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A unified mathematical framework to extension and convexification methods for full waveform inversion

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Main objectives Make a general review of different reformulation of the FWI problem intended to mitigate the non-linearity and ill-posedness of this large scale inverse problem.

New aspects covered A general unifying mathematical framework is proposed to analyze the similarities and differences between the reviewed formulations.

Summary Full waveform inversion (FWI), despite its wide adoption as part of the seismic imaging workflow in the hydrocarbon exploration industry, relies on the solution of a nonlinear and ill-posed inverse problem. For this reason, successful applications of FWI still require non-negligible human intervention to design proper initial models and adequate hierarchy in the data ensuring the convergence towards a geologically meaningful minimum of the misfit function. For specific targets, the complexity of the wavefield can still prevent FWI to converge towards such a minimum. This is the motivation for more than two decades to propose alternative formulations to FWI, alleviating its ill-posedness nature. In this study, we attempt to propose a general review of these formulations, based on a unifying mathematical framework, which makes possible to highlight similarities and differences between the different approaches. A discussion summarizes this comparison, with a focus on the applicability of the proposed methods to the inversion of field data.

Introduction

Full waveform inversion (FWI) is now widely adopted as part of the seismic imaging workflow in the hydrocarbon exploration industry. This is mainly due its high resolution power, yielding accurate velocity models leading to better focused migrated images or even, more recently, to interpretable reflectivity images. From a mathematical standpoint, FWI is an ill-posed inverse problem. The success of its application usually relies on the construction of an accurate velocity model through tomography strategies, and the definition of an adequate hierarchy in the data (time/offset windowing and weighting, frequency filtering) to make the method converge towards a geologically meaningful minimum. Many different strategies have been proposed in the past decades trying to overcome this difficulty, aiming at enabling a more automated application of FWI, less dependent on human expertise. In this study we attempt to review these approaches within a unified mathematical framework, making possible to highlight what they fundamentally share and what make them distinct.

A unified mathematical framework for extension and convexification strategies

We propose to cast the FWI problem as follows

$$\min_m f(d_{cal}[m], d_{obs}), \text{ s.t. } A(m)d_{cal} = b, \quad (1)$$

where d_{obs} is the observed data, f is a generic comparison function, involving any kind of potential pre-processing on the data, $A(m)$ is a wave equation operator, relating the seismic source term b and the calculated data $d_{cal}[m]$. Taking $f(d_{cal}, d_{obs})$ as the least-squares norm between $d_{cal}[m]$ and d_{obs} yields the conventional FWI formulation (Fig. 1).

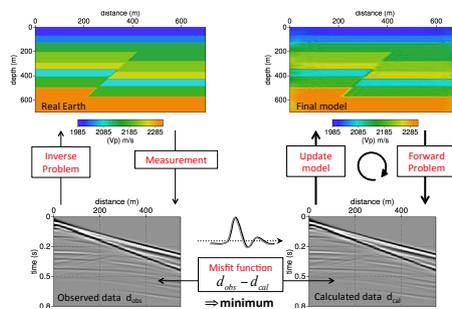


Figure 1 Sketch of the full waveform inversion principle.

Under this framework, the two modifications one can bring to mitigate the ill-posedness of the FWI problem are: 1) the definition of the comparison function f , 2) the way the constraint relating the calculated data d_{cal} to the subsurface model m is enforced. We review in the following how the alternative formulations of FWI can be cast based on these generic notations.

Misfit function modifications

Misfit function modifications act on the definition of the comparison function f while keeping the constraint relating the calculated data d_{cal} to the model m unchanged. Many of such modifications have been proposed since the seminal work of Luo and Schuster (1991) on cross-correlation misfit functions. We focus here on the Adaptive Waveform Inversion (AWI) (Warner and Guasch, 2016) and the Graph Space Optimal Transport (GSOT) strategy (Métivier et al., 2019). These two examples show that complex comparison functions can be used, which can themselves be defined through additional constraints (linear deconvolution for AWI) or minimization problems (linear assignment problem for GSOT).

The Adaptive Waveform Inversion (AWI) can be cast in the proposed framework, with

$$f(d_{cal}, d_{obs}) = \frac{\int_0^T |P(t)w(t)|^2 dt}{\int_0^T |w(t)|^2 dt}, \text{ s.t. } d_{cal} * w = d_{obs}, \quad (2)$$

where $P(t)$ is a weighting operator focusing the energy at zero time-lag, and $*$ denotes the linear convolution operator. For GSOT, the comparison function is formulated as

$$f(d_{cal}, d_{obs}) = \min_{\sigma \in S(N)} \frac{1}{2} \sum_{i=1}^N \frac{B}{\tau} |t_i - t_{\sigma(i)}|^2 + |d_{cal,i} - d_{obs,\sigma(i)}|^2, \quad (3)$$

where N is the number of time samples, $S(N)$ is the ensemble of permutation of $[1, \dots, N]$, B is an amplitude normalization factor, and τ is the maximum expected time shifts between the observed and calculated data.

For both strategies, the actual comparison function f involves a summation over all the seismic traces which we do not explicit here to keep the notations simple.

Wavefield reconstruction inversion

Wavefield reconstruction inversion (WRI) has been introduced by van Leeuwen and Herrmann (2013). As opposed to misfit function modification, the motivation is here to reduce the non-linearity of the FWI problem by relaxing the constraint associated with the computation of the calculated data. Instead of imposing this constraint to machine precision, a quadratic penalty strategy is used, such that the FWI problem becomes

$$\min_{m, d_{cal}} f(d_{cal}, d_{obs}) + \frac{\mu}{2} \|A(m)d_{cal} - b\|^2, \quad (4)$$

where μ is a relaxation parameter. Typically a small value for μ makes it possible to relax the constraint and better fit the data which is useful in the early FWI iterations. An increasing sequence of μ values is then chosen to converge towards the FWI solution.

Aghamiry et al. (2018) further develop this strategy using an augmented Lagrangian approach instead of a quadratic penalty approach, yielding the problem

$$\min_{m, d_{cal}, \lambda} f(d_{cal}, d_{obs}) + (\lambda, A(m)d_{cal} - b) + \frac{\mu}{2} \|A(m)d_{cal} - b\|^2, \quad (5)$$

where λ is an adjoint wavefield and (\cdot, \cdot) a scalar product in the wavefield space. This algorithm has better convergence properties, and does not require a sequence of μ reaching very large values to ensure satisfaction of the constraint. A review of the WRI strategy and its links with the Lippmann-Schwinger equation is proposed in Operto et al. (2023).

Source-space extensions

At the same time wavefield reconstruction inversion was introduced, a source-space extension strategy, named as matched source waveform inversion (MSWI) was introduced by Huang et al. (2018). The essence of the method is to absorb the inaccuracy of the data-fit in the early stages of FWI in an extended source \tilde{b} such that the FWI problem becomes

$$\min_{m, \tilde{b}} f(d_{cal}[m, \tilde{b}], d_{obs}) + \mu \mathbb{A}(\tilde{b}), \quad s.t. \quad A(m)d_{cal} = \tilde{b}, \quad (6)$$

where \mathbb{A} is an annihilator term penalizing the discrepancy between the extended source \tilde{b} and the true source b . MSWI and WRI are closely related. By choosing the annihilator as $\mathbb{A}(\tilde{b}) = \frac{1}{2} \|\tilde{b} - b\|^2$, one recovers exactly the quadratic penalty version of WRI, which can be seen by replacing \tilde{b} with $A(m)d_{cal}$. In this sense, MSWI can be seen as a generalization of WRI.

Receiver-space extensions

An extension at the receiver level is proposed in Métivier and Brossier (2022). The inaccuracy of the data-fit in the early stages of FWI is this time absorbed by a virtual displacement of the receivers from their true position. This makes it possible to correct for the incorrect kinematic of the initial velocity model which would generate cycle skipping with conventional FWI. This receiver-extension strategy can be formulated as

$$\min_{m, \tilde{r}} f(d_{cal}[m, \tilde{r}], d_{obs}) + \mu \mathbb{A}(\tilde{r}), \quad s.t. \quad A(m, \tilde{r})d_{cal} = b, \quad (7)$$

where \tilde{r} is the virtual receiver position, $A(m, \tilde{r})$ is an extended wave equation operator solving for the wave equation and extracting the value of the wavefield at the virtual receiver position \tilde{r} to generate the synthetic data, and \mathbb{A} is an annihilator term which enforces \tilde{r} to converge towards the exact, known position, of the receiver.

Reflection waveform inversion

At depth where the model is not sampled by diving waves (transmitted energy), FWI can only provide information on the high wavenumber component part of the model. Reflection waveform inversion (RWI) is an alternative formulation dedicated to overcome this issue and extends the convergence regime of FWI beyond the Fresnel zone. To do so, it exploits a prior information on the reflectivity, to generate complementary Fresnel zones between the reflectors and the sources and receivers, usually named as “rabbit ears”. Based on a scale separation assumption between a background velocity model m_0 and a reflectivity model δm , RWI reconstructs δm and m_0 in an alternate fashion, solving the problem

$$\min_{m_0, \delta m} f(d_{cal}[m_0, \delta m], d_{obs}), \text{ s.t. } A(m_0, \delta m)d_{cal} = b, \quad \delta m = \mathcal{M}(m_0)d_{obs}, \quad (8)$$

where $\mathcal{M}(m_0)$ is a migration operator mapping the observed data in depth to generate a reflectivity model δm . Precursor to this strategy is the Migration Based Travel Time (MBTT) approach (Clément, 1994), revisited later by Xu et al. (2012) as RWI.

Model-space extensions

Finally, a full set of model-space extension have been proposed to mitigate the ill-posedness of FWI. These extensions were actually the first to be proposed, in the seminal work of Symes and Carazzone (1991). A review of these methods is proposed in Symes (2008), under the name of Migration Velocity Analysis (MVA) methods. The core idea is to exploit the redundancy of the data and the uniqueness of the subsurface model to design a semblance criterion. As for RWI, it exploits a scale separation assumption, where the model parameter m is split into a smooth background model m_0 and a “reflectivity” model δm . An extended migration operator $\mathcal{M}(m_0, h)$ is then introduced, where h can correspond to an offset selection of the data, a subsurface offset, or a time-lag. For the correct background velocity model, the energy should concentrate around 0 in the extension space. This amounts to reformulate the FWI problem as

$$\min_{m_0} \mathbb{A}(\delta m[m_0, h]), \quad \delta m = \mathcal{M}(m_0, h)d_{obs}, \quad (9)$$

where the annihilator \mathbb{A} quantifies how the energy focuses in the extension dimension h .

Discussion

All the presented methods share the following common feature: for each of them, at least one secondary variable is introduced, subject to an additional constraint. This is true (as expected) for extensions strategies, but also for RWI, and for misfit function modifications: GSOT requires an assignment problem to be solved while AWI adds a linear deconvolution constraint to estimate the Wiener filter. To deal with such auxiliary constraints and variable, using variable projection techniques is a standard approach, which is favored when the inner problem is easy to solve (for instance quadratic optimization problems).

Misfit function modifications have been applied successfully to 3D field data (i.e. Guasch et al., 2019; Górszczyk et al., 2021). This is also the case for RWI, combined with misfit function modification strategies to overcome cycle skipping along reflector to source or receiver paths (i.e. Vigh et al., 2019).

Extensions methods are still mostly experimented on synthetic cases. Despite interesting on the paper and relying on a well developed theory, MVA methods suffer from the drawback of the computation cost, high dimensional reflectivity cubes being required to be computed at each iteration of the background model reconstruction. In this perspective, source extension and WRI strategies seem more likely to be efficient on field data. The inner problem on the source/wavefield is a quadratic optimization problem which can be solved in one iteration through Newton strategies. However while its solution can be easily computed in the frequency domain, it requires numerical approximation in the time-domain. Therefore compromises need to be found in terms of computation cost and effectiveness of the strategy to prevent from cycle skipping (Guo et al., 2024).

On the other hand, receiver-extension strategies have been developed to be applicable directly in the time-domain, with an inner loop on the receiver position correction which corresponds to the solution of a nonlinear inverse problem, implemented thanks to global optimization algorithms. The drawback of such strategy tested on 2D synthetic data seems to be its slow convergence, an issue that might be related to the prior assumption that the same time shift affects all the recorded events the same. An extension to a time-dependent receiver position correction is ongoing to overcome this issue (Benziane et al., 2023).

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