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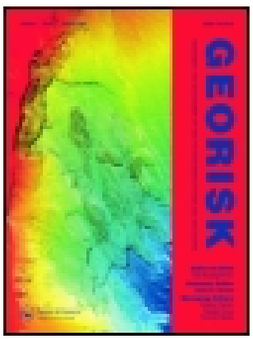

Georisk: Assessment and Management of Risk for Engineered Systems and Geohazards

ISSN: 1749-9518 (Print) 1749-9526 (Online) Journal homepage: https://www.tandfonline.com/loi/ngrk20

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To cite this article: S. Imanzadeh, D. Breysse, J. Baroth, D. Dias, N. Piegay & S.M. Elachachi (2019): Probabilistic comparative analysis between design rules of a shallow foundation safety, Georisk: Assessment and Management of Risk for Engineered Systems and Geohazards

To link to this article: https://doi.org/10.1080/17499518.2019.1598561

Published online: 28 Mar 2019.

GEORISK
https://doi.org/10.1080/17499518.2019.1598561

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Probabilistic comparative analysis between design rules of a shallow foundation safety

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ABSTRACT

A fact that is generally overlooked in many geotechnical uncertainty analyses is that input data of the model may be correlated. While this correlation may influence the system response, epistemic uncertainties i.e. lack of knowledge of this correlation appears as a risk factor. This paper discusses how a negative correlation between cohesion (*c'*) and friction angle (Ø') with their associated uncertainties can influence both the bearing resistance of a shallow strip foundation footing and the estimation of its safety. A probabilistic approach that considers both the negative correlation and the uncertainty is used in this work as a reference. This method is compared to Eurocode 7 variants that do not for the correlation. These variants, resistance and material factoring methods appears to be more or less conservative depending on the negative correlation degree between (*c'*–Ø), their associated uncertainties and soil configurations. Finally, the proposed probabilistic comparison shows that the material factoring method is more conservative than the resistance one.

ARTICLE HISTORY
Received 8 July 2018
Accepted 21 January 2019

KEYWORDS
Shallow foundation; bearing resistance; uncertainty; correlated parameters; Eurocode model; probabilistic approach

- This paper has discussed design rules of the bearing resistance of shallow foundations
- Negative correlation (*c'*, Ø') and soil configuration effect on foundation safety
- Material factoring method is more conservative than the resistance factoring method.
- Eurocode 7 is more or less conservative based on correlation and soil configuration.

1. Introduction

The design of the bearing capacity of a shallow strip foundation footing is a classical geotechnical problem. Effective cohesion (*c'*) and effective friction angle (Ø') are the driving parameters regarding the bearing capacity, but their knowledge is only partial. While safety analyses have been generally focused on the significant effect of the distribution of random soil properties, the correlation between these two properties (*c'* and Ø') has been considered only in few studies (Russelli 2008). Ades and Lu (2003) and Bauduin (2003) have addressed the correlation between input variables. Breysse (2011) highlighted the issue of correlation, particularly between cohesion and friction angle in the Mohr-Coulomb model. Recent studies (Ching, Li, and Phoon 2016; Ching, Phoon, and Wu 2016; Ching, Phoon, and Pan 2017) have confirmed that correlations are common between soil parameters.

Studying the effect of correlation is relevant, but the fact is that this correlation itself remains uncertain. How this epistemic uncertainty (i.e. lack of knowledge about properties) can affect the foundation safety is a complex problem. Breysse (2011) and Tang et al. (2013) discussed the negative correlation between *c'* and Ø' for both soils and rocks, due to the linearisation of the Mohr-Coulomb failure envelope. This negative correlation was confirmed by measurements made on different materials (Cherubini 2000; Tang et al. 2013; Yucemen, Tang, and Ang 1973). Booth, Marshall, and Stace (2016) studied the probabilistic analysis of a coal mine roadway including correlation between the model input parameters (*c'* and Ø'). In their paper, the correlation between parameters was shown to be very important.

The correlation coefficient gives a measure of the dependence of one random variable on another. The correlation coefficient (r) between two variables ranges between −1 and 1 (Wu 2015). The closer the value is to −1, the stronger the evidence of negative association between the two variables is. When r is −1, the relationship is said to be perfectly negative correlated.

In a probabilistic study of safety and design of earth slopes, Yucemen, Tang, and Ang (1973) mentioned that not including the negative correlation between *c'*

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and Ø' gives conservative results. This point of view has been confirmed by other authors (Breysse 2011; Cherubini 2000; Chiwaye and Stacey 2010; Fenton and Griffiths 2003; Li et al. 2011).

In a probabilistic study of shallow foundations reliability with respect to their bearing capacity on a soil characterised by effective cohesion (c') and friction angle (Ø'), Cherubini (2000) showed that the reliability indexes were much larger and hence the safety level was much greater if the negative correlation r between c' and Ø' was considered. Li and Low (2010) studied the behaviour of a circular tunnel in rock subjected to a hydrostatic stress and compared two assumptions: negative and no correlation between c' and Ø'. They found that the results of reliability analysis are conservative if negative correlation among strength parameters is not considered. Fenton and Griffiths (2003) worked on the bearing capacity of a spatially random (c'–Ø') soil. They noted that there was an effect of negative correlation in (c'–Ø') but that it was insignificant, since the results were dominated by other factors (Fenton and Griffiths 2004). Some authors also considered cross-correlated shear strength parameters, studying the effect of various autocorrelation distances in the vertical and horizontal directions on the bearing capacity of a strip footing (Al-Bittar and Soubra 2013; Cho and Park 2010). They both highlighted the decrease of the probability of failure with an increase of the negative correlation coefficient r. They also observed for the range of their studies, that the decrease of the vertical autocorrelation distance leads to a decrease of the uncertainty in the bearing capacity. Youssef Abdel Massih, Soubra, and Low (2008) added that the failure probability is very sensitive to the variation of the angle of internal friction compared to that of the cohesion.

Geotechnical design codes like Eurocode 7 (Frank et al. 2005) still do not account for the uncertainty and correlation between cohesion and friction angle, which would in consequence make them conservative. Youssef Abdel Massih, Soubra, and Low (2008) presented a reliability based design (RBD) approach aiming at providing optimised values of the partial factors. To the best of our knowledge, no previous work has analyzed how Eurocode 7 approaches are conservative depending on the correlation level and on the soil type. This is why it was decided that in this work the (c'–Ø') correlation and their associated uncertainties should be considered, so as to evaluate their possible importance and effects on the bearing capacity of a shallow strip foundation footing, for different soil configurations. The model uncertainty in the cases of a footing on a purely frictional soil (Breysse 2011) and on a purely cohesive soil (Orr and Breysse 2008) has already been studied and thus is not studied in this paper. The focus is, for the case of a footing on a sandy clay soil, on the effects of the change of the correlation degree between cohesion and frictional angle, being considered that these two properties are also described as random parameters. Analyzing the effect of this change has a double interest: checking confirmation about previous results, and quantifying the consequence of epistemic uncertainty, i.e. of a lack of knowledge regarding this issue. The investigation domain covers several sets of (c'–Ø') pairs in order to check the stability of conclusions for different types of soil conditions. The results of two analyses are presented, one with a full probabilistic model, which will consider both the probabilistic distribution of the soil properties and their correlation, and the other with two variants of the Eurocode 7 that do not consider them. The comparison of the results leads to some proposals for a better assessment of the safety of foundation designs in current codes.

In Eurocode 7, the term "bearing resistance" is used instead of "bearing capacity". Because this work highlights Eurocode 7 proposals, in the following, the bearing resistance is used instead of bearing capacity. The next section presents materials and methods, then results are displayed and commented, finally some conclusions are drawn.

2. Material and methods

2.1. Bearing resistance of shallow foundations on soil

The bearing *resistance* of a shallow strip foundation footing (q) depends on the effective friction angle (Ø') and on the effective cohesion (c') (mechanical characteristics identified in laboratory) such that (Equation 1; Terzaghi 1943):

$$q = 0.5\,\gamma.B\,N\gamma(Ø') + \bar{q}\,N_q(Ø') + c'\,N_c(Ø') \qquad (1)$$

where B is the width of the footing, $\bar{q}$ is the effective overburden pressure at the level of the footing base, γ is the self-weight of the soil. Equation 1 has surface, depth and cohesive terms. The functions N_γ, N_q and N_c all depend on the friction angle (Ø'), but can be derived from different models (Brinch Hansen 1970; Meyerhof 1963; Tajeri, Sadrossadat, and Bazaz 2015; Terzaghi 1943; Vesic 1973). In the following work, the given formulas are used:

$$N_c = (N_q - 1)\mathrm{cotan}\,Ø' \quad \text{(Terzaghi 1943)} \qquad (2)$$

$$N_q = \exp(\pi \tan \varnothing'). \tan^2(\pi/4 + \varnothing'/2) \quad \text{(Meyerhof 1963)} \tag{3}$$

$$N_g = 2(N_q - 1). \tan(\varnothing') \quad \text{(Vesic 1973)} \tag{4}$$

The effect of model uncertainty has been studied by several researchers and will not be analyzed here. For example, Breysse (2011) showed that, for a shallow foundation footing, the effects of model errors are negligible compared to those of the variability of the soil properties. Recent research (Lesny and Paikowsky 2011; Paikowsky et al. 2010; Tang et al. 2017; Tang and Phoon 2017) also discussed the influence of the internal friction angle on the bearing resistance of shallow foundations.

For the practical application, this study considers an infinite length footing of width $B = 1$ m, depth $D = 1$ m underground, in a soil with $\gamma = 22$ kN/m^3. The case of a cohesive and frictional soil (sandy clay soil) will be considered, accounting for different combinations of cohesion and friction angle.

2.2. Uncertainties and correlations of soil properties

2.2.1. Nature and origin of uncertainties

Soil exhibits spatial heterogeneities resulting from the history of its deposition and aggregation processes, which can occur in different physical and chemical environments. Some uncertainty is associated with each of the soil parameters due to this variability (Imanzadeh, Denis, and Marache 2014). This natural variability can commonly be modelled through some characteristics of the statistical distribution (mean, variance) and of the covariance function in the case of a spatial correlation of the property (Charlton and Rouainia 2017; Yanez-Godoy, Mokeddem, and Elachachi 2017). Uncertainties can usually be divided into two groups: aleatory or active uncertainty and epistemic or passive uncertainty (Fellin et al. 2010; Lacasse and Nadim 1996; Motra, Stutz, and Wuttke 2016; Uzielli et al. 2008). Aleatory uncertainty mainly includes natural variability of a property (spatial and temporal variability). Epistemic uncertainty consists of statistical uncertainty (Phoon et al. 2016), model uncertainty and measurement uncertainty, which are all classified as a type of uncertainty associated with limited, insufficient or imprecise knowledge.

2.2.2. Correlation between soil properties

Two very different reasons can be given to explain the correlation between cohesion and friction angle (Breysse 2011):

- *A statistical reason:* the Mohr-Coulomb failure criterion is commonly adopted to model the failure envelope of soils and rocks. The intercept of the failure envelope with the shear strength axis is the cohesion, and the slope of the failure envelope is the internal friction angle. Hence, the Mohr-Coulomb criterion is described by two parameters: cohesion (c'), and friction angle ($\varnothing$'). From shear tests on a given soil, one identifies c' and $\varnothing$', building Mohr's circles and a straight line between c' and $\varnothing$' identified values. However, uncertainties related to this kind of test (including sampling uncertainties, disturbance of soil conditions, measurement uncertainties etc.) may lead to many different Mohr-Coulomb limit lines. Therefore, uncertainties between c' and $\varnothing$' are dependent: if a lower value of friction is taken, a larger value of cohesion will compensate it. Finally, a negative correlation between the two parameters results from the whole process.
- A physical reason: let consider a pool of local samples from in situ surveys. Depending on a more or less important part of clayey particles, if the part of clayey particles increases, the friction angle tends to decrease and cohesion tends to increase. As a result, for the soil, one observes a dispersion between cohesion and friction, with a negative correlation.

As it was described above, the two different sources of negative correlation between cohesion and friction angle are very different: the first one is associated with the statistical analysis of the data and therefore originates from a "passive" type of uncertainty while the second one is associated with the physical behaviour of the soil and therefore originates from an "active" type of uncertainty. This paper is mainly concerned with soil variability and therefore with an active type of uncertainty.

Whatever its origin, it is widely reported in the literature and accepted that there exists a negative correlation between cohesion and friction angle (Baecher and Christian 2003; Cherubini 2000; Chiwaye and Stacey 2010; Fenton and Griffiths 2003; Hoek 2007; Li et al. 2011; Li and Low 2010; Low 2007; Lumb 1970; Wolff 1985, 1996; Young 1986). Another issue is the magnitude of this correlation: Are cohesion and friction slightly or strongly correlated? Answering this issue is more difficult and the authors of this paper did not found guidelines telling how to choose a relevant correlation coefficient. From the literature, four negative correlation coefficients r between ($\varnothing$', c') is arbitrarily considered: $r = 0$ (independent parameters); $r = -0.25$; $r = -0.5$; $r = -0.75$ (Russelli 2008). They correspond to a large interval regarding the magnitude of negative correlation between the two driving soil properties.

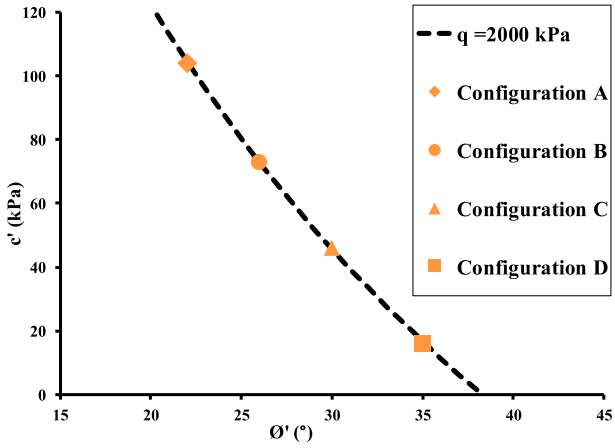

Figure 1. Position of the four soil configurations in the *c'* and *Ø'* diagram on the same value of bearing resistance of 2000 kPa.

2.3. Soil configurations

In order to derive practical conclusions that could apply to a large variety of soil configurations, four soil configurations of a cohesive and frictional soil (sandy clay soil) are considered according to the LPC-USCS (ASTM D2487-11 2012) standard:

- Configuration A: clays of high plasticity-compacted (CH)
- Configuration B: inorganic silts of high plasticity (MH)
- Configuration C: loamy sand, sandy clay loam-compacted (SM, SC)
- Configuration D: silty sand (SM)

It should be mentioned that this work is first an academic research. Nevertheless, the set of soil parameters used for the study were found in the literature and correspond to real soil parameters values (Briaud 2013; NAVFAC Design Manual 1986; Pavement Design, 2007; Swiss Standard 1999). Then it will be useful in engineering. The set of values for mean cohesion and mean friction angle have been carefully chosen, such as they correspond to the same value of bearing resistance (2000 kPa) calculated with Equation 1. The four configurations are presented in Figure 1 and the corresponding properties are given in Table 1.

When soil variability is considered, the friction angle *Ø'* is supposed to follow a distribution such that *tan Ø'* follows a Gaussian law with mean μ [tan Ø'] and standard deviation σ [tan Ø']. The cohesion *c'* is supposed to follow a lognormal law, thus *ln(c')* becomes Gaussian with mean μ [ln(*c'*)] and standard deviation σ [*ln(c')*]. The considered coefficients of variation for tan *Ø'* and *c'* are respectively $CV_{\tan\ Ø'} = 15\%$ and $CV_{c'} = 30\%$, in agreement with what had been considered in previous studies (Cherubini 2000; Mollon, Dias, and Soubra 2013). The following equations are used to transform the lognormal parameters ($\mu_{\log}$ and $\sigma_{\log}$) into the associated Gaussian parameters (μ_G and σ_G).

$$\sigma_G = \sqrt{\ln\left(1 + CV_{\log}^2\right)} \tag{5}$$

$$\mu_G = \ln\left(\mu_{\log}\right) - 0.5\,\sigma_G^2 \tag{6}$$

Statistical properties of probabilistic laws for these configurations are presented in Table 1, where constancy of σ [ln(*c'*)] for all sets directly derives from a same value of $CV_{c'}$.

2.4. Relation between Eurocode 7 models and probabilistic models

2.4.1. Methodology for analyzing the foundation capacity

The foundation bearing resistance is analyzed using different approaches which are illustrated on Figure 2 and it is described in the following. When the variability of soil properties is considered, the foundation response can be plotted in the friction angle-cohesion diagram, in which iso-*q* curves can be drawn.

On the one hand, section 2.4.2 explains, using the Eurocode 7, how it is possible to define characteristic and design values of the soil properties and to calculate, via either a resistance factoring method (design approach 2) or a material factoring method (design approach 3), two design values of the foundation resistance, respectively denoted $q_{R\text{-}EC\ d}$ and $q_{M\text{-}EC\ d}$.

On the other hand, section 2.4.3 proposes to estimate the variability of the bearing resistance using Equation 1 in which soil properties are modelled using random variables. It is thus possible to derive two alternative design values of the bearing resistance from the probabilistic model. These are respectively denoted $q_{R\text{-}p\ d}$ and $q_{M\text{-}p\ d}$.

Table 1. Statistical properties of probabilistic laws.

Configuration A (CH)	Configuration B (MH)	Configuration C (SM, SC)	Configuration D (SM)
Ø' = 22°, *c'* = 104 kPa	*Ø'* = 26°, *c'* = 73 kPa	*Ø'* = 30°, *c'* = 46 kPa	*Ø'* = 35°, *c'* = 16 kPa
μ [tan *Ø'*] = 0.404	μ [tan *Ø'*] = 0.488	μ [tan *Ø'*] = 0.577	μ [tan *Ø'*] = 0.700
σ [tan *Ø'*] = 0.061	σ [tan *Ø'*] = 0.073	σ [tan *Ø'*] = 0.087	σ [tan *Ø'*] = 0.105
μ [ln(*c'*)] = 4.60	μ [ln(*c'*)] = 4.25	μ [ln(*c'*)] = 3.79	μ [ln(*c'*)] = 2.73
σ [ln(*c'*)] = 0.294	σ [ln(*c'*)] = 0.294	σ [ln(*c'*)] = 0.294	σ [ln(*c'*)] = 0.294

$CV_{\tan\ Ø'} = 15\%$, $CV_{c'} = 30\%$ are respectively the coefficients of variation of *Ø'* et *c'*.

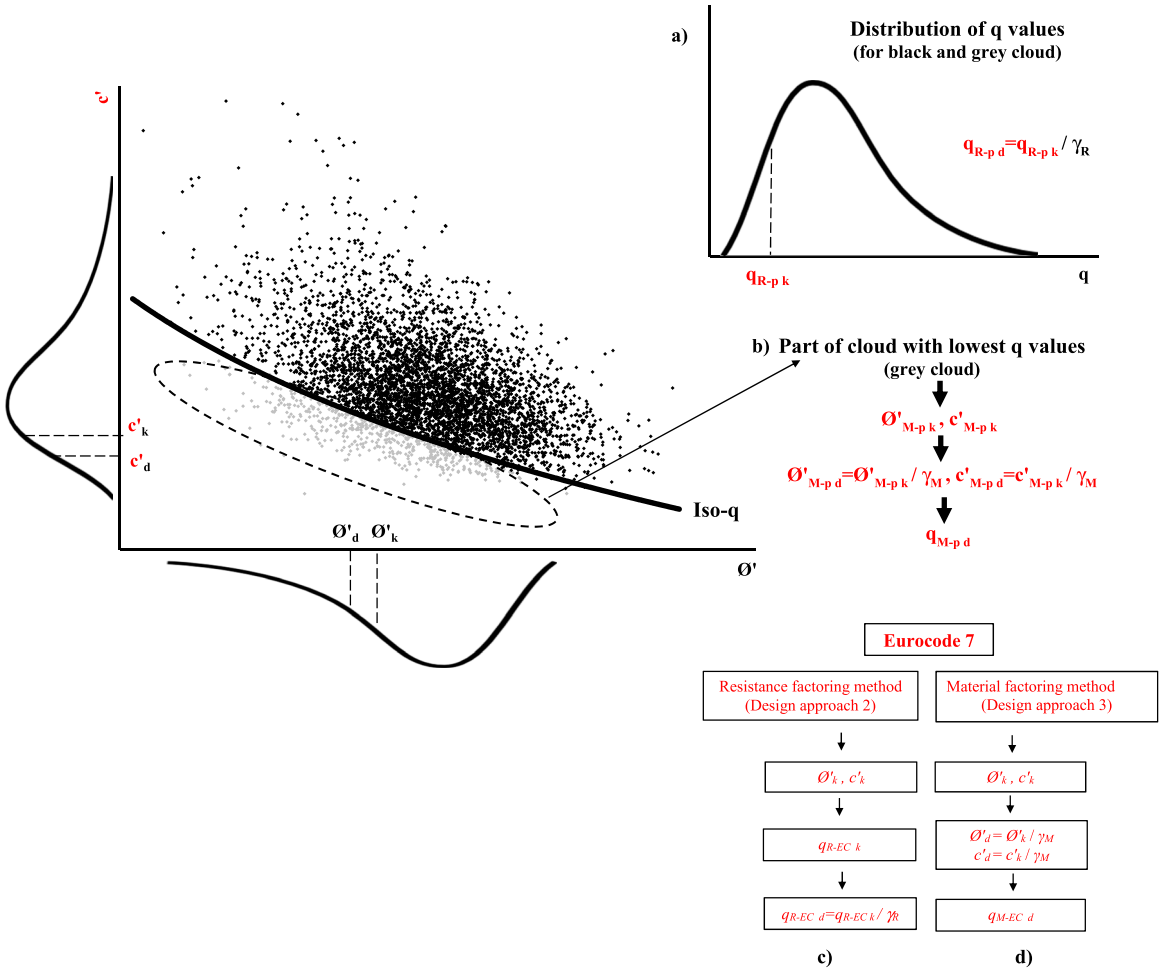

Figure 2. Strategies for analyzing the foundation bearing resistance q, using resistance and material factoring methods, using a probabilistic model: (a) $q_{R\text{-}p\ d}$ and (b) $q_{M\text{-}p\ d}$ respectively, or the Eurocode 7: (c) $q_{R\text{-}EC\ d}$ and (d) $q_{M\text{-}EC\ d}$ respectively. γ_R and γ_M are respectively partial factors for resistance and material factoring methods. Details for the first two parameters are given at section 3.2.2.

2.4.2. Using the Eurocode model

Applying Eurocode 7, the designer can choose between two factoring methods to determine the design bearing resistance of a shallow foundation (Frank et al. 2005):

- a resistance factoring method, which involves applying a partial resistance factor (γ_R) of 1.40 to the resistance calculated using the bearing resistance equation and the characteristic values of the soil properties to obtain the design bearing resistance.
- a material factoring method which involves applying partial material factors (γ_M) of 1.25, to the characteristic soil properties and then using these factored values in the bearing resistance equation to calculate the design bearing resistance.

Partial resistance and material factors primarily aim to provide the required level of safety and to account for uncertainty in the resistance model. The characteristic value of the resistance or a material property may be chosen as corresponding to the lower 5% percentile of the resistance or strength distribution to account for uncertainties in the soil properties. For each soil configuration (i.e. parameters distribution c' and Ø'), characteristic and design values do not consider the fact that material properties can be correlated. Thus, since the case study considers four soil configurations (Table 1) and since one can apply either resistance factoring method or material factoring method, Eurocode 7 delivers 8 values for the foundation bearing resistance.

2.4.3. Using the probabilistic model

The full probabilistic description of the two input variables (friction angle and cohesion) enables, when using the same structural model (i.e. that of Equation 1) a probabilistic assessment of the foundation bearing resistance to be carried out. This probabilistic model is also fully adapted to the consideration of correlated soil properties, since the only change is in the pair of values (friction angle and cohesion) that enter Equation 1. When

Table 2. Indices: R/M for resistance factoring method / material factoring method, EC/p for Eurocode 7/probabilistic models.

	Resistance factoring method	Material factoring method
Eurocode 7 model	$q_{R\text{-}EC\ d}$	$q_{M\text{-}EC\ d}$
Probabilistic model	$q_{R\text{-}p\ d}$	$q_{M\text{-}p\ d}$

this correlation is considered, the bearing resistance distribution is modified.

To derive numerical estimates, these calculations can be carried out with Monte-Carlo simulations (Klammler and Chung 2016) that are fully adapted for simulating the statistical response of problems with correlated variables (for example, the case of two lognormal correlated random variables was explicitly written in Baroth et al. 2006). The random simulation was iterated 5000 times in order to derive relevant statistical estimates (mean value, standard deviation, 5% percentiles). Since it was not looking at very low probabilities, this number of iterations was checked to guarantee convergence and stability of results.

All random simulations can be carried out for the four soil configurations of Table 1 and for the four degrees of correlation between friction angle and cohesion, which corresponds to 16 combinations.

2.4.4. Synthesis of the two approaches

The following table (Table 2) synthesises notations of design values of the bearing resistance that are estimated and compared for the two approaches.

The probabilistic model offers a more accurate view of the foundation response, since it uses a more refined model of data. Our analyses have several objectives:

- from the results of the probabilistic model, quantify how the foundation resistance varies when input data are changed, i.e. different soil configurations or different correlations,
- from both probabilistic and Eurocode 7 models, study how the resistance factoring method differs from the material factoring method,
- Analyze the consistency of the Eurocode 7 model, in order to see if a semi-probabilistic formalism, including partial factors, offers a comparable safety level for all contexts.

3. Results

3.1. Semi-probabilistic Eurocode model

The results obtained using Eurocode 7 (section 2.4.2) are presented in Table 3 for both resistance factoring method and material factoring method. One should note that the four configurations correspond to mean soil properties that would lead to an identical bearing resistance with a deterministic model. It can be seen that characteristic values as well as design values for the two methods are clearly decreasing from configuration A to configuration D, whatever the approach, resistance factoring method or material factoring method. A second feature that can be seen is that the material factoring method leads to a lower bearing resistance than the resistance factoring method, with an increasing relative difference from configuration A (7%) to configuration D (20%). The latter feature has been pointed out earlier in the study carried out by Vogt and Schuppener (2006) for geotechnical verifications used in Germany.

As the Eurocode 7 models do not account for correlation between soil parameters, it is proposed in the following section to use Monte-Carlo simulations to analyze the influence of this correlation.

3.2. Probabilistic models

3.2.1. Combined effects of variability and correlation on the bearing resistance

The probabilistic foundation bearing resistance can be estimated by using Equation 1 in which soil properties are considered as being random variables, which can be correlated or not. Monte-Carlo simulations are used to simulate the probabilistic response, which exhibits, for each soil configuration, a large scatter in the (c', Ø') diagram of Figure 3. This figure represents the four clouds corresponding to the respective responses for the four soil configurations of Table 1. Iso-q values curves are also plotted, on the same diagram.

Table 3. Characteristic and design values of cohesion (in kPa), friction angle and bearing resistance (in kPa), for the resistance and material factoring methods using Eurocode 7 for the considered configurations (k/d for characteristic/design).

Eurocode 7 model		Configuration A	Configuration B	Configuration C	Configuration D
Resistance factoring method	$(\tan Ø')_{R\text{-}EC\ k}$	0.304	0.367	0.435	0.527
	$c'_{R\text{-}EC\ k}$	61.5	43.1	27.2	9.5
	$q_{R\text{-}EC\ k}$	883	834	787	712
	$q_{R\text{-}EC\ d}$	631	595	562	508
Material factoring method	$(\tan Ø')_{M\text{-}EC\ d}$	0.243	0.294	0.348	0.422
	$c'_{M\text{-}EC\ d}$	49.2	34.5	21.7	7.6
	$q_{M\text{-}EC\ d}$	590	532	474	394

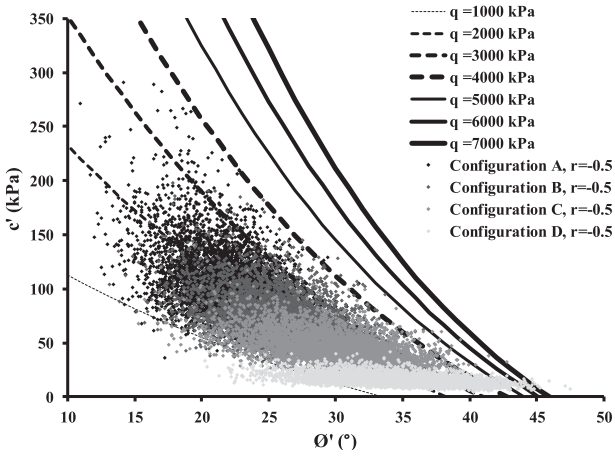

Figure 3. Correlation between parameters Ø′ and *c*′, with iso-values of *q*, and for a correlation $r = -0.5$, for the four configurations.

While the deterministic bearing resistance for the four soil configurations is identical ($q = 2000$ kPa), the points which were distributed along the iso-*q* curve of Figure 1 are changed in to large clouds on Figure 3. Here, the Monte-Carlo simulation corresponds to the case where the correlation coefficient between cohesion and friction angle is taken at −0.5.

Some qualitative statements can be done from the figure:

- The extent of the clouds is due to the coefficients of variation of cohesion and friction angle, given in Table 1. It induces a high scatter of the bearing resistance, which is governed by both variabilities of the friction angle and of the cohesion. Thus, each cloud extends over several iso-*q* curves. Lowest bearing resistances correspond to points located in the low-left part of the clouds.
- The negative correlation between cohesion and friction angle induces a negative "slope" of the cloud.
- The iso-*q* curves have all a negative slope, corresponding to the fact that, for a constant bearing resistance, some increase in friction can be compensated by some decrease in cohesion. However, there is no coincidence between the slope of curves and the slope of clouds. In some cases, the slope of the cloud is less negative than that of the curve, while in other cases, the reverse statement can be true.

When these statements are combined, it appears that the lowest bearing resistances are governed by the thickness of the cloud (i.e. scatter of soil properties) and the relative slopes of the iso-*q* curves and the clouds. Since the latter depends on the cohesion-friction correlation, and since the former depends on the soil configurations (from A to D), there is no simple way to address the foundation safety. It is why more attention is paid in the following to the respective effect of each influencing factor.

To better understand what can be the influence of the correlation between soil properties on relative positions of clouds, one can look at Figure 4(a–d) which show how the cloud is modified, for the four soil configurations (soil A, Figure 4(a), to soil D, Figure 4(d)), when the negative correlation between cohesion and friction angle changes from a moderate correlation ($r = -0.25$, black clouds) to a strong correlation ($r = -0.75$, grey clouds).

Figure 4(a–d) show that the thickness of a cloud around its average regression curve is smaller for a strong correlation ($r = -0.75$) than for a moderate correlation ($r = -0.25$). Thus, the probability of reaching a bearing resistance value very different from the deterministic one becomes lower.

If one looks at the cases with a moderate correlation ($r = -0.25$), it can be seen from the Figure 4 that worst cases (i.e. lowest bearing resistances that they can be clearly highlighted under $q = 1000$ kPa isovalue curves) always correspond to the low-left part of the cloud. In the other word, the cases when both friction angle and cohesion are lower than their expected average. This is not true when a strong negative correlation is considered between the two soil properties. For instance, the grey cloud on Figure 4(d) (soil D, $r = -0.75$) shows that lowest bearing resistance corresponds to the lowest friction angle values, but also to cohesion values which are above the expected average. The reverse statement can be made with regard to the cloud in Figure 4(a) (soil A, $r = -0.75$) where the lowest bearing resistance is governed by lowest cohesion values, which correspond to friction angles larger than the expected average. These statements show that considering, like it is done in Eurocode 7 with "material factoring method", that the bearing resistance characteristic (thus design) values can be deduced from the soil properties characteristic (thus design) values is probably misleading when the effect of correlation is considered.

In the following, the effect of the uncertainties and correlations of soil properties on the bearing resistance and the safety of a shallow foundation is studied more into details.

3.2.2. How the probabilistic models can be used to derive resistance and material factoring values

For each configuration and each value of the friction angle-cohesion correlation, the probabilistic model can be used to derive design bearing resistance values. It is explained here how two values, denoted respectively

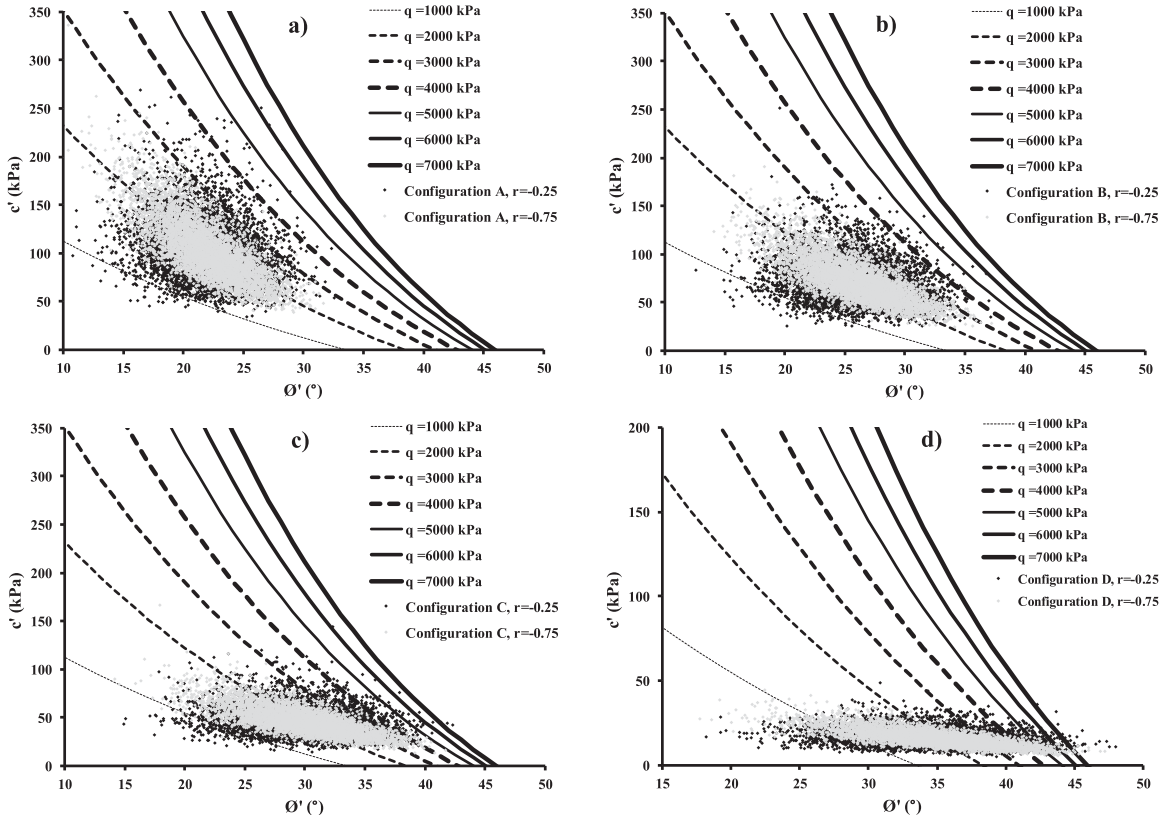

Figure 4. (a–d) Correlation between parameters Ø′ and *c*′, with iso-values of *q* and for two correlations $r = -0.25$ (black cloud) and -0.75 (grey cloud), for the four configurations.

$q_{R\text{-}p\ d}$ for resistance probabilistic design value and $q_{M\text{-}p\ d}$ for material probabilistic design value, are calculated. The basic idea is to propose a method which conforms to the Eurocode 7 philosophy of resistance and material factoring methods while using the probabilistic model. This is the only one, which can consider the correlation between parameters. Figure 2 illustrates how the process works. Figure 2(c–d) show how $q_{R\text{-}EC\ k}$, $q_{R\text{-}EC\ d}$ and $q_{M\text{-}EC\ d}$ values given in Table 3 have been obtained. Figure 2(a–b) detail how additional calculations are carried out.

- Calculation of $q_{R\text{-}p\ d}$ values

Monte-Carlo simulations provided a large number of bearing resistance values, whose statistical analysis is straightforward with a resistance factoring method. The characteristic bearing resistance of the resistance factoring method, denoted $q_{R\text{-}p\ k}$, is directly inferred from simulations, by simply taking the value corresponding to the 5% – percentile (see item (a) on Figure 2). Then the design value is given by $q_{R\text{-}p\ d} = q_{R\text{-}p\ k} / 1.4$.

- Calculation of $q_{M\text{-}p\ d}$ values

The process with the material probabilistic approach is more complex. The difficulty comes from the fact that characteristic material properties entering the mechanical model in Eurocode 7 does not consider the correlation between material properties whereas it was shown at section 3.2.1 that the range of material properties which are the most unfavourable regarding the foundation safety depend on this correlation.

Based on this statement, the process is as follows (see item (b) on Figure 2):

(a) Selecting from the Monte-Carlo simulation results the sample that corresponds to all values below the $2 \times k$ lower percentile of the *q* distribution, i.e. the 10% percentile if $k = 5\%$. This selection process can be more easily seen on Figure 5 which plots the *q* value as a function of the friction angle only.
(b) Considering the distributions of cohesion and friction angle within this sample, let $c'_{M\text{-}p\ k}$ and $Ø'_{M\text{-}p\ k}$ respectively denote their mean values. These values approximatively correspond to the central values of the 10% – percentile, thus being consistent with the 5% – percentile usually considered for characteristic values.
(c) Applying the common Eurocode partial factor on cohesion and friction angle to calculate respectively $c'_{M\text{-}p\ d}$ and $Ø'_{M\text{-}p\ d}$ values,

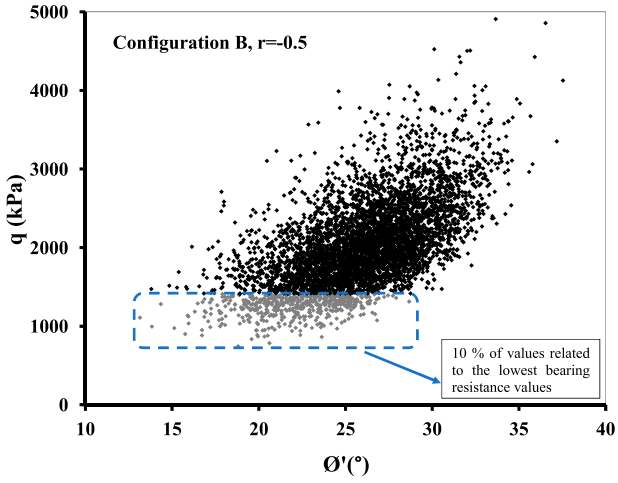

Figure 5. Relation between the bearing resistance and the friction angle, for the configuration *B*, $r = -0.5$ (illustration of 10% of values related to the lowest bearing resistance values).

(d) Applying finally the bearing resistance model of Equation 1 to derive the probabilistic bearing resistance for the material factoring method ($q_{M\text{-}p\ d}$).

It must be denoted that this process, by the selection of an adequate sample that considers both the effects of the correlation between material properties and how the bearing resistance is affected applies the formalism of material factoring method to the probabilistic modelling.

Table 4 synthesises the $c'_{M\text{-}p\ k}$ and $Ø'_{M\text{-}p\ k}$ values corresponding to the four soil configurations and to the four levels of correlation. While the deterministic tendency from the A to D configurations is expected, as it was given in Table 1, the dependency on the r value is a new phenomenon, which corresponds to the complex dependencies that has been identified above.

The characteristic friction angle tends to increase slightly when the correlation is less negative, while the cohesion significantly increases in the same conditions. This confirms that this interaction between the mechanical model, the scatter of soil properties and their correlation is significant. Since design values of soil properties are simply derived from characteristic values after a division by partial factor, they would exhibit a similar behaviour.

3.2.3. Probabilistic assessment results with resistance and material factoring methods

Table 5 synthesises the results of Monte-Carlo simulations with the probabilistic models and all the possible combinations of soil configuration and correlation magnitude, which comes to 16 sets of results.

These simulations have several objectives:

(a) To analyze if conclusions drawn from deterministic calculations may be applicable generally in the case of correlated parameters,
(b) To analyze if the Eurocode model, with partial factors, may be adapted to account for the influence of correlation. This issue is studied when using the partial resistance factoring method or the partial material factoring method.

For each soil configuration and correlation magnitude configuration, the results provided in Table 5 are summarised in the following:

- The mean value μ_q and standard deviation σ_q of bearing resistance q, which corresponds to the location and extent of the cloud with regards to iso-q curves. These estimates result from 5000 simulations, which are enough for ensuring a good convergence except for the configurations A and B with $r = -0.75$. Only for these two cases, the number of simulations was increased in order to estimate correctly the lower failure probabilities of about 0.001%.
- .Characteristic and design values of the bearing resistance (respectively noted $q_{R\text{-}p\ k}$ and $q_{R\text{-}p\ d}$) for the resistance factoring method. These estimates result from 10% of values related to the lowest bearing resistance values (500 simulations, see Figure 5),
- Design values of the bearing resistance $q_{M\text{-}p\ d}$ for the material factoring method. These estimates result from 10% of values related to the lowest bearing resistance values (500 simulations, see Figure 5),

Table 4. Characteristic values for friction angle and cohesion with the material factoring method of the probabilistic model.

Correlation coefficient (r)		Configuration A	Configuration B	Configuration C	Configuration D
0	$(\tan Ø')_{M\text{-}p\ k}$	0.331	0.386	0.438	0.518
−0.25	$(\tan Ø')_{M\text{-}p\ k}$	0.350	0.399	0.449	0.518
−0.50	$(\tan Ø')_{M\text{-}p\ k}$	0.360	0.407	0.450	0.520
−0.75	$(\tan Ø')_{M\text{-}p\ k}$	0.386	0.420	0.453	0.517
0	$c'_{M\text{-}p\ k}$ (kPa)	68.8	50.9	35.7	14.2
−0.25	$c'_{M\text{-}p\ k}$ (kPa)	70.3	53.8	38.5	15.9
−0.50	$c'_{M\text{-}p\ k}$ (kPa)	74.7	59.6	44.3	18.2
−0.75	$c'_{M\text{-}p\ k}$ (kPa)	78.1	67.1	51.0	21.5

Table 5. Characteristic and design values of bearing resistance for the resistance and material factoring methods for the 16 combinations of correlation between (c', $Ø'$) and soil configurations (all values are in kPa).

r	Approaches		Config. A	F_q (%)	Config. B	F_q (%)	Config. C	F_q (%)	Config. D	F_q (%)
0		μ_q	2029	57.8	2092	56.8	2156	57.9	2240	59.9
	σ_q	722	–	793	–	913	–	1217		
	Resistance factoring	$q_{R\text{-}p\ k}$	1070	4.44	1020	4.17	957	4.16	798	4.02
		$q_{R\text{-}p\ d}$	764	0.3	728	0.45	684	0.58	570	0.78
	Material factoring	$q_{M\text{-}p\ d}$	701	0.14	665	0.22	608	0.24	447	0.14
−0.25		μ_q	2011	55.9	2047	56.3	2119	57.0	2191	59.9
	σ_q	607	–	658	–	796	–	1146	–	
	Resistance factoring	$q_{R\text{-}p\ k}$	1167	4.26	1119	4.28	1055	4.12	840	3.80
		$q_{R\text{-}p\ d}$	833	0.22	799	0.29	753	0.33	600	0.62
	Material factoring	$q_{M\text{-}p\ d}$	753	0.07	720	0.09	672	0.2	468	0.13
−0.5		μ_q	2005	55.2	2042	55.1	2081	56.2	2186	58.9
	σ_q	496	–	541	–	653	–	1023	–	
	Resistance factoring	$q_{R\text{-}p\ k}$	1292	4.56	1253	4.08	1173	4.32	905	4.04
		$q_{R\text{-}p\ d}$	923	0.10	895	0.21	838	0.20	646	0.38
	Material factoring	$q_{M\text{-}p\ d}$	822	0.04	805	0.05	743	0.05	505	0.05
−0.75		μ_q	1967	53.7	2000	53.8	2064	56.3	2155	59.8
	σ_q	343	–	378	–	531	–	968	–	
	Resistance factoring	$q_{R\text{-}p\ k}$	1438	4.11	1420	4.15	1320	4.30	973	3.96
		$q_{R\text{-}p\ d}$	1027	0.008	1014	0.025	943	0.08	695	0.43
	Material factoring	$q_{M\text{-}p\ d}$	910	0.001	873	0.003	839	0.02	546	0.07

Note: μ_q and σ_q are respectively mean value and standard deviation of bearing resistance (q), $q_{R\text{-}p\ d} = q_{R\text{-}p\ k}/\gamma_M$.

Table 5 also provides the estimated probability F_q that the random bearing resistance, as it is simulated, is respectively lower than the values of μ_q, $q_{R\text{-}p\ k}$, $q_{R\text{-}p\ d}$ and $q_{M\text{-}p\ d}$.

The first result of the probabilistic model relates to the μ_q and σ_q values. It can be seen that the mean value of the random bearing resistance is not equal to 2000 kPa and varies with the soil configuration. This is due to the highly non-linear character of Equation 1, which leads to a non-symmetric right skewed bearing resistance distribution, with a mean value different of the median. The estimated probability F_q values, where the random bearing resistance is lower than the μ_q values, range from 53.7% to 59.9%, with a slight dependency to the correlation and to the soil configuration. Furthermore, the estimated probability F_q values, where the random bearing resistance is smaller than the values of $q_{R\text{-}p\ k}$, $q_{R\text{-}p\ d}$ and $q_{M\text{-}p\ d}$, range from 0.001% to 4.56%. These values were presented in Table 5.

4. Discussion

The following analyses focus on design values, since the material factoring method only delivers a material value, and since for the resistance factoring method, the characteristic to material values ratio is a constant. Thus, any statement on characteristic values would be redundant.

4.1. Regarding the influence of soil configuration and correlation

The $q_{R\text{-}p\ d}$ values given in Table 5 depend both on the soil configuration and on the correlation magnitude, as it is illustrated on Figure 6. How the soil configurations affect the $q_{R\text{-}p\ d}$ value is illustrated by the four points visible for each value of the correlation, with the bearing resistance much lower for configuration D while the results are close for the three other configurations. This figure clearly illustrates the beneficial influence of the negative correlation. Indeed, the $q_{R\text{-}p\ d}$ value monotonously increases, as the correlation becomes more negative, whatever the soil configuration. This effect is lower for soil D, i.e. for soil with a low cohesion and high friction. The increase is about 21% for configuration A, 23% for configurations B or C, and 13% for configuration D, using the resistance factoring method, with a correlation coefficient of −0.5 when compared to uncorrelated properties.

Figure 7 provides the same type of results for the case of the material factoring method, which follow the same tendencies. The effect of the negative correlation

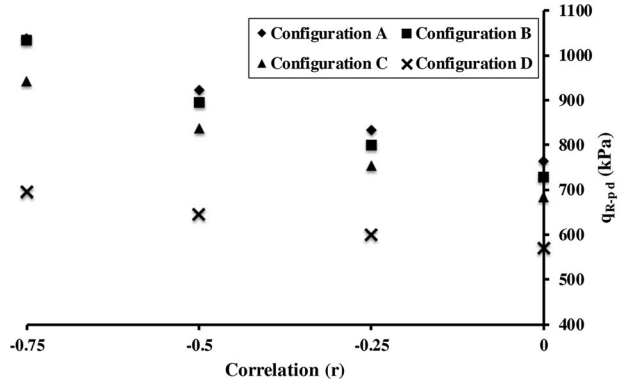

Figure 6. Effect of the correlation between (c', $Ø'$) on the bearing resistance for the resistance factoring method.

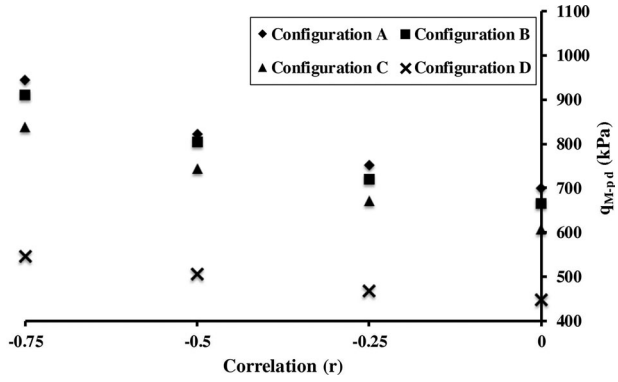

Figure 7. Effect of the correlation between (c', $\emptyset'$) on the bearing resistance for the material factoring method.

amounts to 17, 21, 22, 13% increase in bearing resistance for configurations A, B, C and D respectively when $r = -0.5$ compared to when assuming uncorrelated properties.

By comparing values in Table 5, it can also be checked that the material factoring method always leads to lower values than those obtained with the resistance factoring method, with or without correlation. This result was also obtained for the Eurocode 7 models (see Table 3).

All results demonstrate that not accounting for the negative correlation leads to conservative results, which is consistent with former conclusions of Forrest and Orr (2010) and Breysse (2011). Thus, geotechnical designs are conservative when they are based to the Eurocode 7, in accordance with Eurocode 0 (EN 1990 2002) that do not consider this correlation.

One consequence of not considering this correlation is that the safety level may therefore be much larger than that which is aimed in standards. This difference should be analyzed to check if and how the Eurocode 7 partial factors could be adapted in order to better correspond to the target safety.

4.2. Regarding the safety level in Eurocode 7

Before discussing about safety, it must be pointed that our results offer only a partial view on the safety level of the foundation, since they do not consider the variability of loading. The estimated probability values F_q provided in Table 5 do not correspond to any failure probability, but only correspond to the probability than some q bearing resistance is not reached. The effective failure probability of the foundation will probably be lower than the F_q value because of the additional contribution of safety offered by the semi-probabilistic model of loading.

One can however examine the on F_q values as given in Table 5 to see what the probability of exceedance of the design values is for all soil configurations and if the r value is lower. For all soil configurations and whatever the value of r, the value of F_q for the design values is lower than 1% (Table 5). It is lower for material factoring method than for resistance factoring method (the former being more conservative), and also decreases as r becomes more negative. Thus, the bearing resistance is analyzed independently of any consideration about a more global failure probability.

An adaptation of Eurocode 7 would mean to propose a simple change in partial factors values (either γ_R for resistance factoring method or γ_M for material factoring method). This would lead to the same level of safety in a real situation where soil properties are correlated than that is obtained with the current values which neglects the (positive effects) of this correlation. One has however, some main difficulty since the real value of correlation factor remains uncertain.

4.2.1. Case of resistance factoring method

The results in Table 5 show that neglecting correlation is conservative. A simple adaptation would be to consider a new $\gamma_{R\ \mathrm{adapt}}$ value which accounts for r, such as:

$$\gamma_{R\,\mathrm{adapt}}(r) = \gamma_{R\,\mathrm{ref}} \times q_{R-p\,d}(r=0)/q_{R-p\,d}(r) \qquad (7)$$

where $\gamma_{R\ \mathrm{ref}}$ is the current value of the partial factor on bearing resistance (=1.40 for the shallow foundation) and where $q_{R\text{-}p\ d}\ (r=0)$ and $q_{R\text{-}p\ d}\ (r)$ are available in Table 5. Since neglecting the correlation is conservative, and $q_{R\text{-}p\ d}\ (r=0) < q_{R\text{-}p\ d}\ (r)$, Equation (7) leads to lower values for the partial resistance factor.

If one takes the case $r = -0.50$, the $\gamma_R\mathrm{ref} \times q_{R-p\,d}(r=0)/q_{R-p\,d}(r)$ ratio varies from 1.13 to 1.24 depending on the soil configuration, with a mean value of 1.17. These mean values are respectively 1.30 for $r = -0.25$ and 1.05 for $r = -0.75$.

While true values of r are unknown, taking the results when $r = -0.25$ is more cautious than assuming $r = -0.5$ and will result in the 1.40 value being replaced by a value of about 1.30. In a situation where the designer would have enough information confirming a stronger correlation, an even lower value could be chosen for the same partial factor.

4.2.2. Case of material factoring method

Defining modified partial material factors that would be applied to the cohesion and friction angle, to account for the effect of their correlation in the Eurocode 7 is not so simple, since the dependency of the bearing resistance q on these two soil properties is not linear.

One can see from results in Table 5 and Figures 6 and 7 that the rate of increase of the design value of the bearing resistance when the correlation coefficient decreases (from 0 to −0.75) is very similar for resistance and material factoring methods. Thus, partial factors for friction angle and cohesion (originally equal to 1.25) can probably be adapted in the same way and reduced, to account for the correlation.

To analyse this aspect in greater detail would require to better capture how the most unfavorable combinations of *c'* and *Ø'* change with *r*. This issue has been briefly referred to in section 3.2.1 and would deserve further attention.

5. Conclusions

This paper has discussed design rules for the bearing resistance of shallow foundations. The influence of the negative correlation has been analyzed for four soil configurations corresponding to the same bearing resistance if variability is neglected, using Monte-Carlo simulations with correlated lognormal and Gaussian random variables.

These four configurations have been tested, using the "resistance factoring" and "material factoring" methods, allowed by the Eurocode 7. This strategy leads to clear conclusions:

- Accounting for a negative correlation between cohesion and friction angle always increases the bearing resistance of a shallow foundation. Thus, a Eurocode 7 design, which does not account for this correlation is always conservative,
- While leading to different bearing resistance, resistance and material factoring methods show a similar sensitivity to soil parameter correlation,
- Eurocode 7 design could be adapted to account for the effect of correlation by simply reducing the partial factors. Tentative values for the reduced partial resistance factor have been proposed in a particular case study and a similar proposal would also work for the partial material factors. However, this case will deserve a further analysis that will be detailed in a future paper.

After such a work at the ultimate limit state, another step will be to focus on the serviceability limit state, i.e. on the settlement of the shallow foundations. In such case, uncertainty propagation will also be applied to the soil properties, accounting for its spatial variability.

Disclosure statement

No potential conflict of interest was reported by the authors.

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