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Repetitive Control: Concept, Limitations, Potential[★]

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Abstract: Repetitive control represents a control scheme aimed at following or rejecting periodic exogenous signals. Its essence relies on incorporating a time-delayed signal model capable of expressing every periodic signal with a known period. Nevertheless, stabilization becomes a major issue due to the dynamics introduced to the loop by the signal model leading to delay-differential equations of neutral type for the closed-loop system. This paper briefly reviews the elements of the original repetitive control concept and the underlying issues from a time-delay system theory viewpoint. Later on, based on the highlighted facts, potential modifications to make the control scheme practically applicable are explored and briefly presented.

Keywords: repetitive control, time-delay, delay-differential equations, periodic signal.

1. INTRODUCTION

Repetitive control is one of the methods to actively regulate the periodic behavior of a dynamical system. In fact, it was first considered in 1980s to regulate power supply Inoue et al. (1981) and later for instance, in vibration suppression, Hillerstrom (1996) and ventilators, Reinders et al. (2023), showing that the control strategy has gained wide audience covering a large spectrum of applications.

Repetitive control (RC) is based on the Internal Model Principle (Francis and Wonham (1976)) which states that a regulation problem can be turned into a stabilization problem if the model of the signal is incorporated in the control-loop (see, for instance, Figure 1). The signal model considered by the repetitive control scheme directly stems from the definition of periodic signal. A signal is periodic if it satisfies the condition given by:

$$\exists T \in \mathbb{R}^+ \forall t \in \mathbb{R} \forall k \in \mathbb{Z}^*, p(t) = p(t + kT) \quad (1)$$

where the smallest T is referred to as the period of the signal. One can regard this equality as a continuous-time difference equation and can simply derive a universal model readily satisfied by all signals with period T by taking the Laplace transform:

$$P(s) = \frac{1}{1 - e^{-sT}}. \quad (2)$$

Without any lack of generality we consider $k = 1$ in (2). Nevertheless, further past periods can be included in the

model to improve the overall performance of the control, see for instance, Pipeleers et al. (2008).

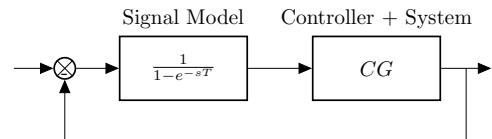


Fig. 1. Ideal Repetitive Control.

2. STABILITY PROBLEM

The signal model corresponds to a delay-difference equation in continuous-time, and such operators appear in systems represented by delay-differential equations of neutral type. More precisely, the closed-loop system can be represented by coupled delay-difference with delay-differential equations (see, e.g. Michiels and Niculescu (2014) and the references therein) corresponding to the neutral delay systems mentioned above. In the linear case, excepting the point spectrum, such systems possess also essential spectrum and, in particular, one has a chain of poles asymptotically approaching to the imaginary axis. If the augmented system CG is strictly proper, creating a closed-loop as shown in the Figure 1 makes the corresponding sensitivity transfer function also attain a chain of poles that converge to the imaginary axis, making the stability of the loop questionable. Indeed as stated in the paper of Partington and Bonnet (2004), regardless of the fact that closed-loop poles converge to the imaginary axis from the LHP, the H_∞ norm of the sensitivity can possibly become infinitely large at high frequencies, rendering the closed-loop to be unstable. Same conclusion can be arrived

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at by considering the small-gain of the loop as in, Hara et al. (1988). In other words, if a frequency outside the augmented system's bandwidth is used to excite the loop, the output of the signal model that has the tendency to magnify, due to positive feedback within, can be stabilized by the augmented system until the output of $P(s)$ reaches to a certain magnitude. However, the upper bound of this magnitude increases as the excitation frequency increases, leading to an unbounded H_∞ norm. Thus, to keep the H_∞ norm bounded, the augmented system must be bi-proper.

3. PRACTICAL MODIFICATIONS

On the other hand, the suggested idea to deal with periodic signals by the repetitive control is still worth consideration as it inherently exploits a feature of periodic signals with the use of a lumped delay. Also the original concept is aimed at addressing all periodic signals which may be unnecessarily demanding from the application point of view.

Hence, by taking into account practical facts and tweaking the original concept, the repetitive control scheme can be still put into use. Some of these methods are as follows: (i) Adjusting the signal model with a low-pass filter to limit the bandwidth. This turns the neutral system into a retarded system, see for instance, Hara et al. (1988). (ii) Utilizing more delays in the signal model and tuning it to place harmonic zeros, Yuksel et al. (2023). Similar techniques were also used to improve the performance of the low-pass filter approach (see Pipeleers et al. (2008)). (iii) Regarding the neutral signal model as a PDE and applying different discretization methods to the delay variable to form approximations, Astolfi et al. (2021). (iv) Reviewing the scheme from a disturbance observer perspective and utilizing only the errors that repeat itself in the control action generation, Chen and Tomizuka (2013).

Moreover, fused with other control approaches, the original idea can also be carried to greater extent. For instance RC can be made adaptive to period variations (Hillerstrom (1996)) and system uncertainties (Kaneko and Horowitz (1997)). Furthermore it can be made applicable to time-delay systems through Equivalent-Input-Disturbance approach Yu et al. (2017).

Nevertheless, the approach of applying RC to a system turned conceptually into a bi-proper form has not received attention. For instance in mass-spring-damper systems, one ideally achieves a bi-proper system with the use of ideal accelerometers and can form a stable neutral closed-loop with the feedback shown in Figure 2. Even though practically no accelerometers are ideal, the implications of the ideal sensory configuration can reflect positively in realizing RC when taken into account with other facts such as noise, sampling and dissipation.

REFERENCES

- Astolfi, D., Marx, S., and van de Wouw, N. (2021). Repetitive control design based on forwarding for nonlinear minimum-phase systems. *Automatica*, 129, 109671.
- Chen, X. and Tomizuka, M. (2013). New repetitive control with improved steady-state performance and

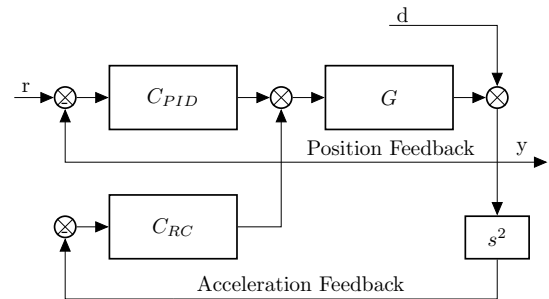


Fig. 2. Repetitive Control realized with Acceleration Feedback.

- accelerated transient. *IEEE Transactions on Control Systems Technology*, 22(2), 664–675.
- Francis, B.A. and Wonham, W.M. (1976). The internal model principle of control theory. *Automatica*, 12(5), 457–465.
- Hara, S., Yamamoto, Y., Omata, T., and Nakano, M. (1988). Repetitive control system: A new type servo system for periodic exogenous signals. *IEEE Transactions on automatic control*, 33(7), 659–668.
- Hillerstrom, G. (1996). Adaptive suppression of vibrations—a repetitive control approach. *IEEE Trans. Control Syst. Technol.*, 4(1), 72–78.
- Inoue, T., Nakano, M., Kubo, T., Matsumoto, S., and Baba, H. (1981). High accuracy control of a proton synchrotron magnet power supply. *IFAC Proceedings Volumes*, 14(2), 3137–3142.
- Kaneko, K. and Horowitz, R. (1997). Repetitive and adaptive control of robot manipulators with velocity estimation. *IEEE Transactions on Robotics and Automation*, 13(2), 204–217.
- Michiels, W. and Niculescu, S.I. (2014). *Stability, Control, and Computation for Time-Delay Systems. An Eigenvalue-Based Approach*. SIAM, Philadelphia.
- Partington, J.R. and Bonnet, C. (2004). H and bibo stabilization of delay systems of neutral type. *Systems & Control Letters*, 52(3-4), 283–288.
- Pipeleers, G., Demeulenaere, B., De Schutter, J., and Swevers, J. (2008). Robust high-order repetitive control: Optimal performance trade-offs. *Automatica*, 44(10), 2628–2634.
- Reinders, J., Giaccagli, M., Hunnekens, B., Astolfi, D., Oomen, T., and van De Wouw, N. (2023). Repetitive control for lur'e-type systems: application to mechanical ventilation. *IEEE Trans. Control Syst. Technol.*
- Yu, P., Wu, M., She, J., Liu, K.Z., and Nakanishi, Y. (2017). An improved equivalent-input-disturbance approach for repetitive control system with state delay and disturbance. *IEEE Trans. Industrial Electronics*, 65(1), 521–531.
- Yuksel, C.K., Bušek, J., Anderle, M., Vyhlídal, T., and Niculescu, S.I. (2023). A distributed delay based controller for simultaneous periodic disturbance rejection and input-delay compensation. *Mechanical Systems and Signal Processing*, 197, 110364.