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► **To cite this version:**

Lucas Brunel, Mathieu Balesdent, Loïc Brevault, Rodolphe Le Riche, Bruno Sudret. A review of multi-fidelity surrogate models for high dimensional field outputs. 6th international workshop on model reduction techniques (MORtech, an ECCOMAS thematic conference), Nov 2023, Paris-Saclay, France. hal-04793505

HAL Id: hal-04793505

<https://hal.science/hal-04793505v1>

Submitted on 20 Nov 2024

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A review of multi-fidelity metamodels for field predictions

Lucas Brunel^{1,2}, Mathieu Balesdent¹, Loïc Brevault¹, Rodolphe Le Riche², and Bruno Sudret³



¹ONERA, DTIS, Université Paris-Saclay, 91123, Palaiseau, France

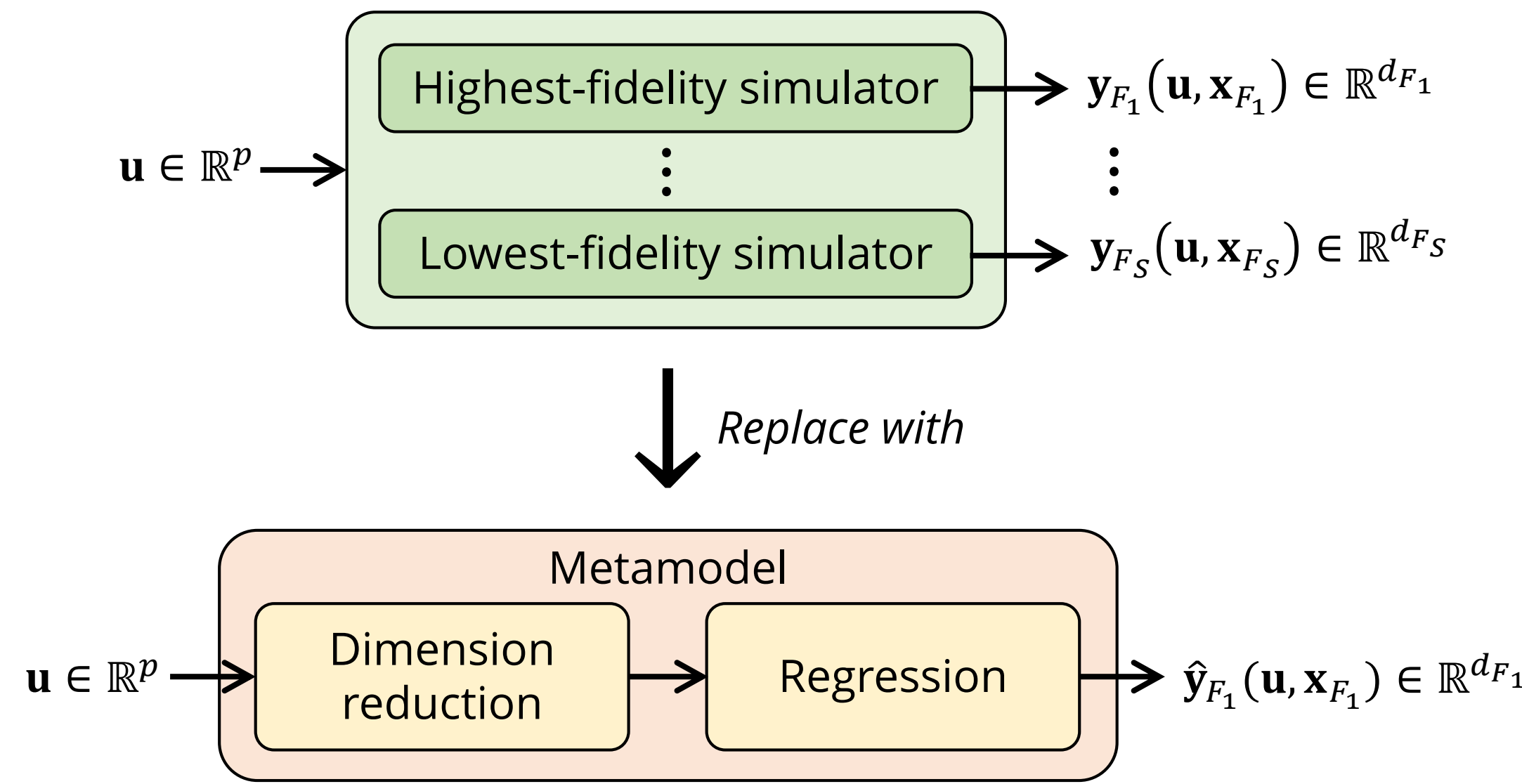
²CNRS LIMOS, Mines Saint-Etienne and Université Clermont Auvergne, France

³ETH Zurich, Chair of Risk, Safety and Uncertainty Quantification, Stefano-Francini Platz 5, Zurich 8093, Switzerland



CONTEXT

- ▶ Simulations with a **high computational cost** (e.g., Computational Fluid Dynamics (CFD))
- ▶ Need to build a **metamodel** for applications that require a **large number of calls to a simulator** (e.g., uncertainty quantification, optimization)
- ▶ The quantity of interest is a **field** (e.g., pressure at the wall of a launch vehicle) **discretized on a mesh** (temporal, spatial, frequential, etc.)
- ▶ Simulators of **variable fidelity** are available (e.g., simplified physics, mesh resolution)



NOTATIONS

- ▶ $\mathbf{u} \in \mathbb{R}^p$: vector of **input variables** (e.g., length of the vehicle, angle of attack)
- ▶ $F_j, j = 1, \dots, S$: **fidelity level**, 1 corresponds to the highest-fidelity
- ▶ \mathbf{x}_{F_j} : **mesh** at fidelity F_j
- ▶ $\mathbf{y}_{F_j}(\mathbf{u}, \mathbf{x}_{F_j}) \in \mathbb{R}^{d_{F_j}}$: a **field discretized on the mesh** \mathbf{x}_{F_j} of dimension d_{F_j}
- ▶ $\hat{\square}$: **prediction** of the field \square

BUILDING BLOCKS

Dimension reduction

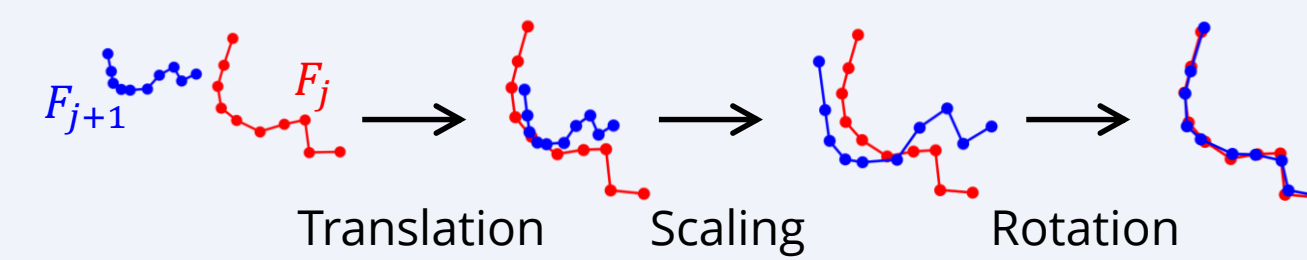
Compression of a high-dimensional field into a lower-dimensional space (latent space), for instance with Principal Component Analysis (PCA) [1], Kernel PCA [2].

Orthogonal part

Information lost when performing dimension reduction. For instance, this can be modeled by a kriging with tensorized covariance [3].

Manifold alignment by Procrustes analysis [4]

Operations on the low-dimensional representation of the fields to make them better match between fidelities.

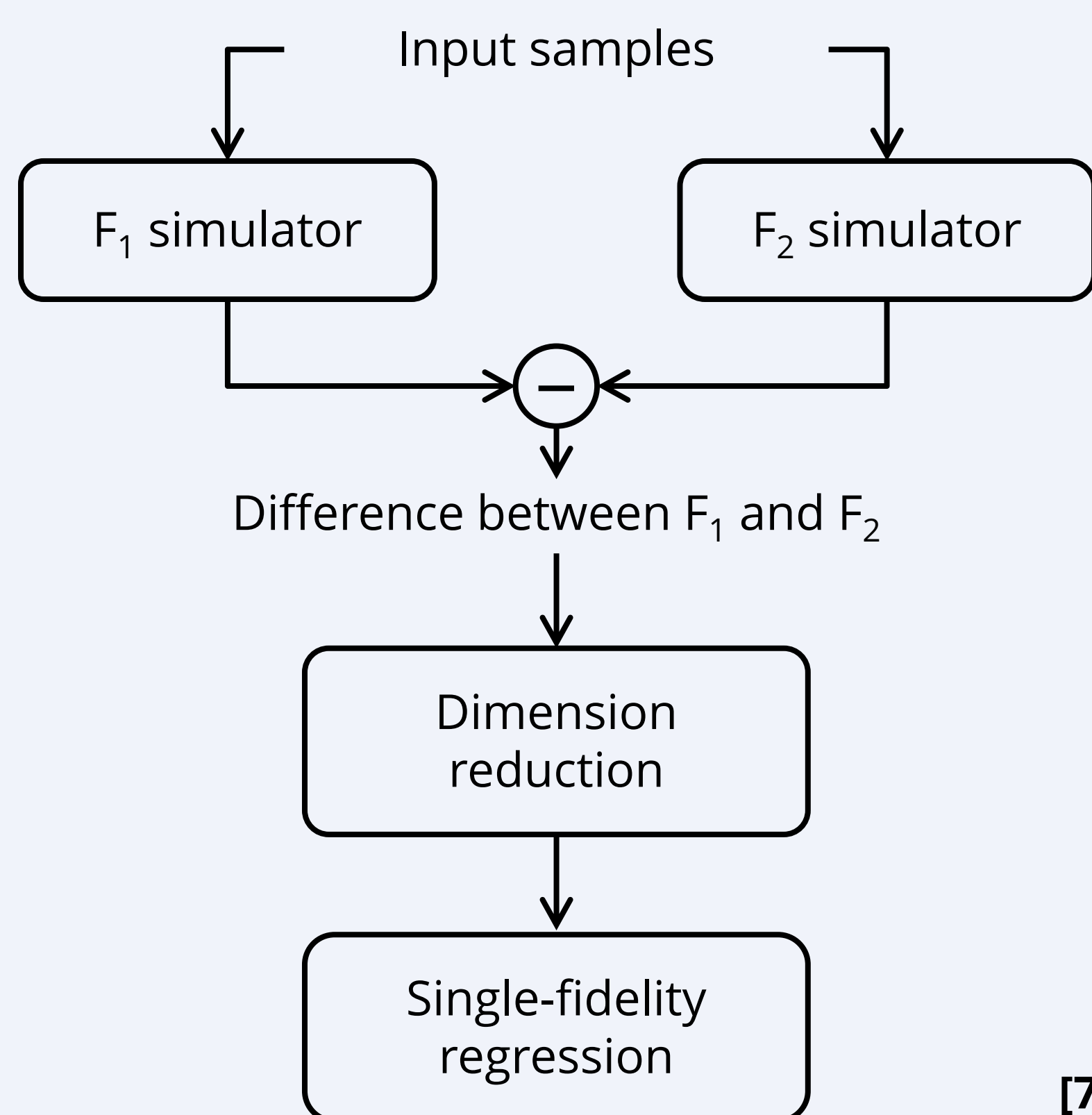


Regression

Mapping between the input variables and the low-dimensional representation of the fields, can be mono-fidelity (e.g., gaussian process regression [5]) or multi-fidelity (e.g., autoregressive cokriging [6]).

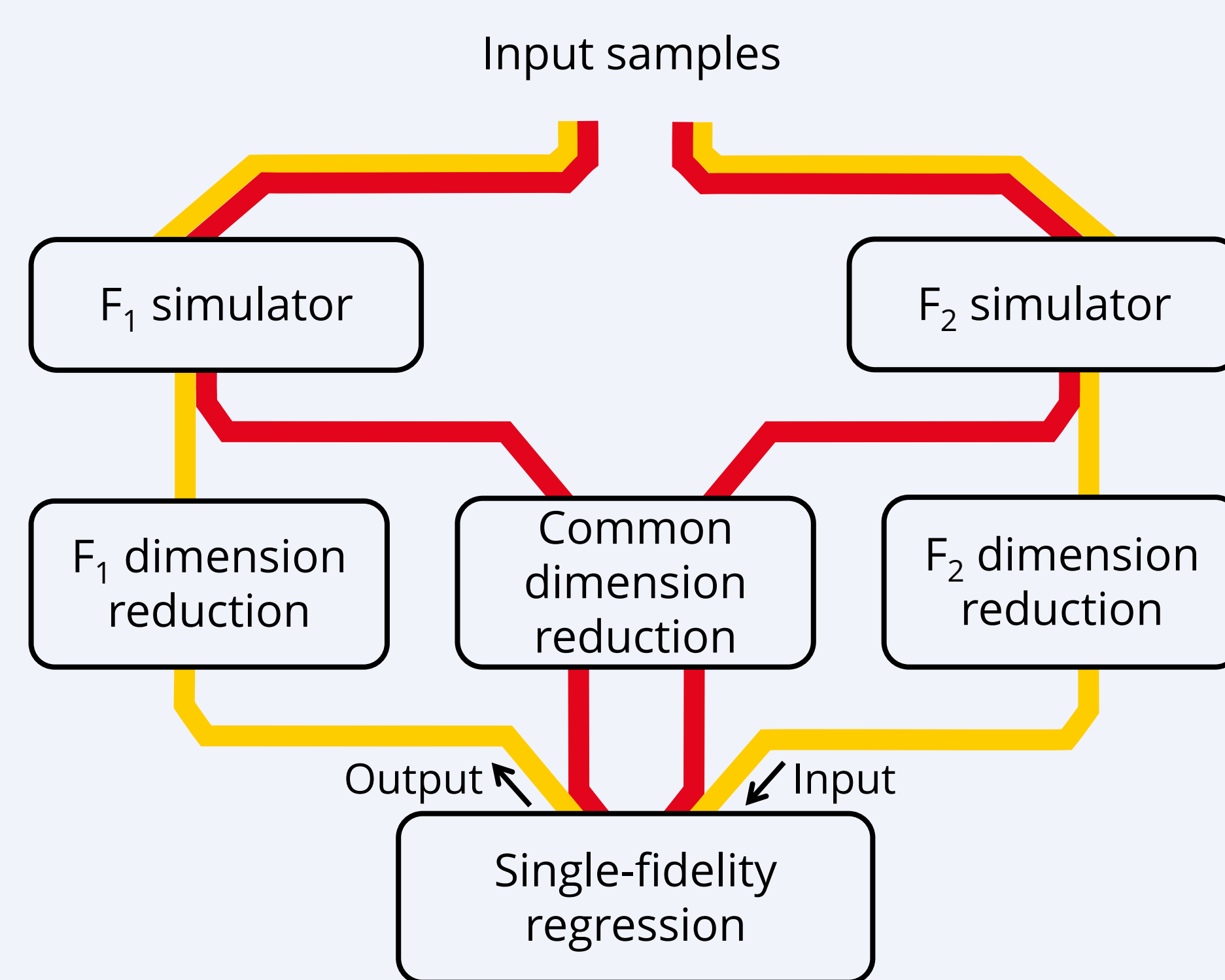
UNIFIED FRAMEWORK OF METAMODEL FAMILIES (ILLUSTRATED FOR 2 FIDELITIES)

Corrective



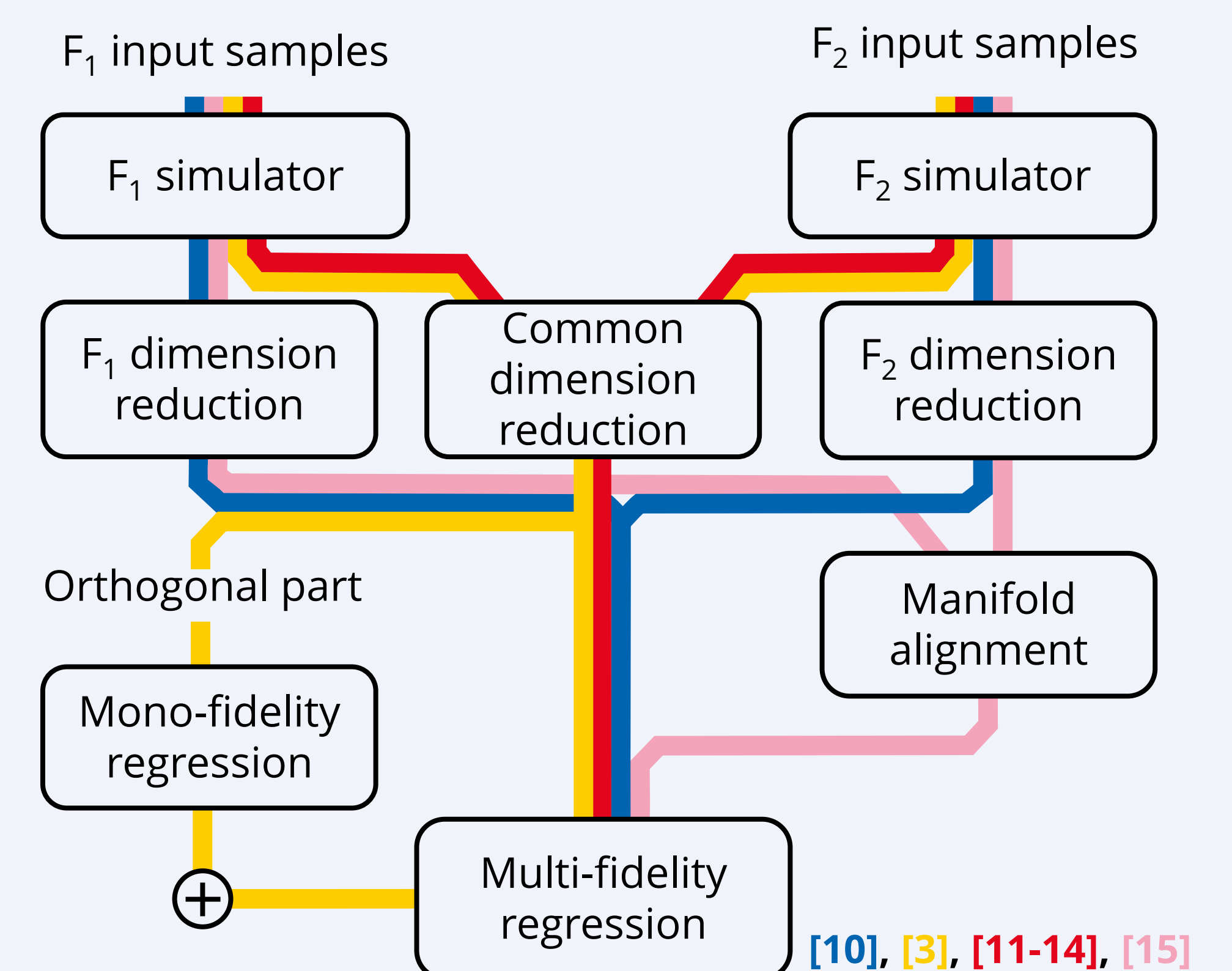
[7]

Mapping



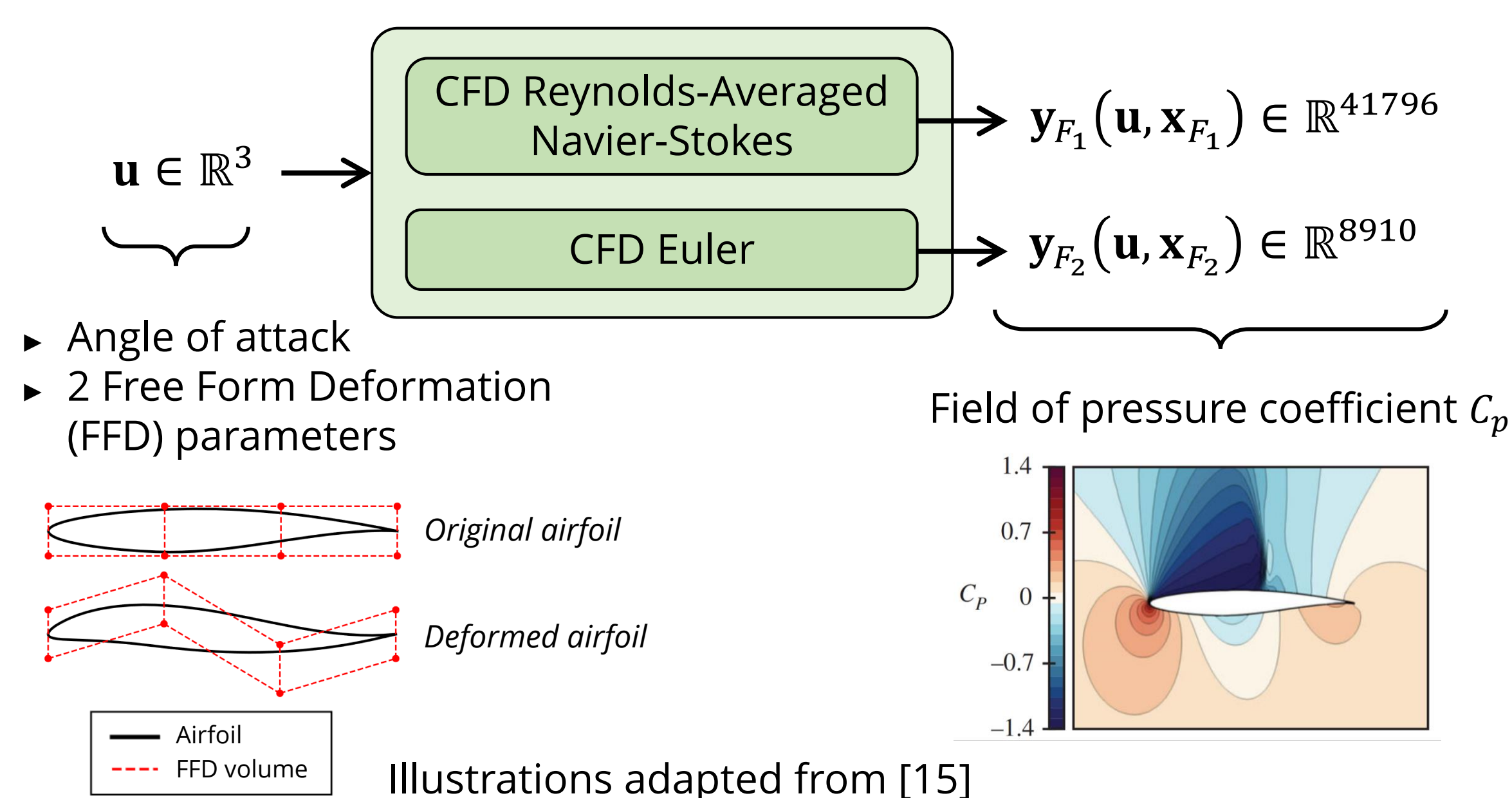
[8], [9]

Fusion



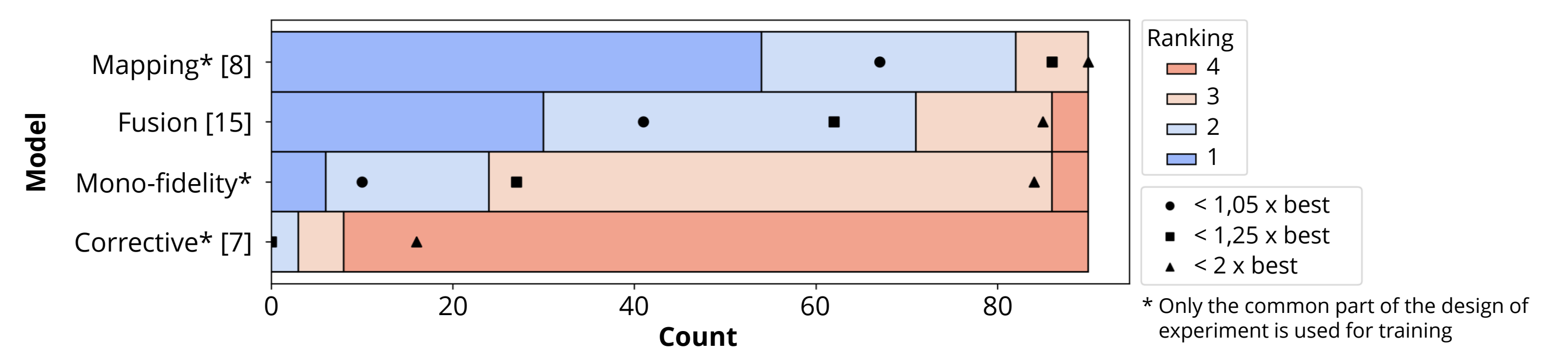
[10], [3], [11-14], [15]

NUMERICAL COMPARISON



Illustrations adapted from [15]

- ▶ Prediction capabilities measured for **different numbers of high-fidelity samples, different ratios of number of low-fidelity samples over the number of high-fidelity samples τ , and multiple repetitions**
- ▶ In the following figure, only the **best performing method of each family** is retained in addition to the mono-fidelity



CONCLUSIONS

- ▶ Existing metamodels in the literature can be reformulated into a **unified framework** with **building blocks** that can be **freely composed**
- ▶ **Most multi-fidelity metamodels yield better results** than a mono-fidelity metamodel
- ▶ Mapping and corrective metamodels require the same number of snapshots for each fidelity and require running the lower-fidelity simulator to make a high-fidelity prediction
- ▶ **The best performing method and the associated ranking very much depend on the application case**, justifying the need for a benchmark with various tests
- ▶ **All referenced methods have been tested on 4 different test cases** but the results are not displayed here due to a space constraints

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