfy the condition (S44) due to the structure of the dissipation. To proceed it is convenient to evaluate few commutators between  $S$  and different operators, namely

$$[S, n_\sigma] = \sum_{\mathbf{k}} X_{\mathbf{k}\sigma} \left( c_{\mathbf{k}\sigma}^\dagger d_\sigma + hc \right) \quad (\text{S51})$$

$$[S, n_\uparrow n_\downarrow] = \sum_{\mathbf{k}\sigma} X_{\mathbf{k}\sigma} \left( c_{\mathbf{k}\sigma}^\dagger d_\sigma + hc \right) n_{\bar{\sigma}} \quad (\text{S52})$$

$$[S, n_{\mathbf{k}\sigma}] = -X_{\mathbf{k}\sigma} \left( c_{\mathbf{k}\sigma}^\dagger d_\sigma + hc \right) \quad (\text{S53})$$

Similar relations, upon sending tilde-operators into non-tilde and viceversa, hold for the commutator with  $\tilde{n}_\sigma, \tilde{n}_\uparrow \tilde{n}_\downarrow, \tilde{n}_{\mathbf{k}\sigma}$ . Finally we need to evaluate the commutator between  $S$  and the dissipation term coupling the two sectors, namely

$$[S, n\tilde{n}] = \sum_{\mathbf{k}\sigma} X_{\mathbf{k}\sigma} \left( c_{\mathbf{k}\sigma}^\dagger d_\sigma + hc \right) \tilde{n} + \sum_{\mathbf{k}\sigma} \tilde{X}_{\mathbf{k}\sigma} \left( \tilde{c}_{\mathbf{k}\sigma}^\dagger \tilde{d}_\sigma + hc \right) n \quad (\text{S54})$$

Using these results we can now evaluate the commutator between  $S$  and  $\mathcal{L}_0$  and obtain the conditions that fix the generator  $S$ . From this equation, using the commutators above, we get the following conditions

$$(\varepsilon_{\mathbf{k}} - \varepsilon_d + i\gamma/2) X_{\mathbf{k}\sigma} - (U - i\gamma) X_{\mathbf{k}\sigma} n_{\bar{\sigma}} - i\gamma X_{\mathbf{k}\sigma} \tilde{n} = V_{\mathbf{k}} \quad (\text{S55})$$

$$(\varepsilon_{\mathbf{k}} - \varepsilon_d - i\gamma/2) \tilde{X}_{\mathbf{k}\sigma} - (U + i\gamma) \tilde{X}_{\mathbf{k}\sigma} \tilde{n}_{\bar{\sigma}} + i\gamma \tilde{X}_{\mathbf{k}\sigma} n = V_{\mathbf{k}} \quad (\text{S56})$$

which we can solve to obtain the coefficients

$$s_{\mathbf{k}} = \frac{V_{\mathbf{k}}}{(\varepsilon_{\mathbf{k}} - \varepsilon_d + i\gamma/2)} \quad (\text{S57})$$

$$t_{\mathbf{k}} = \frac{(U - i\gamma)s_{\mathbf{k}}}{(\varepsilon_{\mathbf{k}} - \varepsilon_d - U + 3i\gamma/2)} = \frac{(U - i\gamma)V_{\mathbf{k}}}{(\varepsilon_{\mathbf{k}} - \varepsilon_d - U + 3i\gamma/2)(\varepsilon_{\mathbf{k}} - \varepsilon_d + i\gamma/2)} \quad (\text{S58})$$

$$x_{\mathbf{k}} = \frac{i\gamma s_{\mathbf{k}}}{(\varepsilon_{\mathbf{k}} - \varepsilon_d - i\gamma/2)} = \frac{i\gamma V_{\mathbf{k}}}{(\varepsilon_{\mathbf{k}} - \varepsilon_d)^2 + \gamma^2/4} \quad (\text{S59})$$

$$w_{\mathbf{k}} = \frac{(U - i\gamma)x_{\mathbf{k}} + i\gamma t_{\mathbf{k}}}{(\varepsilon_{\mathbf{k}} - \varepsilon_d - U + i\gamma/2)} \quad (\text{S60})$$

$$h_{\mathbf{k}} = \frac{2i\gamma x_{\mathbf{k}}}{(\varepsilon_{\mathbf{k}} - \varepsilon_d - i3\gamma/2)} = -\frac{2\gamma^2}{(\varepsilon_{\mathbf{k}} - \varepsilon_d)^2 + \gamma^2/4} \frac{V_{\mathbf{k}}}{(\varepsilon_{\mathbf{k}} - \varepsilon_d + i3\gamma/2)} \quad (\text{S61})$$

$$g_{\mathbf{k}} = \frac{(U - i\gamma)h_{\mathbf{k}} + 2i\gamma w_{\mathbf{k}}}{(\varepsilon_{\mathbf{k}} - \varepsilon_d - U - i\gamma/2)} \quad (\text{S62})$$

$$(\text{S63})$$

Similarly, solving for the coefficients entering in the tilde-space part  $\tilde{X}_{\mathbf{k}\sigma}$  we discover that  $\tilde{s}_{\mathbf{k}} = s_{\mathbf{k}}^*$  and similarly for the other coefficients.

### Derivation of the Effective Lindbladian

The effective Lindbladian is obtained from the expression

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_0 + \frac{1}{2}[S, \mathcal{L}_{\text{hyb}}] = \mathcal{L}_0 + \frac{1}{2}[S, \mathcal{L}_{\text{hyb}}] = \mathcal{L}_0 - \frac{i}{2}[S, H_{\text{hyb}} - \tilde{H}_{\text{hyb}}] \quad (\text{S64})$$

We now evaluate each of the two commutator separately, involving  $H_{\text{hyb}}, \tilde{H}_{\text{hyb}}$  respectively. First we have

$$[S, H_{\text{hyb}}] = \sum_{\mathbf{k}\mathbf{p}} \sum_{\sigma\sigma'} X_{\mathbf{k}\sigma} V_{\mathbf{p}} \left[ \left( c_{\mathbf{k}\sigma}^\dagger d_\sigma - hc \right), \left( c_{\mathbf{p}\sigma'}^\dagger d_{\sigma'} + hc \right) \right] \quad (\text{S65})$$

$$+ \sum_{\mathbf{k}\mathbf{p}} \sum_{\sigma\sigma'} V_{\mathbf{p}} [X_{\mathbf{k}\sigma}, \left( c_{\mathbf{p}\sigma'}^\dagger d_{\sigma'} + hc \right)] \left( c_{\mathbf{k}\sigma}^\dagger d_\sigma - hc \right) \quad (\text{S66})$$

$$+ \sum_{\mathbf{k}\mathbf{p}} \sum_{\sigma\sigma'} V_{\mathbf{p}} [\tilde{X}_{\mathbf{k}\sigma}, \left( c_{\mathbf{p}\sigma'}^\dagger d_{\sigma'} + hc \right)] \left( \tilde{c}_{\mathbf{k}\sigma}^\dagger \tilde{d}_\sigma - hc \right) \quad (\text{S67})$$

Similarly, for the tilde term we have

$$[S, \tilde{H}_{\text{hyb}}] = \sum_{\mathbf{k}\mathbf{p}} \sum_{\sigma\sigma'} \tilde{X}_{\mathbf{k}\sigma} V_{\mathbf{p}} \left[ \left( \tilde{c}_{\mathbf{k}\sigma}^\dagger \tilde{d}_\sigma - hc \right), \left( \tilde{c}_{\mathbf{p}\sigma'}^\dagger \tilde{d}_{\sigma'} + hc \right) \right] \quad (\text{S68})$$

$$+ \sum_{\mathbf{k}\mathbf{p}} \sum_{\sigma\sigma'} V_{\mathbf{p}} [\tilde{X}_{\mathbf{k}\sigma}, \left( \tilde{c}_{\mathbf{p}\sigma'}^\dagger \tilde{d}_{\sigma'} + hc \right)] \left( \tilde{c}_{\mathbf{k}\sigma}^\dagger \tilde{d}_\sigma - hc \right) \quad (\text{S69})$$

$$+ \sum_{\mathbf{k}\mathbf{p}} \sum_{\sigma\sigma'} V_{\mathbf{p}} [X_{\mathbf{k}\sigma}, \left( \tilde{c}_{\mathbf{p}\sigma'}^\dagger \tilde{d}_{\sigma'} + hc \right)] \left( c_{\mathbf{k}\sigma}^\dagger d_\sigma - hc \right) \quad (\text{S70})$$

We see that we have therefore to evaluate two types of commutators:

$$\left[ \left( c_{\mathbf{k}\sigma}^\dagger d_\sigma - hc \right), \left( c_{\mathbf{p}\sigma'}^\dagger d_{\sigma'} + hc \right) \right] = [c_{\mathbf{k}\sigma}^\dagger d_\sigma, d_{\sigma'}^\dagger c_{\mathbf{p}\sigma'}] - [d_\sigma^\dagger c_{\mathbf{k}\sigma}, c_{\mathbf{p}\sigma'}^\dagger d_{\sigma'}] \quad (\text{S71})$$

$$= (c_{\mathbf{k}\sigma}^\dagger c_{\mathbf{p}\sigma'} \delta_{\sigma\sigma'} + hc) - \delta_{\mathbf{k}\mathbf{p}} \delta_{\sigma\sigma'} (d_{\sigma'}^\dagger d_\sigma + hc) \quad (\text{S72})$$

and equivalently for the tilde space

$$\left[ \left( \tilde{c}_{\mathbf{k}\sigma}^\dagger \tilde{d}_\sigma - hc \right), \left( \tilde{c}_{\mathbf{p}\sigma'}^\dagger \tilde{d}_{\sigma'} + hc \right) \right] = (\tilde{c}_{\mathbf{k}\sigma}^\dagger \tilde{c}_{\mathbf{p}\sigma'} \delta_{\sigma\sigma'} + hc) - \delta_{\mathbf{k}\mathbf{p}} \delta_{\sigma\sigma'} (\tilde{d}_{\sigma'}^\dagger \tilde{d}_\sigma + hc) \quad (\text{S73})$$

as well as

$$[X_{\mathbf{k}\sigma}, (c_{\mathbf{p}\sigma'}^\dagger d_{\sigma'} + hc)] = (t_{\mathbf{k}} + w_{\mathbf{k}}\tilde{n} + g_{\mathbf{k}}\tilde{n}_\uparrow\tilde{n}_\downarrow) \delta_{\bar{\sigma}\sigma'} (c_{\mathbf{p}\sigma'}^\dagger d_{\sigma'} - hc) \quad (\text{S74})$$

$$[\tilde{X}_{\mathbf{k}\sigma}, (\tilde{c}_{\mathbf{p}\sigma'}^\dagger \tilde{d}_{\sigma'} + hc)] = (\tilde{t}_{\mathbf{k}} + \tilde{w}_{\mathbf{k}}n + \tilde{g}_{\mathbf{k}}n_\uparrow n_\downarrow) \delta_{\bar{\sigma}\sigma'} (c_{\mathbf{p}\sigma'}^\dagger d_{\sigma'} - hc) \quad (\text{S75})$$

$$[X_{\mathbf{k}\sigma}, (\tilde{c}_{\mathbf{p}\sigma'}^\dagger \tilde{d}_{\sigma'} + hc)] = -(x_{\mathbf{k}} + w_{\mathbf{k}}n_{\bar{\sigma}}) (\tilde{c}_{\mathbf{p}\sigma'}^\dagger \tilde{d}_{\sigma'} - hc) \quad (\text{S76})$$

$$+ (h_{\mathbf{k}} + g_{\mathbf{k}}n_{\bar{\sigma}}) (\tilde{c}_{\mathbf{p}\sigma'}^\dagger \tilde{d}_{\sigma'} - hc) (\delta_{\sigma'\downarrow}\tilde{n}_\uparrow + \delta_{\sigma'\uparrow}\tilde{n}_\downarrow) \quad (\text{S77})$$

$$[\tilde{X}_{\mathbf{k}\sigma}, (c_{\mathbf{p}\sigma'}^\dagger d_{\sigma'} + hc)] = -(\tilde{x}_{\mathbf{k}} + \tilde{w}_{\mathbf{k}}\tilde{n}_{\bar{\sigma}}) (c_{\mathbf{p}\sigma'}^\dagger d_{\sigma'} - hc) \quad (\text{S78})$$

$$+ (\tilde{h}_{\mathbf{k}} + \tilde{g}_{\mathbf{k}}\tilde{n}_{\bar{\sigma}}) (c_{\mathbf{p}\sigma'}^\dagger d_{\sigma'} - hc) (\delta_{\sigma'\downarrow}\tilde{n}_\uparrow + \delta_{\sigma'\uparrow}\tilde{n}_\downarrow) \quad (\text{S79})$$

We can then evaluate the commutators between  $S$  and  $H_{\text{hyb}}, \tilde{H}_{\text{hyb}}$  entering Eq. (S64). We obtain

$$[S, H_{\text{hyb}}] = \sum_{\mathbf{k}\mathbf{p}\sigma} X_{\mathbf{k}\sigma} V_{\mathbf{p}} (c_{\mathbf{k}\sigma}^\dagger c_{\mathbf{p}\sigma} + hc) - 2 \sum_{\mathbf{k}\sigma} X_{\mathbf{k}\sigma} V_{\mathbf{p}} d_{\sigma}^\dagger d_{\sigma} \quad (\text{S80})$$

$$- \sum_{\mathbf{k}\mathbf{p}\sigma} V_{\mathbf{p}} (t_{\mathbf{k}} + w_{\mathbf{k}}\tilde{n} + g_{\mathbf{k}}\tilde{n}_\uparrow\tilde{n}_\downarrow) (c_{\mathbf{p}\sigma}^\dagger d_{\bar{\sigma}} - hc) (c_{\mathbf{k}\sigma}^\dagger d_{\sigma} - hc) \quad (\text{S81})$$

$$- \sum_{\mathbf{k}\mathbf{p}} \sum_{\sigma\sigma'} V_{\mathbf{p}} (\tilde{x}_{\mathbf{k}} + \tilde{w}_{\mathbf{k}}\tilde{n}_{\bar{\sigma}}) (c_{\mathbf{p}\sigma'}^\dagger d_{\sigma'} - hc) (\tilde{c}_{\mathbf{k}\sigma}^\dagger \tilde{d}_{\sigma} - hc) \quad (\text{S82})$$

$$+ \sum_{\mathbf{k}\mathbf{p}} \sum_{\sigma\sigma'} V_{\mathbf{p}} (\tilde{h}_{\mathbf{k}} + \tilde{g}_{\mathbf{k}}\tilde{n}_{\bar{\sigma}}) (c_{\mathbf{p}\sigma'}^\dagger d_{\sigma'} - hc) n_{\sigma'} (c_{\mathbf{k}\sigma}^\dagger \tilde{d}_{\sigma} - hc) \quad (\text{S83})$$

while for the one involving  $\tilde{H}_{\text{hyb}}$  is sufficient to swap all tilde operators into non-tilde ones in the equation above and take complex conjugation of the coefficients. Using these results and Eq. (S64) we can rewrite the effective Lindbladian in a more compact form. To this extent it is useful to introduce the spinors  $\Phi_{\mathbf{k}}^\dagger = (c_{\mathbf{k}\uparrow}^\dagger, c_{\mathbf{k}\downarrow}^\dagger)$  and  $\Phi_d^\dagger = (d_{\uparrow}^\dagger, d_{\downarrow}^\dagger)$ . The effective Lindbladian is composed of different contributions,

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_0 + \mathcal{L}_{\text{bath}} + \mathcal{L}_{\text{Kondo}} + \mathcal{L}_{\text{scatt}} + \mathcal{L}_{\text{pair}} + \mathcal{L}_{\text{diss}} \quad (\text{S84})$$

which we now discuss in detail.  $\mathcal{L}_{\text{Kondo}}$  describes a Kondo coupling between the dot spin and the spin of the bath,

$$\mathcal{L}_{\text{Kondo}} = i \sum_{\mathbf{k}\mathbf{q}} J_{\mathbf{q}\mathbf{k}} \left( \Phi_{\mathbf{q}}^\dagger \frac{\vec{\sigma}}{2} \Phi_{\mathbf{k}} \right) \cdot \vec{S}_d + \text{TildeVersion} \quad (\text{S85})$$

with a coupling

$$J_{\mathbf{q}\mathbf{k}} = V_{\mathbf{q}} (t_{\mathbf{k}} + w_{\mathbf{k}}\tilde{n}) + V_{\mathbf{k}} (t_{\mathbf{q}} + w_{\mathbf{q}}\tilde{n})$$

which is in general complex.  $\mathcal{L}_{\text{scatt}}$  describes a scattering potential for the conduction electrons,

$$\mathcal{L}_{\text{scatt}} = -i \sum_{\mathbf{k}\mathbf{q}} \left( \frac{1}{2} W_{\mathbf{q}\mathbf{k}} + \frac{1}{4} J_{\mathbf{q}\mathbf{k}} (\Psi_d^\dagger \Psi_d) \right) \Phi_{\mathbf{q}}^\dagger \Phi_{\mathbf{k}} + \text{TildeVersion} \quad (\text{S86})$$

with coupling constant

$$W_{\mathbf{q}\mathbf{k}} = V_{\mathbf{q}} (s_{\mathbf{k}} + x_{\mathbf{k}}\tilde{n}) + V_{\mathbf{k}} (s_{\mathbf{q}} + x_{\mathbf{q}}\tilde{n})$$

$\mathcal{L}_{\text{pair}}$  describes a pair tunneling term

$$\mathcal{L}_{\text{pair}} = \frac{i}{2} \sum_{\mathbf{q}\mathbf{k}\sigma} T_{\mathbf{q}\mathbf{k}} (c_{\mathbf{q}\sigma}^\dagger c_{\mathbf{k}\bar{\sigma}}^\dagger d_{\bar{\sigma}} d_{\sigma} + h.c) + \text{TildeVersion} \quad (\text{S87})$$

with amplitude

$$T_{\mathbf{q}\mathbf{k}} = V_{\mathbf{k}} [t_{\mathbf{k}} + g_{\mathbf{k}}\tilde{n}_\uparrow\tilde{n}_\downarrow + w_{\mathbf{k}}\tilde{n}]$$

$\mathcal{L}_{\text{dot}}$  describes a local Lindblad contribution

$$\begin{aligned} \mathcal{L}_{\text{dot}} = & i \sum_{\mathbf{k}} V_{\mathbf{k}} (2t_{\mathbf{k}} + 2w_{\mathbf{k}}\tilde{n} + 2g_{\mathbf{k}}\tilde{n}_{\uparrow}\tilde{n}_{\downarrow}) n_{\uparrow}n_{\downarrow} - i \sum_{\mathbf{k}} V_{\mathbf{k}} (2\tilde{t}_{\mathbf{k}} + 2\tilde{w}_{\mathbf{k}}\tilde{n} + 2\tilde{g}_{\mathbf{k}}\tilde{n}_{\uparrow}\tilde{n}_{\downarrow}) \tilde{n}_{\uparrow}\tilde{n}_{\downarrow} \\ & + i \left( \sum_{\mathbf{k}} V_{\mathbf{k}} [s_{\mathbf{k}} + x_{\mathbf{k}}\tilde{n} + h_{\mathbf{k}}\tilde{n}_{\uparrow}\tilde{n}_{\downarrow}] \right) \left( \sum_{\sigma} n_{\sigma} \right) - i \left( \sum_{\mathbf{k}} V_{\mathbf{k}} [\tilde{s}_{\mathbf{k}} + \tilde{x}_{\mathbf{k}}\tilde{n} + \tilde{h}_{\mathbf{k}}\tilde{n}_{\uparrow}\tilde{n}_{\downarrow}] \right) \left( \sum_{\sigma} \tilde{n}_{\sigma} \right) \end{aligned} \quad (\text{S88})$$

and finally  $\mathcal{L}_{\text{diss}}$  describes a dissipative term

$$\mathcal{L}_{\text{diss}} = i \left[ \sum_{\mathbf{qk}} H_{\mathbf{qk}} \left( \Phi_{\mathbf{q}}^{\dagger} \frac{\vec{\sigma}}{2} \Phi_{\mathbf{k}} \right) \cdot \vec{S}_d + \sum_{\mathbf{qk}} \left( \frac{1}{2} I_{\mathbf{qk}} + \frac{1}{4} H_{\mathbf{qk}} \left( \Psi_d^{\dagger} \Psi_d \right) \right) \Phi_{\mathbf{q}}^{\dagger} \Phi_{\mathbf{k}} \right] \tilde{n}_{\uparrow}\tilde{n}_{\downarrow} + \text{TildeVersion} \quad (\text{S89})$$

$$+ \frac{i}{2} \sum_{\mathbf{qk}\sigma\mu} \Gamma_{\mathbf{qk}} \left( c_{\mathbf{k}\sigma}^{\dagger} d_{\sigma} + c_{\mathbf{k}\sigma} d_{\sigma}^{\dagger} \right) \left( \tilde{c}_{\mathbf{q}\mu}^{\dagger} \tilde{d}_{\mu} + \tilde{c}_{\mathbf{q}\mu} \tilde{d}_{\mu}^{\dagger} \right) \quad (\text{S90})$$

with coupling constants

$$\begin{aligned} I_{\mathbf{qk}} &= V_{\mathbf{q}} h_{\mathbf{k}} + V_{\mathbf{k}} h_{\mathbf{q}} \\ H_{\mathbf{qk}} &= V_{\mathbf{q}} g_{\mathbf{k}} + V_{\mathbf{k}} g_{\mathbf{q}} \\ \Gamma_{\mathbf{qk}} &= [x_{\mathbf{k}} V_{\mathbf{q}} - \tilde{x}_{\mathbf{q}} V_{\mathbf{k}}] + \tilde{n}_{\bar{\mu}} [h_{\mathbf{k}} V_{\mathbf{q}} - \tilde{w}_{\mathbf{q}} V_{\mathbf{k}}] + \tilde{n}_{\bar{\mu}} n_{\bar{\sigma}} [g_{\mathbf{k}} V_{\mathbf{q}} - \tilde{g}_{\mathbf{q}} V_{\mathbf{k}}] \end{aligned} \quad (\text{S91})$$

It is first of all useful to comment on the structure of the effective Lindbladian as compared to the equilibrium case, when the Schrieffer-Wolff on the AIM leads to a Kondo model. Indeed we notice that some of the effective terms which are generated in Eq. (S84) would be also present in equilibrium. This is the case for example of the Kondo coupling  $\mathcal{L}_{\text{Kondo}}$ , the scattering potential  $\mathcal{L}_{\text{scatt}}$ , the pair-hopping term  $\mathcal{L}_{\text{pair}}$  or the local term  $\mathcal{L}_{\text{dot}}$ . The effect of dissipation here is to renormalize the coupling constants  $J_{\mathbf{qk}}, W_{\mathbf{qk}}, T_{\mathbf{qk}}$ , leading to either complex (dissipative) interactions or to introduce coupling between Hilbert space sectors (tilde/non-tilde operators), which as we know describe dissipative processes. For example, we can see in Eq. (S88) that dissipation induces a renormalization to the bare Hubbard repulsion  $U$ , compatible with the spectral function in Fig. (S2) which shows a drift of the Hubbard bands with  $\gamma$ , as well as an effective doublon-doublon dissipation.

Dephasing is however also responsible for generating intrinsically new terms such as  $\mathcal{L}_{\text{diss}}$ , which couples operators in the two Hilbert spaces and introduce new dissipative processes which for example create or destroy a doublon/holon with a rate  $\Gamma_{\mathbf{qk}}$ . These dissipative terms couple the singly occupied spin sector to the charge fluctuations and therefore allow the system to go out from the singly occupied manifold. In the equilibrium low-energy limit of the AIM, one then proceeds by projecting the effective theory in the singly-occupied manifold, which is the low-energy one, to obtain a Kondo model. In the next section we show that the dynamics in this subspace is described by a non-Hermitian Kondo model. Our analysis on the effective Lindbladian, as well as our NCA results, show however that this projection can only be valid in certain regimes of dephasing, roughly for  $\gamma < U$ , before heating due to doublon production take over.

### Effective Lindbladian and non-Hermitian Kondo model

We now discuss the structure of the effective Lindbladian  $\mathcal{L}_{\text{eff}}$  in the singly occupied half-filled sector. To this extent we introduce projectors for the degrees of freedom in each Hilbert space  $\mathcal{H}/\tilde{\mathcal{H}}$

$$P = \sum_{\sigma} n_{\sigma} (1 - n_{\bar{\sigma}}) \quad \text{and} \quad Q = 1 - P \quad (\text{S92})$$

$$\tilde{P} = \sum_{\sigma} \tilde{n}_{\sigma} (1 - \tilde{n}_{\bar{\sigma}}) \quad \text{and} \quad \tilde{Q} = 1 - \tilde{P} \quad (\text{S93})$$

with  $P^2 = P$  and  $[P, \tilde{P}] = [P, \tilde{Q}] = 0$ . We note that  $P$  projects on the dot single occupied states (either spin up or down) while  $Q$  on the empty/double occupied states, and similarly for  $\tilde{P}, \tilde{Q}$  in  $\tilde{\mathcal{H}}$ . The effective Lindbladian reads

$$\mathcal{L}_{\text{eff}} = \left( \tilde{P} + \tilde{Q} \right) (P + Q) \mathcal{L}_{\text{eff}} (P + Q) \left( \tilde{P} + \tilde{Q} \right) \quad (\text{S94})$$

Since the total Lindbladian of the system is particle hole symmetric, the low energy sector in the symmetric case is characterized by  $\Psi_d^\dagger \Psi_d = 1$  and  $\epsilon_d = -U/2$ . In this limit many of the terms entering  $\mathcal{L}_{eff}$  cancel out and we obtain

$$\mathcal{L}_{eff} = \mathcal{L}_{bath} - i \left( H_{eff} - \tilde{H}_{eff} \right) \quad (S95)$$

where the  $H_{eff}$  describes a non-Hermitian Kondo model with a potential scattering term

$$H_{eff} = - \sum_{\mathbf{kq}} J_{\mathbf{qk}} \left( \Phi_{\mathbf{q}}^\dagger \frac{\vec{\sigma}}{2} \Phi_{\mathbf{k}} \right) \cdot \vec{S}_d + \sum_{\mathbf{kq}} \left( \frac{1}{2} W_{\mathbf{qk}} + \frac{1}{4} J_{\mathbf{qk}} \left( \Psi_d^\dagger \Psi_d \right) \right) \Phi_{\mathbf{q}}^\dagger \Phi_{\mathbf{k}} \quad (S96)$$

where the complex spin-exchange coupling and the scattering potential are given by,

$$J_{\mathbf{qk}} = V_{\mathbf{q}}(t_{\mathbf{k}} + w_{\mathbf{k}}) + V_{\mathbf{k}}(t_{\mathbf{q}} + w_{\mathbf{q}}) = \left( \frac{V_{\mathbf{q}}V_{\mathbf{k}}(U - i\gamma)}{(\epsilon_{\mathbf{k}} - \epsilon_d - U + i\gamma/2)(\epsilon_{\mathbf{k}} - \epsilon_d - i\gamma/2)} + \frac{V_{\mathbf{q}}V_{\mathbf{k}}(U - i\gamma)}{(\epsilon_{\mathbf{q}} - \epsilon_d - U + i\gamma/2)(\epsilon_{\mathbf{q}} - \epsilon_d - i\gamma/2)} \right)$$

$$W_{\mathbf{qk}} = V_{\mathbf{q}}(s_{\mathbf{k}} + x_{\mathbf{k}}) + V_{\mathbf{k}}(s_{\mathbf{q}} + x_{\mathbf{q}}) = \left( \frac{V_{\mathbf{q}}V_{\mathbf{k}}}{\epsilon_{\mathbf{k}} - \epsilon_d - i\gamma/2} + \frac{V_{\mathbf{q}}V_{\mathbf{k}}}{\epsilon_{\mathbf{q}} - \epsilon_d - i\gamma/2} \right) \quad (S97)$$

If one evaluate the Kondo coupling  $J_{\mathbf{qk}}$  for bath momenta at the Fermi energy, i.e.  $\epsilon_{\mathbf{k}} = \epsilon_{\mathbf{q}} = 0$ , which is the relevant limit at long-times and for  $\epsilon_d = -U/2$  as required by particle-hole symmetry the expressions of the couplings become

Moreover the complex-valued spin-exchange coupling constant  $J$  at the Fermi level is obtained by setting  $\epsilon_{\mathbf{k}} = \epsilon_{\mathbf{q}} = 0$  as well as  $\epsilon_d = -U/2$  in the equation above, this approximation is valid because only the physics close to the Fermi energy is studied

$$J_{\mathbf{qk}} = J_R + iJ_I = - \frac{8V^2}{(U - i\gamma)} \quad (S98)$$

$$W_{\mathbf{qk}} = W = \frac{4V^2}{(U - i\gamma)} \quad (S99)$$

Moreover, in this limit, the potential scattering terms cancel out and the dissipative part of  $\mathcal{L}_0$  do not contribute to the low energy physics. The behavior of the real and imaginary part of the Kondo coupling is plotted in Fig. S4 showing that the real-part is quickly suppressed with  $\gamma$  while the imaginary part  $J_I$  displays the characteristic Zeno crossover around  $\gamma \sim U$ .

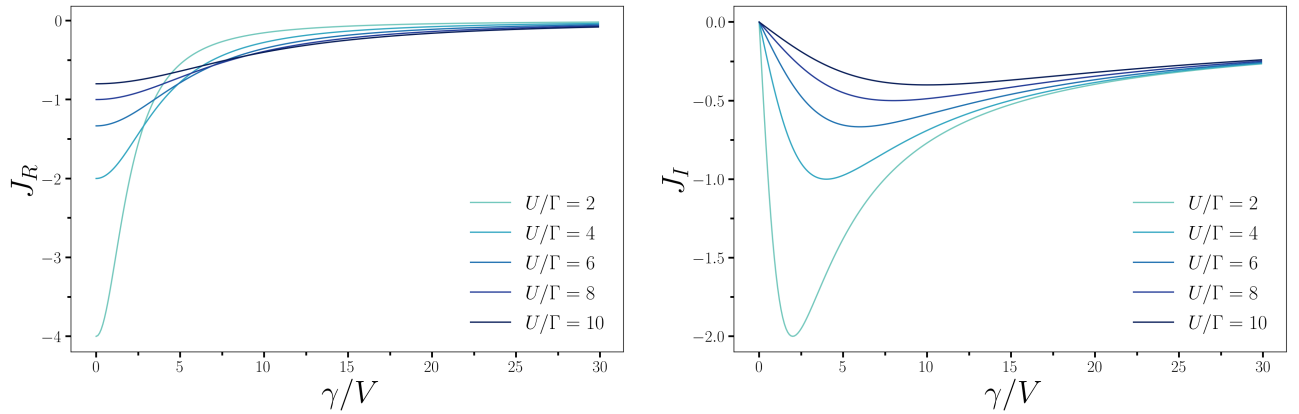


FIG. S4. Spin-exchange complex coupling in the Half filling case  $\epsilon_d = -U/2$  and at the Fermi level  $\epsilon_{\mathbf{k}} = 0$  - (Left Panel) Real part of Kondo coupling (Right Panel) imaginary part of the Kondo coupling