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► To cite this version:

Chinnaraji Annamalai. Novel Method to compute the Sum of Geometric Series and its Factors. 2024. <hal-04787154>

HAL Id: hal-04787154

<https://hal.science/hal-04787154v1>

Preprint submitted on 17 Nov 2024

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Novel Method to compute the Sum of Geometric Series and its Factors

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Abstract: Nowadays, the growing complexity of mathematical and computational modelling demands the simplicity of mathematical equations to solve today's scientific problems and meet challenges. In this paper, the author introduces a novel technique to compute the sum of geometric series and its factors. This computational technique can enable the students and scholars to enhance their research knowledge in the area of computing and mathematical sciences.

MSC Classification codes: 40A05 (65B10)

Keywords: computation, geometric progression, calculus

1. Introduction

Geometric series [1-10] played a vital role in differential and integral calculus at the earlier stage of development and still continues as an important part of the study in science, engineering, and management. In this article, the author introduces the sum of geometric series and its factors in a new way.

2. Computation of Geometric Series

Let x be a variable and it is assumed to a real number. Then, the sums of geometric series are computed on the real number in a novel way.

$$x = x \Rightarrow x = (x - 1) + 1 \Rightarrow x = \frac{x - 1}{x^0} + \frac{x - 1}{x} + \frac{1}{x} \Rightarrow x = \frac{x - 1}{x^0} + \frac{x - 1}{x} + \frac{x - 1}{x^2} + \frac{1}{x^2}.$$

We can further expand the above expression as follows:

$$x = x \Rightarrow x = \frac{x - 1}{x^0} + \frac{x - 1}{x} + \frac{x - 1}{x^2} + \frac{x - 1}{x^3} + \frac{x - 1}{x^4} + \dots + \frac{x - 1}{x^{n-2}} + \frac{x - 1}{x^{n-1}} + \frac{x - 1}{x^n} + \frac{1}{x^n}.$$

$$x = x \Rightarrow \frac{x - 1}{x^0} + \frac{x - 1}{x} + \frac{x - 1}{x^2} + \frac{x - 1}{x^3} + \dots + \frac{x - 1}{x^{n-1}} + \frac{x - 1}{x^n} = x + \frac{1}{x^n} = \frac{n^{n+1} - 1}{x^n}.$$

$$x = x \Rightarrow \frac{1}{x^0} + \frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^3} + \frac{1}{x^4} + \frac{1}{x^5} + \frac{1}{x^6} + \dots + \frac{x}{x^{n-2}} + \frac{1}{x^{n-1}} + \frac{1}{x^n} = \frac{n^{n+1} - 1}{x^n (x - 1)}.$$

$$\text{Therefore, } x = x \Rightarrow \sum_{i=0}^n \frac{1}{x^i} = \frac{n^{n+1} - 1}{x^n (x - 1)}.$$

Next, let us compute the sum of geometric series based on the equation $x^n = x^n$.

$$x^n = x^n \Rightarrow x^n = (x - 1)x^{n-1} + x^{n-1} \Rightarrow x^n = (x - 1)x^{n-1} + (x - 1)x^{n-2} + x^{n-2}.$$

We can further expand the above expression as follows

$$x^n = x^n \Rightarrow x^n = (x - 1)x^{n-1} + (x - 1)x^{n-2} + (x - 1)x^{n-3} + \dots + (x - 1)x + (x - 1) + 1.$$

$$x^n = x^n \Rightarrow (x - 1)x^{n-1} + (x - 1)x^{n-2} + (x - 1)x^{n-3} + \dots + (x - 1)x + (x - 1) = x^n - 1.$$

$$x^n = x^n \Rightarrow x^{n-1} + x^{n-2} + x^{n-3} + \dots + x + 1 = \frac{x^n - 1}{x - 1}.$$

$$\text{Therefore, } x^n = x^n \Rightarrow \sum_{i=0}^{n-1} x^i = \frac{x^n - 1}{x - 1}.$$

3. Factors of Geometric Series

The following theorem indicates the factors of geometric series [7].

Theorem 2.3: $\prod_{k=0}^n (x^{2^k} + 1) = \frac{x^{2^{n+1}} - 1}{x - 1}, x \neq 1.$

Proof. Let's prove this theorem as follows.

$$\frac{x^2 - 1}{x - 1} = \frac{(x + 1)(x - 1)}{x - 1} = x + 1. (\because 1^n = 1).$$

$$\frac{x^{2^2} - 1}{x - 1} = \frac{x^4 - 1}{x - 1} = \frac{(x^2 + 1)(x^2 - 1)}{x - 1} = (x^2 + 1)(x + 1).$$

$$\frac{x^{2^3} - 1}{x - 1} = \frac{x^8 - 1}{x - 1} = (x^4 + 1)(x^2 + 1)(x + 1) = (x^{2^2} + 1)(x^{2^1} + 1)(x^{2^0} + 1).$$

We can continue the same process up to 2^{n+1} .

$$\frac{x^{2^3} - 1}{x - 1} = \frac{x^8 - 1}{x - 1} = (x^{2^n} + 1)(x^{2^{n-1}} + 1) \dots (x^{2^1} + 1)(x^{2^0} + 1).$$

$$\therefore \prod_{k=0}^n (x^{2^k} + 1) = \frac{x^{2^{n+1}} - 1}{x - 1}, x \neq 1. (\because 1^n = 1).$$

Hence proved.

Corollary 2.3: $\sum_{i=0}^{2^{n+1}-1} x^i = \prod_{k=0}^n (x^{2^k} + 1) = \frac{x^{2^{n+1}} - 1}{x - 1}, x \neq 1.$

4. Conclusion

In this article, the author has introduced the sums of geometric series and its factors in a novel way. This idea can enable the researchers to be involved in the scientific research on computing and mathematical sciences.

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