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Modal complexity as a metric for Anderson localization

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We present a thorough study of the complexity of optical localized modes in two-dimensional disordered photonic crystals. Direct experimental measurements of complexity were made using an interferometric setup that allowed for extraction of phases and, hence, complex-valued wavefunctions. The comparison of experimental and theoretical results allows us to propose a metric for Anderson localization based on the average value and statistical distribution of complexity. Being an alternative to other known criteria of localization, the proposed metric exploits the openness of the disordered medium and provides a quantitative characterization of the degree of localization allowing for determining the localization length.

The mathematically rigorous definition of Anderson localization (AL) [1–3] requires exponential decay of the eigenstate intensity $|\psi(\mathbf{r})|^2$ in a disordered medium with the distance from its “localization center” \mathbf{r}_c : $|\psi(\mathbf{r})|^2 \leq a \exp(-b|\mathbf{r} - \mathbf{r}_c|)$ for sufficiently large $|\mathbf{r} - \mathbf{r}_c|$ and with positive a, b . AL is expected to take place when the scattering mean free path ℓ becomes of order or shorter than the wavelength λ , which is known as the Ioffe-Regel criterion of localization [4–6]. Both the exponential decay of $|\psi(\mathbf{r})|^2$ and the condition $\ell \lesssim \lambda$ are difficult to use in experimental and numerical studies of AL because ℓ is not easy to determine and the investigated disordered samples are always of finite extent, so that, sufficiently large distances $|\mathbf{r} - \mathbf{r}_c|$ may not be accessible. To circumvent this difficulty, Edwards and Thouless proposed a localization criterion that *exploits* the finite size and the openness of a sample: the dimensionless conductance of the sample g should be small ($g < g_c \sim 1$) if the states $\psi(\mathbf{r})$ are spatially localized and large ($g > g_c$) otherwise [7, 8]. The scaling theory of localization relaxed the requirement of knowing g_c by moving the focus on the evolution of g with the sample size L rather than on its precise value: the exponential decrease of g with L in the regime of AL is contrasted with its power-law growth for extended states [9]. Despite the scaling theory being a milestone in our understanding of AL and its major role in theoretical studies of AL [10], it remains difficult to use this theory to interpret experiments because this would require fabrication of a series of disordered samples of different sizes but with exactly the same statistical properties of disorder. As a result, other less rigorous but easier-to-apply criteria of AL have been used in recent experiments and numerical simulations: enhanced fluctuations of the scattered intensity [11–14], non-exponential decay of the time-of-flight distribution in transmission [12, 14, 15], and arrest of transverse ex-

pansion of a transmitted beam [12, 14, 16].

In this Letter, we propose and experimentally test a new metric of AL that exploits the finite size and the openness of the disordered sample in the spirit of Thouless criterion, and relies on the statistical properties of wave field fluctuations beyond intensity statistics. The criterion is based on the probability distribution $P(q^2)$ and the average value $\langle q^2 \rangle$ of the complexity $q^2 = \int \psi_r^2(\mathbf{r}) d\mathbf{r} / \int \psi_i^2(\mathbf{r}) d\mathbf{r}$ of the wave function $\psi(\mathbf{r}) = \psi_r(\mathbf{r}) + i\psi_i(\mathbf{r})$. In general, q^2 characterizes the transition from real/static wavefunctions $\psi(\mathbf{r})$ to complex/propagating ones due to the non-Hermiticity of the considered physical system [17–19]. It was extensively studied in open chaotic billiards [17, 20, 40] but very few works exist for disordered media [22]. With increasing disorder, Anderson localization effects come into play and suppress the coupling of modes to the outside world, thereby inducing a certain degree of mode stationarity. This observation allows us to propose the complexity q^2 as a measure of degree of localization. As we demonstrate below, such an approach is a viable alternative to using Ioffe-Regel or Thouless criteria of localization. Its main advantage is that it requires neither measuring $k = 2\pi/\lambda$ and ℓ (which, in addition, may be loosely defined when Anderson localization sets in [6]) nor relying on arbitrary critical values of $k\ell$ or g , both of order 1 but unknown otherwise. Compared to the direct application of scaling theory, using complexity eliminates the need for varying the sample size L in order to determine the dependence $g(L)$, which is a great simplification for most of experiments. In addition, the proposed approach allows for quantitative characterization of localized states giving access to the localization length ξ .

To demonstrate the use of complexity to reveal and characterize Anderson localization, we perform optical experiments in two-dimensional (2D) disordered photonic crystals with open boundaries. Previously, disordered photonic crystal samples were employed for studying various aspects of one-dimensional Anderson localization within PC waveguides. [23–26]. We address

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two-dimensional localization over deliberately disordered lattices, as described further. We produce our samples by lithographically writing air holes on thin GaAs membranes (thickness 340 nm). A triangular lattice (lattice constant $a = 630$ nm and radius of holes $r = 139$ nm) is used as the underlying periodic structure. The disordered samples are made by deliberately displacing each hole from its original lattice site. See Supplementary Information for details. Structure factor calculations reveal no short range correlations in our disordered samples at any disorder [27, 28]. Two representative scanning electron microscope (SEM) images of crystalline and disordered samples are shown in Figs. 1(a) and (b). The overall sample dimensions are $20\mu\text{m} \times 20\mu\text{m}$ and the experiment is performed in the near infrared regime ($\lambda \sim 1550$ nm). Phase measurements are implemented through an interferometric setup, as illustrated in Fig 1(c). We use the interferometric phase measurement technique that is well-documented in the literature [29–31], and adopt the Fourier transform method (FTM). FTM involves Fourier-space filtering of the recorded interference pattern in order to isolate the cross-term of interest as explained later. Further, we employ an efficient computational method which allows us to determine the carrier-fringe frequency to a sub-pixel accuracy, so that, the measured phase map is free of any undesirable tilt phase background [32, 33]. This technique allows rapid high-fidelity phase retrieval and makes it possible to measure optical wavefunctions over several configurations and various degrees of disorder in a unified manner.

Light output from a tunable NIR diode laser is tightly focused onto the edge of the free-standing GaAs membrane. While the light is essentially transversely confined within the membrane, the inherent surface roughness allows a fraction of the light to scatter out of the plane. Owing to the miniscule roughness, this out-of-plane scatter does not alter the mode structure, and hence acts as a non-invasive probe of the modes. The scattered light is imaged onto an InGaAs CCD using a 100X objective, and is made to interfere with the reference beam, as shown. The mode structure, in absence of the reference beam, is hereafter referred to as the ‘object’ beam. At every modal wavelength, three images corresponding to the object, reference and their interference are recorded and used to calculate the phase as discussed below.

Denoting the object beam and the reference beam at the CCD sensor plane by $O(x, y)$ and $R(x, y)$ respectively, the interference pattern $g(x, y)$ recorded by the sensor may in general be described by:

$$g = |O|^2 + |R|^2 + (O^*R + OR^*). \quad (1)$$

The problem of interest to us is to estimate the phase map $\phi_O(x, y)$ associated with the object beam $O(x, y)$. To this end, we first subtracted the individually recorded object and reference beam intensities $|O(x, y)|^2$ and $|R(x, y)|^2$ from the interference record $g(x, y)$ to retain information about the cross terms alone as follows:

$$h(x, y) = g(x, y) - |R(x, y)|^2 - |O(x, y)|^2. \quad (2)$$

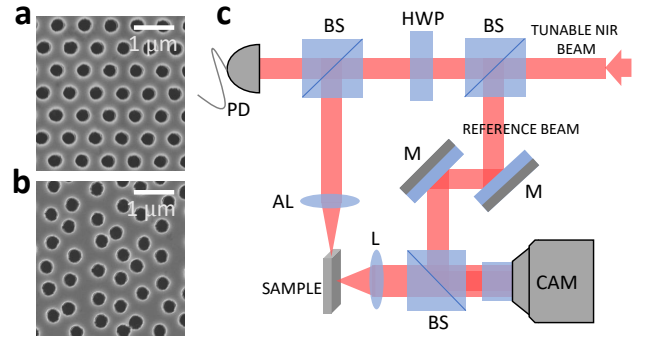


FIG. 1: (a) and (b): SEM images of a crystalline and a disordered sample. The scalebar is $1\mu\text{m}$. (c): Experimental setup used in this study. HWP: Half wave plate; BS: Beam splitter; PD: Photo detector; M: Mirror; AL: Aspheric lens; L: Lens; CAM: Infrared camera.

Since the reference wave is a tilted plane wave as illustrated in Fig. 1, the phase map of the reference wave $R(x, y)$ has the typical form $\phi_R(x, y) = 2\pi(f_{0x}x + f_{0y}y)$. A two-dimensional Fourier transform of $h(x, y)$ is observed to have two well-separated energy lobes that are centered on the carrier fringe frequencies $(\pm f_{0x}, \pm f_{0y})$. We estimated the spatial frequencies f_{0x} and f_{0y} by localizing the centroids of the cross-term lobes in the 2-dimensional Fourier transform of $h(x, y)$ with sub-pixel accuracy [32, 33]. The next processing step involves low-pass filtering of the product $[h(x, y) \times R(x, y)]$ by a circle shaped filter in 2-dimensional Fourier transform domain (with a Hamming window for avoiding any ringing artifacts) in order to separate out the cross-term $h_1(x, y) = |R|^2 O$. The phase of $h_1(x, y)$ is the same as $\phi_O(x, y)$ that we wish to recover. Due to the imaging configuration, the interference pattern is available only over a small region of the sensor array and the subtraction step in Eq. (2) is essential in order to clearly identify the carrier fringe frequency. For more details see Supplementary Information (Section I) [34].

The experimental measurements for a near-periodic sample are shown in Fig 2. Subplot (a) illustrates the standing mode formed from counter-propagating perturbed Bloch waves at $\lambda = 1575$ nm with the corresponding extracted phase-map $\phi_O(x, y)$, wrapped between $-\pi$ and π , shown in (b). Linear cross-sections at two positions in the phase map (as marked with dashed lines) are plotted in the inset. Next, we show the data for disordered samples. Two representative modes, one weakly localized and the other strongly localized are depicted in images (c) and (e) respectively, with the corresponding extracted phase maps illustrated in (d) and (f) respectively. The randomness in the spatial phases is very apparent in the images. Given the intensity $I(x, y)$ and phase $\phi_O(x, y)$, the wavefunction $\psi(x, y)$ is reconstructed as $\sqrt{I} \exp[i\phi_O(x, y)]$.

As a verification of the experimental procedure, the distribution of phases in localizing systems can be anal-

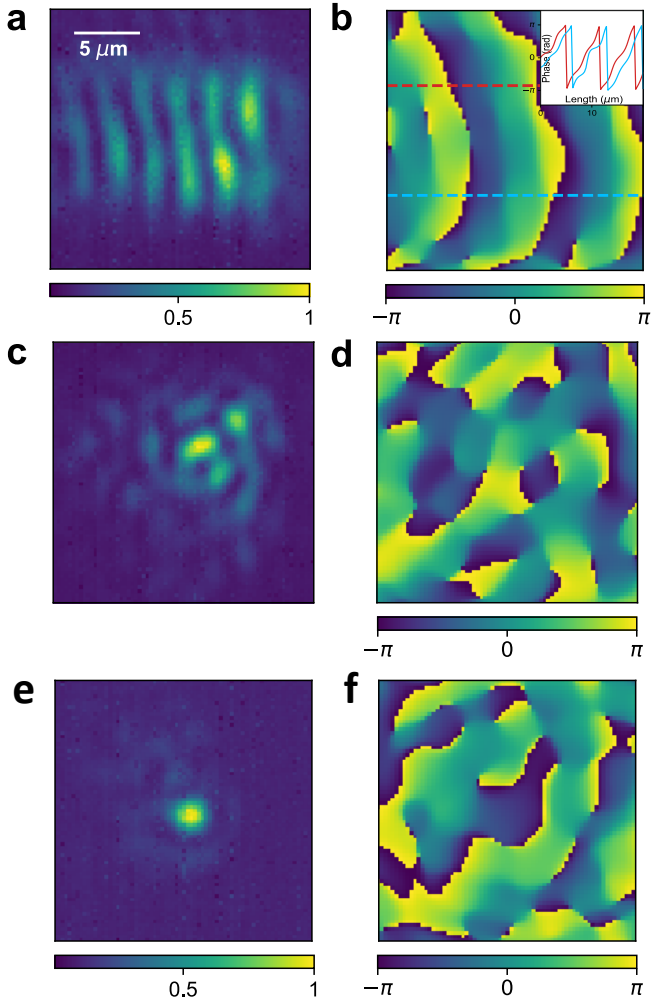


FIG. 2: (a) Standing mode formed from counter-propagating (perturbed) Bloch waves in a nearly-periodic structure and (b) the extracted phase map therefrom. Inset shows cross-sections along the dashed red and blue lines. (c,d) and (e,f) show the same for weakly-localized and Anderson-localized modes.

ysed, as they are a well-discussed feature in mesoscopic physics. Strongly localized modes are expected to exhibit a sharply peaked distribution of phase, with the peaks at 0 and π rads. With decreasing localization, the peaks broaden out to yield a flat phase distribution [22]. We characterise the spatial extents of modes in our system by their inverse participation ratio (IPR) $q_2 = N_p \sum_{i=1}^{N_p} I_i^2 / (\sum_{i=1}^{N_p} I_i)^2$, where N_p is the total number of pixels over which the mode is measured, and I_i is the intensity at the i^{th} pixel. Depending on the spatial extent of a mode, q_2 ranges from 1 for an ideally extended mode to N_p for a mode localized within a single pixel. Figure 3 displays the measured distribution of phases for four different q_2 . Clearly, as the localization strengthens, the distribution peaks at 0 and π radians, in very good agreement with the theoretical predictions, as elaborated in SI [34, 35].

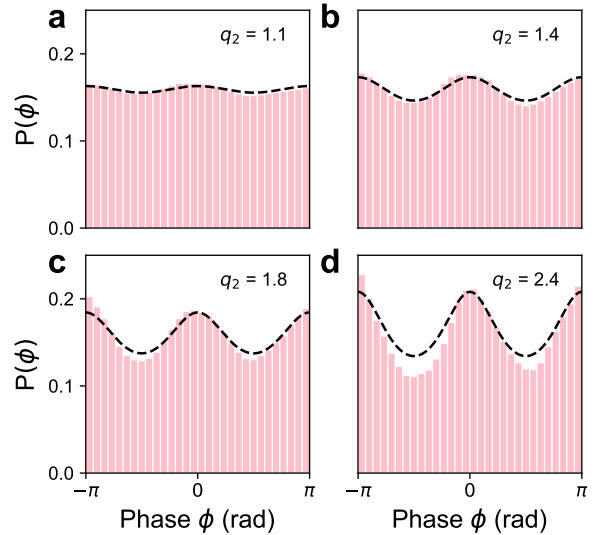


FIG. 3: Measured phase distributions for varying degree of localization (histograms) compared with theoretical predictions (dashed lines) for four values of IPR q_2 . Anderson localization leads to sharply peaked distributions about 0 and π .

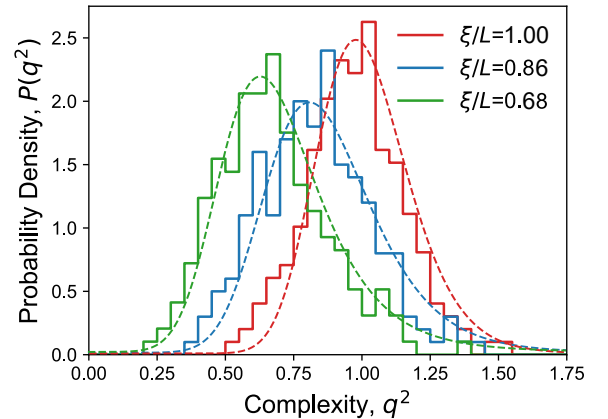


FIG. 4: Distribution of complexities at different disorder strengths parameterized by the localization length ξ (histograms). Dashed lines show theoretical fits (see Eq. (S14) of SI [34]) with $\beta = \langle q^2 \rangle$ and N as fit parameters ($\langle q^2 \rangle = 1, 0.86, 0.68$ and $N = 140, 68, 47$ for the red, blue and green lines, respectively).

Subsequently, we addressed the behavior of the extracted complexities. To this end, we determined the mode localization length ξ as an average over multiple modes categorized as per their IPR q_2 into 10 bins. The mode profiles within each bin are averaged and an exponential decay is fit to the average profile. The complexity q^2 is also averaged over the same bins. Figure 4 illustrates the probability distribution of q^2 for modes with three different localization lengths ξ . For a nearly-periodic system (red histogram), the distribution is asymmetric and peaks around $q^2 = 1$. With decreasing ξ , the distribution shifts towards the lower values of q^2 . Experimental data

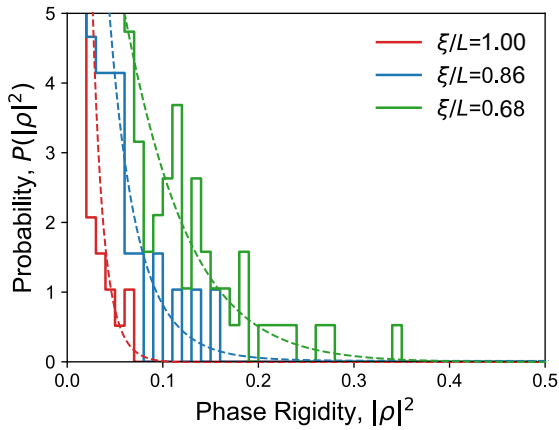


FIG. 5: Probability distribution of phase rigidity $|\rho|^2$ for the same values of ξ as in Fig. 5 (histograms). Dashed lines show theoretical predictions (see SI Section II [34]).

are fitted by a theoretical model that assumes $N \gg 1$ and treats both the numerator and the denominator of the definition of q^2 as Gaussian random variables [34, 36–39]. The first fit parameter of the model $\langle q^2 \rangle$ is discussed later, while the second fit parameter $N \sim 60$ estimates the effective number of independent terms in the definition of q^2 , which should be of the order of the number of independent speckle spots in our measurements. In addition to inducing the complexity of the wavefunction, the openness of a disordered system also leads to fluctuations of the phase rigidity $|\rho|^2$. In open quantum systems, the probability distribution $P(|\rho|^2)$ determines the effective number of open channels through which the system is coupled to the environment [20]. In our disordered samples, $P(|\rho|^2)$ can be obtained directly from $P(q^2)$ by a simple transformation. The resulting expressions are quite lengthy [34] but can be readily compared to the experimental data. Such a comparison is shown in Fig. 5. Note that the theory predicts an integrable divergence of $P(|\rho|^2) \propto 1/|\rho|$ for $|\rho|^2 \rightarrow 0$, in contrast to the predictions of finite $P(0)$ that exist for chaotic billiards [17, 20].

Finally, we reveal a novel method to identify the localization length by using the average complexity in the system. Figure 6 discusses the variation of complexity with localization length. We first note that, as shown in the inset, the first fit parameter $\langle \beta \rangle$ of the above model nicely follows the expected dependence on ξ/L . The main plot depicts the variation of the measured $\langle q^2 \rangle$ with localization length. A theoretical model for this quantity can be obtained by assuming that in our disordered samples, $\langle q^2 \rangle$ is related to the typical mode width Γ and the average mode spacing Δ in the same way as in an open chaotic system [40]: $\langle q^2 \rangle = C \times \Gamma/\Delta$ where C is a model-specific constant. The ratio $g = \Gamma/\Delta$ is known as the dimensionless conductance in the context of mesoscopic physics. In the regime of Anderson localization, it is $g \simeq \exp(-L/\xi)$. We thus have $\langle q^2 \rangle \simeq \exp(c - L/\xi)$

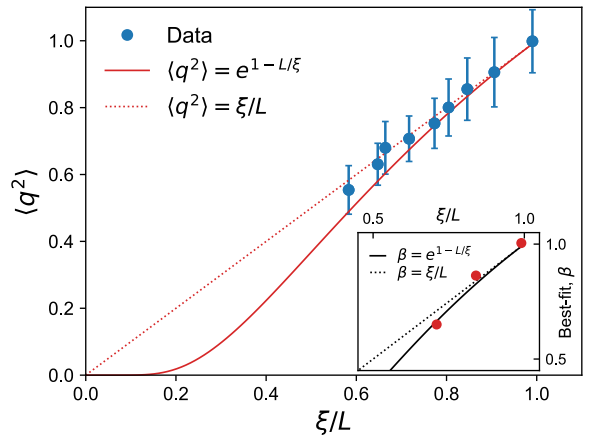


FIG. 6: Average complexity as a function of the localization length ξ . A clear linear behavior is observed in the data. Solid and dotted lines show two theoretical predictions for $\langle q^2 \rangle$. Inset shows the best-fit values of $\beta = \langle q^2 \rangle$ obtained from the fits in Fig. 4 (symbols).

with an unknown constant $c \sim 1$. Setting $c = 1$ yields a reasonable fit to the experimental data in Fig. 5. The fit is even better if we use a power series expansion of the above results around $\xi/L = 1$: $\langle q^2 \rangle \simeq \xi/L$. The simple yet powerful relation $\langle q^2 \rangle = \exp(1 - L/\xi) \simeq \xi/L$ for localized modes established above offers an immediate diagnostic into the localization parameters, such as ξ or g . The extrapolation of this relation to tighter localization (not accessible experimentally in our samples) leads to $\langle q^2 \rangle = 0$ for $\xi = 0$, which carries a very plausible implication that a δ -localized wavefunction should have no propagating component.

In conclusion, we present a novel criterion given by modal complexity to estimate the degree of light localization in disordered structures. We perform, to our knowledge, the first direct experimental measurement of the complexity of Anderson localized wavefunctions in open 2D disordered systems. Theoretical models developed for both the average complexity and its full statistical distribution allow us to estimate the localization length, thus demonstrating that complexity is an appropriate quantity for revealing and characterizing the Anderson localization phenomenon. Our work suggests a complexity-based metric of Anderson localization that may be an interesting alternative to other known criteria such as those proposed by Ioffe and Regel [4] or Thouless [7, 8]. Although we demonstrate its validity for localization of light in 2D, we believe that it can be readily applied to any type of waves in disordered media because the theoretical arguments behind it are not specific for light or for two spatial dimensions. It would be particularly interesting to analyze complexity of localized modes in 3D experiments with ultrasound [12, 41] and matter waves [42, 43] as well as in recent electromagnetic simulations [14]. Our work promises to reveal hitherto-unknown rela-

tions between complex fields and mesoscopic parameters of transport. It also emphasizes the need to address field statistics and not that of intensities, on which the current literature predominantly relies.

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