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# Stability and Transient Analysis in the Modelling of Railway Disc Brake Squeal

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## Summary

The paper deals with friction induced vibrations and especially with railway disc brake squeal. The first part of the paper is devoted to the strategy used to model the general problem of self-excited vibrations of a rotating disc in frictional contact with two pads. Unilateral contact conditions with Coulomb friction and constant friction coefficient are considered. In order to predict the occurrence of self-excited vibrations, a classical stability analysis is performed, which consists on computing the complex modes associated to the linearised problem. A common interpretation of the stability analysis is that frequencies of unstable complex modes correspond to squeal frequencies. To check this assumption, the behaviour of the solution far from the sliding equilibrium is determined by using a non linear transient analysis. Moreover, an expansion of the transient solution on the complex modes provided by the stability analysis helps us to highlight the role of the unstable modes. The second part of the paper focuses on the application of the approach to the TGV disc brake system. Results on complex modes are presented and compared with measurements. The interest of both analyses is discussed.

## 1 Formulation of the Problem

The mechanism of the simplified disc brake system represented on figure 1 is considered. The rotation speed  $\Omega$  of the disc is assumed to be constant and sufficiently small so that the gyroscopic terms and the volume forces induced by rotation may be neglected. An “eulerian” description is adopted. Unilateral contact with Coulomb friction conditions are taken into account. By using a finite element method, the nonlinear dynamics problem may be written in a discrete form as follows (see details in reference [3]):

$$\begin{aligned}
 [M]\{\ddot{U}\} + [C]\{\dot{U}\} + [K]\{U\} &= \{F\} + [P_n]^T \{r_n\} + [P_t]^T \{r_t\} \\
 \{r_n\} &= \text{Proj}_{\mathbb{R}^-} (\{r_n\} - \alpha_n ([P_n]\{U\} - \{g_0\})) \\
 \{r_t\} &= \text{Proj}_C (\{r_t\} - \alpha_t ([P_t]\{\dot{U}\} + \{V\}))
 \end{aligned} \tag{1}$$

where  $[M]$ ,  $[C]$  and  $[K]$  denote the mass, Rayleigh damping and stiffness matrices whereas  $\{U\}$  and  $\{F\}$  represent the vectors of nodal displacements and external nodal forces. In addition,  $\{r_n\}$  and  $\{r_t\}$  denote the vectors of normal and tangential reactions forces at the contact nodes whereas  $[P_n]$  and  $[P_t]$  are projection matrices on the normal and tangential relative displacements between the disc and the pads at the contact nodes.  $C$  is the Coulomb cone and coefficients  $\alpha_n$  and  $\alpha_t$  must be positive. Finally,  $\{g_0\}$  is the vector of initial gaps whereas  $\{V\}$  is the vectors of imposed sliding velocities due to the disc rotation speed  $\Omega$  at the contact nodes.

## 2 Stability and Transient Analysis

### 2.1 Linear Stability Analysis

System (1) is a set of non-linear differential equations characterised by a sliding equilibrium. Considering small regular perturbations that does not break the contact (bilateral contact), the frictional forces may be linearised and the evolution of the perturbations  $\{U^*\}$  verifies (see [2]):

$$\begin{aligned} & ([M] + f[M_f])\{\ddot{U}^*\} + ([C] + f[C_f] + f[C_e])\{\dot{U}^*\} + ([K] + f[K_f])\{U^*\} = [P_n]^T \{r_n^*\} \\ & [P_n]\{U^*\} = 0 \end{aligned} \quad (2)$$

where  $[M_f]$ ,  $[C_f]$ ,  $[K_f]$  and  $[C_e]$  are non symmetrical matrices provided by the linearisation of the frictional forces,  $f$  is the friction coefficient and  $\{r_n^*\}$  denote the vector of perturbed normal reactions forces at the contact nodes. By eliminating the bilateral contact constraints, a non symmetrical linear system of equations is obtained and the stability of the equilibrium may then be deduced from a complex eigenvalue analysis of the system, providing complex modes and complex eigenvalue  $\lambda_i$ . A complex mode  $i$  is unstable if  $\text{Re}(\lambda_i) > 0$ , which may happen since the system is non symmetric. A modal growth rate  $\zeta_i = \text{Re}(\lambda_i) / \text{Im}(\lambda_i)$  may also be defined (physically equivalent to a negative modal damping).

### 2.2 Non Linear Transient Analysis

In addition to the stability analysis, an implicit numerical resolution of the system of equations (1) may be performed. A time discretisation method is used here, resting on a former work of M. Jean and J.J. Moreau (see [3]). The  $\theta$ -method allows one to avoid numerical problems at the time of an impact. Indeed, the instability of the sliding equilibrium may lead to strongly non-linear events like a separation followed by a shock, as well as stick/slip transitions. In order to introduce an inelastic shock law, one uses the modified version of the  $\theta$ -method (see [4]).

### 2.3 Relations between Stability and Transient Analyses

In order to understand the role of the unstable modes in the non linear part of the transient behaviour, it is proposed to expand the numerical transient solution on the basis of the complex modes. Taking into account the non symmetrical character of the matrices involved in the problem (2), it is necessary to introduce the complex modes

provided by transposed linearised problem. Indeed, the generalisation of the usual orthogonality conditions in the case of symmetrical matrices leads to bi-orthogonality conditions, and given by:

$$\forall i \neq j \quad \{L_i\}^T [A] \{\Phi_j\} = 0 \quad (3)$$

where  $\{\phi_j\}$  are the complex modes of the direct problem (2) and  $\{L_i\}$  the complex modes of the transposed problem of (2).  $[A]$  is the mass matrix in state space variable (see [5]).

This bi-orthogonality property allows one to compute the evolution of the complex amplitude of the mode  $j$ ,  $\beta_j(t)$ , from the transient perturbation written in state-space variables  $\{\alpha^*(t)\}$ :

$$\beta_j(t) = \frac{\{L_j\}^T [A] \{\alpha^*(t)\}}{\{L_j\}^T [A] \{\Phi_j\}} \quad (4)$$

with  $\{\alpha^*(t)\}$  provided by numerical resolution of system (1).

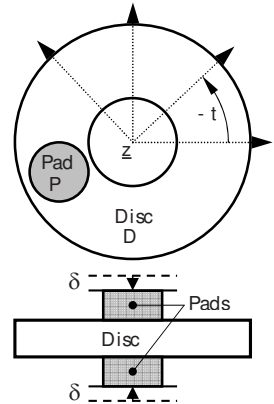
The contribution of the mode  $j$  to the perturbed displacement and velocity fields can then be computed from  $\beta_j(t)$ . Finally, the variation of the total perturbed energy of a mode  $j$  can be computed from these contributions.

### 3 Application to a Simplified Disc Brake System

In this part, the simplified disc brake system of figure 1 is considered. The geometric and physical characteristics of the structures are given in table 1. The other parameters are  $f=0.35$ ,  $\Omega=2.5$  rad/s and  $\delta=3.33 \times 10^{-6}$  m. A stability analysis is performed in the  $[0 \text{ } 15\text{kHz}]$  frequency range. Among the 100 computed complex modes, three modes are found unstable, called M1 (8583 Hz,  $\zeta=0.05 \%$ ), M2 (9288 Hz,  $\zeta=0.13 \%$ ) and M3 (10130 Hz,  $\zeta=0.17 \%$ ). The transient solution is also calculated with  $\theta = 0.5$

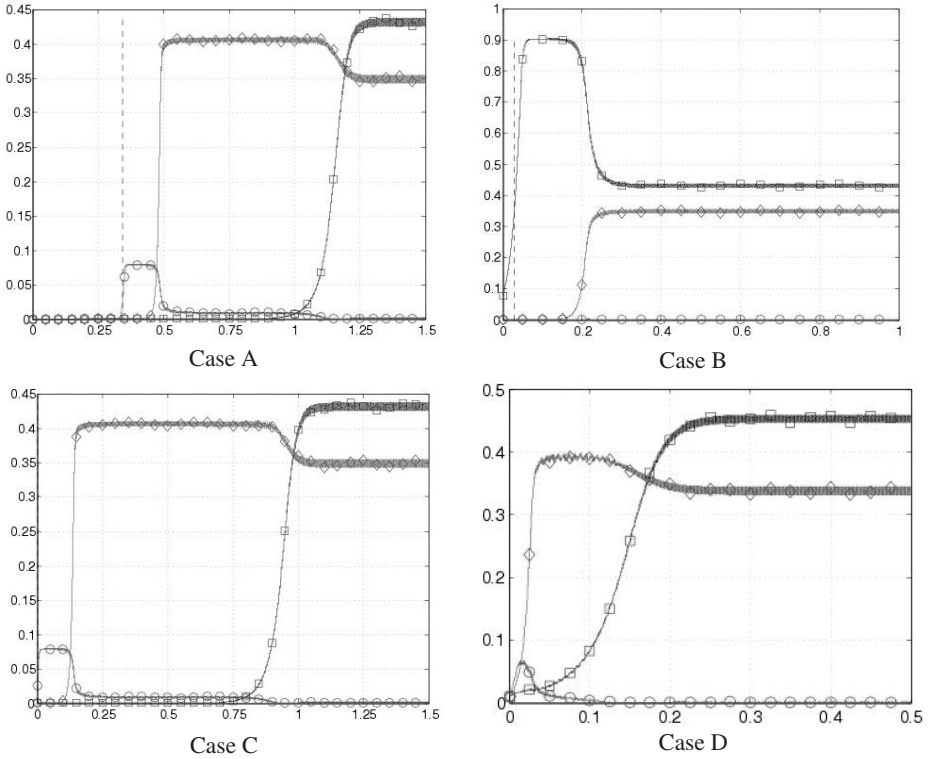
**Table 1.** Physical and geometric characteristics

	Disc	Pads
Young's modulus $E$	$2.02 \times 10^{11}$ Pa	$4.5 \times 10^9$ Pa
Poisson's ratio $\nu$	0.29	0.3
Density $\rho$	$7850 \text{ kg.m}^{-3}$	$5250 \text{ kg.m}^{-3}$
Damping param. $\alpha$	$0 \text{ s}^{-1}$	$108 \text{ s}^{-1}$
Damping param. $\beta$	$7.5 \cdot 10^{-9} \text{ s}$	$1.44 \cdot 10^{-6} \text{ s}$
Ext. diameter	0.6 m	0.16 m
Int. diameter	0.2 m	
Thickness	0.04 m	0.04 m



**Fig. 1.** Simplified disc brake system

and time step  $\Delta t = 5 \times 10^{-6}$  s. In order to study the influence of the initial conditions on the transient solution, 4 cases are considered (A to D) for which the initial contributions of the unstable modes are different. The contributions of the unstable modes to the total perturbed energy  $E_{TM1}(t)$ ,  $E_{TM2}(t)$  and  $E_{TM3}(t)$  are represented on the figure 2 for the 4 cases. The different figures show that the stabilised solution is not dependent on the initial conditions. It may be observed that this stabilised solution is made up of 2 of the 3 unstable modes (M1 and M2). These computations show that the stability analysis is able to predict the prone-squeal modes but that a transient analysis is necessary to predict which modes remain in the stabilised solution. It also highlights the possible coexistence of several unstable modes in the self-sustained vibrations.

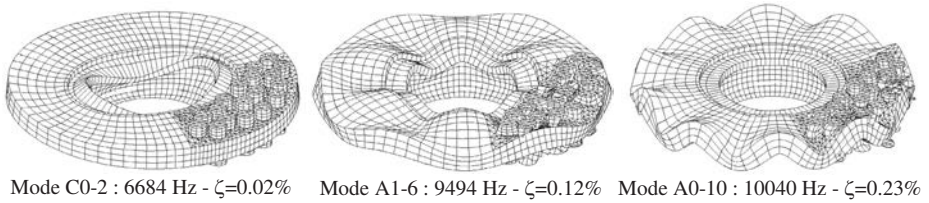


**Fig. 2.** Evolution of modal contribution to total perturbed energy  $E_T(t)$  [J] for the different cases as a function of time [s].  $\square$   $E_{TM1}(t)$ ,  $\diamond$   $E_{TM2}(t)$ ,  $\circ$   $E_{TM3}(t)$ .

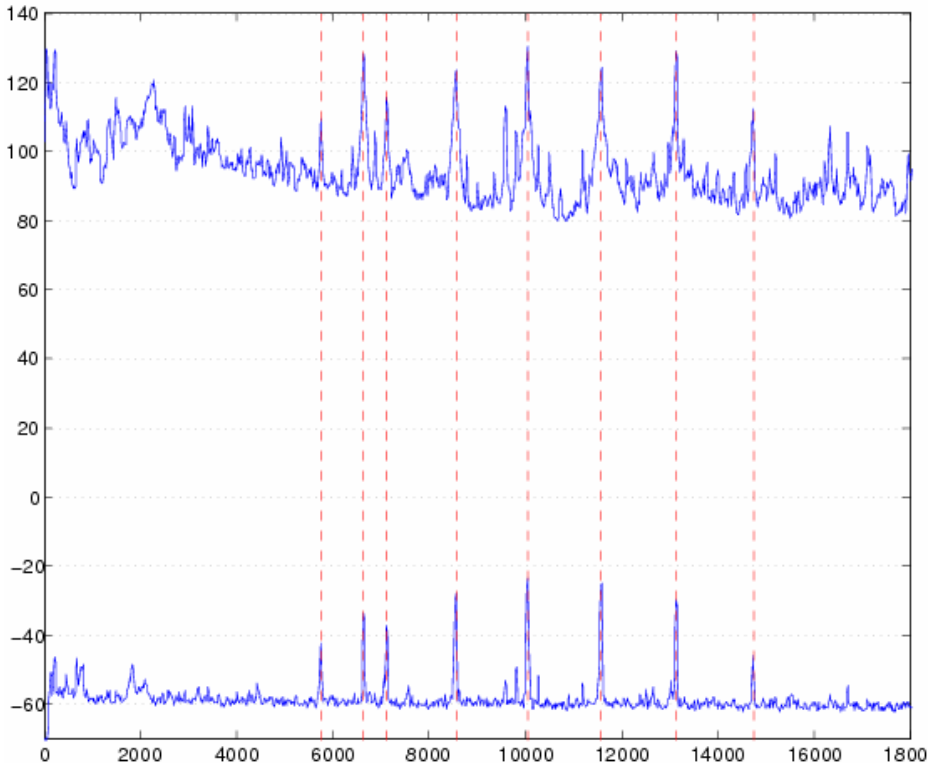
## 4 Stability Analysis of a TGV Brake System

In this part, a finite element model of the TGV brake system (cf. fig 3) is considered. First, the modes of the disc in free conditions have been calculated and have been classified according to the direction of the dominant deformations. In particular, in-plane circumferential modes  $Cn-m$ , in plane radial modes  $Rn-m$  and out-of-plane axial modes  $An-m$  may be distinguished where  $n$  and  $m$  denote respectively the number of nodal circles and nodal diameters.

A stability analysis has been performed. More than 800 complex modes have been calculated to reach an upper limit frequency of about 14 kHz. The corresponding growth factors are represented on figure 5. Two kinds of modes may be distinguished: the pad modes, for which the disc vibrations are very small, and the disc modes for which the vibrations of the disc are dominating. The corresponding mode shapes and frequencies of the disc modes are close to the modes of the disc in free conditions but rotate along the disc. Most of these modes are axial modes without nodal circles and with one nodal circle (see fig 3). Another mode is rather an in-plane mode (C0-2) but with some axial components.

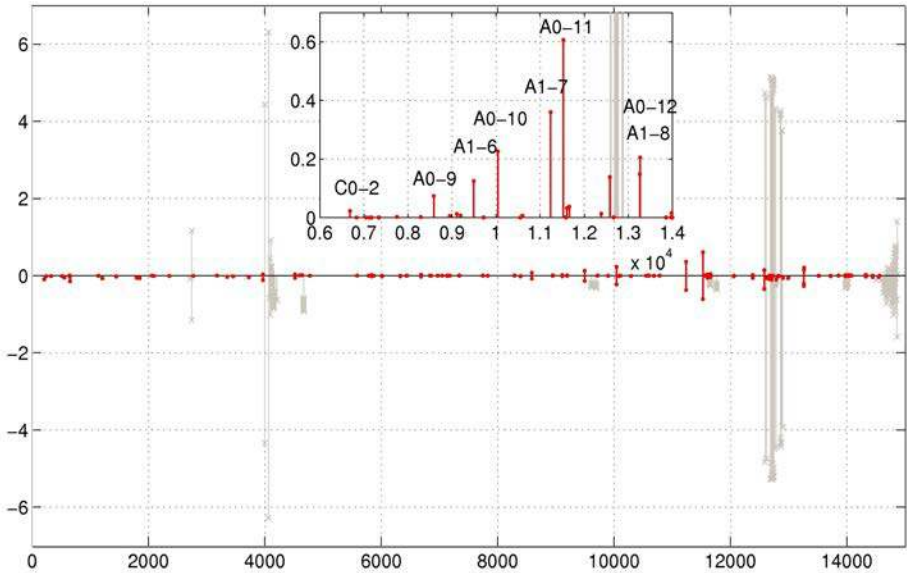


**Fig. 3.** Some unstable modes of the TGV brake F.E. model



**Fig. 4.** Experimental power Spectrum of the normal velocity [dB] (below, ref 1 [m/s]) and acoustic pressure [dB] (above, ref 20 [ $\mu$ Pa])

Some experimental results have been obtained from a TGV in braking operation at about 10 km/h. The acoustic pressure at one centimetre from the disc and the axial vibratory velocity at a point on the disc surface have been measured during braking. Figure 4 shows that the disc is responsible for the emitted noise and that the vibrations are composed of 8 high frequencies from 5000 to 15000 Hz. Except the axial modes with one nodal circle, all the unstable modes are close to the experimental squeal frequencies.



**Fig. 5.** Growth rates  $\zeta$  [%] of complex modes as a function of frequency [Hz] (O: disc mode  $\times$ : Pad modes)

## 5 Conclusion

In this paper, the modelling strategy of disc brake squeal has been studied. In order to investigate the relations between stability and transient classical analyses, a new method has been proposed, which consists on expanding the transient vibratory field on the complex modes provided by the stability analysis. This method has been tested on a simplified disc brake model for various initial conditions. Results have shown that the stabilised solution is made up of two coexisting unstable modes and that this solution is the same for different initial conditions. A stability study has also been performed on a finite element TGV brake system and compared with vibration measurements. It has been found that the frequencies of the unstable disc modes correspond to most of the vibration frequencies.

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