



HAL
open science

A review of multi-fidelity surrogate models for high dimensional field outputs

Lucas Brunel, Mathieu Balesdent, Loïc Brevault, Rodolphe Le Riche, Bruno Sudret

► **To cite this version:**

Lucas Brunel, Mathieu Balesdent, Loïc Brevault, Rodolphe Le Riche, Bruno Sudret. A review of multi-fidelity surrogate models for high dimensional field outputs. 6th international workshop on model reduction techniques (MORtech, an ECCOMAS thematic conference), Nov 2023, Paris-Saclay, France. hal-04793505

HAL Id: hal-04793505

<https://hal.science/hal-04793505v1>

Submitted on 20 Nov 2024

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

A review of multi-fidelity metamodels for field predictions

Lucas Brunel^{1,2}, Mathieu Balesdent¹, Loïc Brevault¹, Rodolphe Le Riche², and Bruno Sudret³



¹ONERA, DTIS, Université Paris-Saclay, 91123, Palaiseau, France

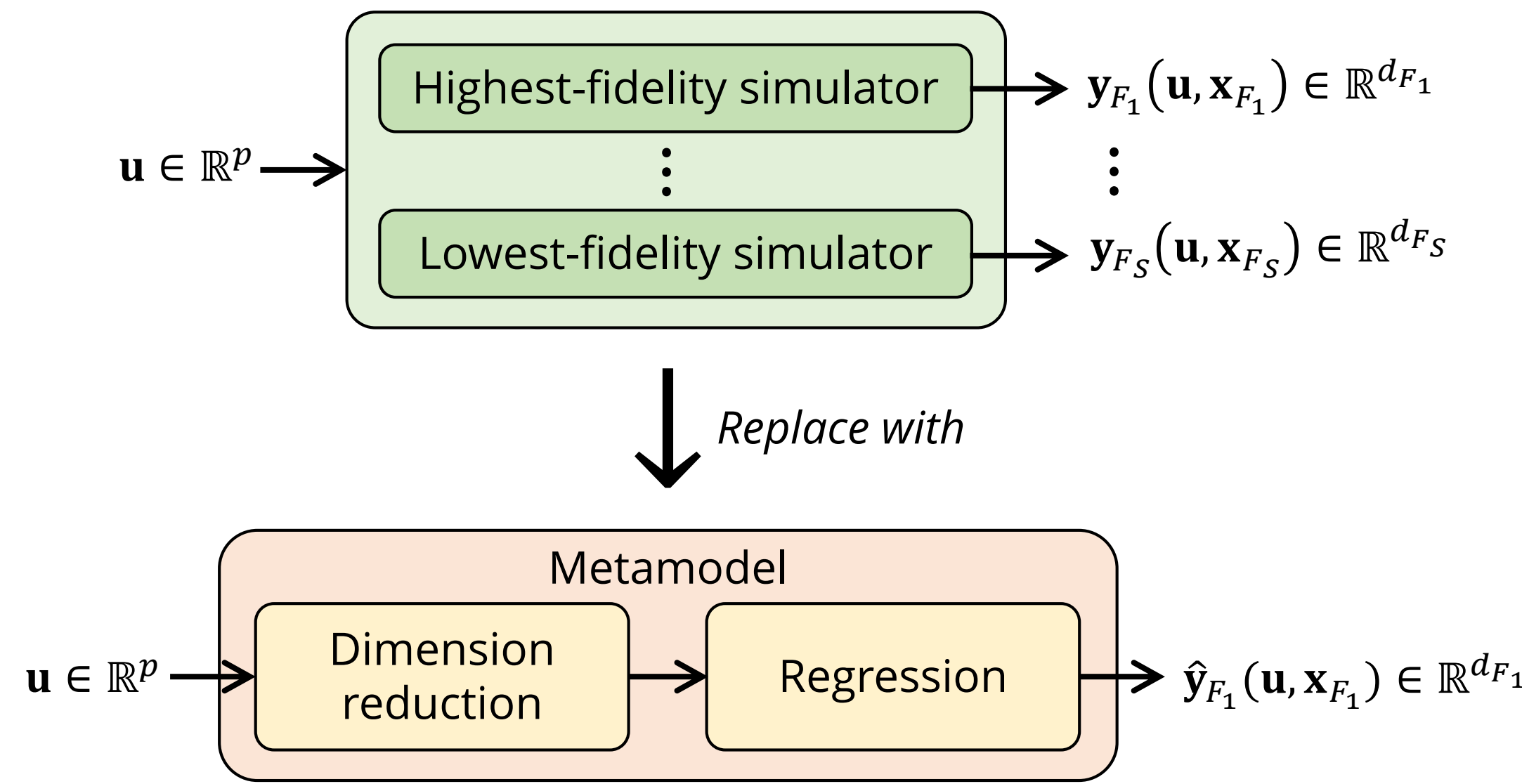
²CNRS LIMOS, Mines Saint-Etienne and Université Clermont Auvergne, France

³ETH Zurich, Chair of Risk, Safety and Uncertainty Quantification, Stefano-Francini Platz 5, Zurich 8093, Switzerland



CONTEXT

- ▶ Simulations with a **high computational cost** (e.g., Computational Fluid Dynamics (CFD))
- ▶ Need to build a **metamodel** for applications that require a **large number of calls to a simulator** (e.g., uncertainty quantification, optimization)
- ▶ The quantity of interest is a **field** (e.g., pressure at the wall of a launch vehicle) **discretized on a mesh** (temporal, spatial, frequential, etc.)
- ▶ Simulators of **variable fidelity** are available (e.g., simplified physics, mesh resolution)



NOTATIONS

- ▶ $\mathbf{u} \in \mathbb{R}^p$: vector of **input variables** (e.g., length of the vehicle, angle of attack)
- ▶ $F_j, j = 1, \dots, S$: **fidelity level**, 1 corresponds to the highest-fidelity
- ▶ \mathbf{x}_{F_j} : **mesh** at fidelity F_j
- ▶ $\mathbf{y}_{F_j}(\mathbf{u}, \mathbf{x}_{F_j}) \in \mathbb{R}^{d_{F_j}}$: a **field discretized on the mesh** \mathbf{x}_{F_j} of dimension d_{F_j}
- ▶ $\hat{\square}$: **prediction** of the field \square

BUILDING BLOCKS

Dimension reduction

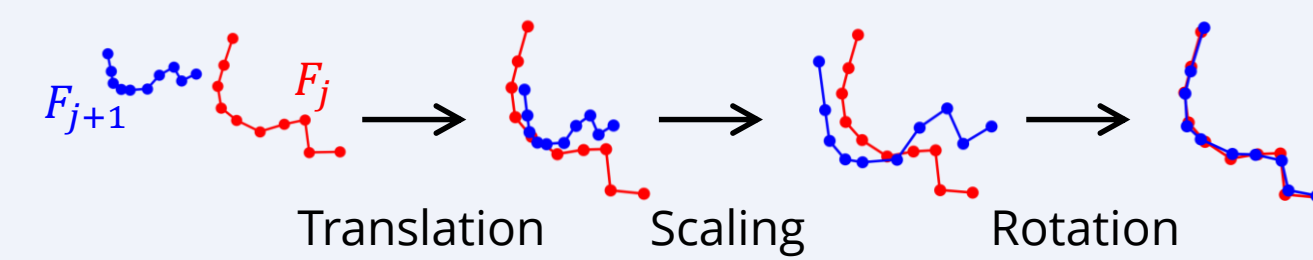
Compression of a high-dimensional field into a lower-dimensional space (latent space), for instance with Principal Component Analysis (PCA) [1], Kernel PCA [2].

Orthogonal part

Information lost when performing dimension reduction. For instance, this can be modeled by a kriging with tensorized covariance [3].

Manifold alignment by Procrustes analysis [4]

Operations on the low-dimensional representation of the fields to make them better match between fidelities.

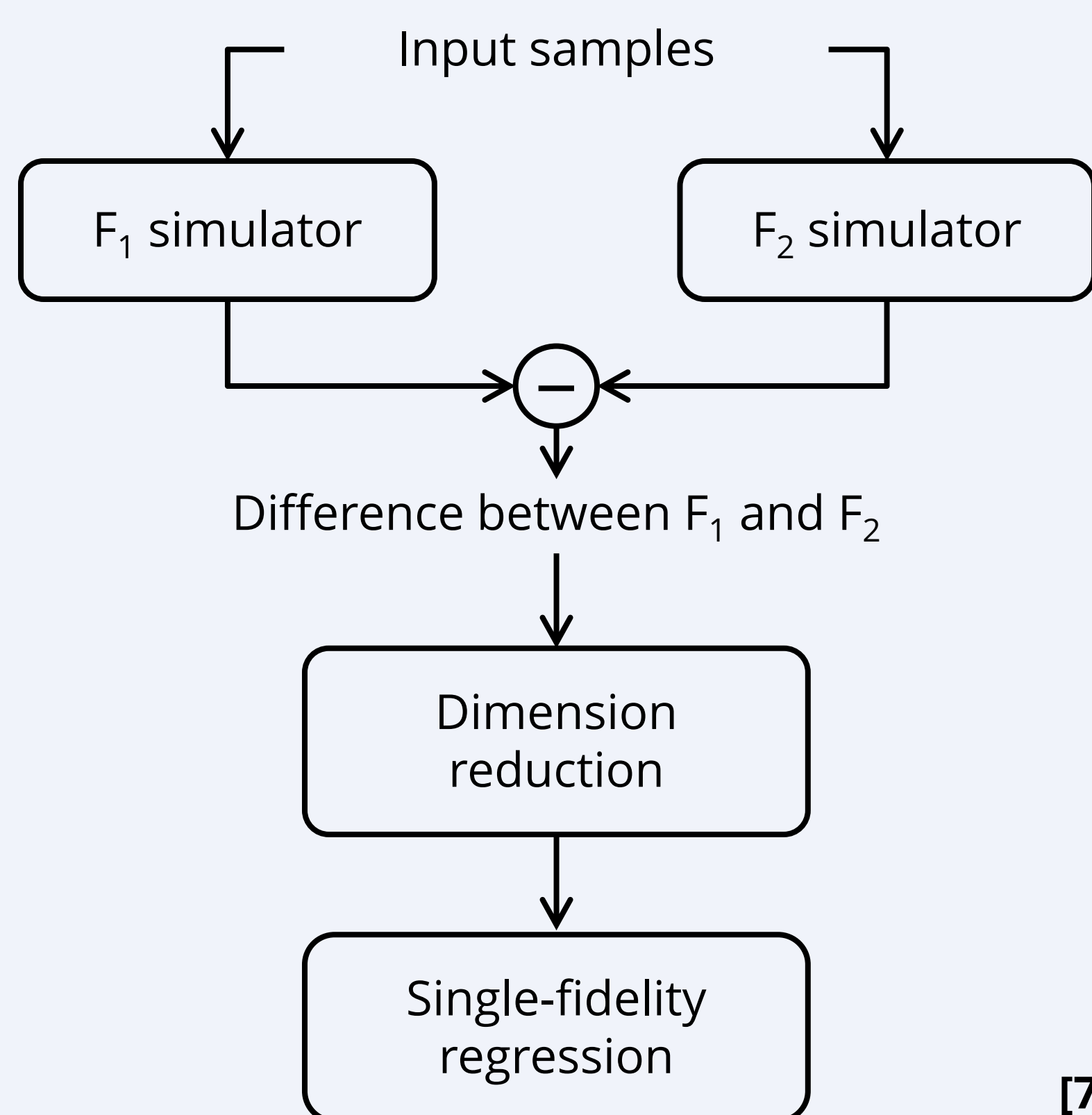


Regression

Mapping between the input variables and the low-dimensional representation of the fields, can be mono-fidelity (e.g., gaussian process regression [5]) or multi-fidelity (e.g., autoregressive cokriging [6]).

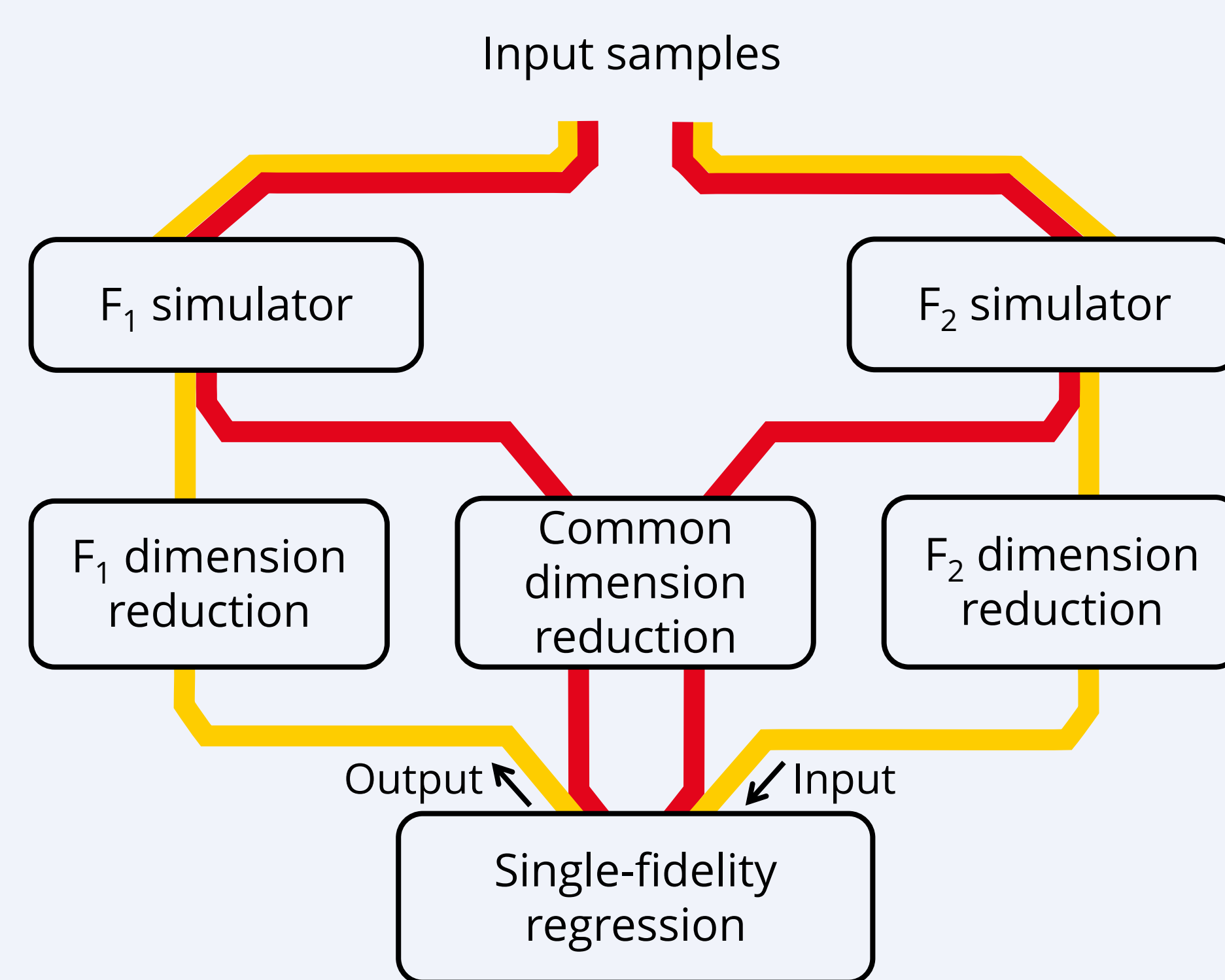
UNIFIED FRAMEWORK OF METAMODEL FAMILIES (ILLUSTRATED FOR 2 FIDELITIES)

Corrective



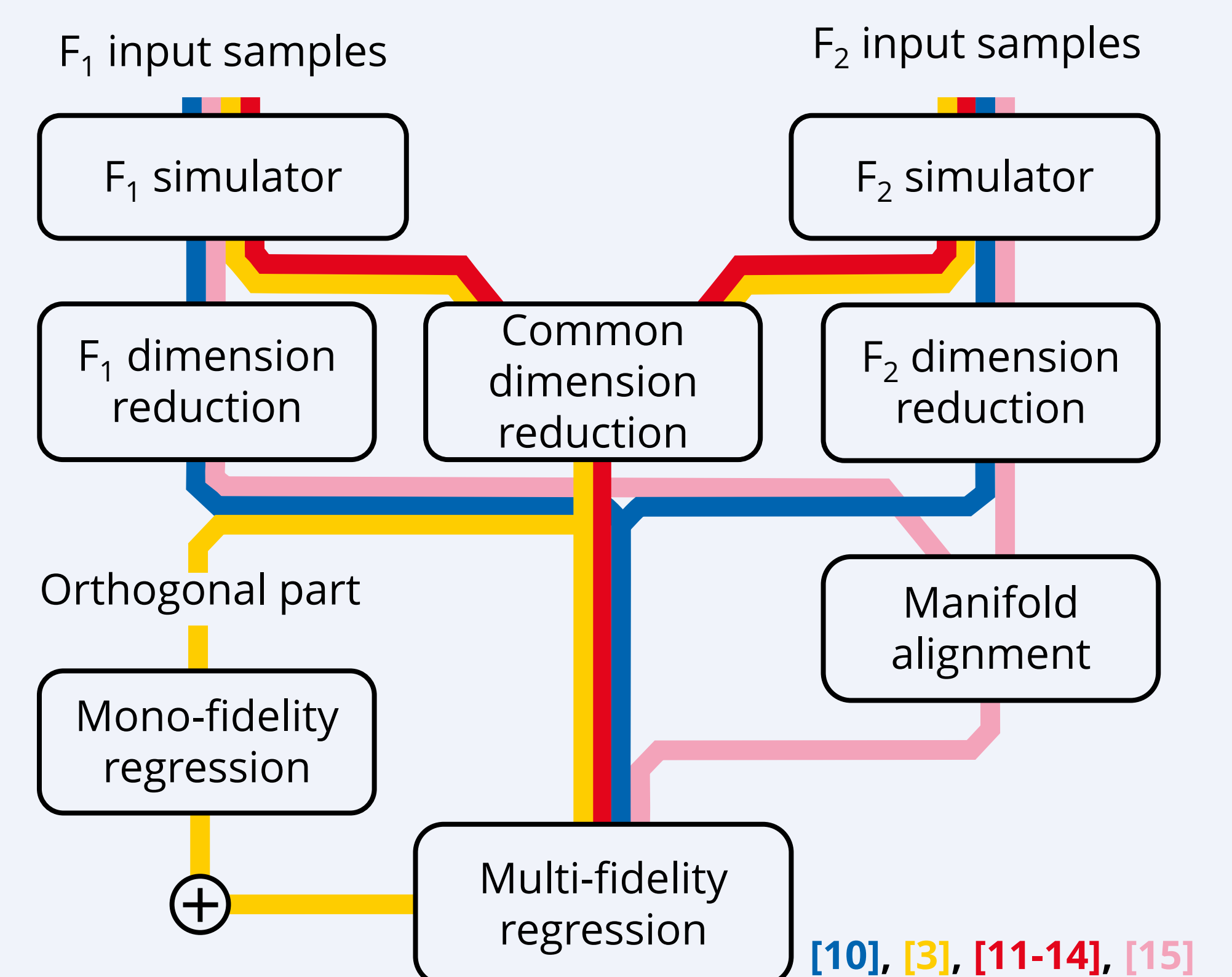
[7]

Mapping



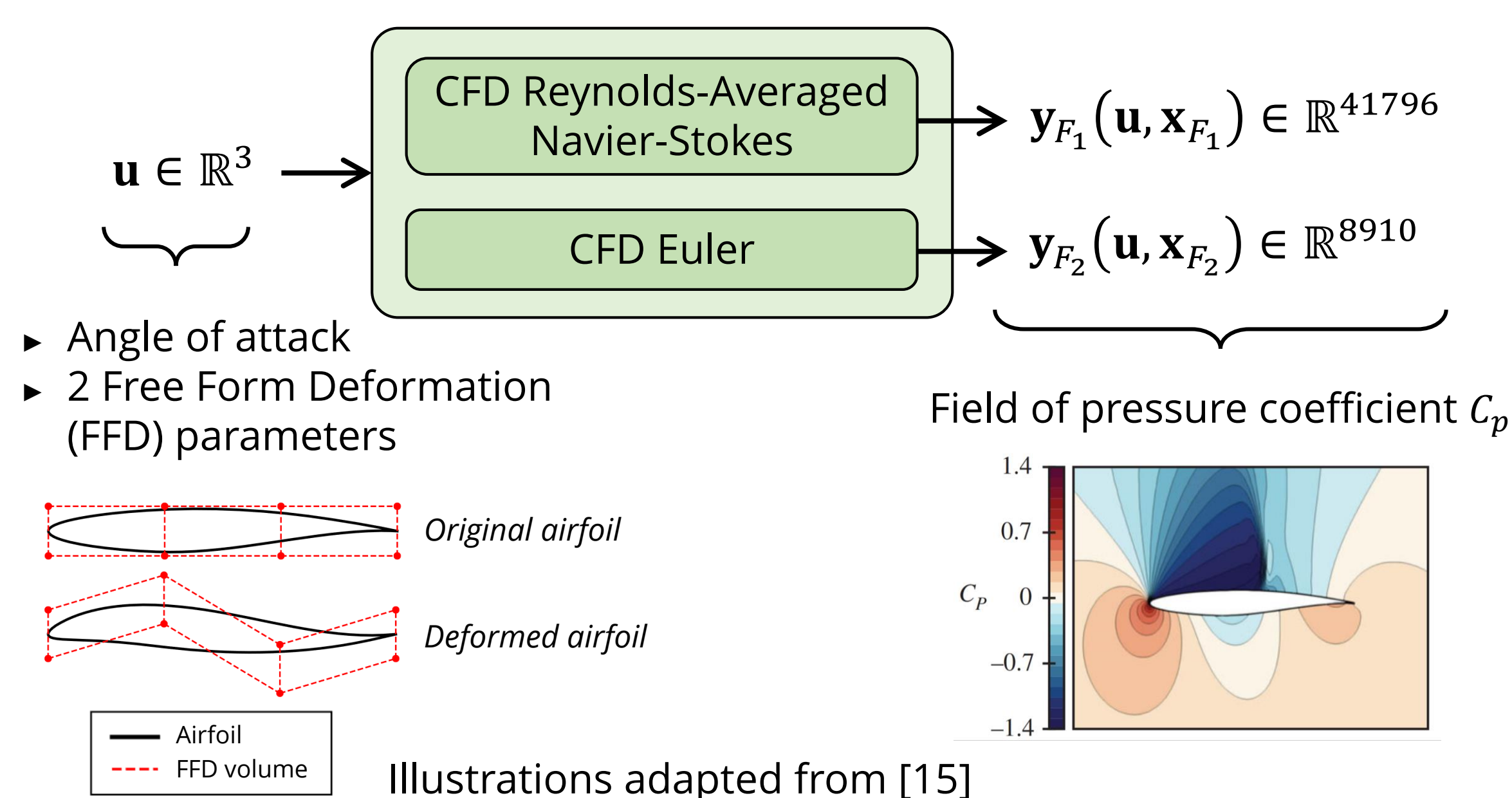
[8], [9]

Fusion



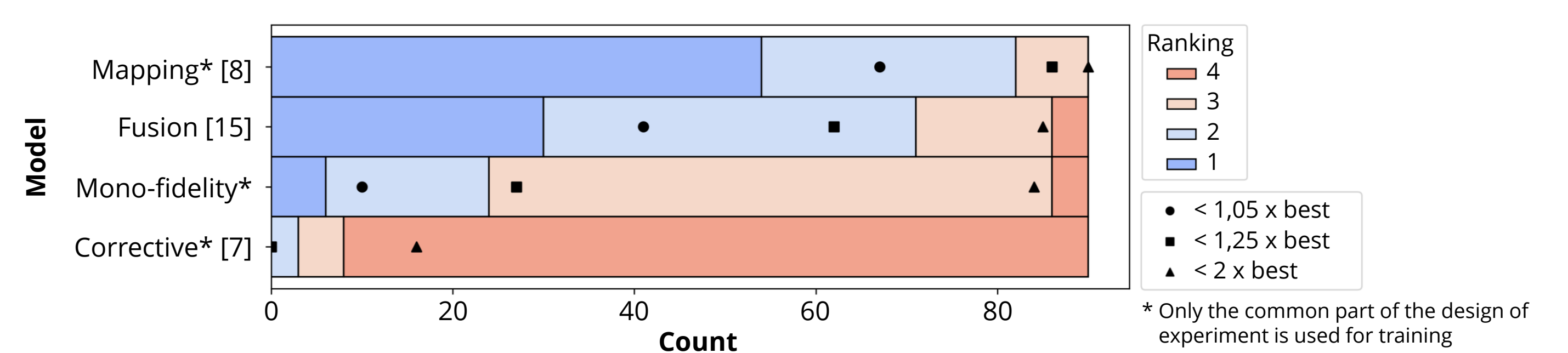
[10], [3], [11-14], [15]

NUMERICAL COMPARISON



Illustrations adapted from [15]

- ▶ Prediction capabilities measured for **different numbers of high-fidelity samples, different ratios of number of low-fidelity samples over the number of high-fidelity samples τ , and multiple repetitions**
- ▶ In the following figure, only the **best performing method of each family** is retained in addition to the mono-fidelity



* Only the common part of the design of experiment is used for training

CONCLUSIONS

- ▶ Existing metamodels in the literature can be reformulated into a **unified framework** with **building blocks** that can be **freely composed**
- ▶ **Most multi-fidelity metamodels yield better results** than a mono-fidelity metamodel
- ▶ Mapping and corrective metamodels require the same number of snapshots for each fidelity and require running the lower-fidelity simulator to make a high-fidelity prediction
- ▶ **The best performing method and the associated ranking very much depend on the application case**, justifying the need for a benchmark with various tests
- ▶ **All referenced methods have been tested on 4 different test cases** but the results are not displayed here due to a space constraints

REFERENCES

- [1] Y. C. Liang et al., *J. Sound Vib.*, vol. 252, n° 3, p. 527-544, 2002.
- [2] B. Schölkopf et al., *Neural Comp.*, vol. 10, no. 5, pp. 1299-1319, 1998.
- [3] B. Kerleguer, *SIAM/ASA J. Uncertain.*, vol. 11, no. 2, pp. 514-539, 2023.
- [4] C. Wang et al., *25th Int. Conf. on Machine learning*, Jul. 2008, pp. 1120-1127.
- [5] G. Matheron, *Econ. Geo.*, vol. 58, no. 8, pp. 1246-1266, 1963.
- [6] L. Le Gratiet et al., *Int. J. Uncertain. Quantif.*, vol. 4, no. 5, pp. 365-386, 2014.
- [7] B. Malouin et al., *Int. J. Aerosp. Eng.*, vol. 2013, 2013.
- [8] D. J. Toal, *ASME Turbo Expo 2014: Turbine Technical Conference and Exposition*, 2014.
- [9] X. Wang et al., *Int. J. Numer. Methods Fluids*, vol. 92, no. 12, pp. 1826-1844, 2020.
- [10] S. Bunnell et al., *Struct. Multidiscipl. Optim.*, vol. 63, no. 5, pp. 2177-2190, 2021.
- [11] A. Thenon et al., *Comput. Geosci.*, vol. 20, no. 6, pp. 1231-1250, 2016.
- [12] T. Rokita et al., *AIAA J.*, vol. 56, no. 8, pp. 3060-3070, 2018.
- [13] T. Benamara et al., *Struct. Multidisc. Optim.*, vol. 56, no. 6, pp. 1387-1412, 2017.
- [14] M. J. Mifsud et al., *Int. J. Numer. Methods Fluids*, vol. 82, no. 10, pp. 646-663, 2016.
- [15] C. Perron et al., *Proc. Math. Phys. Eng. Sci.*, vol. 477, no. 2253, p. 20210495, 2021.