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# 3D seismic acquisition geometry optimization based on wavenumber sampling regularity for target-oriented full waveform inversion

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## Main objectives

In this work we extend in 3D a previous work on the optimal design of acquisition geometry for the improvement of imaging quality in the context of Full Waveform Inversion.

## New aspects covered

1. A study of the wavenumber content of the approximate Full Waveform Inversion gradient in the 3D case.
2. Optimal Design of the acquisition for 3D target-oriented Full Waveform Inversion.
3. Optimization that takes into account the effects of the velocity model on the illumination.
4. Optimization with constraints on the deployment area of the acquisition.

## **Summary**

We propose a method for the optimal design of acquisition geometry in terms of imaging quality by improving the distribution of the wavenumber content of the approximate full waveform inversion gradient at a specified target. It has been successfully developed in 2D and we present here the extension to the 3D case. A study through diffraction tomography shows that for a regular layout of the acquisition, the distribution of the wavenumbers is not regular and for a fixed maximum offset those wavenumbers will vary inside an envelope which can be approximated by a spherical dome. We are interested in finding the positions of sources and receivers that give a regular wavenumbers sampling inside this envelope. As in 2D, we express this problem as an instance of Centroidal Voronoi Tessellation. By minimizing a newly designed objective function we can find acquisition geometries that improve our regularity criterion. It is also possible to add constraints on the deployment area and take into account the effects on the illumination from the heterogeneity of the initial model. We believe that those new developments open up new perspectives for target-oriented full waveform inversion and optimal post-acquisition data selection.

### 3D seismic acquisition geometry optimization based on wavenumber sampling regularity for target-oriented full waveform inversion

Optimal experimental design (OED) is a general topic shared by practically all fields in experimental sciences, with the global objective to optimize the collection of data for a given scientific purpose. In the frame of seismic imaging, we develop OED to find the best position for sources and receivers in order to optimize the illumination of a particular zone or target to image.

Recent work using full waveform inversion (FWI) focuses on the conditioning of the Hessian operator (Krampe et al., 2020). We propose a new method for the optimization of the imaging quality based on the resolution analysis of diffraction imaging at a specific point. Our novel approach expresses the problem of regularity of the wavenumber content of the approximate FWI gradient as an instance of Centroidal Voronoi Tessellation problem. This method has been implemented and tested in 2D (Abdellaziz et al., 2023) and gives satisfying results in terms of reconstruction of a target region. In this paper we discuss the extension of this method in realistic 3D geometries and the following developments associated with.

#### Wavenumbers in 3D

Suppose we are interested in imaging a target at position  $\mathbf{x}$  in the subsurface. At that point we focus on the illumination provided by the acquisition. To analyze the resolution of the FWI gradient we can use diffraction tomography : this gives an estimation the wavenumber sampling (Wu and Toksöz, 1987). Let  $\mathbf{s}$  be the position of a source and  $\mathbf{r}$  of a receiver. The wavenumber associated to those devices is given by :

$$\vec{k}(\mathbf{s}, \mathbf{r}) = k_0 (\vec{\mathbf{p}}_s + \vec{\mathbf{p}}_r), \quad (1)$$

where  $\vec{\mathbf{p}}_s$  and  $\vec{\mathbf{p}}_r$  are the unit vectors giving the direction of the rays connecting respectively the source and the receiver to the diffraction point, here  $k_0 = \frac{2\pi f_0}{c_0}$  with  $f_0$  the reference frequency and  $c_0$  the local velocity.

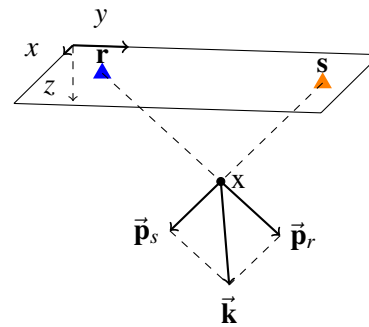
For an acquisition of  $N_s$  sources and  $N_r$  receivers we would have  $N_s \times N_r$  wavenumbers. Using formula (1) we can define a function  $G : (S, R) \mapsto K$  which would compute the wavenumbers associated to an acquisition  $(S, R)$ , where  $S$  and  $R$  denote respectively the set of sources and receivers and  $K$  is the set of associated wavenumbers. We make the assumption that each source sees the same receiver deployment, representative to fixed-spread acquisitions, so we have  $N_s + N_r$  degrees of freedom. Figure 2 shows an example of acquisition (a) and corresponding wavenumber content (b). We notice that for a regularly-spaced acquisition on the surface we have an irregular distribution of the points in the wavenumber space and we would like to find an acquisition on the surface which provides a more regular sampling.

If we fix the source  $\mathbf{s}$  in equation (1) and let the receivers around the target move then only the vector  $\vec{\mathbf{p}}_r$  will vary. Because it is a unit vector all the wavenumbers will be situated in a part of sphere of center  $\mathbf{c} = k_0 \vec{\mathbf{p}}_s$  and radius  $r = k_0$ . The size of this dome is linked to the maximum offset available. If we do this for every source  $\mathbf{s}$  then we can describe the cloud of wavenumber points as a union of spherical domes and this allows us to approximate the whole envelope as a spherical dome like in Figure 2 (b).

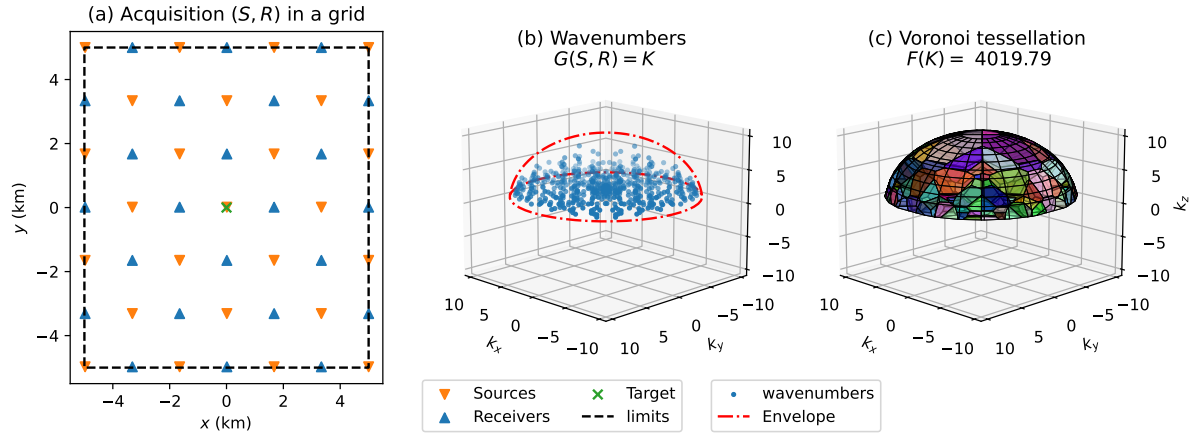
Our idea is to find the positions of a fixed number of sources and receivers on the surface that would yield a regular distribution of the points inside this envelope. The envelope has a fixed size if we consider a fixed area of deployment and would represent the domain of possible wavenumbers. Similarly as in our previous work, we express the problem of regularity of the wavenumbers as an instance of Centroidal Voronoi Tessellation, extending it this time to a 3D framework.

#### Centroidal Voronoi Tessellation

We define the Voronoi tessellation of a domain  $\Omega \in \mathbb{R}^3$  as a partition  $\{\Omega_i, i = 1, \dots, n\}$  of Voronoi cells generated by seeds  $X = (\mathbf{x}_i)_{i=1, \dots, n} \subset \Omega$ . Each cell is generated by a seed  $\mathbf{x}_i$  and is defined as the set of points of  $\Omega$  that are closer to  $\mathbf{x}_i$  than any other seed.



**Figure 1** Schematic representation of equation (1). We have a source and a receiver on the surface. The medium is homogeneous so the rays connecting the devices to the target  $\mathbf{x}$  are straight lines. The directions of those rays are given by vectors  $\vec{\mathbf{p}}_s$  and  $\vec{\mathbf{p}}_r$ , the sum of which yields the wavenumber  $\vec{k}(\mathbf{s}, \mathbf{r})$ .



**Figure 2** An example of regularly-spaced acquisition on a grid (a) with the corresponding wavenumbers (b) and Voronoi tessellation (c). Notice that the points in (b) sample a spherical dome portion of the wavenumber space and the regular distribution of the sources and receivers yield more wavenumbers in the lower part than at the top of the dome. The Voronoi tessellation in (c) is generated by the wavenumbers inside the envelope. It has cells of varying sizes with noticeably larger ones on top. This discrepancy yields a high value of the CVT energy function  $F$  (equation 2).

In Figure 2 (c) we can see the Voronoi tessellation generated by the wavenumbers inside the envelope. For irregularly distributed seeds we get Voronoi cells of irregular sizes. We are interested in tessellations where the cells are homogeneous in size which would yield points which regularly sample the domain. The following energy function

$$F(X) = \sum_{i=1}^n \int_{\mathbf{y} \in \Omega_i} \|\mathbf{y} - \mathbf{x}_i\|^2 d\mathbf{y}, \quad (2)$$

gives a measure of the homogeneity of the cells: The smaller its value, the more similar in size are the cells. A local minimizer of  $F$  provides us with a special kind of Voronoi tessellation, called Centroidal Voronoi Tessellation (CVT). It is a tessellation where the seeds coincide with the centers of mass of the corresponding Voronoi cells. In 3D this function is  $C^2$  if the domain  $\Omega$  is convex and a Newton method can be used to find local minima (Liu et al., 2009).

### Numerical strategy

The composition of  $F$  and  $G$  evaluates the quality of an acquisition in terms of our criterion of homogeneous distribution of the wavenumbers using the CVT characterization. We take the spherical dome envelope as domain  $\Omega$  for the wavenumbers. The envelope is convex. We propose to express the question of finding an optimal acquisition as a problem of minimization :

$$\begin{aligned} \min_{(S,R)} \quad & F \circ G(S, R) \\ \text{subject to:} \quad & h_i(\mathbf{s}) \leq 0, \quad \mathbf{s} \in S \\ & h_i(\mathbf{r}) \leq 0, \quad \mathbf{r} \in R \end{aligned} \quad (3)$$

Since we need to fix the shape of the wavenumber envelope to use the CVT then we should delimit the area of deployment. We can add simple boundary constraints  $h_i(\mathbf{x}) \leq 0$  on the positions of sources and receivers to restrict them inside a square. However we are sometimes more restricted on where to deploy the acquisition, be it because of topography, safety or local laws. We can take those limitations into account by considering more complex geometries of area of deployment. To that end we approximate those limits by a polygon and describe them using linear inequality constraints  $h_i(\mathbf{x}) \leq 0$  that express the fact of whether a point  $\mathbf{x} \in \mathbb{R}^2$  lies inside the polygon or not.

In a homogeneous medium the rays connecting the sources and receivers to the target are straight lines and the computation of the vectors  $\vec{\mathbf{p}}_s$  and  $\vec{\mathbf{p}}_r$  is straightforward. Consequently the function  $G$  has a simple analytical expression and so does its Jacobian  $J_G$ . However when running FWI we usually have an initial guess about the velocity model. Thus we can take into account the effects of the heterogeneity of the model on the illumination at the target. In a heterogeneous medium the rays are more complex.

In a smooth enough medium we can compute  $\vec{p}_s$  and  $\vec{p}_r$  through two-point ray-tracing by numerically solving eikonal equations. In this project we used a Fast Marching algorithm (Mirebeau and Portegies, 2018) and the Jacobian is calculated using finite-differences.

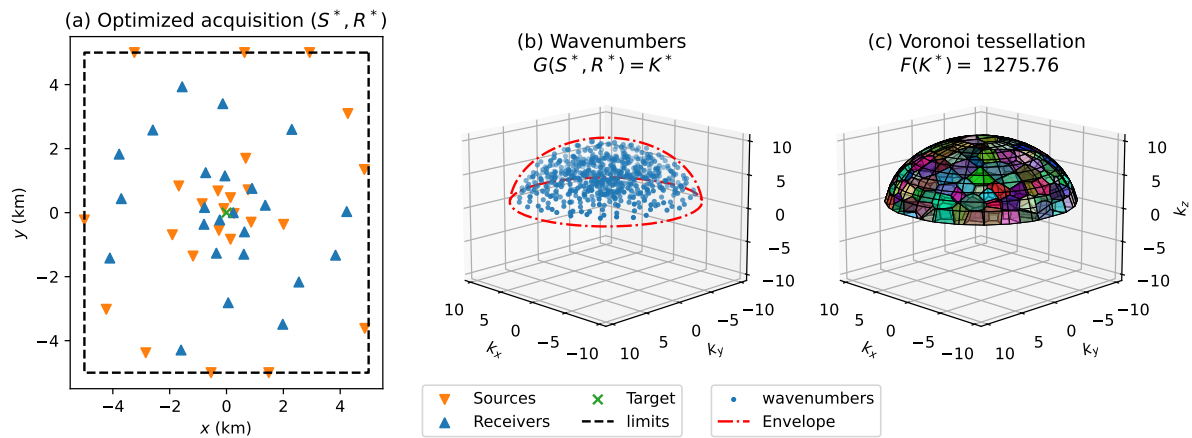
For the generation of Voronoi tessellations we used the Voro++ library (Rycroft, 2009). From the Voronoi cells we evaluate the volume integrals in  $F$  (2) using Gaussian quadratures. The partial derivatives of  $F$  with regards to a seed  $\mathbf{x}_i$  is given by  $\frac{\partial F}{\partial \mathbf{x}_i} = 2m_i(\mathbf{x}_i - \mathbf{c}_i)$  where  $m_i$  and  $\mathbf{c}_i$  are respectively the volume and centroid of the corresponding Voronoi cell (Liu et al., 2009). The evaluation of the gradient is then possible once we have the tessellation.

Given those elements we can apply the chain rule to compute the gradient  $\nabla(F \circ G) = J_G^T \nabla F(G)$  and use a quasi-Newton method to solve (3). To solve this inequality constrained minimization problem we use the NLOPT library (<https://github.com/stevengj/nlopt>) which provides a sequential quadratic programming (SQP) method (Kraft, 1994).

### Numerical examples

To illustrate the applicability of our method in 3D we consider two scenarios: the first one is an acquisition for a target in a homogeneous medium inside a simple square region, the second one in a heterogeneous medium with a particular constraint on the shape of the deployment area.

#### Homogeneous medium



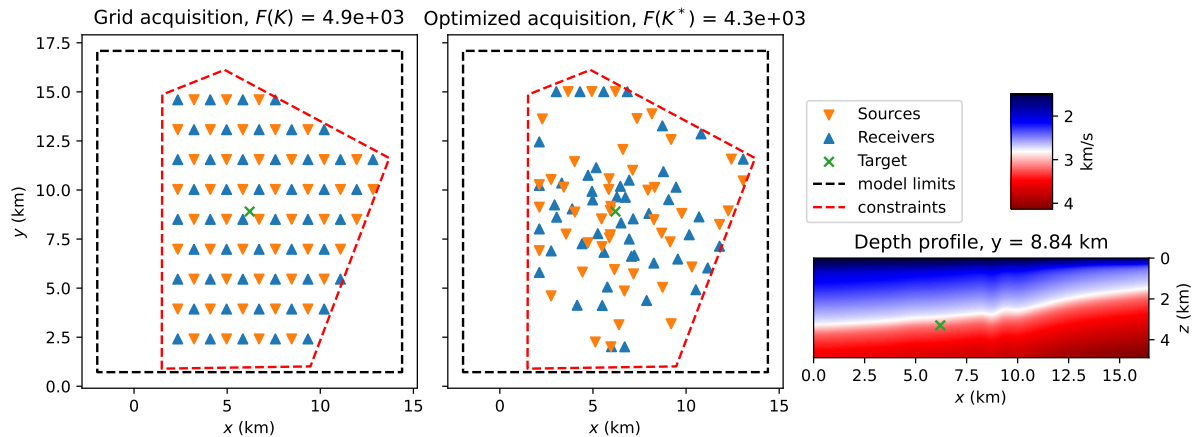
**Figure 3** An example of optimized acquisition, to be compared with the grid acquisition in Figure 2. Here the uniformity of the acquisition geometry is lost. We have a regular distribution of the wavenumbers inside the envelope as can be seen in the center panel. This improvement is also suggested by the more homogeneous Voronoi tessellation and the decrease in  $F$ .

We would like to optimize the acquisition of the example shown in Figure 2. We are allowed to position 25 sources and 24 receivers on the surface inside a  $10 \text{ km} \times 10 \text{ km}$  square area around a target situated at depth 1 km. The velocity model is homogeneous. Our optimized acquisition is shown in Figure 3 (a). We notice the particular geometry which, as expected, is denser in the region above the target and sparser the further from it. With this geometry we are able to improve the sampling of the wavenumber space in Figure 3 (b) compared to the grid acquisition in Figure 2 (b). The positioning of the receivers suggests an optimal shape close to a spiral-like design (Ridyard et al., 2023).

#### Heterogeneous medium with deployment constraints

In this second example we optimize the acquisition for a smoothed synthetic heterogeneous velocity model and the area of deployment has the shape of a polygon. The acquisition is composed of 47 sources and 49 receivers while the target is situated at the center of the polygon at a depth of 3.3 km. The synthetic model and constraint geometry have been provided by ACTeQ.

Figure 4 compares a regular grid acquisition with an optimized acquisition and shows a profile of the velocity model. We were able to again reduce the value of the objective function and get an optimal acquisition that concentrates around the target while respecting the deployment constraints.



**Figure 4** A second example of optimized acquisition. The limits of the allowed area of deployment is a polygon and a profile of the medium is shown. Notice how the optimized acquisition is pushed until the edges of the polygonal limits.

## Conclusions

In this work we are able to extend our approach for the optimal design of an acquisition from the 2D to the 3D case. It is based on the sampling of the wavenumber space by the approximate FWI gradient at a specific target and uses a CVT characterization of the problem to design a measure of imaging quality.

We can add constraints on the acquisition and take into account the effects of the heterogeneity of the medium on the illumination. Additional features are planned for implementation (exclusion zones, topography). Another extension will be multi-target optimization to consider volume regions of interest. In the near future we will study the practical impact of the optimized acquisition on the image reconstruction through FWI.

All those developments are promising and can lead to new algorithms for acquisition design in the context of target-oriented FWI or optimal post-acquisition data selection.

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## References

- Abdellaziz, A., Brossier, R., Métivier, L. and Oudet, E. [2023] Optimal Experimental Design in Full Waveform Inversion. In: *84<sup>th</sup> Annual EAGE Meeting (Vienna)*, 2023. European Association of Geoscientists & Engineers.
- Kraft, D. [1994] Algorithm 733: TOMP—Fortran modules for optimal control calculations. *ACM Transactions on Mathematical Software*, **20**, 262–281.
- Krampe, V., Edme, P. and Maurer, H. [2020] Optimized experimental design for seismic full waveform inversion: A computationally efficient method including a flexible implementation of acquisition costs. *Geophysical Prospecting*, **69**(1), 152–166.
- Liu, Y., Wang, W., Lévy, B., Sun, F., Yan, D.M., Lu, L. and Yang, C. [2009] On Centroidal Voronoi Tessellation—Energy Smoothness and Fast Computation. *ACM Trans. Graph.*, **28**(4).
- Mirebeau, J.M. and Portegies, J. [2018] Hamiltonian Fast Marching: A numerical solver for anisotropic and non-holonomic eikonal PDEs. Preprint.
- Ridyard, D., Hite, D., Monk, D. and Mattox, B. [2023] A novel approach to OBN survey design for Full Waveform Inversion. **2023**(1), 1–5.
- Rycroft, C.H. [2009] VORO++: A three-dimensional Voronoi cell library in C++. *Chaos: An Interdisciplinary Journal of Nonlinear Science*, **19**(4), 041111.
- Wu, R.S. and Toksöz, M.N. [1987] Diffraction tomography and multisource holography applied to seismic imaging. *Geophysics*, **52**, 11–25.