

Wind speed turbulence system identification and signal generator

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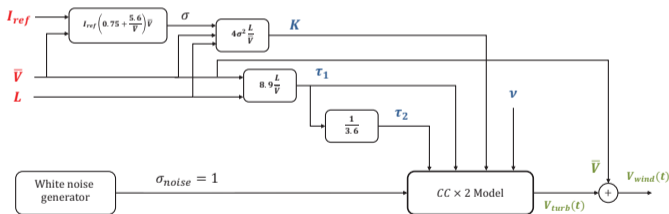
Séminaire du
Groupe
Automatique



M. Hajjem, S. Victor, P. Melchior, P. Lanusse, and L. Thomas (2023), "Wind turbulence modeling for real-time simulation." *Fractional Calculus & Applied Analysis* vol. 26, pp.1632–1662 doi: 10.1007/s13540-023-00165-0.

Summary

- 1 Introduction
- 2 State of the art
 - Wind speed components
 - von Kármán model
 - Model identification
- 3 Improved fractional models
 - Davidson-Cole model
 - Cole-Cole model
 - Improved Cole-Cole model
- 4 Wind turbulence model design
 - Model parameter tuning
 - Time simulation
 - Wind speed generator
 - Application on a 3-axis antenna
- 5 Conclusion



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Introduction

- Wind turbulence = main disturbance for many mechanical systems operating outdoors
wind turbines, wind response of solar panels, kites for traction or electric power generation, sailing, ...
- A need to study wind effects in order to design a suitable control law capable of rejecting them



Introduction

- Wind turbulence = main disturbance for many mechanical systems operating outdoors
wind turbines, wind response of solar panels, kites for traction or electric power generation, sailing, ...
- A need to study wind effects in order to design a suitable control law capable of rejecting them
- Wind turbulence = non-stationary and stochastic process
⇒ Models based on spectral characteristics
- Design of models capable of time simulating wind turbulence effects on systems

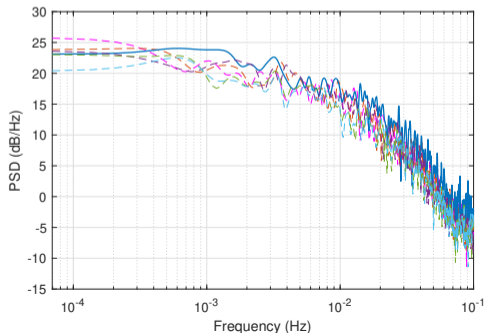
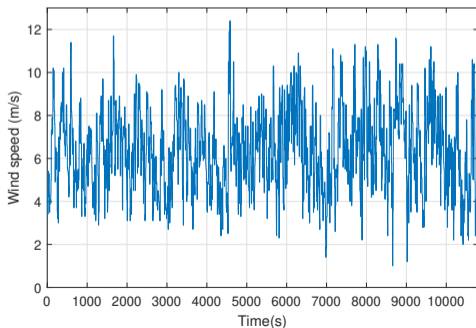


Objectives

Design an accurate model to well simulate realistic wind speed and provide a wind turbulence generator.

Experimental protocol

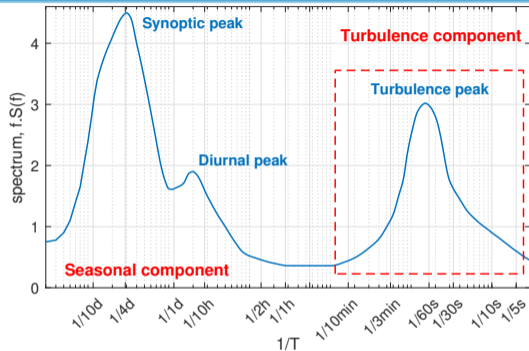
- **28 datasets** recorded near Montpellier, France
- Ultrasonic anemometer integrated into a Vaisala WXT520 weather transmitter
range: $[0, 60]$ m/s; accuracy: $\pm 3\%$ in $[0, 36]$ m/s and $\pm 5\%$ in $[36, 60]$ m/s
- Wind speed studied over a period of 3h
⇒ modeling only short-term fluctuations
- Study focused on spectral characteristics (PSD)
at frequencies below 0.1 Hz with a quasi-constant average speed



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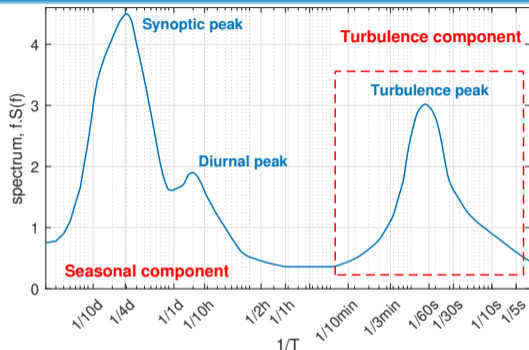
Wind speed components

- Spectral analysis of wind power over a long period of time \Rightarrow Several scales of fluctuations [van der Hoven (1957)]
- Spectrum divided into 3 parts
 - Seasonal: long periods ($T > 3h$)
 - Energy gap ($15min < T < 2h$)
 - Turbulence: short periods ($T < 10min$)
- Turbulence peak has a period of 76s
- Frequency range $[\frac{1}{10min}, \frac{1}{5s}] = [1.6 \cdot 10^{-3}Hz, 0.2Hz]$



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Wind speed components

Wind speed can be expressed as:

$$V(t) = \bar{V}(t) + V_{turb}(t) \quad (1)$$

where $\bar{V}(t)$ is the long-term component and $V_{turb}(t)$ the turbulence component.



I. van der Hoven (1957), "Power Spectrum of Horizontal Wind Speed in the Frequency Range from 0.0007 to 900 Cycles Per Hour."

Journal of Atmospheric Sciences 14(2), 160–164. v9(4), 341–351.

Wind turbulence PSD approximation

- The von Kármán model approximates the wind speed turbulence PSD [Kármán (1948)]:

$$S_{VK}(f) = \frac{4\sigma^2 \frac{L}{\bar{V}}}{\left(1 + 70.8 \left(\frac{L}{\bar{V}} f\right)^2\right)^{5/6}} \quad (2)$$

where σ is the turbulence standard deviation, L is the length scale and \bar{V} is the mean speed

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- The model is tuned by 2 parameters:
 - length scale L = average size of the fluctuations in terms of distance
from Engineering Sciences Data Unit (ESDU) report:

$$L_{ESDU} = 25z^{0.35} z_0^{-0.063} \quad (3)$$

where z is the altitude and z_0 is a surface roughness parameter.



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11, pp.530 LP – 539.

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- turbulence intensity $I = \frac{\sigma}{\bar{V}}$
from International Electrotechnical Commission (IEC) report is:

$$I_{IEC} = I_{ref} \left(0.75 + \frac{5.6}{\bar{V}}\right) \quad (4)$$

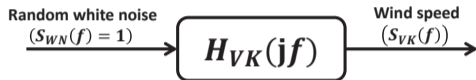


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Shaping filter

Wind PSD approximation \Rightarrow Time simulation ☹

\Rightarrow Need to use **shaping filters**



$H_{VK}(jf)$ = the filter transfer function stemming from the von Kármán model

$$S_{VK}(f) = \frac{K}{(1 + (\tau f)^2)^{5/6}} \quad (5)$$

where $K = 4\sigma^2 \frac{L}{V}$ and $\tau = \sqrt{70.8} \frac{L}{V}$

$$S_{VK}(f) = S_{WN}(f) |H_{VK}(f)|^2 = |H_{VK}(f)|^2 = H_{VK}(jf) H_{VK}^*(jf) \quad (6)$$

The resulting transfer function:

$$H_{VK}(jf) = \frac{\sqrt{K}}{(1 + j\tau f)^{5/6}} \quad (7)$$

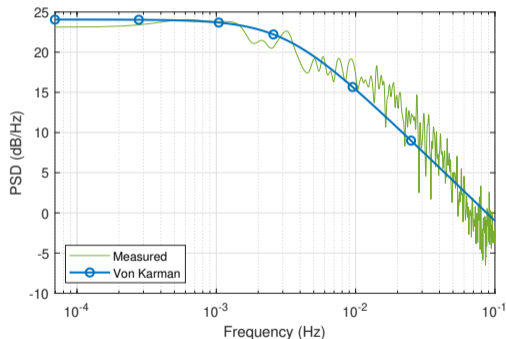
Identification results

- Minimization of the cost function:

$$J = \frac{1}{N} \sum_{i=1}^N |S_{dB}(f_i) - \widehat{S}_{dB}(f_i, K, \tau)|^2 \quad (8)$$

by using the nonlinear simplex algorithm

- The von Kármán model provides a good fit in low frequency
- Need to improve the fit in middle and high frequency
High frequency slope fixed to $5/6$
⇒ Constraint that can be released according to real data



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Davidson-Cole model

A direct extension is to use a Davidson-Cole form with a **variable non-integer order** ν :

$$H_{DC}(s) = \frac{\sqrt{K}}{\left(1 + \frac{\tau}{2\pi} s\right)^\nu}. \quad (9)$$

The resulting PSD expression:

$$S_{DC}(f) = H_{DC}(jf)H_{DC}(jf)^*$$

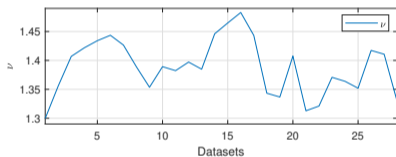
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Davidson, D. W. and Cole, R. H. (1951), "Dielectric relaxation in glycerol, propylene glycol, and n-propanol." *The Journal of Chemical Physics*, 19(12):1484–1490, (1951).

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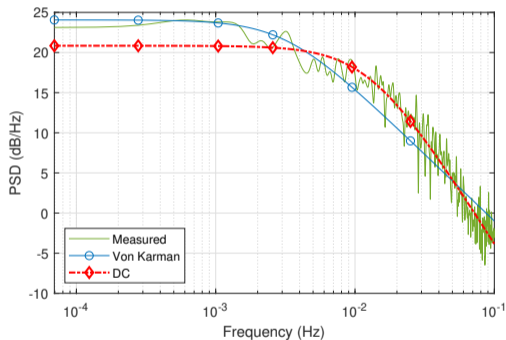
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Identification results :



- The DC model fits well the measurements in high frequency ($f > 0.01\text{Hz}$) thanks to the estimated $\nu \in [1.30, 1.48] \neq 5/6 (\approx 0.833)$

Cole-Cole model

An improved model transfer function based on the Cole-Cole form (CC model):

$$H_{CC}(s) = \frac{\sqrt{K}}{1 + \left(\frac{\tau}{2\pi} s\right)^\nu}. \quad (10)$$

The resulting PSD expression:

$$\begin{aligned} S_{CC}(f) &= H_{CC}(jf)H_{CC}(jf)^* \\ &= \frac{K}{1 + 2 \cos\left(\nu \frac{\pi}{2}\right) (\tau f)^\nu + (\tau f)^{2\nu}} \end{aligned}$$



K.S. Cole and R.H. Cole (1941), "Dispersion and absorption in dielectrics I. alternating current characteristics" *The Journal of chemical physics* v9(4), 341–351.

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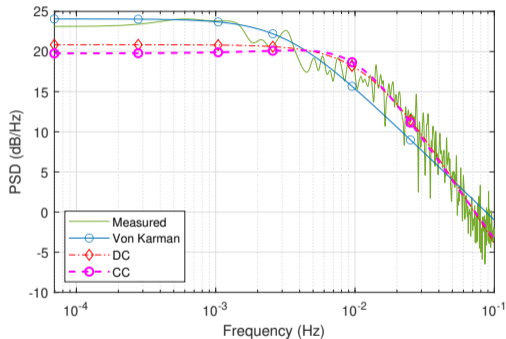
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Identification results :



- The CC model fits well the measurements in high frequency ($f > 0.01\text{Hz}$) thanks to the estimated $\nu \in [1.16, 1.25] \neq 5/6 (\approx 0.833)$

Improved Cole-Cole model: CC×2 model

Addition of a second fractional order cell to the Cole-Cole model (CC×2 model):

$$H_{CC \times 2}(s) = \frac{\sqrt{K}}{\left(1 + \left(\frac{\tau_1}{2\pi} s\right)^\nu\right) \left(1 + \left(\frac{\tau_2}{2\pi} s\right)^{2\nu}\right)} \quad (11)$$

⇒ **A second slope for a better fit in middle frequency**

The resulting PSD expression:

$$S_{CC \times 2}(f) = \frac{K}{D_1(f)D_2(f)} \quad (12)$$

with

$$D_1(f) = 1 + 2 \cos\left(\nu \frac{\pi}{2}\right) (\tau_1 f)^\nu + (\tau_1 f)^{2\nu}$$

$$D_2(f) = 1 + 2 \cos(\nu\pi) (\tau_2 f)^{2\nu} + (\tau_2 f)^{4\nu}$$

Identification results :

Improved Cole-Cole model: CCx2 model

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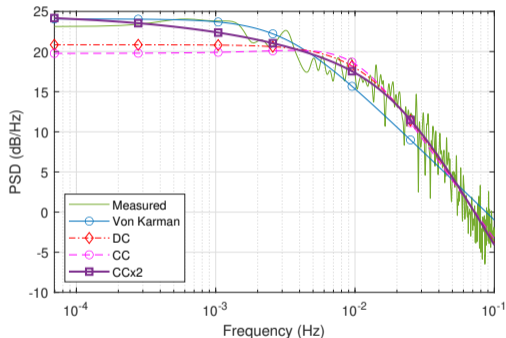
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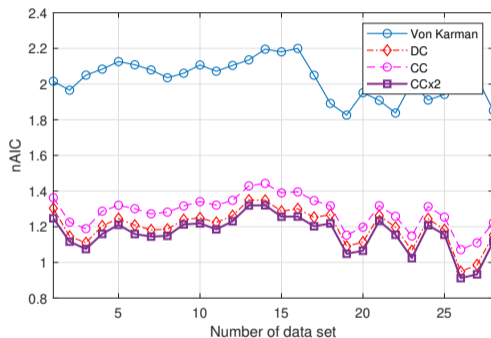


- Good fit in high frequency
- Static gain equal to the von Kármán one
- **Improved fit in middle frequency**

Identification results

Table: Comparison between the mean nAIC of the different models

Model	Number of parameters	Mean nAIC
von Kármán	2	2.027
Davidson-Cole	3	1.200
Cole-Cole	3	1.283
Cole-Cole×2	4	1.162



⇒ **The CC×2 model reproduces accurately the wind turbulence spectral characteristics**

$$nAIC = N \log\left(\det\left(\frac{1}{N} E^t E\right)\right) + \frac{2n_p}{N} \quad (13)$$

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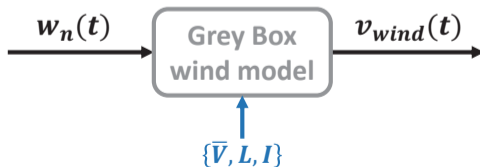
Wind grey box model



Wind grey box model:

- same transfer function form as the black box model
- parameters obtained from the wind known physical characteristics

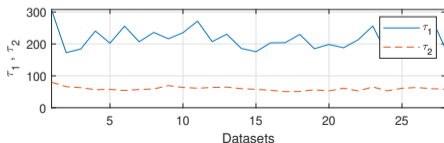
⇒ **Generating random realistic wind speed turbulence without reference data**



System Identification for 28 datasets

Selected model: **Wind CC×2 model**:

$$H_{CC \times 2}(s) = \frac{\sqrt{K}}{\left(1 + \left(\frac{\tau_1}{2\pi} s\right)^\nu\right) \left(1 + \left(\frac{\tau_2}{2\pi} s\right)^{2\nu}\right)}$$



System Identification for 28 datasets

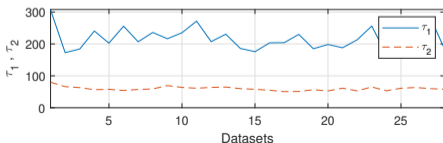
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- Same static gain expression as the Von Kármán model:

$$K = 4\sigma^2 \frac{L}{V}$$

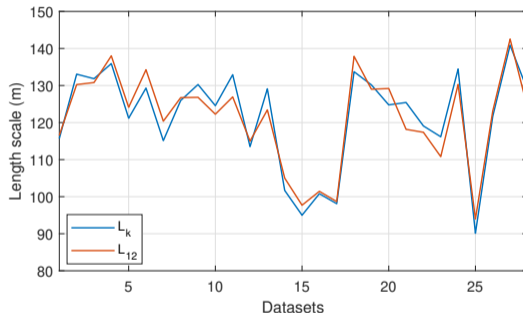
- The mean value of $\nu = 0.516$
($\nu \in [0.49, 0.53]$)
- The ratio between τ_1 and τ_2 is in
[2.6, 4.6]: average value **3.6**



Model parameter tuning

(Von Kármán model: $K = 4\sigma^2 \frac{L}{\bar{V}}$ and $\tau = 8.41 \frac{L}{\bar{V}} \implies L_K = \frac{K\bar{V}}{4\sigma^2}$.)

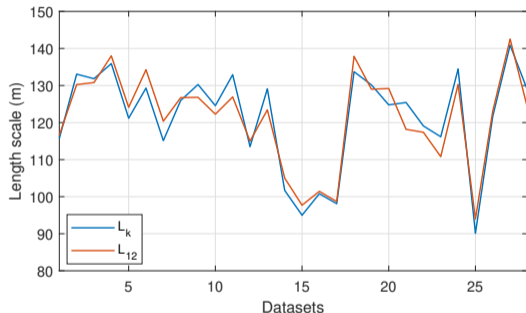
- **Validation** by comparing the length scales L_k and $L_{12} = \frac{\bar{V}\sqrt{\tau_1\tau_2}}{4.7}$ obtained from τ_1 and τ_2 for 28 different datasets



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Grey-box wind turbulence model

Wind CC×2 model defined by:

$$\mathbf{K} = 4\sigma^2 \frac{\mathbf{L}}{\bar{V}}, \quad \tau_1 = 8.9 \frac{\mathbf{L}}{\bar{V}}, \quad \tau_2 = \frac{\tau_1}{3.6}, \quad \nu = 0.516 \quad (14)$$

Rational filter design: Oustaloup's approximation

Recursive distribution of zeros and poles

The frequency band $[\omega_A, \omega_B] = [10^{-1}, 10^1]$. Therefore, the rational approximation will be carried out on $[10^{-2}, 10^2]$. s^ν is approximated by:

$$s^\nu \rightarrow s_{[\omega_A, \omega_B]}^\nu = C_0 \left(\frac{1 + \frac{s}{\omega_b}}{1 + \frac{s}{\omega_h}} \right)^\nu \approx C_0 \prod_{k=1}^N \frac{1 + \frac{s}{\omega'_k}}{1 + \frac{s}{\omega_k}} \quad (15)$$

where:

$$\omega'_1 = \eta^{1/2} \omega_b, \quad \omega_1 = \alpha \omega'_1,$$

$$\omega'_{k+1} = \alpha \eta \omega'_k, \quad \omega_{k+1} = \alpha \eta \omega_k,$$

$$\nu = \frac{\log \alpha}{\log(\alpha \eta)}, \quad \alpha = \left(\frac{\omega_h}{\omega_b} \right)^{\frac{\nu}{N}}, \quad \eta = \left(\frac{\omega_h}{\omega_b} \right)^{\frac{1-\nu}{N}}$$

and

$$C_0 = \left| \frac{1 + j \frac{1}{\omega_h}}{1 + j \frac{1}{\omega_b}} \right|^\nu = \left(\frac{\omega_h}{\omega_b} \right)^\nu \left(\frac{1 + \omega_b^2}{1 + \omega_h^2} \right)^{\nu/2}.$$



A. Oustaloup, F. Levron, B. Mathieu, and F. M. Nanot (2000), "Frequency-band complex noninteger differentiator: characterization and synthesis" *IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications*, 47(1):25–39, DOI:10.1109/81.817385.

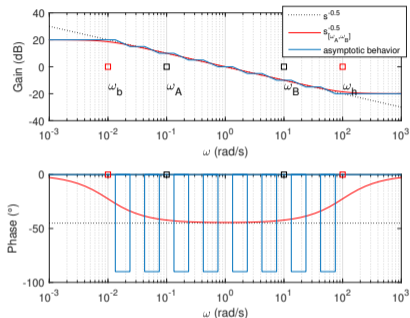


Figure: Fractional integrator $s^{-0.5}$ and its approximation $s_{[0.1, 10]}^{-0.5}$ based on Oustaloup's recursive poles and zeros approximation

Time simulation

- Measured wind characteristics:
 $\bar{V} = 6.6\text{m/s}$, $\sigma = 1.92$, $L = 120\text{m}$
- Identified CCx2 model:

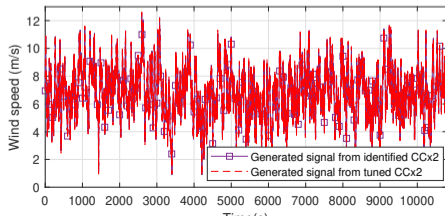
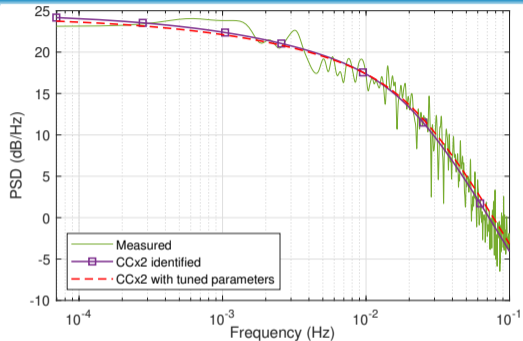
$$H_{BB}(s) = \frac{17.35}{48.72s^{1.554} + 8.59s^{1.036} + 5.67s^{0.518} + 1}$$

- Tuned grey box model:

$$K = 269.94, \quad \tau_1 = 161.74\text{s}, \quad \tau_2 = 44.93\text{s}$$

$$H_{GB}(s) = \frac{16.43}{40.7s^{1.548} + 7.62s^{1.032} + 5.34s^{0.516} + 1}$$

⇒ Both models provide wind speed with a PSD similar to the measured wind PSD



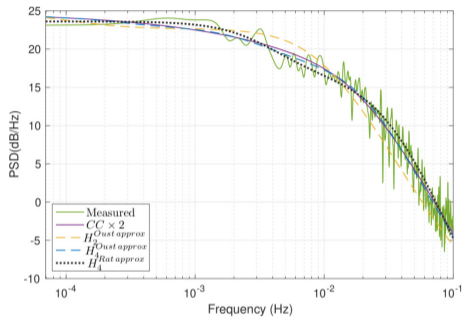
Comparison with rational models

- Rational approximation from Oustaloup's approximation:

$$H_{N_{cells}}^{Oust\ approx} = H_0 \prod_{k=1}^N \frac{1 + \frac{s}{\omega'_k}}{1 + \frac{s}{\omega_k}}, \quad (16)$$

- Rational approximation:

$$H_4^{Rat\ approx} = H_0 \frac{1 + \frac{s}{\omega_2}}{\left(1 + \frac{s}{\omega_1}\right) \left(1 + \frac{s}{\omega_3}\right)}, \quad (17)$$



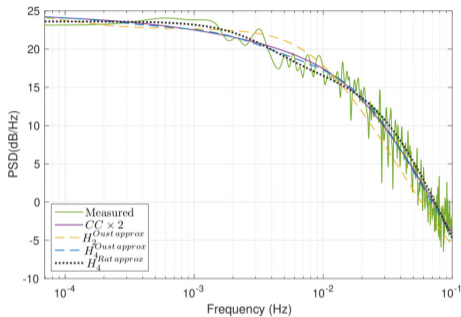
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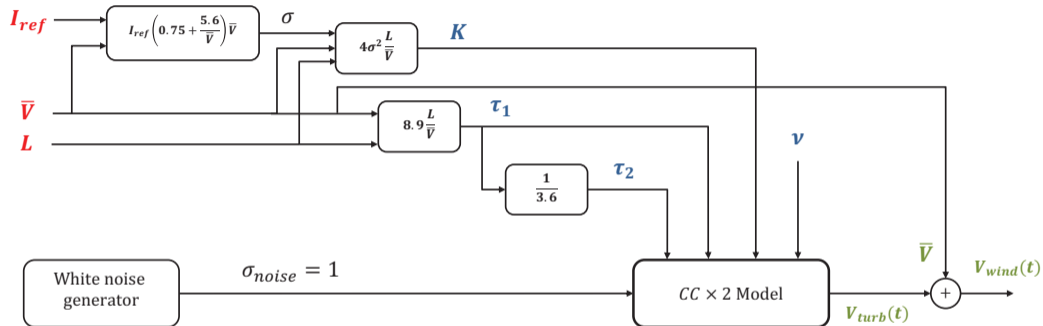
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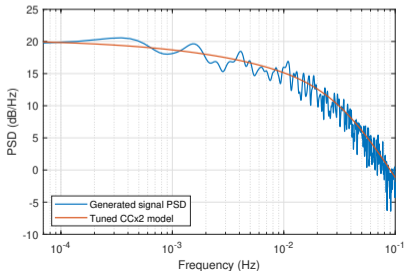
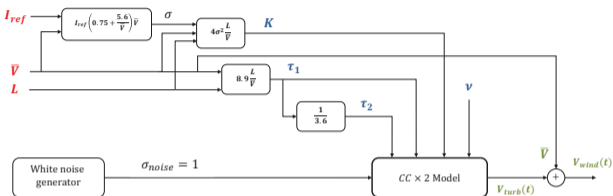


Model	Number of parameters	Fit (%)	nAIC
Cole-Cole×2	4	96.473	1.193
Oust. Approx $H_2^{Oust\ approx}$	14	91.514	2.083
Oust Approx $H_4^{Oust\ approx}$	26	96.502	1.214
Rat. Approx $H_4^{Rat\ approx}$	4	73.166	3.222

Wind speed generator



Wind speed generator



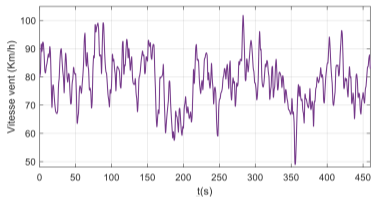
```

1 %% Model inputs
2 L = 110; %Length scale (m)
3 Vm = 80/3.6; %Mean speed (m/s)
4 I_ref = 0.12; %Turbulence intensity reference (low: 0.12, mean: 0.14,
5             % high: 0.16)
6 Ts = 1;
7 %% Tuning model parameters
8 I = I_ref.*(0.75+5.6/Vm);
9 sigma = I*Vm;
10 K = 4.*(sigma)^2.*L./Vm;
11 Taul = 8.*L./Vm;
12 Tau2 = Taul./3;
13 nu = 0.51;
14
15 %% Fractional transfer function
16 num=sqrt(K);
17 den1=frac.poly_exp([(Taul/2/pi)^nu 1],[nu 0]);
18 den2=frac.poly_exp([(Tau2/2/pi)^(2*nu) 1],[2*nu 0]);
19
20 sys = frac.tf(num,den1)*frac.tf(1,den2);
21
22 %% Approximated rational transfer function
23 set(sys,'N',10); %Cell number
24 set(sys,'band',[1e-4,10]); %Frequency range + 2 decades
25 set(sys,'sim','Oustaloup');
26
27 H_CCx2 = frac2int(sys);

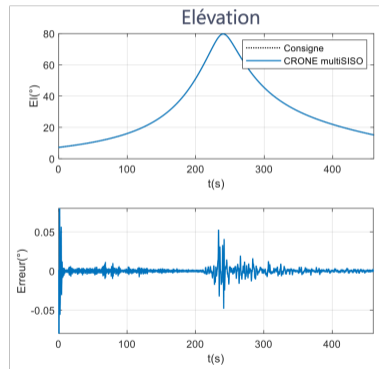
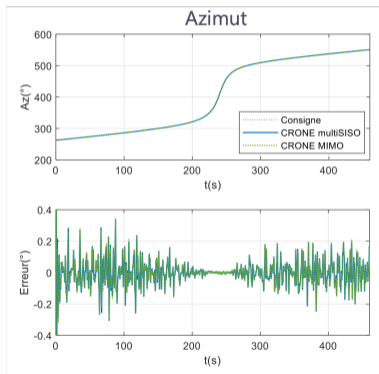
```

<http://cronetoolbox.ims-bordeaux.fr>

Application on a 3-axis antenna




- Wind turbulence average:
 $\bar{V} = 80\text{km/h}$
- Wind turbulence speed:
 $[50, 100]\text{km/h}$
- MIMO CRONE control for turbulence rejection



- 1 Introduction
- 2 State of the art
 - Wind speed components
 - von Kármán model
 - Model identification
- 3 Improved fractional models
 - Davidson-Cole model
 - Cole-Cole model
 - Improved Cole-Cole model
- 4 Wind turbulence model design
 - Model parameter tuning
 - Time simulation
 - Wind speed generator
 - Application on a 3-axis antenna
- 5 Conclusion

Conclusion

- Wind models based on the spectral characteristics and not on time series data, then shaping filters are synthesized for time simulation
- Study and identification of the Von Kármán approximation and its limits
- Improved fractional models with: Davidson-Cole, Cole-Cole and Cole-Cole \times 2 models
- Tuning methodology to transform the CC \times 2 black box model into a grey box model \Rightarrow not limited by access to reference data
- **A long period wind mean speed generator can be developed for more realistic scenarios**

 **M. Hajjem, S. Victor, P. Melchior, P. Lanusse, and L. Thomas (2023)**, “Wind turbulence modeling for real-time simulation.” *Fractional Calculus & Applied Analysis* vol. 26, pp.1632–1662 doi: 10.1007/s13540-023-00165-0.