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► **To cite this version:**

Louis-Marie Harpedanne de Belleville. Bank's skin-in-the-game and hold-up by illiquid firms: strategic bargaining, dynamic inconsistency and credit constraints. 2024. hal-04792104

HAL Id: hal-04792104

<https://hal.science/hal-04792104v1>

Preprint submitted on 19 Nov 2024

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Bank's skin-in-the-game and hold-up by illiquid firms: strategic bargaining, dynamic inconsistency and credit constraints¹

Louis-Marie Harpedanne²

November 19th, 2024

Abstract

The loan literature analyzes the hold-up problem from the bank monopolistic information perspective, but if only the firm can fully repay the bank, the loan relationship is actually a bilateral monopoly. Then, if a firm borrows short to finance a long-term project, non-cooperative bargaining occurs at loan renewal. If, regardless of the firm's second-period quality, the perfect equilibrium partition derived from this bargaining grants the bank less than the break-even condition, she declines to lend *ex-ante*. That is, expected hold-up by the firm induces credit constraints. If the firm gets more by defaulting than by borrowing from another bank, the initial bank cannot break even by filing for the firm bankruptcy; that is, the bank has a weak outside option. Then, even if this option is binding, the previous credit constraints result holds. Such hold-up by illiquid firms provides a new foundation for long-term lending to finance long-term projects.

Keywords

Hold-up, credit constraints, subgame perfection, non-cooperative bargaining, strategic default

JEL

C72, G21, G32, G33, D53

¹ I thank Christophe Amat, Tobias Broer, Alex Clymo, Nuno Coimbra, Eustache Elina, François Fontaine, Philippe Jehiel, Jean Imbs, Matt Leduc, Yann Perdereau, Noémie Pinardon-Touati, Normann Rion, Gilles Saint-Paul, Enrico Sette, Olivier Tercieux, Gisèle Umbhauer, Frédéric Vinas and participants in workshops at Banque de France (Oct. 2023 and May 2024), Paris School of Economics (Feb. and Oct. 2024) and the Augustin Cournot Doctoral Days (Strasbourg, May 2024) for useful comments. Clémence Charavel and Florence Lépicier provided useful institutional and legal elements. Estelle Colasson provided invaluable explanation and advice on ANACREDIT data. All errors are mine.

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1. Introduction

Why are long-term loans needed to finance long-term investment? In 2023, in France, the median initial maturity of a construction investment loan was 82 months but was only 2.5 months for export/import credit and 4 months for working capital facility lending³.

I propose a new answer: *ex post*, firms can hold up banks, which therefore decline to lend *ex ante*. I describe the mechanism, analyze the frictions supporting it, and contrast it with traditional answers.

The new mechanism is as follows: if bank b lends short, she has skin-in-the-game (SITG) and may be forced to grant a new loan at abnormally favorable rates to have the initial loan repaid⁴. That is, firm f may hold up b ; then, *ex-ante*, b declines to lend short. This is described in the following paragraphs.

The literature underlines that b has monopolistic information on f and has therefore hold-up power. But f alone can fully repay the loan; therefore, he has monopoly power. Thus, when the initial loan must be rolled over, a bilateral monopoly exists, such that b and f must bargain to share the surplus.

Agents are self-interested, and this bargaining process is non-cooperative. Moreover, the game is not repeated, which rules out “Folk theorem” solutions: b and f cannot implement threats that are not optimal *ex-post*. In other (game-theoretic) words, their actions must be supported by subgame-perfect strategies. The solution is well-known: in an alternating-offers bargaining model, a unique Perfect Equilibrium Partition (PEP) exists ([Rubinstein, 1982](#)). This PEP is bracketed by outside options: b or f call an outside option if this is more profitable than the PEP. Conversely, these outside options do not affect the PEP (outside option principle, [Binmore et al. 1989](#)). This paper introduces in the loan literature the distinction between PEP and the outside options (financial frictions affect the latter).

Negotiating powers are derived from discount factors, regardless of who plays first. b 's power is weakened since she must pay interest on what she borrowed from financial markets to lend to f , whereas what f pays b is part of the negotiation: it is an outcome of the negotiation, not an input.

To analyze this situation, I extend the existing PEP results ([Binmore, 1986](#), [Sutton, 1986](#)) to encompass interest-bearing or interest-costing assets. I bridge the gap between macroeconomic practice, which routinely considers income flows during a negotiation, and the game-theoretic literature, which provides microeconomic foundations. I also extend the outside-option principle to options that are binding (*i.e.*, better than the PEP for one player) and/or encompass a loss threat.

Finally, b declines to lend short if two conditions are satisfied: (1) the PEP does not grant b a break-even income (this must hold regardless of f 's quality over the second period, which prevents the bank

³ Computations based on ANACREDIT data, which do not single out non-construction investment loans.

⁴ Throughout the analysis, I use “she” for the initial bank b and “he” for the firm/entrepreneur f .

from compensating losses in bad states with profits in good states); (2) b has a weak outside option. The latter condition is determined by reaction when b declines to roll over; if f is better off defaulting than borrowing from another bank, then, and only then, will b have a weak outside option.

The mechanism relies on the lending bank having **skin-in-the-game**: b has a vested interest in f 's ability to repay the loan and is therefore likely to roll over, even at a loss. This phenomenon is pervasive. [Caballero et al. \(2008\)](#) show that during the Japanese 1990s crisis, large banks financed insolvent borrowers in the hope of recovery. This depressed job dynamics, dampened productivity, and prolonged stagnation. Banks' SITG also generalizes the soft-budget-constraint analyzed by [Dewatripont and Maskin \(1995\)](#): when b has a weak option, she must roll over without another bank's support. In the property rights theory ([Grossman and Hart, 1986](#), [Hart and Moore, 1990](#)), SITG corresponds to a specific investment made by the bank by lending to f , giving f the hold-up power. Finally, SITG is the complement of the free-rider problem identified by [Bulow and Shoven \(1978\)](#), [Gertner and Scharfstein \(1991\)](#) and [Bolton and Scharfstein \(1996\)](#). Existing creditors benefit from helping distressed borrowers (SITG) but each creditor declines to bear the full costs (free-riding).

SITG interacts with three **frictions**: firms' illiquidity, the unverifiability of the firm's probability of default, and the possibility of strategic default. SITG and frictions do not affect negotiating powers, but they affect outside options and create the conditions for bilateral monopoly and therefore hold-up.

Hold-ups can occur after a short-term loan but because f 's liquidity position of the firm is tight when the loan must be rolled over. Conversely, over the latter period, firm-specific (*i.e.*, illiquid) productive assets are turned into fully versatile sales income ([Kiyotaki and Moore, 2001](#), [Kermani and Ma, 2023](#)).

Asset specificity gives monopoly power to the firm which, together with other frictions, create bilateral monopoly, *i.e.* relationship specificity. [Caballero and Hammour \(1998\)](#) detail the link between specific relationships and hold-up. This articulation is detailed in Section 5.

The unverifiability of the firm's situation by a court is a common assumption. I assume more specifically that the firm's probability of default is unverifiable. This is corroborated by institutional evidence and is realistic because, even *ex-post*, neither default nor survival provides compelling evidence on *ex-ante* risk. This cause of incompleteness of loan contracts is immune to the critique of [Maskin and Tirole \(1999\)](#). It complements the views of [Hart and Moore \(1998\)](#) who rather assume income' unverifiability.

Finally, firms can strategically default, even if moral hazard is assumed away. I provide innovative empirical results that entrepreneurs quickly default in case of a positive shock to private benefits. This is consistent with benefits extraction at the time of bankruptcy, while [Giroud et al. \(2012\)](#) find effects over the years before default. This is also consistent with a direct effect, while [Schoenherr and Starmans \(2022\)](#) find that private benefits affect risk-taking, which causes default only after a while.

These frictions are instrumental in explaining why long-term loans are not affected by this new credit constraints mechanism. Indeed, a firm with a long-term loan may intend to default to force a renegotiation. However, f cannot default since f 's liquidity position is verifiable by a court (unlike f 's probability of default). Even if assumptions were slightly modified and f were able to default, that would indicate to the court that f is either unlikely to pay his future debt (making him unable to extort a low rate on a new loan) or attempting to hold up b . That is, long-term debt prevents illiquidity, unverifiability of f 's probability of default, and limits (directly or not) his ability to default strategically.

This new mechanism explains long-term lending more convincingly than **traditional approaches**. First, firms may fear banks' inability to roll over short-term loans ([Diamond and Dybvig, 1983](#)). However, this encouraged governments and central banks to establish deposit insurance and loan refinancing, strongly limiting banks' default risk (mostly below 1% at a one-year horizon). Additionally, recent public resolution mechanisms ensure loans are rolled over to continue illiquid projects in case a bank fails.

Second, hedging by firms against interest rate changes explains the preference for long-term lending; the share of variable-rate loans drops from 54% for maturities below one year to 9% for maturities between three and four years (French ANACREDIT data, 2023). Nevertheless, proper derivatives, rather than long-term loans, would suffice to hedge against macroeconomic-related rate changes.

However, agents may use long-term loans to hedge against macroeconomic-related rate changes. Namely, firms may hedge against hold-ups when loans are rolled over. A lending bank accumulates inside information and may use loan renewal to extract a monopoly rate ([Rajan, 1992](#), [Houston and James, 1996](#), [Farinha and Santos, 2002](#), [Santos and Winton, 2008](#), [Hale and Santos, 2009](#), [Schenone, 2010](#), [Bird et al., 2019](#)). Firms may favor long-term loans to avoid dependence on extortive banks. Still, hold-up by banks relies crucially on costly information acquisition. Conversely, the French Central bank rated the largest 300.000 firms in 2023, likely limiting banks' informational monopoly and their ability to hold up firms. Many European economies exhibit similar conditions.

Overall, traditional approaches may no longer work well in many economies, while the mechanism identified here is consistent with current institutional features. This mechanism explains why long-term projects cannot be funded without long-term financing ([Choudhary and Limodio, 2022](#)).

The findings present several policy implications including fostering long-term bank lending (as *e.g.* [Diamond and Dybvig, 1983](#), but for very different reasons), and providing courts with independent external ratings to alleviate unverifiability frictions.

Also, the contracts proposed by [von Thadden \(1995\)](#) to protect firms from hold-up by banks (with a rate fixed *ex ante* but a decision to roll over by the bank) could be adapted to protect banks from hold-up by firms. Interestingly, such contracts would provide for low interest rates in the first period and

high interest rates in the second period, to maximize the probability of rollover. That is, these contracts would be observationally equivalent to hold-up by banks. Increasing rates over the course of the loan relationship are often used to provide empirical support for the importance of asymmetric information in banking. This result shows that more precise research on this important topic is needed.

The remainder of this paper is organized as follows. The model is presented in Section 2. Section 3 extends the results of [Binmore \(1986\)](#) and [Sutton \(1986\)](#) by including interest-bearing assets. It also extends the outside option principle to the binding option and loss threat cases. Section 4 proposes a model solution through backward induction using the tools developed in Section 3. Section 5 identifies the financial frictions supporting these results. Section 6 concludes.

2. Structure of the economy

In a two-period production economy, firm f builds productive capacity over the first period (from T^0 to T^1) and produces over the second period (from T^1 to T^2). A loan negotiation game between the bank and the firm occurs at T^1 , at the juncture of both periods. Following [Diamond and Dybvig \(1983\)](#) long-term production is profitable but liquidation at T^1 induces a loss related to a lack of liquidity of the asset built over the first period.

2.1 Firms

Production process. Firm f builds a factory over period 1. Then, it produces over the second period. The (raw and undiscounted) value y of the project, conditional on firm survival in both periods, is known *ex-ante*. y measures the production and productivity of the project.

The literature indicates that capital misallocation is primarily due to the extensive margin ([Buera et al., 2011](#), [Midrigan and Xu, 2014](#)). Accordingly, a project has unit size and is financed or not binarily, which avoids issues related to scale effects. Specifically, this avoids the discrepancy between productivity and profitability induced by downsizing production ([Agrell and West, 2001](#)). The investment decision is based on project characteristics but not on other firm characteristics, that do not influence yield. That is, the firm is equivalent to the project. It is also equivalent to the entrepreneur.

Prices are constant and fixed at unity, enabling focus on productivity issues by eliminating the discrepancy between productivity and profitability documented empirically by [Foster et al. \(2008\)](#) when prices vary depending on demand or market power. Constant prices can be rationalized by assuming that the firms and projects are atomistic.

Default risk. At the start of each period t , risk-neutral firm f can be in a good state (the period t probability of default - PD - is p_G^t) or in a bad state ($PD = p_B^t$) with $p_G^t < p_B^t$. p_B^t and p_G^t are exogenous

or “fundamental” *failure rates* of f . Additionally, f can choose bankruptcy or suffer from a liquidity shortage, but these outcomes are out-of-equilibrium. The transition process of the PD is

$$[1] \quad \begin{bmatrix} P(p_G^2) \\ P(p_B^2) \end{bmatrix} = \begin{bmatrix} 1-p & q \\ p & 1-q \end{bmatrix} \begin{bmatrix} P(p_G^1) \\ P(p_B^1) \end{bmatrix}$$

where q is the probability that a firm with high credit risk over period 1 (“bad”) has low credit risk over period 2 (“good”), and p is the probability that a firm with low credit risk in period 1 (“good”) has high credit risk in period 2 (“bad”). The mechanism examined in this study can hold true for constant or even null PDs. However, PD is naturally stochastic, justifying the limitations to information transferability. Also, if second-period PD were known at T^0 , [von Thadden \(1995\)](#) contracts could be implemented and would address the disequilibrium credit constraints identified here.

2.2 Banks

Banks are risk-neutral. Following [Petersen and Rajan \(1995\)](#) and [von Thadden \(1995\)](#), I assume that banks are the only source of external finance. At T^0 , the bank is not liquidity-stressed *ex-ante* and is ready to provide short-term (one-period) financing for projects with a positive NPV. At T^1 , each bank can borrow from financial markets to refinance a first-period loan.

Banks' cost of funds over period i is the risk-free rate r_i . All banks have the same cost structure. *Ex-ante* (at T^0), all banks have the same information about the firm and the banking system is competitive. *Ex-post* (at T^1), if the firm has borrowed short, the bank that bore the one-shot monitoring cost at $T^0 + \varepsilon$ has a cost advantage if screening costs are positive.

The bank never defaults. This assumption is consistent with capital regulations on banks and other regulations and supervisions to prevent bank failures.

2.3 Information

I assume symmetric information between the bank and the firm, with a few reservations for generality and comparison with the literature. A bank that lends to a new borrower faces a one-shot screening cost c_s , while the firm that wants to borrow from a new bank faces a one-shot cost c_f to transmit information to the bank (based on [Mazet-Sonilhac, 2022](#)). The case where $c_s = c_f = 0$ corresponds to symmetric information and perfect competition among banks⁵.

I assume that cost c_s is borne differently over both periods. For the second-period loan, c_s is a screening cost borne at T^1 . Conversely, for the first-period loan, the probability of default is assumed to be known and c_s is a monitoring cost necessarily paid at $T^0 + \varepsilon$, just after the loan has been granted.

⁵ Even when c_s is positive, I assume that it is sufficiently low for the bank to incur this cost. In the absence of other obstacle to lending, and if able to lend long (which avoids the constraints mechanism analyzed below), the bank will decide to incur this cost.

Assuming symmetric information, I rule out costly state verification ([Townsend, 1979](#), [Gale and Hellwig, 1985](#)); the final production y is costlessly observed by the bank⁶.

Finally, also by the symmetry assumption, I rule out moral hazard. Firms always exert effort (this is a crucial difference from, e.g., [von Thadden, 1995](#)) and cannot increase risk to increase expected private benefits. However, in the short-term loan case, the firm can (threaten to) decline to borrow from another bank if the existing bank declines to roll over. This enables the firm to extort low rates from this initial bank. The firm can also impose costly early repayment on the bank.

2.4 Firms failures

The project requires an investment of size 1 to build or buy an asset. In case of failure over period 1, the bank recovers Γ^1 with $0 < \Gamma^1 \leq 1$, where $1 - \Gamma^1$ may be interpreted as a measure of asset specificity ([Kermani and Ma, 2023](#)). Alternative interpretations are possible.

Alternatively, $1 - \Gamma^1$ may represent profits diverted by the firm, which may look surprising with a well-functioning legal system given the symmetric information assumption. Nevertheless, an entrepreneur can legally appropriate private benefits, for instance when including entrepreneur's wage in these private benefits. Also, in § 5.1, I show that the net private benefits *at the time of default* matter. To better distinguish between these different mechanisms, private benefits are identified as such and are labelled PB^1 with:

$$[2] \quad 0 \leq PB^1 \leq 1 - \Gamma^1$$

The right-hand-side constraint derives from the assumption that no production occurs over period 1. Similarly, I define Γ^2 (what the bank gets in case of failure over period 2) and PB^2 with:

$$[3] \quad 0 \leq PB^2 \leq y - \Gamma^2$$

Notably, the interpretation of Γ^2 may differ from that of Γ^1 . Γ^1 is the asset's collateral value before production starts. Conversely, Γ^2 is not related to asset value if the asset is exhausted at T^2 (if not, production continues after T^2 and there is more than two periods). Γ^2 may rather be the value of a blanket lien providing the bank with a guarantee based on the firm's overall production. Thus, at T^1 , Γ^1 may be interpreted as the liquidation value of the firm for the banks and Γ^2 , at least to some extent, as its going-concern value ([Kermani and Ma, 2020](#)). Finally, a fixed failure cost F is borne by the firm in case of default.

⁶ For simplicity, I consider "German loans" with principal's repayment at maturity. Still, frequent payments could be used by the bank to identify firm failure earlier (which would imply a limited departure from the symmetry assumption) or to prevent the bank from voluntarily failing to report firm default to authorities (cf. [Caballero et al., 2008](#)), which would be fully consistent with the symmetry assumption.

3. Method

Here, I explain why bank b and firm f implement non-cooperative bargaining with subgame-perfect strategies when the loan must be renewed and build the tools necessary to analyze it.

3.1 Non-cooperative bargaining, subgame-perfection, and their consequences

b and f are rational and self-interested. If they agreed on an unconditional second-period loan, that would be a long-term loan. A conditional contract for the second-period loan would not be enforceable (Section 5.3) because crucial aspects of such a contract (probabilities of default) cannot be observed by a court. At T^0 , f has committed to repay the loan, but at T^1 he prefers to renegotiate if he makes more money, indicating dynamic inconsistency.

Self-interested players implement non-cooperative bargaining because they cannot get more by cooperating. Indeed, the surplus (defined by equation [36]) cannot be increased through cooperation. Interestingly, if b or f files for f 's bankruptcy, this incurs cost F , which decreases this surplus; however, b and f avoid such an outcome at equilibrium in the non-cooperative bargaining game.

The game is not repeated, preventing bargainers from using "Folk Theorem" strategies (*i.e.*, implementing loss-loss threats to affect their opponent's strategy)⁷. Thus, rational agents with common knowledge of their rationality implement subgame-perfect strategies.

Thus, the model must be solved backwards, and even off-equilibrium outcomes must be considered carefully. Consequently, I consider what happens after b or f filed for f 's bankruptcy, even if they actually avoid such a costly outcome.

Under usual conditions in the bargaining literature, there exists a unique subgame-perfect equilibrium (§ 3.2). This solution is independent of who plays first if the time between a rejection and a counterproposal tends towards zero, which is justified in § 4.1.

Existing literature neglects the possibility that b and f accumulate interest during negotiations⁸. I show that the unicity result extends to the case in which a player has an asset that brings or costs interest during the negotiation, but PEP is affected by these assets (§ 3.3)⁹.

⁷ I thank Gisèle Umbhauer for underlying this point.

⁸ Conversely, the macroeconomic literature considers that bargainers get a specific income during the negotiation (typically unemployment benefits during negotiations for a new job's wage) but, to the best of my knowledge, without explicit consideration of subgame-perfection. Accordingly, the macroeconomic literature often relies on Nash-bargaining, which is axiomatic rather than strategic (Rubinstein, 1982).

⁹ Note that it would also be possible to consider multiple solutions; see *e.g.* Hart and Moore (1998). Here, I have sufficient conditions to get uniqueness. In particular, following the literature, I consider that outside options can be used by agents when they reject their opponent's offer. Shaked (1994) finds that multiple equilibria are possible *in specific cases*, when a bargainer can opt out just after his offer has been rejected. However, his analysis relies crucially on the fact that backward solving the Rubinstein (1982)'s alternating offers game causes the profit of each bargainer to converge towards a limit close to $\frac{1}{2}$. This feature relates directly to "long"

Then, I consider the case in which the non-binding outside option induces a loss for the other player and show that this is irrelevant: the outside option principle still applies (§ 3.4). Similarly, the outside option principle fully applies when a normal outside option (with no loss threat) is binding (§ 3.5). Conversely, when an outside option is binding *and* induces a loss to the other player, the player that has this option can affect his opponent's actions, but gets almost nothing more from that (§ 3.6).

3.2 Modeling and solving negotiations with subgame-perfect strategies

Models of non-cooperative (strategic) bargaining with alternating offers were introduced by [Rubinstein \(1982\)](#) who shows that any split of a surplus is a Nash equilibrium but that there exists only two subgame-perfect equilibria (here, one if the bank makes the first offer and one if the firm makes it), which depend on the time preference of both agents.

The original Rubinstein model assumes constant length "one" between two offers. However, a player can immediately make a counteroffer after rejecting an offer. Thus, letting Δ , the time-interval between two offers, tend towards zero is reasonable. This further simplifies the analysis by making the identity of the first player irrelevant: a unique subgame-perfect equilibrium exists ([Binmore, 1986](#), [Sutton, 1986](#), [Binmore, Rubinstein and Wolinski, 1986](#), [Binmore, 1987](#), [Binmore, Shaked and Sutton, 1989](#), [Osborne and Rubinstein, 1990](#), proposition 4.5). When δ_f (resp. δ_b) is the discount factor for player f (resp. b), the share of player f (resp. b) is $s_f^* = \frac{\ln \delta_b}{\ln \delta_f + \ln \delta_b}$ (resp. $s_b^* = \frac{\ln \delta_f}{\ln \delta_f + \ln \delta_b}$) ([Sutton, 1986](#), p. 711, [Binmore, 1987](#), p. 73, [Osborne and Rubinstein, 1990](#), p. 84)¹⁰. These limits form a time-preference version of Nash's (1950) (non-strategic) risk-aversion-based bargaining equilibrium. Noteworthy, rates play a role, although no interest is paid or received during negotiations.

Outside options do not affect the computation of this equilibrium (outside option principle). However, a bargainer chooses the outside option if it is more profitable, and this will replace the aforementioned equilibrium¹¹. That is, outside options are only used as constraints on the validity range of the bargaining solution based on the surplus and discounting factors.

negotiation intervals, whereas my analysis realistically assumes that negotiation intervals length tends toward zero. Also, the multiplicity of equilibria relates directly to exogenous heterogeneity in the number n of periods over which agents are forced to bargain. This feature is inherited from the analysis of the firm-insider relationship during wage negotiations in [Shaked and Sutton \(1984\)](#) and makes sense in such a framework. Conversely, agents can clearly not be forced to maintain a relationship: $n=0$. Thus, the outside option timing assumption used here is conservative and the unique perfect equilibrium results are likely to hold in a more general framework.

¹⁰ If r_b (resp. r_f) is the rate facing b (resp. f), then $\delta_b = \frac{1}{1+r_b}$; $\delta_f = \frac{1}{1+r_f}$.

¹¹ [Binmore, Shaked and Sutton, 1989](#) , p. 756: ("Deal-me out" means using the outside option) "how can a bargainer use his outside option to gain leverage? By threatening to play the deal-me-out card. When is such a threat credible? Only when dealing himself out gives the bargainer a bigger payoff than dealing himself in. It follows that the agreement that would be reached *without* outside options is *immune* to deal-me-out threats, unless the deal assigns one of the bargainers less than he can get elsewhere."

A unique result exists: either the equilibrium aforementioned or the outside option, depending on what is more profitable for the bargainers ([Binmore, Rubinstein and Wolinsky, 1986](#), proposition 6, [Sutton, 1986](#), § 3, [Binmore, 1986](#), [Binmore, Shaked and Sutton, 1989](#)).

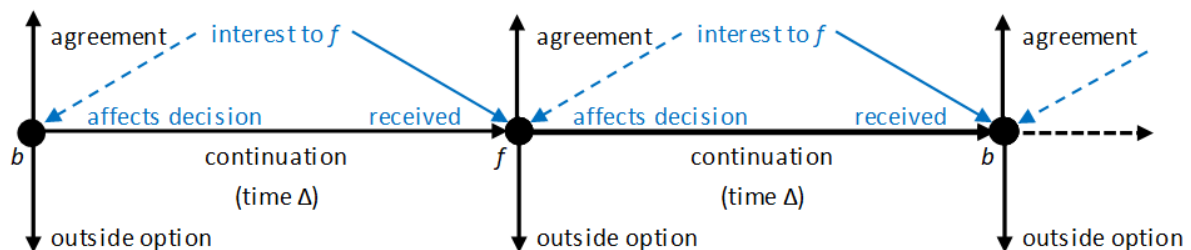
As stated in Section 2, for simplicity, I assume that the refinancing rate r is positive but arbitrarily small. Thus, when both b and f have access to financial markets at riskless rate, $\delta_b = \delta_f$ and, therefore, $s_b = s_f = 0.5$. Bank b , which never defaults, always has access to financial markets at the riskless interest rate. This is also the case for f when he has excess liquidity that he can invest in financial markets or when a positive profit is discounted.

3.3 Bargaining with interest-bearing or interest-costing assets

Classical game-theoretic models of alternative bargaining rely on three components: the surplus to be shared and discounting factors that together determine a central equilibrium and outside options. Crucially, this surplus is not delimited by the outside options ([Binmore, Rubinstein and Wolinsky, 1986](#), proposition 6, [Sutton, 1986](#), § 3, [Binmore, 1986](#), [Binmore, Shaked and Sutton, 1989](#)). That is, the outside options are only used as constraints on the validity range of the bargaining solution geted without outside options and not as limits of the surplus to be shared.

Macroeconomists rightly connect disagreement points to income flows during negotiations, but often resort to cooperative bargaining ([Hall and Milgrom, 2008](#), being a well-known exception) and fail to distinguish between these disagreement points and outside options. Also, they do not consider the subgame-perfection of supporting strategies. On the game-theoretic side, [Binmore, Shaked and Sutton \(1989\)](#), who consider wages negotiations, correctly define the disagreement point as “income flows during a strike” (emphasis added) and outside options as the best income flows if collaboration stops. Still, to the best of my knowledge, the effect of interest-bearing or interest-costing assets during negotiations has not been formalized in the game-theoretic literature yet. Here, I show that the assets that bring (or cost) interest during negotiations affect the partition of the (unaffected) surplus.

Graph 1: Bargaining game of alternating offers with interest income during the negotiation



Let S be the total surplus (including the value of interest-bearing assets) to be shared between b and f . The bargaining process consists of alternating offers with immediate responses. The time between two offers is Δ , which tends towards zero. Discounting factors are determined by the interest rate to

which each player is exposed: $\delta_f = \frac{1}{1+r_f}$ and $\delta_b = \frac{1}{1+r_b}$. s_f is the share of the surplus that f offers to keep, and s_b is the share that b offers to keep. I consider first the case in which b offers first (Table A3-1). Unlike the case analyzed in § 3.2, f has an asset P that pays interest at rate r_f .

Let consider the supremum $M_{b,2\Delta}$ that b can get at $T=2\Delta$. Assuming no deal has been struck before, if a deal is struck at $T=2\Delta$, b makes the offer and receives at most $s_b S$. Hence, at $T=\Delta$, when f makes the offer, b accepts any offer in which he gets more than $\delta_b^A s_b S$. That is, f gets at least $S - \delta_b^A s_b S$ if a deal is struck at $T=\Delta$. But a deal at $T=\Delta$ means that no deal was struck at $T=0$. Thus, at $T=\Delta$, f also receives (in any case actually) the income flow $P\{(1+r_f)^A - 1\}$ from asset P . Thus, at $T=0$, f requires an offer (defined as $m_{f,\Delta}$) that brings him at least $\delta_f^A [S - \delta_b^A s_b S + P\{(1+r_f)^A - 1\}]$. That is, b can get at most $S - m_{f,\Delta}$ at $T=0$. In turn, this happens to be the supremum of the $T=0$ proposal $s_b S$ which would be accepted by f since the forthcoming game is the same at $T=0$ and at $T=2\Delta$ ¹². Therefore:

$$[4] \quad s_b S \leq M_{b,0} = S - \delta_f^A [S - \delta_b^A s_b S + P\{(1+r_f)^A - 1\}]$$

Table A3-1: Summary of the computation of s_b when b makes the first offer

T	f	b
0	$m_{f,0} = \delta_f^A [S - \delta_b^A s_b S + P\{(1+r_f)^A - 1\}]$	$s_b S \leq M_{b,0}$ and $M_{b,0} = S - m_{f,0}$
Δ	$m_{f,\Delta} = S - M_{b,\Delta}$ $= S - \delta_b^A s_b S$ $(+P\{(1+r_f)^A - 1\})$	$M_{b,\Delta} = \delta_b^A s_b S$
2Δ		$M_{b,2\Delta} = s_b S$

This table presents the different steps of the computation of s_b when b makes the first offer. At $T=2\Delta$, I determine the maximal amount b can get (supremum), which determines successively what b can get at most at $T=\Delta$, then what f can get at least at that time (infimum), then what f can get at least at $T=0$ and finally what b can get at most at that time. This last amount is in turn equal to $s_b S$. Switching the infimum m and supremum M in all steps proves unicity of s_b when b makes the first offer. The blue component is determined by choices over the previous periods and does not affect the current-period behavior.

Switching the infimum m and supremum M , “at least” and “at most”, in all steps, we get:

$$[5] \quad S - \delta_f^A [S - \delta_b^A s_b S + P\{(1+r_f)^A - 1\}] = m_{b,0} \leq s_b S$$

From [4] and [5], we get:

$$[6] \quad s_b S (1 - \delta_f^A \delta_b^A) = (1 - \delta_f^A) (S - P)$$

Proof: $s_b S (1 - \delta_f^A \delta_b^A) = S - \delta_f^A [S + P\{(1+r_f)^A - 1\}] = S - \delta_f^A S - P(1 - \delta_f^A)$ where the first equality follows from [4] and [5] and the second from the fact that $\delta_f^A (1+r_f)^A = 1$. QED.

¹² I approximate when assuming that the forthcoming game is identical at 0 and 2Δ . S remains unchanged indeed. But between 2Δ and 3Δ , f receives $P\{(1+r_f)^{3\Delta} - (1+r_f)^{2\Delta}\}$ against only $P\{(1+r_f)^A - 1\}$ between 0 and 2Δ . The difference, $\{(1+r_f)^{2\Delta} - 1\}P\{(1+r_f)^A - 1\}$, converges to zero when Δ tends towards zero, so that the approximation is good, especially when r_f is very small, which is shown in Section 4 to flow directly from the assumption that the riskless refinancing rate is almost zero. Thus, this “local” analysis suffices here. In a more general setting, the cumulated interests of asset P could exceed what f could get by making a deal or calling an outside option. In such an “explosive” case, b would call her own outside option. b would also call her outside option if she were burdened with debt. In both cases, the results presented in § 4.4 would be reinforced.

This proves the unicity of the partition determined by s_b , conditional on b making the first offer. This partition is supported as a perfect equilibrium (Sutton, 1986, p. 711). The subgame-perfect strategies are as follows: at each step, a bargainer makes an offer that corresponds to this perfect equilibrium and the other bargainer accepts any offer that does not exceed that amount. As Sutton (1986) notes, this proof does not assume that the strategies supporting the PEP are stationary, but rather exploits the stationarity of the underlying structure.

Now, I consider the case in which f makes the first offer (Table A3-2). Assuming no deal has been struck yet, at $T=2\Delta$, f receives in any case $P(1+r_f)^{2\Delta} - P(1+r_f)^\Delta$ (compounded interests; flow between Δ and 2Δ). If a deal is struck at $T=2\Delta$, f additionally receives $s_f S$. Hence, f receives at least:

$$[7] \quad m_{f,2\Delta} = s_f S + P(1+r_f)^\Delta \{(1+r_f)^\Delta - 1\}$$

At $T=\Delta$, b makes the offer. In any case, f gets $P(1+r_f)^\Delta - P$ (this amount is obtained whether f accepts b 's offer or not; therefore, it is not relevant to the comparison). Furthermore, f gets at least $m_{f,2\Delta}$ in the following period; thus, f will not accept any offer from b lower than $\delta_f^\Delta \cdot m_{f,2\Delta}$ (the value $m_{f,2\Delta}$ actualized at $T=\Delta$). That is, at $T=\Delta$, b can get at most the supremum $M_{b,\Delta} = S - \delta_f^\Delta \cdot m_{f,2\Delta}$

$$[8] \quad M_{b,\Delta} = Z - \delta_f^\Delta [s_f Z + P(1+r_f)^\Delta \{(1+r_f)^\Delta - 1\}]$$

$$[9] \quad S - \delta_b^\Delta \left[Z - \delta_f^\Delta [s_f S + P(1+r_f)^\Delta \{(1+r_f)^\Delta - 1\}] \right] = m_{f,0} \leq s_f S$$

Switching the infimum m and supremum M , "at least" and "at most", in all steps, we get:

$$[10] \quad s_f S \leq M_{f,0} = S - \delta_b^\Delta \left[Z - \delta_f^\Delta [s_f S + P(1+r_f)^\Delta \{(1+r_f)^\Delta - 1\}] \right]$$

From [9] and [10], we get:

$$[11] \quad s_f S (1 - \delta_b^\Delta \delta_f^\Delta) = S - \delta_b^\Delta [S - P\{(1+r_f)^\Delta - 1\}]$$

This proves the unicity of the perfect partition determined by s_f , conditional on f making the first offer.

Table A3-2: Summary of the computation of s_f when f makes the first offer

T	f	b
0	$s_f S \geq m_{f,0}$ and $m_{f,0} = S - M_{b,0}$	$M_{b,0} = \delta_b^\Delta M_{b,\Delta}$
Δ	$m_{f,\Delta} = \delta_f^\Delta \cdot m_{f,2\Delta}$ $= \delta_f^\Delta [s_f S + P(1+r_f)^\Delta \{(1+r_f)^\Delta - 1\}]$	$M_{b,\Delta} = S - \delta_f^\Delta \cdot m_{f,2\Delta}$ $= S - \delta_f^\Delta \cdot [s_f S + P(1+r_f)^\Delta \{(1+r_f)^\Delta - 1\}]$
2Δ	$m_{f,2\Delta} = s_f S \quad (+P(1+r_f)^\Delta \{(1+r_f)^\Delta - 1\})$	

This table presents the different steps of the computation of s_f when f makes the first offer. At $T=2\Delta$, I determine the minimal amount f can get (infimum), which determines successively what f can get at least at $T=\Delta$, then what b can get at most at that time (supremum), then what b can get at most at $T=0$ and finally what f can get at least at $T=0$. This last amount is in turn equal to $s_f S$. Switching the infimum m and supremum M in all steps proves unicity of s_f when f makes the first offer. The blue component is determined by choices over the previous periods and does not affect the current-period behavior.

I now analyze what this implies for the net surplus. From [11], letting $\Delta \rightarrow 0$:

$$[12] \quad s_b S = (S - P) \frac{\ln \delta_f}{\ln \delta_f + \ln \delta_b} = (S - P) s_b^*$$

That is, to determine the amount $s_b S$ accruing to b when b makes the first offer, one must apply the share $s_b^* = \frac{\ln \delta_f}{\ln \delta_f + \ln \delta_b}$ to a net surplus excluding the interest-bearing asset P .

For the case in which f makes the first offer, from [11]

$$[13] \quad \begin{aligned} s_f S (1 - \delta_b^A \delta_f^A) &= S (1 - \delta_b^A) + \delta_b^A P \{(1 + r_f)^A - 1\} \\ &= S (1 - \delta_b^A) + P (1 - \delta_b^A) - P \left(1 - \frac{(1 + r_f)^A}{(1 + r_b)^A} \right) \\ &= S (1 - \delta_b^A) + P (1 - \delta_b^A) - P \left(1 - \frac{\delta_b^A}{\delta_f^A} \right) \end{aligned}$$

Letting $\Delta \rightarrow 0$:

$$[14] \quad s_f S = (S + P) \frac{\ln \delta_b}{\ln \delta_f + \ln \delta_b} - P \frac{\ln \delta_b - \ln \delta_f}{\ln \delta_f + \ln \delta_b} = (S + P) s_f^* - P \frac{\ln \delta_b - \ln \delta_f}{\ln \delta_f + \ln \delta_b}$$

If $\delta_f = \delta_b$ (symmetric case as regards interest rates), then $\frac{\ln \delta_b - \ln \delta_f}{\ln \delta_f + \ln \delta_b} = 0$, $s_f = \left(1 + \frac{P}{Z}\right) s_f^* = \left(1 + \frac{P}{Z}\right) \frac{1}{2}$ and $s_b = \left(1 - \frac{P}{Z}\right) s_b^* = \left(1 - \frac{P}{Z}\right) \frac{1}{2}$. Evidently, this distorts the partition of surplus Z in favor of interest-receiving player f at the expense of b . More generally, even when $\delta_f \neq \delta_b$, we get

$$[15] \quad s_b S + s_f S = S + P \frac{\ln \delta_b - \ln \delta_f}{\ln \delta_f + \ln \delta_b} - P \frac{\ln \delta_b - \ln \delta_f}{\ln \delta_f + \ln \delta_b} = S$$

In other words, when compared to the canonical case without interest-bearing assets, surplus S is unchanged (it is the gain when a deal is struck), and interest-bearing assets only modify the partition.

3.4 The outside option principle with a loss threat

In this paragraph and the followings, I consider extensions of the outside option principle. As a principle, it cannot be “proven” and actually, [Binmore, Shaked and Sutton \(1989\)](#) do not try to prove it but rather show its empirical relevance. Still, it is clear that the underlying reason for the outside principle is the lack of subgame-perfect strategy supporting the threat to use the option when it is not “binding”, that is when the PEP provides more to the option holder. Thus, in this paragraph and the followings, I examine the extent to which the threat of using an outside option is supported by a subgame-perfect strategy.

First, I consider a special case of the outside principle: what happens when calling the outside option incurs a loss on the other bargainer? Such a situation typically occurs when the outside option of a creditor consists in filing for the debtor's bankruptcy, since then, the debtor will suffer (at least) the administrative costs of bankruptcy. The answer is: the outside option principle fully applies and if the outside option is not binding, a bargainer cannot get more by threatening to deal himself out.

I consider the case in which the PEP provides bargainer b with the utility (or amount) P^b and bargainer f with the utility $S_f = S - S_b$ where S is the surplus to be shared between b and f . I assume that P^b is higher than the utility b can get by playing the "deal-me-out" card:

$$[16] \quad Out_b < S_b$$

When b chooses the outside option, he incurs a specific positive loss F on f :

$$[17] \quad Out_f = S_f - F$$

The question is: can b exert pressure on f and extort more than S_b by threatening f to implement the outside option? To answer this question, I refer to [Binmore, Shaked and Sutton, \(1989\)](#), p. 756; "Deal-me out" means using the outside option): "how can a bargainer use his outside option to gain leverage? By threatening to play the deal-me-out card. When is such a threat credible? Only when dealing himself out gives the bargainer a bigger payoff than dealing himself in. It follows that the agreement that would be reached *without* outside options is *immune* to deal-me-out threats, unless the deal assigns one of the bargainers less than he can get elsewhere."

Principle A: A player with a non-binding outside option encompassing the threat of loss to the other player can neither extort strictly more than the option nor affect the other player's actions.

Proof:

Here, by assumption, $Out_b < S_b$, and $[S_b, S_f]$ is the only PEP. If b offers a partition $[\tilde{S}_b, \tilde{S}_f]$ with $S_b < \tilde{S}_b$ (and consequently $\tilde{S}_f < S_f$) and threatens to implement the outside option if f rejects this offer, then:

- f is willing to reject the offer as $\tilde{S}_f < S_f$
- if f rejects the offer, b is worse off implementing the threat than yielding as $Out_b < S_b$.

That is, the threat is not subgame-perfect. Thus, b can neither extort anything nor affect f 's actions by threatening to incur a loss on f . QED.

In other words, the outside option principle fully applies in this case.

3.5 The outside option principle with a binding option

Now, I consider the case in which the outside option provides b with more utility than the PEP determined by the surplus, discount factors, and interest-bearing or interest-costing assets. Such an

option can be labeled as “binding” by making an analogy between the option and a condition. The questions are: can the bargainer who has this option gain more than this option, and can he affect his opponent? The answer is no because

$$[18] \quad S_b < Out_b$$

Can b get more than Out_b ? We must distinguish the case in which f makes the first offer from that in which b makes the first offer. For simplicity, I assume that the surplus is 1, so that $s_f = S_f$ and $s_b = S_b$ with $s_f = 1 - s_b$

Principle B: Even when an outside option is binding, the player who has this outside option can neither get more than the outside option nor affect the other player's actions.

Proof: (I assume w.l.o.g. that player b has the outside option. The other player is f).

- *When b makes the first offer:*

I assume that player b offers $s_b = s^$ with $Out_b < s^*$. If b can get s^* , this means that $s^* \leq M^*$ where M^* is the supremum of what b can get at the first step. Thus:*

$$[19] \quad Out_b < M^*$$

First step: f accepts the offer $[s^, 1 - s^*]$ iff he cannot get more by rejecting and making a counteroffer at the next step ($T=\Delta$). b accepts this counteroffer iff he cannot get more by rejecting and making a counteroffer at the next step ($T=2\Delta$). As past actions are irrelevant, M^* is also the supremum of what b can get at $T=2\Delta$.*

Second step (Sutton method): As M^ is the supremum of what b can get at $T=2\Delta$. This means that at $T=\Delta$, b accepts any offer at least equal to $\delta_b^\Delta M^*$ so that f can get at least $1 - \delta_b^\Delta M^*$. That is, at $T=0$, f can get at least $\delta_f^\Delta (1 - \delta_b^\Delta M^*)$ and b can get at most $M_{b,0} = 1 - \delta_f^\Delta (1 - \delta_b^\Delta M^*)$. However, by definition of M^* , $M_{b,0} = M^*$. Thus:*

$$[20] \quad M^* = 1 - \delta_f^\Delta (1 - \delta_b^\Delta M^*)$$

that is,

$$[21] \quad M^* = \frac{1 - \delta_f^\Delta}{1 - \delta_f^\Delta \delta_b^\Delta}$$

with $\Delta \rightarrow 0$:

$$[22] \quad M^* = \frac{\ln \delta_f^\Delta}{\ln \delta_f^\Delta + \ln \delta_b^\Delta}$$

From [Sutton, 1986](#), we recognize the PEP. Then, [AY-4] contradicts the assumption that the outside option is binding. QED.

- When f makes the first offer:

Repeating the second step of the “ b makes the first offer” case and replacing $T=0$ (resp. $T=\Delta$, resp $T=2\Delta$) with $T=\Delta$ (resp. $T=2\Delta$, resp. $T=3\Delta$) shows that at $T=\Delta$, b cannot get more than M^* .

Then at $T=0$, f makes an offer $[\delta_b^\Delta M^*, 1 - \delta_b^\Delta M^*]$ and b accepts it. Thus, b cannot get more than $\delta_b^\Delta M^*$, which is lower than M^* . QED.

The proof extends directly to interest-bearing or interest-costing assets.

3.6 Binding option with a loss threat: a limit to the outside-option principle

A distinct question is: what happens when an outside option is binding *and* induces a loss to the other player? The proof above for the loss threat case relies on the fact that the outside option is not binding.

When b chooses the outside option, it incurs a positive loss F on f :

$$[23] \quad (S^f | Out_b) = P^f - F$$

Additionally, b 's option is binding

$$[24] \quad S_b \leq Out_b$$

Principle C: A player who has a binding outside option that includes a positive loss threat can affect the other player's actions and extract strictly more than this outside option. However, the additional amount is positive but can be made arbitrarily small by the other player.

Proof (I assume w.l.o.g. that $S = 1$, b has the option, and f is the other player):

- When f makes the first offer:

First step (b can affect f 's action and get strictly more than Out_b)

I assume that the maximum M^ that b can get is the outside option:*

$$[25] \quad M^* = Out_b$$

Under [24] and [38] and since $\delta_b^\Delta < 1$, implementing the following strategy is clearly subgame-perfect for b : (1) b accepts any offer $[\widehat{s}_b, 1 - \widehat{s}_b]$ when $Out_b < \widehat{s}_b$; (2) b calls the outside option when $\widehat{s}_b < Out_b$.

However, if b implements this strategy, f suffers the loss F if he offers $\widehat{s}_b = Out_b$ but can avoid this loss by offering $Out_b + \varepsilon$ rather than \widehat{s}_b , with $0 < \varepsilon$. This deviation is profitable for f as long as $0 < \varepsilon < F$. Thus, b can get $Out_b + \varepsilon$, contradicting the assumption [38].

Second step (f can make the additional amount granted to b arbitrarily small)

I assume that f cannot make ε arbitrarily small. Then, there exists $\bar{\varepsilon} > 0$ such that b can get $Out_b + \bar{\varepsilon}$. The following strategy is subgame perfect for b : (1) b accepts any offer $[\widehat{s}_b, 1 - \widehat{s}_b]$ when $Out_b + \bar{\varepsilon} \leq \widehat{s}_b$; (2) b calls the outside option when $\widehat{s}_b < Out_b + \bar{\varepsilon}$ ¹³.

If f offers $Out_b + \bar{\varepsilon}/2$, b is worse off calling the outside option rather than accepting this offer. Thus, the strategy described above is not subgame-perfect for b ¹⁴ (noteworthy, I do not use the condition that $\Delta \rightarrow 0$).

- When b makes the first offer:

From the “ f makes the first offer” case, f knows that if he rejects b 's offer at $t = 0$, the supremum (which is not a maximum) of what he can get at $T = \Delta$ is $1 - Out_b$. Thus, implementing the following strategy is subgame-perfect for f : (1) f accepts any offer in which he gets at least $\delta_f^\Delta(1 - Out_b)$ (2) f rejects any offers in which he gets strictly less than $\delta_f^\Delta(1 - Out_b)$.

Then, b offers $[1 - \delta_f^\Delta(1 - Out_b), \delta_f^\Delta(1 - Out_b)]$, f accepts and b gets $1 - \delta_f^\Delta(1 - Out_b)$, which is higher than Out_b . Indeed, $1 - \delta_f^\Delta(1 - Out_b) - Out_b = (1 - \delta_f^\Delta)(1 - Out_b) > 0$.

When $\Delta \rightarrow 0$, $(1 - \delta_f^\Delta) \rightarrow 0$ and, therefore, $1 - \delta_f^\Delta(1 - Out_b) \rightarrow Out_b$; thus, the additional amount that b can get tends towards zero. QED

4. Results

To solve the model backwards, I consider two cases that constitute outside options for f and b during the second-period loan negotiation: default (§ 4.1) and repayment with a loan from a new bank (§ 4.2). Then, I study the negotiation (§ 4.3) and identify sufficient conditions for hold-up by the firm (§ 4.4).

To simplify computations and facilitate comparison with most of the literature, I assume that firms' information costs are null ($c_f = 0$). Screening/monitoring costs are non-negative and perfect bank competition/symmetric information corresponds to $c_s = 0$. I must compute various “fair” rates that enable a bank to break even, depending on fixed costs and loan size.

4.1 Post-default operations

After a default, the firm no longer has to repay the loan. b has seized the asset produced by f over the first period but cannot use it: an entrepreneur/firm is needed to use it and produce. Since the asset is f -specific, what another firm produces with this asset is lower than y (what f can produce). This should

¹³ I assume that $Out_b + \bar{\varepsilon}$ is a maximum; if it is a supremum and not a maximum, \leq is replaced with $<$ in (1) and $<$ is replaced with \leq in (2)

¹⁴ Notably, b can also reject the offer and make a counteroffer. However, I only show that the strategy described is not subgame perfect. Under the assumption that $\bar{\varepsilon}$ exists, this strategy would be subgame perfect.

open the way to a bilateral negotiation to share a surplus between f and b ; then, the collateral value would not be defined *ex-ante* as Γ^1 but would be the negotiated value.

This case would rely on the assumption that after a petition had been filed, the court would not do the job properly, and f and b would keep bargaining directly. Here, I consider a more traditional case: once default is delivered, a court takes over the process and organizes an auction, preventing direct bargaining between f and b and ensuring a competitive outcome in which b always gets Γ^1 ¹⁵.

If the project fails in the first period, the firm retains only the net private benefits of default, $PB^1 - F$ (see § 2.4). Conversely, if the default is strategic (implying that the project was successful over the first period) and the asset is f -specific, f values the asset more than other potential buyers. Then, f can buy the asset back¹⁶ at the competitive price Γ^1 during the auction process¹⁷. Buying the asset back requires borrowing $\Gamma^1 - PB^1 + F$ ¹⁸ at the "fair" rate $r_X^{2,f}$. Then, f gets the net income:

$$[26] \quad Z_f = PB^1 - F + \frac{1}{1+r_2} \left\{ (1 - p_X^2)(y - (\Gamma^1 - PB^1 + F)r_X^{2,f}) + p_X^2(PB^2 - F) \right\} - \Gamma^1$$

where $\frac{1}{1+r_2}$ recalls that if r_2 (the second-period riskless rate in financial markets) were not almost null, revenues at T^2 should be discounted¹⁹. For simplicity, I assume that r_2 is arbitrarily small. Then²⁰:

$$[27] \quad r_X^{2,f} = \frac{p_X^2(\Gamma^1 - PB^1 + F - \Gamma^2)}{(\Gamma^1 - PB^1 + F)(1 - p_X^2)} + \frac{c_s}{(\Gamma^1 - PB^1 + F)(1 - p_X^2)}$$

The last term means that the fixed screening cost is distributed over the second-period loan ($\Gamma^1 - PB^2 + F$) and that the bank repays her own credit even if the firm defaults ($1 - p_X^2$). Finally:

$$[28] \quad Z_f = PB^1 - F + (1 - p_X^2)y - (\Gamma^1 - PB^1 + F)p_X^2(\Gamma^1 - PB^1 + F - \Gamma^2) - c_s + p_X^2(PB^2 - F) - \Gamma^1$$

¹⁵ It is usually considered optimal that the court organize a second-price auction so that f reveals his true value (but pays only Γ^1 , the value of the asset to other entrepreneurs). Alternatively, here, in case of first-price auction one may assume that f knows his competitors' valuation and f bids $\Gamma^1 + \epsilon$ (with ϵ positive but arbitrarily small).

¹⁶ This case may be particularly realistic if at the previous step, the bank had decided not to roll over the loan (or to offer an unacceptable rate) and to fill a petition for bankruptcy. The firm may disagree and attempt to obtain the asset back (*e.g.* by borrowing from another bank), even after the bankruptcy petition.

¹⁷ One may wonder if a third party would buy the asset (at a price higher than the competitive price) to negotiate with f . Actually, a third party would not accept to pay more than the competitive (*i.e.*, non-specific user) price. Indeed, at equilibrium, most defaults may be non-strategic. Crucially, the "illiquid firm hold-up" mechanism described in this paper is off-equilibrium path, and many banks would have to diverge from equilibrium path simultaneously for the situation described here to become relevant.

¹⁸ The firm must pay Γ^1 to get the asset back and has $PB^1 - F$ from the default.

¹⁹ If defaults happened uniformly over period 2 instead, the discount factor would rather be close to $\frac{1}{1+r_2/2}$.

²⁰ Here, $r_X^{2,f}$ is computed for a loan of size $\Gamma^1 - PB^1 + F$, collateral value Γ^2 and information costs c_s .

The firm must borrow from a bank to pay the initial bank. However, the initial bank has a price advantage for such a loan because it has already borne the screening costs. *Ex-post*, it is optimal neither for the firm to borrow from another bank nor for the bank to decline lending. Thus:

Result A: If the firm must borrow to buy the asset back, it borrows from the initial bank at rate $r_X^{2,f}$, and the bank obtains the collateral value Γ^1 and the profit on the second-period loan c_s :

$$[29] \quad Z_b = \Gamma^1 + c_s$$

4.2 The other bank option

A^2 is the amount borrowed over period 2, *i.e.* from T^1 to T^2 , if f repays the first-period loan. As no production occurs over period 1, A^2 covers the repayment of the (unit) principal and period-1 interest:

$$[30] \quad A^2 = 1 + r_W^1$$

where r_W^1 is the first-period interest rate with $W = G$ if f was *ex-ante* relatively safe over the first period and $W = B$ if f was relatively risky.

f has the option to borrow from another bank. Depending on whether being good or bad, f could borrow at the *ex-post* "fair" price:

$$[31] \quad r_X^{2,o} = \frac{A^2 r + p_X^2 (A^2 - \Gamma^2) + c_s (1 + r)}{(1 - p_X^2) A^2} = \frac{(1 + r_W^1) r + p_X^2 (1 + r_W^1 - \Gamma^2) + c_s (1 + r)}{(1 - p_X^2) (1 + r_W^1)}$$

where superscript o denotes the outside option, $X = G$ if the firm is relatively safe in the second period, and $X = B$ if it is risky.

Proof:

The expected income of the new bank at T^2 must cover the amount this bank must repay, that is $(A^2 + c_s)(1 + r)$. Given bank competition, it cannot be higher. Thus:

$$(A^2 + c_s)(1 + r) = A^2 (1 - p_X^2) (1 + r_X^{2,o}) + p_X^2 \Gamma^2$$

$$(1 + r_X^{2,o}) = \frac{(A^2 + c_s)(1 + r) - p_X^2 \Gamma^2}{A^2 (1 - p_X^2)}$$

$$r_X^{2,o} = \frac{(A^2 + c_s)(1 + r) - p_X^2 \Gamma^2 - A^2 (1 - p_X^2)}{A^2 (1 - p_X^2)} = \frac{A^2 r + p_X^2 (A^2 - \Gamma^2) + c_s (1 + r)}{A^2 (1 - p_X^2)}$$

QED.

Since r_2 is arbitrarily small to simplify the analysis, $r_X^{2,o} = \frac{p_X^2 (A^2 - \Gamma^2) + c_s}{A^2 (1 - p_X^2)}$ where $p_X^2 (A^2 - \Gamma^2)$ is the expected cost of firm's default for the bank. In the denominator, $(1 - p_X^2)$, which inflates $r_X^{S,o}$, reflects bank refunding its own creditor(s) even if the firm defaults.

The second period “fair” interest rate *for the initial bank*, $r_X^{2,F}$, corresponds to the case where $c_S = 0$. It can be deducted directly from the second-period zero profit condition for the initial bank: $0 = (1 - p_X^2)A^2r_X^{2,F} + p_X^2(\Gamma^2 - A^2)$, which gives $r_X^{2,F} = \frac{p_X^2(A^2 - \Gamma^2)}{A^2(1 - p_X^2)}$

Thus:

$$[32] \quad r_X^{2,O} \geq r_X^{2,F}$$

If f borrows from another bank to repay the loan, he gets:

$$[33] \quad Y_f = \frac{1}{1 + r_2} \{ (1 - p_X^2)(y - (1 + r_W^1)r_X^{2,O}) + p_X^2(PB^2 - F) \}$$

Assuming, as before, that r_2 is null (or positive but arbitrarily small), we get:

$$[34] \quad Y_f = (1 - p_X^2)y - p_X^2(1 + r_W^1 - \Gamma^2) - c_S + p_X^2(PB^2 - F)$$

4.3 The second-period credit negotiation: perfect equilibrium partition and outside options

The bank and the firm must bargain for the potential surplus, given a bilateral monopoly at T^1 . Agents are forward-looking and fully rational, and *ex-ante* contracts are not enforceable (§ 5.3); thus, a dynamic inconsistency exists, and agents bargain rather than implement *ex-ante* contracts.

Here, I consider the negotiation of second-period loan at T^1 . At T^1 , both bargainers learn if the first-period investment is successful (no exogenous failure) and know the second-period PD. During the negotiation, failure is either the bank's or the firm's decisions because according to bankruptcy law, either a creditor or the debtor must fill the petition for bankruptcy. This simplifies the analysis. Real-life characteristics correspond to a classical bargaining framework. For simplicity, I assume that the firm has fully paid other creditor (suppliers), so that these other creditors will not interfere with the negotiation process. If they did, b and f would face an exogenous risk of breakdown, which would make the analysis slightly more complex ([Binmore, Rubinstein and Wolinsky, 1986](#)).

I assume alternative offers bargaining ([Rubinstein, 1982](#)). Bargainers can opt out after rejecting their opponent's offer. In the present framework of loan negotiation, it is realistic to assume that the time Δ between a rejection and a counteroffer tends towards zero. Either f and b agree on the interest rate for the new loan, or one of them exercises an outside option.

The surplus in the second-period loan negotiation

Since the project is profitable, the surplus becomes:

$$[35] \quad S_{raw}^{nego} = (1 - p_X^2)y + p_X^2(\Gamma^2 + PB^2 - F)$$

This surplus is obtained at the end of the second period, that is over a period of length “one” after an agreement. S_{raw}^{nego} is the certainty equivalent to the surplus at that time. That is, by committing to

repay the actual end-of-period income (y or $\Gamma^2 + PB^2 - F$), the bank and the firm could borrow $S_{raw}^{nego} / (1 + r_2)$ at the time of the agreement from risk averse financial markets. And since r_2 is arbitrarily small, S_{raw}^{nego} is the surplus at the time of the agreement.

From this raw surplus, the bank and firm must deduct what is paid to the outside financial system: $S_{net}^{nego} = S_{raw}^{nego} - NFP^{nego}$ where NFP denotes net payments to the outside financial system (other banks and financial markets). The objective is to ensure that these various components are constant so that S_{net}^{nego} is constant, and the classical bargaining framework applies.

From **Result A**, the amount paid to the outside financial system is the (limited) amount paid by b to financial markets at the riskless rate, which is arbitrarily small. Therefore, $NFP^{nego} \approx 0$ and:

$$[36] \quad S_{net}^{nego} \approx S_{raw}^{nego} = (1 - p_X^2)y + p_X^2(\Gamma^2 + PB^2 - F)$$

Discounting factors

During the bargaining that starts at T^1 , b and f face the same risk-free interest rate r_2 . This result is clear as regards b , which borrows at this rate in financial markets.

Since the bargaining process has a stationary structure, f receives the same equilibrium profit whenever a deal is struck. This profit will be obtained at the end of the second period, which lasts "one" after the deal. But under the simplifying assumption that the second-period riskless interest rate is constant (actually, stationary would suffice), the discounted value of this profit at the time of the deal is independent of this time. Finally, the value of this profit at agreement time is discounted at T^1 (the start of the bargaining) at this same rate r_2 , which is what was to be shown. In other words, and in particular, the interest rate paid by f to b over the second period, after the negotiation, is an outcome of this negotiation and not an input that affects the choice of f . Finally, the risk margin does not enter the discounting factor because what matters for f is the certainty equivalent.

Interest-bearing or interest-costing assets in the second-period loan negotiation

f has neither interest-bearing nor interest-costing assets during negotiations. The absence of interest-costing assets is unintuitive as f is b 's debtor. But any amount paid by f to b is part of the negotiation, and agents are forward-looking, so that any previous commitment is irrelevant. In other words, what f pays b is an outcome of the bargaining process and not an input in the process.

Since f does not pay interest on his debt to b , b has no interest-bearing assets. Conversely, b must pay interest at the risk-free interest rate r_2 (although b initially borrowed at the first-period risk-free rate r_1) on the first-period loan principal (which is 1), since b borrows from financial markets to lend to b .

Perfect-Equilibrium Partition

Since the discounting factors δ_b and δ_f are equal, the formula in § 3.3 simplifies, so that at the perfect-equilibrium partition, b and f respectively obtain S_b and S_f with:

$$[37] \quad S_b = 0.5[S_{net}^{nego} - 1] = 0.5[(1 - p_X^2)y + p_X^2(\Gamma^2 + PB^2 - F) - 1]$$

$$[38] \quad S_f = 0.5[S_{net}^{nego} + 1] = 0.5[(1 - p_X^2)y + p_X^2(\Gamma^2 + PB^2 - F) + 1]$$

Determination of the outside option of the firm in the second-period loan negotiation

f may either file for bankruptcy and obtain Z_f or borrow from another bank to repay the initial bank and get Y_f ²¹. The potentially binding option (i.e., the most profitable for f) must be determined first.

$$Z_f > Y_f \leftrightarrow$$

$$[39] \quad PB^1 - F + (1 - p_X^2)y - (\Gamma^1 - PB^1 + F)p_X^2(\Gamma^1 - PB^1 + F - \Gamma^2) - c_S + p_X^2(PB^2 - F) \\ - \Gamma^1 > (1 - p_X^2)y - p_X^2(1 + r_W^1 - \Gamma^2) - c_S + p_X^2(PB^2 - F)$$

i.e.

$$[40] \quad Z_f > Y_f \leftrightarrow (PB^1 - F - \Gamma^1)(1 + p_X^2(\Gamma^1 - PB^1 + F - \Gamma^2)) + p_X^2(1 - \Gamma^2) + p_X^2 r_W^1 > 0$$

f prefers default to repayment if:

- private benefits over the first period (PB^1) increase (the firm is incentivized to default);
- failure costs decrease;
- first-period rate r_W^1 is higher. Due to liquidity constraints (no production over period 1), it increases the amount that the firm must borrow for the second period on which it must pay interest;
- first-period collateral value Γ^1 is lower as f buys this asset back at that price in case of default;
- second-period collateral value Γ^2 is lower. Indeed, lower collateral increases the other bank's option interest rate, $r_X^{S,O}$, which makes this option less interesting.

Information costs do not affect a firm's choice of the optimal outside option. It is clear that these costs are paid by f if he borrows from another bank to repay the initial bank. But if he files for bankruptcy, he will eventually buy the asset back – actually from the initial bank, who will extract the information costs as a profit; see § 4.1).

Interestingly, one could imagine that a higher y would make the “other bank” option comparatively more attractive for f . This is actually not the case since in case of default, if y is high enough, f will buy the asset back and produce (with probability $1 - p_X^2$, as in the “other bank” case).

Determination of the outside option of the bank in the second-period loan negotiation

For b , there exists two potential outside options:

- (1) filing for bankruptcy and get Γ^1 ;
- (2) declining to roll over the first period loan.

²¹ *Ex-post*, f actually borrows from the initial bank since, if f can borrow from another bank at rate $r_X^{2,f}$, it is mutually beneficial for f and b to make a loan contract at rate $r_X^{2,f} - \varepsilon_X^{2,f}$ with $\varepsilon_X^{2,f}$ positive but arbitrarily small.

What b gets in the second case depends on f 's reaction. If f chooses to default, then b gets Γ^1 . Conversely, if f is better off borrowing from another bank, then the initial loan is repaid and b gets $1 + r_W^1$. Thus, the outcome for b of declining to roll over depends on whether [40] holds. If [40] holds, b has a weak outside option (2) which amounts to filing for bankruptcy (*i.e.*, option (1)). Conversely, if [40] does not hold and the firm is better off borrowing from another bank than defaulting, b has a strong outside option and is always fully repaid, even if the PEP does not provide so. Thus, [40] is labeled “weak option condition”.

4.4 Hold-up by the firm

In the present sub-section, I identify the sufficient conditions for hold-up by the firm. I focus on the case in which $S_b < \bar{L}$ where S_b is the amount (or utility) accruing to b at the PEP in the negotiation that occurs at T^1 , and \bar{L} corresponds to the value of repaying the first-period debt (capital and “fair” interest), borrowing from the initial bank at a “fair” second-period rate and repaying at the second period. In total, this means repaying the capital and interest of the first-period loan (interest and capital) and the interest of the second-period loan.

$$[41] \quad \bar{L} = 1 + r_W^{1,F} + (1 - p_X^2)(1 + r_W^1)r_G^{2,f}$$

where $r_W^{1,F}$ is the “fair” first-period rate that covers the expected expenses borne by b over the first period, including screening costs:

$$[42] \quad r_W^{1,F} = \frac{p_W^1(\Gamma^1 - 1) + c_S}{1 - p_W^1}$$

and $r_G^{2,f}$ is the “fair” second-period rate that covers the expected b 's expenses incurred (screening costs are borne in the first period and not afterwards) when the firm is good over the second period:

$$[43] \quad r_G^{2,f} = \frac{p_G^2(1 + r_W^{1,F} - \Gamma^2)}{(1 - p_G^2)(1 + r_W^{1,F})}$$

Since $r_G^{2,f} < r_B^{2,f}$, $S_b < \bar{L}$ and [41] ensure that *ex-post*, irrespective of whether f is in state G or B, f will repay the initial loan only if b grants a second-period rate at which she suffers a loss.

Hereafter, I assume that $c_S = 0$, both for simplicity and to show that the mechanism described here does not rely on screening costs. Thus, I define the “fair” first-period rate without screening costs:

$$[44] \quad r_W^{1,f} = \frac{p_W^1(\Gamma^1 - 1)}{1 - p_W^1}$$

Again, I consider what happens when the PEP provides b with a share S_b lower than \bar{L} , *i.e.*:

$$[45] \quad 0.5 \left[\frac{(1 - p_X^2)y + p_X^2(\Gamma^2 + PB^2 - F)}{1 + r_X^2} - 1 \right] \leq 1 + r_W^{1,F} + (1 - p_G^2)(1 + r_W^1)r_G^{2,f}$$

Below, I show that when [45] holds, f repays the first-period loan but forces b to grant an abnormally favorable rate on the second-period loan. It may be useful to recall that b can neither force f to repay the first-period loan nor fix the conditions for the second-period loan. From a game-theory perspective, b 's negotiating power is not sufficiently high. From a legal and economic viewpoint, the first-period contract is incomplete: f can (falsely) pretend that the first-period interest rate was "too high" and the current risk is low to justify a low second-period rate. As discussed below (§ 5.3) Loan contracts are not fully enforceable at T^1 but would be fully enforceable at T^2 because f is illiquid at T^1 but not at T^2 . This is discussed in § 5.2.

If b played the outside option and filed for f 's bankruptcy, f would incur a loss F due to the administrative costs of bankruptcy (even if f buys the asset back at the price Γ^1). Can b leverage her outside options to affect f 's actions? I consider two subcases of the " $S_b < \bar{L}$ " situation.

First, I consider the case $Out_b < S_b < \bar{L}$. In this case, b 's outside option (filing for f 's bankruptcy) affords b less than the PEP, S_b , induced by b 's and f 's negotiating powers, which itself brings less than the "fair" amount necessary for b to cash in on. Then, from **Principle A** (§ 3.4), b cannot affect f 's actions. In other words, during the negotiation that starts at Γ^1 , b cannot credibly threaten to implement her outside option and will rather roll over the loan at a loss.

The intuition is as follows: at the PEP determined by b 's and f 's negotiating powers, b accepts an expected loss on the second-period loan to avoid a larger loss on the first-period loan.

This case constitutes a hold-up by the firm, which profits from condition [45] and repays the first-period loan but extorts a second-period interest rate below the fair interest rate which would cover the expected expenses of the bank. Consequently, *ex-ante*, the bank will decline to lend: productive projects are not financed; in other words, firm's hold-up induces credit constraints if the bank is unable to lend long and can only offer one-period loans.

Now, I consider the case $S_b < Out_b < \bar{L}$. I focus on the case in which b has a **weak outside option** because f is better off defaulting when b declines to roll over (weak option condition).

Since $S_b < Out_b$, b 's outside option is binding and induces a specific loss. From § 3.6, b can therefore affect f 's actions, so we must consider the possibility that b affects the first- and/or second-period interest rate to recoup her costs. This outside option also relates to the financial literature, which relies on the possibility of the bank seizing assets to force the firm to repay the first-period loan.

Then, from **Principle C**, b forces f to take out a second-period loan at a rate such that b gets $Out_b + \epsilon$, where f can make ϵ arbitrarily small. That is, b gets only (ϵ more than) the two-period income corresponding to the weak outside option (that amounts to filing for the firm's bankruptcy). Then, unless the first-period collateral value fully covers the first-period loan ($\Gamma^1 = 1$), b incurs a loss. Since this is fully predictable, b declines to lend short originally. Once again, if b is unable to lend long, it induces credit constraints, and some productive projects are not financed.

In other words, for a series of productive projects defined by [45], hold-up by firms may occur and prevent banks from lending short to finance long-term projects. This happens either because the bank's outside option is not binding (outside option principle) or, if the bank's outside option is binding (and includes a loss threat to the firm, namely failure costs), because the weak option conditions holds and the collateral value does not fully cover the first-period loan.

Discussion. One may wonder whether the bank may actually accept to lose money on the new loan to get the initial loan repaid. A (wrong) objection would be that the bank's option to file for the firm's bankruptcy would increase her negotiating power and avoid an $S_b < \bar{L}$ situation.

From a game-theoretic perspective, such a misleading view confuses an outside option with a no-deal situation, itself considered as a lower bound on the computation of the equilibrium partition in the "split-the-difference" approach of cooperative bargaining. But from the non-cooperative game-theoretic literature reminded in § 3.2, an outside option (here, recovering the collateral value by filing for f 's bankruptcy) is **not** the lower bound that determines what b would get at the PEP if f had all the bargaining power. This lower bound is rather zero, which explains why b does not break even if f has enough negotiating power. A related but different misconception is that b could leverage her outside option to bargain and eventually get more than this outside option. Actually, that would directly contradict the outside option principle and its extensions described in Section 3.

From an economic perspective, agents are rational and forward looking. Thus, the agreement at T^0 to repay the loan has no value at T^1 . f will not repay if refusing is optimal, and, more generally, his actions will not be determined by his past commitment but rather by optimal choice on future actions. In the words of the property-rights theory ([Grossman and Hart, 1986](#), [Hart and Moore, 1990](#)) and its application to credit ([Hart and Moore, 1998](#)), the loan is, for the bank, a sunk investment cost. Additionally, the lending bank has SITG when the loan must be rolled over. The empirical literature ([Caballero et al., 2008](#)) shows that banks are ready to lose money on new loans to avoid losses on existing loans. That is, in various bargaining processes that occur during loan renewal, a bank's no-loss condition on new loans is definitely not a lower bound.

One may also wonder whether the bank can avoid a loss by playing on the first-period rate. By reducing r_W^1 , b may indeed reduce the right hand side of [45], potentially avoiding the dynamic loss case. However, if b reduces r_W^1 below $r_W^{1,F}$, she suffers an expected loss over the first period and must require a rate higher than the competitive one over the second period. If screening costs are low enough, another bank will then make a better offer, and b will not recoup this loss with a second-period profit. Conversely, if b requires a higher r_W^1 , the incentive for f to default at T^1 increases, and, more generally, [45] is more likely to hold so that *ex-ante*, b is even more willing to decline to lend short²².

5. The nature of frictions

I use a straightforward model to identify a bare minimum set of frictions that can induce hold-up by firms. The usual ingredients of credit constraints models, such as risk aversion, liquidity shocks, debt overhang, creditors' coordination issues, implicit options, adverse selection, and moral hazard, are assumed away, and information asymmetries play only an indirect role. Regarding the relevant frictions, hold-up can occur at T^1 but not at T^2 because the firm's liquidity improves over period 2 (income accumulated over period 2 sales is available and versatile at T^2). **Illiquidity** at T^1 flows not only from delayed funds availability but also from asset specificity at T^1 . Still, as noted by [Caballero and Hammour \(1998\)](#), "specificity acquires a more troublesome dimension when combined with contracting difficulties". Here, the **incompleteness** of short-term loan contracts flows from the unverifiability of credit risk by a court; this is unusual as a foundation for incompleteness, but is supported by the literature on banks' specificity and by institutional evidence. But first, hold-up is based on the possibility of **strategic default**, and I provide empirical evidence for such defaults, even when risk-inducing moral hazard is assumed away. Finally, I discuss the respective roles of information asymmetries and SITG.

5.1 Strategic defaults: net bankruptcy benefits rather than risk-taking

The hold-up mechanism identified above relies on the possibility of firms strategically defaulting. However, I assume away moral hazard and risk-taking because, strategic defaults apart, the credit risk is fully exogenous. Also, according to the weak option condition [40], the strategic default decision depends directly on the net private benefits in the case (and only in the case) of bankruptcy.

²² The fact that measures taken at T^0 to compensate for future losses actually backfire is a form of dynamic inconsistency. Also, in a previous draft, I studied the interaction of first and second-period rates and showed that, unless the first-period loan was fully covered, there was no equilibrium combination of both rates. This is no longer necessary in the present draft, based on non-cooperative bargaining which makes it possible to directly identify conditions for inconsistency. Still, studying the interactions of first and second-period rates could be interesting when the firm repays the bank if "good" over period two.

[Schoenherr and Starmans \(2022\)](#) find that lower failure costs (*i.e.*, higher net bankruptcy benefits) induce more risk-taking and eventually more failures²³, but fail to support the present model because I assume away risk-taking. [Giroud et al. \(2012\)](#) show that entrepreneurs can divert assets over several years, which provides evidence for private benefits but fail to differentiate between default and non-default cases; such a distinction is crucial for the weak option condition [40].

I consider a natural French experiment that affects the managers' failure costs and private benefits. Until September 2018, managers with two failures over the previous five years were "flagged" by the French Central Bank (the flag was available to all banks). On October 1, 2018, this flag was suppressed, and, since then, managers have been flagged only if they have experienced three failures over the previous five years. If net private benefits are relevant to default, the number of managers with two failures relative to those with three failures should have increased after this legal change.

This ratio increased very quickly at that time, over three months, which is not consistent with a risk-taking channel that would induce default after a while, but rather with strategic failures directly linked to private benefits in case (and only in case) of defaults. Therefore, these empirical results are consistent with the assumption in the present model that firms can decide to default strategically based on private benefits, even if moral hazard issues and risk-taking are assumed away, which is innovative *w.r.t.* [Giroud et al. \(2012\)](#) and [Schoenherr and Starmans \(2022\)](#).

Graph 2: Ratio of two- to three-defaults managers



²³ This is especially clear in the first draft, [Schoenherr, 2017, "Managers' Personal Bankruptcy Costs and Risk-Taking"](#). In the published draft, this effect is balanced with the beneficial effects of stronger creditors' rights when firms are credit-constrained. [Rodano et al. \(2016\)](#) also provide interesting results.

5.2 Liquidity: from funds availability to assets specificity

f 's illiquidity is a crucial feature of this analysis. When discussing this issue, one must tackle at least three topics. First, liquidity versus solvency: f is productive (and therefore solvent) if the project is conducted over two periods, but if it is canceled at T^1 due to liquidity problems, f is no longer profitable. This explains why hold-up can occur at T^1 but not at T^2 . Second, liquidity as delayed availability of funds: this raises the question of why f does not borrow from another bank at T^1 to keep the project ongoing and reap a surplus. Third, the linkages between liquidity and specificity: this encompasses the firm-asset specificity, and the bank-firm specificity, the former causing the latter.

The distinction between liquidity and solvency is common for banks. Here, this distinction is used for firm f , which is productive (hence solvent) if the project is conducted until T^2 , but is illiquid at T^1 because he has borrowed short and must repay the initial loan before sufficiently producing to obtain the required cash. This is a crucial difference from the zombie lending literature (e.g. [Caballero et al., 2008](#)) which also considers banks' SITG to explain holdup by firms. Zombie firms are unproductive but can extract low rates from banks because their involuntary failures would hurt them. Here, productive but illiquid firms hold banks up by threatening to default strategically.

A second liquidity-related issue is why f declines to borrow from another bank to repay the first-period loan, keeping the productive project working and reap benefits. The answer is that f finds it more profitable to default if the weak option condition [40] is met. This crucial condition means, essentially, that private benefits are higher than the collateral value of the asset or that the project is hazardous over the second period. That is, f strategically exploits his own illiquidity while he could find liquidity if he wanted to. § 5.3 tells more about this mechanism.

Third, At T^1 , b can seize f 's asset, but it is unlikely to cover the debt one-for-one because it is affected by asset specificity ([Kermani and Ma, 2023](#)) and has a lower value for a new buyer than for the initial firm/entrepreneur f who built it for a specific purpose (the case in which the collateral value is lower than the debt value is the interesting case in which liquidation incurs a loss on the bank : e.g. [Diamond and Dybvig, \(1983\)](#)). Conversely, at T^2 , b can seize income y , which fully covers the two-period debt (for the distinction between collateral and going-concern values, see [Kermani and Ma \(2020\)](#)). The second-period income y is liquid, that is, it has the same value for all agents. This definition of liquidity, as opposed to specificity, comes from [Kiyotaki and Moore \(2001\)](#).

Finally, I consider the consequences of such illiquidity. Because f 's asset is worth less than the debt to the bank (owing to asset specificity), only f can fully repay b . In the words of property rights theory ([Grossman and Hart, 1986](#), [Hart and Moore, 1990](#)), by lending to f , b has made an f -specific investment. In other words, asset-firm specificity induces bank-firm specificity.

This bank-firm specificity induces a bilateral monopoly that forces negotiations at T^1 . Interestingly, the macroeconomic literature on specificity ([Caballero and Hammour, 1996](#), [Caballero and Hammour, 1998](#)) also identifies specificity (here bank-firm) as a key factor forcing bargaining, because classical market mechanisms no longer work²⁴. The above analysis explains why in more detail.

5.3 Exogenous default risk, unverifiability, and incompleteness

The incompleteness of future loan contracts, central to the present analysis, takes two forms. First, it is difficult to establish, *ex-ante*, an unconditional second-period loan contract. Second, the PD over the second period is unverifiable by a third party (typically a court).

The difficulty in drawing up an unconditional second-period loan contract at T^0 has two causes; one is model-specific and the other is more general and realistic. The former is that the information cost c_s is assumed to be a monitoring cost over the first period, such that b obtains information on f 's PD only at $T^0 + \varepsilon$. This simplifying assumption prevents negotiation at T^0 to focus on negotiation at T^1 , which is the core of the present study. If c_s was instead a screening cost, b would know the first-period PD T^0 , and negotiation would still occur at T^1 . Thus, this model-specific feature is not determining.

A more fundamental cause for incompleteness is that at T^1 , the second-period PD, p_X^2 , is not verifiable by a court. That is, f can pretend that b is trying to hold him up and justify his choice to default if b declines to roll over. Conversely, if a court could observe p_X^2 , it could enforce a "fair" loan contract.

The literature on bank specificity supports PD's unverifiability: if banks need special skills to analyze borrowers' PD, courts, which lack these skills, cannot evaluate PD. Also, even *ex-post*, failure or success provides incomplete information on *ex-ante* PD. More generally, the unverifiability of *ex-post* firm outcomes is a central feature of bank-firm relationships. This explains why debt contracts are not contingent on firms' outcomes ([Hart and Moore, 1998](#)) and why firms must pledge collateral to borrow (*e.g.* [Kiyotaki and Moore, 2001](#)). Overall, PD's unverifiability is weaker than the unverifiability assumption in [Hart and Moore \(1998\)](#). It is also immune to the critique of [Maskin and Tirole \(1999\)](#) for whom the costs of describing future states do not necessarily interfere with optimal contracting²⁵.

Crucially, if a liquid entrepreneur with a two-period loan contract defaulted at T^1 , he could not pretend that the bank had underestimated his risk. Indeed, although *ex-post* outcomes do not provide full information on *ex-ante* risk, a failure may discard the assumption that the bank was overestimating the risk. Thus, a liquid firm could not credibly hold up the bank.

²⁴ I thank Gilles Saint-Paul for these references

²⁵ More precisely, here, immunity to this critique is unrelated to information perturbations (unlike [Aghion et al., 2012](#)). I conjecture it is also unrelated to risk aversion (unlike [Segal, 1999](#) or [Hart and Moore, 1999](#)).

The fact that the second-period PD is known (by f and b) only at T^1 , (cf. equation [1]) is less problematic. This delay induces costs because the agents must list all potential cases (here just two), which is the most common mechanism support contracts incompleteness. The three-step solutions proposed by Maskin and Tirole (1999) may fully work here. A even simpler solution would be to implement [von Thadden \(1995\)](#) contracts, in which the interest rate is fixed *ex-ante* but the bank decides whether to roll over, and supplement such contracts with the obligation for the firm to take a loan (at pre-specified conditions) if the bank decide to roll over. Interestingly, if these modified von Thadden contracts were feasible, they would stipulate high rates over period 2, hence low rates over period 1 (by competition), and would therefore be observationally equivalent to hold-up by banks.

5.4 The role (or not) of information asymmetries

The credit constraint literature extensively uses asymmetric information ([Diamond, 1984](#), [Freixas and Rochet, 2008](#)). Unverifiability by a court is distinct from information asymmetries. [Hart and Moore 1998](#), p. 2, assume “symmetric information between the entrepreneur and investor both when the contract is written and once the relationship is under way. However, many of the variables [...] are assumed not to be verifiable by outsiders, *e.g.*, a court; hence contracts cannot be conditioned (directly) on these”.

Information asymmetries are crucial to support hold-up by banks but are not crucial in the SITG mechanism. Beyond the mechanism analyzed in this study, banks' SITG may constitute an alternative to information asymmetries to explain many classical empirical results in the relationship banking literature. First, banks' SITG may explain why firms with an existing bank relationship display a higher probability of obtaining credit ([Cole, 1998](#)): the bank has a stake in the firm and is therefore willing to keep it afloat. Second, the empirical literature has provided mixed evidence on the positive role of longer relationships, but to the extent that they improve access to credit, SITG could be part of the explanation. For instance, long-term loans may benefit from blanket liens and not only from asset collaterals (on this distinction, see [Kermani and Ma, 2020](#)). Thus, banks with long-term loans may be more interested in firms' going-concern value. Third, banks' SITG may also explain the effects of concentration and the number of relationships. [Petersen and Rajan \(1994\)](#) find a positive effect of the number of banks on interest rates. This is often attributed to less intensive monitoring; however, multiple banks also have less SITG and may rely on other banks to keep a borrower afloat. Fourth, [Hoshi et al. \(1990\)](#) find that distressed firms belonging to a large group with a main bank are more likely to invest and grow than their independent counterparts, especially with concentrated borrowing or if the main lender is also a large shareholder. These results are consistent with banks' SITG

interpretation²⁶. Fifth, [Ashcraft \(2005\)](#) uses OLS regressions and finds that (information-sensitive) banks' failures have a greater effect on the local economy than thrifts' failures. However, these results could also point to SITG mechanisms, that no longer work when a bank fails²⁷.

6. Conclusion

This paper provides extensions of the outside option principle and perfect equilibrium partitions in bargaining models with alternating offers. It applies these results to the bargaining that occurs when a loan must be rolled over. Indeed, the loan relationship is a bilateral monopoly only the firm can fully repay the bank. This analysis identifies a new mechanism, hold-up by illiquid firms, which discourages banks from lending short to finance long-term projects. It may even induce credit constraints and capital misallocation when banks are otherwise unable or unwilling to lend long. In addition to banks skin-in-the-game, three types of financial friction underlie this mechanism. First, a firm that defaults reaps specific private benefits; innovative empirical evidence shows private benefits *at the time of default*. Second, hold-up can occur after a short-term loan but not after a long-term loan because of the provisional borrower's illiquidity, which is linked to asset specificity; this link is known, but problems in enforcing loan contracts are usually related to liquid assets' diversion ([Hart and Moore, 1998](#)). Third, a court cannot verify a firm's probability of default: this is supported by the bank specificity literature and by institutional evidence.

Policy implications include fostering long-term bank lending, providing courts with independent external ratings, and adapting [von Thadden \(1995\)](#) contracts by forcing firms to take loans if banks roll over. Interestingly, such optimal contracts would stipulate high second-period rates and would be observationally equivalent to hold-up by banks.

This model is deliberately straightforward to focus on a new mechanism and identify the minimum required frictions. Combining this new mechanism with classical information asymmetry between banks and firms would be interesting. [Grossman and Perry \(1986\)](#) provide interesting results²⁸. This study assumes that other creditors have been refunded before bargaining; however, more agents can be considered. If other creditors are not part of the bargaining but can file for the firm's bankruptcy, the extension is straightforward ([Binmore, Rubinstein and Wolinsky, 1986](#), p. 181). Conversely, a bargaining process involving these creditors would raise specific technical issues. [Compte and Jehiel \(2010\)](#) provide an advanced approach for analyzing such situations. This is left for future research.

²⁶ For [Hoshi et al. \(1990\)](#) (pp. 85-86) "firms with financial structure in which free-rider and information problems are likely to be small perform better than other firms"; the free-rider problems are those avoided by banks' SITG.

²⁷ The 1996 Economic Growth and Regulatory Paperwork Reduction Act prohibits thrifts from lending more than 20% of their assets to corporations. Thus, banks failures are more likely to disrupt the production process.

²⁸ [Fudenberg and Tirole \(1983\)](#) examine bargaining with asymmetric incomplete information but with unilateral offers. [Rubinstein \(1985\)](#) extends bargaining to incomplete information on time preferences only.

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Appendix 1: Notations and abbreviations
in alphabetical order (Latin then Greek) - § of first occurrence

- A^2 : Amount that must be borrowed by the firm when rolling over a short-term loan at T^1 . § 4.2
- b : Agent b in Section 3. Otherwise, b denotes the initial lending bank. Section 1
- c_s : One-shot screening cost by a new bank. Fixed cost. Over period 1, this cost is modeled as a monitoring cost to avoid bargaining issue at T^0 . This specific modeling over period 1 has no effect on bargaining at T^1 , which is the focus of this analysis. § 2.3
- c_f : One-shot cost for the firm of transmitting information to a new bank. Fixed cost. § 2.3
- f : Agent f in Section 3. Otherwise, f denotes the firm/entrepreneur undertaking the project. Section 1
- F : Cost of failure (default, bankruptcy) for firms. § 2.4
- \bar{L} : Minimal amount paid to b over both periods so that she breaks even. § 4.4
- M^* : Supremum of what b can get at a given step - used in the proof of **principle B**. § 3.5
- NFP : Net payments to the external financial system (*i.e.* excluding b). § 4.3
- Out_b : (Best) outside option available to the initial lending bank b . § 3.4
- Out_f : (Best) outside option available to (the firm) f . § 3.4
- p : Probability of f transitioning from “good” to “bad” state from period 1 to period 2. § 2.1
- P : Interest-bearing asset. § 3.3
- PB^1 : Private benefits in the case of a default over period 1 (including negotiations). § 2.4
- PB^2 : Private benefits in the case of a default over period 2. § 2.4
- PD: Firm's probability of default. § 2.1
- PEP: Perfect equilibrium partition. Section 1
- P^f : Amount or utility that f would obtain by giving b just more than her outside option. § 3.6
- p_W^1 : (With $w = G$ or B) exogenous probability of default of the firm over period 1. § 3.2
- p_X^2 : (With $x = G$ or B) exogenous probability of default of the firm over period 2. § 3.2
- q : Probability of f transitioning from “bad” to “good” state from period 1 to period 2. § 2.1
- r_b : Generic notation for the interest rate agent b pays or receives. Used only to compute δ_b . § 3.2
- r_f : Generic notation for the interest rate agent f pays or receives. Used only to compute δ_f . § 3.2
- r_1 : First-period cost of funds for banks (risk-free rate in financial markets). By extension, this is the rate that the bank is supposed to pay during the bargaining that starts at T^1 . § 2.2
- r_2 : Second-period cost of funds for the bank (risk-free rate in financial markets). § 2.2
- r_W^1 : (With $w = G$ or B) interest rate required by b for a short-term loan to a firm of quality W . r_W^1 is a general notation for the effective rate required by b at T^1 ; this rate determines the amount to be repaid at T^1 . § 4.2
- $r_W^{1,F}$: (With $w = G$ or B) fair interest rate necessary for b to break even over the first-period loan. § 4.4
- $r_X^{2,f}$: (With $x = G$ or B) fair interest rate at which a bank breaks even on the second-period loan when the firm strategically defaults and, therefore, borrows a limited amount to buy the asset back; it includes c_s (although the firm actually borrows from the initial bank). § 4.1
- $r_X^{2,F}$: (With $x = G$ or B) fair interest rate at which a bank breaks even in the second period when the firm borrows (the full amount of the initial loan) from the initial bank. § 4.2
- $r_X^{2,O}$: (With $x = G$ or B) interest rate on a second-period loan when the firm borrows (the full amount of the initial loan) from a new bank; this includes c_s . § 4.2
- SITG: skin-in-the-game. Section 1
- s^* : b 's offer when he makes the first offer, in the proof of **principle B**. § 3.5
- s_b : Share of the surplus accruing to b according to the PEP. In § 3.3, this notation is used to *compute* the share accrued to b at the PEP. Otherwise, it is used as the share accrued to b at the PEP. § 3.3

s_b^* : Share of the surplus accruing to b according to the PEP in a model without interest-bearing (or costing) assets, as defined in the existing literature. § 3.2

S : Surplus shared between b and f in a theoretical framework. § 3.3

S_b : Value of the share of surplus accrued to b at a PEP. $S_b = s_b S$. § 3.4

\widetilde{S}_b : Value of the share of the surplus accrued to b in a partition that would differ from the PEP – used in the proof of **principle A**. § 3.4

s_f : Share of the surplus accruing to f according to the PEP. In § 3.3, this notation is used to *compute* the share accruing to b at the PEP. Otherwise, it is used as the share accrued to f at the perfect equilibrium partition. § 3.3

s_f^* : Share of the surplus accruing to f according to the PEP in a model without interest-bearing (or costing) assets, as defined in the existing literature. § 3.2

S_f : Value of the share of the surplus accrued to f at the PEP. $S_f = s_f S$. § 3.4

\widetilde{S}_f : Value of the share of the surplus accruing to f in a partition that would differ from the PEP - used in the proof of **principle A**. § 3.4

S_{raw}^{nego} : Actual raw surplus to be shared between the bank and the firm. §4.3

S_{net}^{nego} : Actual net surplus. Equal to actual raw surplus minus net financial payments to the external financial system. Because the latter tends towards zero, based on the assumption on the refinancing rate, it is approximately equal to the actual raw surplus. §4.3

T^0 : Beginning of period 1 (capacity building); beginning of first loan. § 2.1

T^1 : End of period 1 (capacity building); time to repay the first-period loan, start of bargaining (because the length of bargaining steps tends towards zero, T^1 is approximately the start of the second loan and of period 2 (the production period) § 2.1

T^2 : End of period 2 (production period), which starts at the agreement time; time to repay the second-period loan. § 2.1

W : Denotes G if the firm is “good” over period 1 and for B otherwise. § 2.1

X : Denotes G if the firm is “good” over period 2 and for B otherwise. § 2.1

y : Gross material return, production, or capital productivity (unit-size) of the project. § 2.1

Y_f : Firm income if he turns to another bank to repay the first-period loan. § 4.2

Z_b : Bank income in case of firm’s strategic default. § 4.1

Z_f : Firm income in case of strategic default (if $Z_f > Y_f$, then $Out_f = Z_f$). § 4.1

Δ : Time-interval between an offer and a counteroffer. § 3.2

δ_b : Discount factor for b . § 3.2

δ_f : Discount factor for f . § 3.2

Γ^1 : Collateral value recovered by the bank in case of failure over period 1 (including the negotiation). § 2.4

ε : Positive element tending towards zero in mathematical proofs.

$\bar{\varepsilon}$: Additional amount (w.r.t. the outside option) that the holder of a binding option with a loss threat can get - tends towards zero - used in the proof of **principle C**. § 3.6

$\varepsilon_X^{2,f}$: Positive element close to zero that the initial bank would grant to the firm to lend rather than letting another bank lend at the “fair” rate including screening cost.

Γ^2 : Collateral value recovered by the bank in case of failure over period 2. § 2.4