

# Wind speed turbulence system identification and signal generator

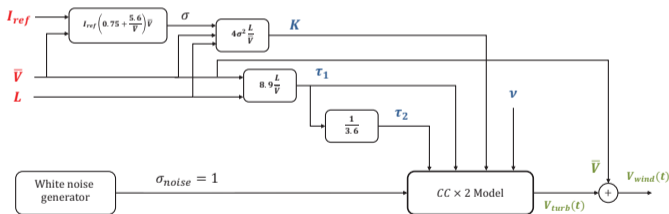
**Stéphane VICTOR**



**M. Hajjem, S. Victor, P. Melchior, P. Lanusse, and L. Thomas (2023)**, "Wind turbulence modeling for real-time simulation." *Fractional Calculus & Applied Analysis* vol. 26, pp.1632–1662 doi: 10.1007/s13540-023-00165-0.

# Summary

- 1 Introduction
- 2 State of the art
  - Wind speed components
  - von Kármán model
  - Model identification
- 3 Improved fractional models
  - Davidson-Cole model
  - Cole-Cole model
  - Improved Cole-Cole model
- 4 Wind turbulence model design
  - Model parameter tuning
  - Time simulation
  - Wind speed generator
  - Application on a 3-axis antenna
- 5 Conclusion



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# Introduction

- Wind turbulence = main disturbance for many mechanical systems operating outdoors  
wind turbines, wind response of solar panels, kites for traction or electric power generation, sailing, ...
- A need to study wind effects in order to design a suitable control law capable of rejecting them



# Introduction

- Wind turbulence = main disturbance for many mechanical systems operating outdoors  
wind turbines, wind response of solar panels, kites for traction or electric power generation, sailing, ...
- A need to study wind effects in order to design a suitable control law capable of rejecting them
- Wind turbulence = non-stationary and stochastic process  
⇒ Models based on spectral characteristics
- Design of models capable of time simulating wind turbulence effects on systems

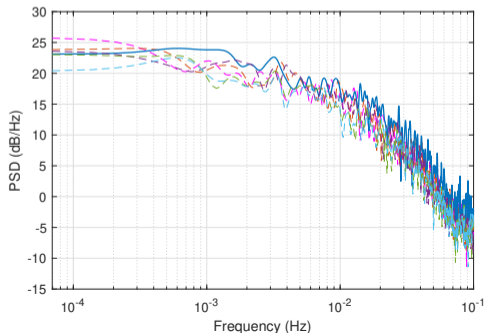
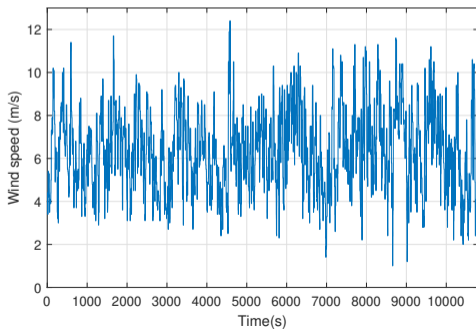


## Objectives

Design an accurate model to well simulate realistic wind speed and provide a wind turbulence generator.

# Experimental protocol

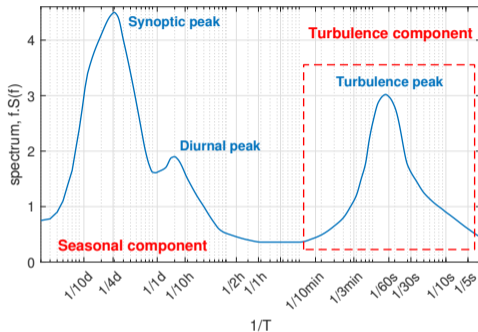
- **28 datasets** recorded near Montpellier, France
- Ultrasonic anemometer integrated into a Vaisala WXT520 weather transmitter  
range:  $[0, 60]$ m/s; accuracy:  $\pm 3\%$  in  $[0, 36]$ m/s and  $\pm 5\%$  in  $[36, 60]$ m/s
- Wind speed studied over a period of 3h  
⇒ modeling only short-term fluctuations
- Study focused on spectral characteristics (PSD)  
at frequencies below 0.1 Hz with a quasi-constant average speed



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# Wind speed components

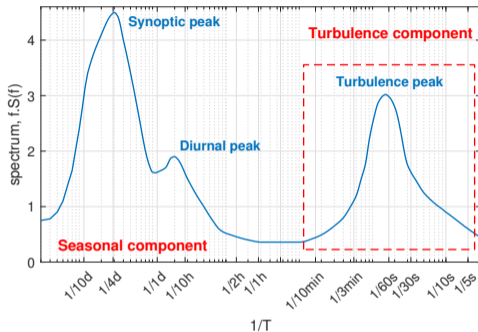
- Spectral analysis of wind power over a long period of time  $\Rightarrow$  Several scales of fluctuations [van der Hoven (1957)]
- Spectrum divided into 3 parts
  - Seasonal: long periods ( $T > 3h$ )
  - Energy gap ( $15min < T < 2h$ )
  - Turbulence: short periods ( $T < 10min$ )
- Turbulence peak has a period of 76s
- Frequency range  $[\frac{1}{10min}, \frac{1}{5s}] = [1.6 \cdot 10^{-3}Hz, 0.2Hz]$





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## Wind speed components

Wind speed can be expressed as:

$$V(t) = \bar{V}(t) + V_{turb}(t) \quad (1)$$

where  $\bar{V}(t)$  is the long-term component and  $V_{turb}(t)$  the turbulence component.



I. van der Hoven (1957), "Power Spectrum of Horizontal Wind Speed in the Frequency Range from 0.0007 to 900 Cycles Per Hour."

*Journal of Atmospheric Sciences* 14(2), 160–164. v9(4), 341–351.

# Wind turbulence PSD approximation

- The von Kármán model approximates the wind speed turbulence PSD [Kármán (1948)]:

$$S_{VK}(f) = \frac{4\sigma^2 \frac{L}{\bar{V}}}{\left(1 + 70.8 \left(\frac{L}{\bar{V}} f\right)^2\right)^{5/6}} \quad (2)$$

where  $\sigma$  is the turbulence standard deviation,  $L$  is the length scale and  $\bar{V}$  is the mean speed

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- The model is tuned by 2 parameters:
  - length scale  $L$  = average size of the fluctuations in terms of distance  
from Engineering Sciences Data Unit (ESDU) report:

$$L_{ESDU} = 25z^{0.35} z_0^{-0.063} \quad (3)$$

where  $z$  is the altitude and  $z_0$  is a surface roughness parameter.



T. von Kármán (1948), "Progress in the Statistical Theory of Turbulence." *Proceedings of the National Academy of Sciences* vol. 34, no.

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- turbulence intensity  $I = \frac{\sigma}{\bar{V}}$   
from International Electrotechnical Commission (IEC) report is:

$$I_{IEC} = I_{ref} \left(0.75 + \frac{5.6}{\bar{V}}\right) \quad (4)$$

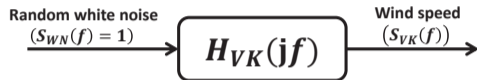


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# Shaping filter

Wind PSD approximation  $\Rightarrow$  Time simulation ☹

$\Rightarrow$  Need to use **shaping filters**



$H_{VK}(jf)$  = the filter transfer function stemming from the von Kármán model

$$S_{VK}(f) = \frac{K}{(1 + (\tau f)^2)^{5/6}} \quad (5)$$

where  $K = 4\sigma^2 \frac{L}{V}$  and  $\tau = \sqrt{70.8} \frac{L}{V}$

$$S_{VK}(f) = S_{WN}(f) |H_{VK}(f)|^2 = |H_{VK}(f)|^2 = H_{VK}(jf) H_{VK}^*(jf) \quad (6)$$

The resulting transfer function:

$$H_{VK}(jf) = \frac{\sqrt{K}}{(1 + j\tau f)^{5/6}} \quad (7)$$

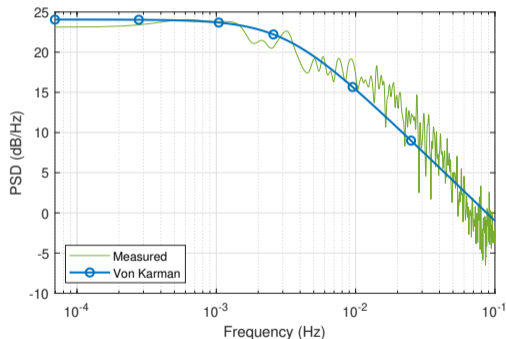
# Identification results

- Minimization of the cost function:

$$J = \frac{1}{N} \sum_{i=1}^N |S_{dB}(f_i) - \widehat{S}_{dB}(f_i, K, \tau)|^2 \quad (8)$$

by using the nonlinear simplex algorithm

- The von Kármán model provides a good fit in low frequency
- Need to improve the fit in middle and high frequency  
High frequency slope fixed to  $5/6$   
⇒ Constraint that can be released according to real data



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# Davidson-Cole model

A direct extension is to use a Davidson-Cole form with a **variable non-integer order**  $\nu$ :

$$H_{DC}(s) = \frac{\sqrt{K}}{\left(1 + \frac{\tau}{2\pi} s\right)^\nu}. \quad (9)$$

The resulting PSD expression:

$$S_{DC}(f) = H_{DC}(jf)H_{DC}(jf)^*$$

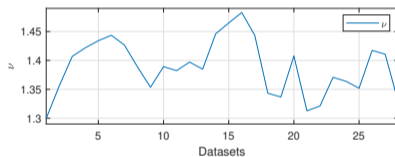
with

$$H_{DC}(jf) = \frac{\sqrt{K}}{(1 + j\tau f)^\nu}$$



**Davidson, D. W. and Cole, R. H. (1951)**, "Dielectric relaxation in glycerol, propylene glycol, and n-propanol." *The Journal of Chemical Physics*, 19(12):1484–1490, (1951).

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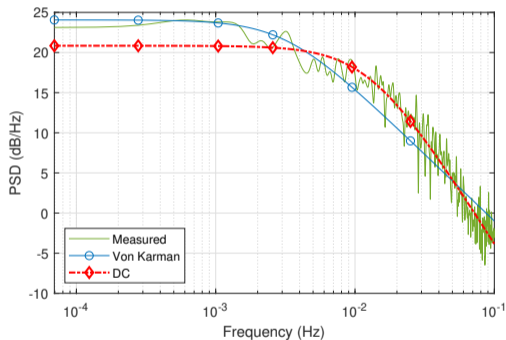
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## Identification results :



- The DC model fits well the measurements in high frequency ( $f > 0.01\text{Hz}$ ) thanks to the estimated  $\nu \in [1.30, 1.48] \neq 5/6 (\approx 0.833)$

# Cole-Cole model

An improved model transfer function based on the Cole-Cole form (CC model):

$$H_{CC}(s) = \frac{\sqrt{K}}{1 + \left(\frac{\tau}{2\pi} s\right)^\nu}. \quad (10)$$

The resulting PSD expression:

$$\begin{aligned} S_{CC}(f) &= H_{CC}(jf)H_{CC}(jf)^* \\ &= \frac{K}{1 + 2 \cos\left(\nu \frac{\pi}{2}\right) (\tau f)^\nu + (\tau f)^{2\nu}} \end{aligned}$$



**K.S. Cole and R.H. Cole (1941)**, "Dispersion and absorption in dielectrics I. alternating current characteristics" *The Journal of chemical physics* v9(4), 341–351.

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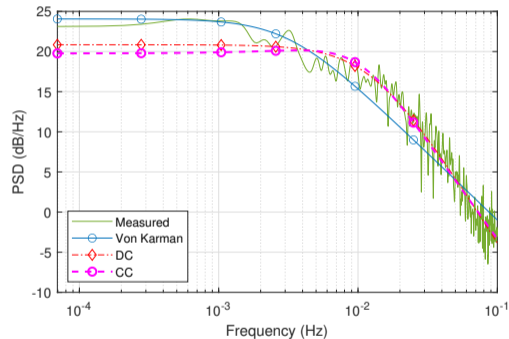
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## Identification results :



- The CC model fits well the measurements in high frequency ( $f > 0.01\text{Hz}$ ) thanks to the estimated  $\nu \in [1.16, 1.25] \neq 5/6 (\approx 0.833)$

# Improved Cole-Cole model: CC×2 model

Addition of a second fractional order cell to the Cole-Cole model (CC×2 model):

$$H_{CC \times 2}(s) = \frac{\sqrt{K}}{\left(1 + \left(\frac{\tau_1}{2\pi} s\right)^\nu\right) \left(1 + \left(\frac{\tau_2}{2\pi} s\right)^{2\nu}\right)} \quad (11)$$

⇒ **A second slope for a better fit in middle frequency**

The resulting PSD expression:

$$S_{CC \times 2}(f) = \frac{K}{D_1(f)D_2(f)} \quad (12)$$

with

$$D_1(f) = 1 + 2 \cos\left(\nu \frac{\pi}{2}\right) (\tau_1 f)^\nu + (\tau_1 f)^{2\nu}$$

$$D_2(f) = 1 + 2 \cos(\nu\pi) (\tau_2 f)^{2\nu} + (\tau_2 f)^{4\nu}$$

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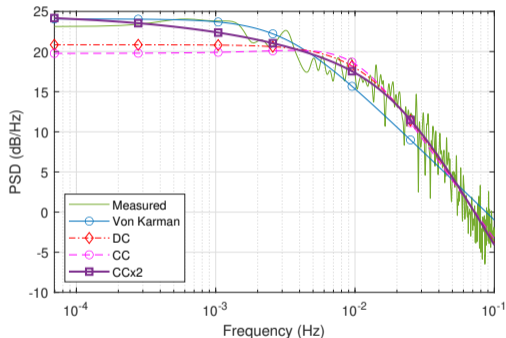
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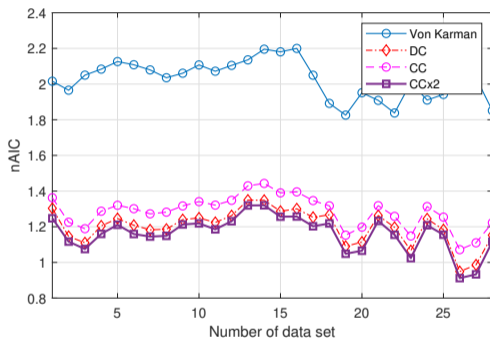


- Good fit in high frequency
- Static gain equal to the von Kármán one
- **Improved fit in middle frequency**

# Identification results

**Table:** Comparison between the mean nAIC of the different models

Model	Number of parameters	Mean nAIC
von Kármán	2	2.027
Davidson-Cole	3	1.200
Cole-Cole	3	1.283
Cole-Cole×2	4	1.162



⇒ **The CC×2 model reproduces accurately the wind turbulence spectral characteristics**

$$nAIC = N \log\left(\det\left(\frac{1}{N} E^t E\right)\right) + \frac{2n_p}{N} \quad (13)$$

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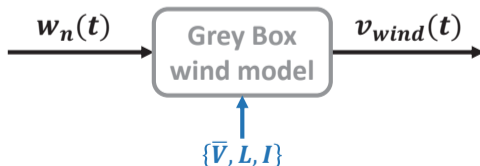
# Wind grey box model



Wind grey box model:

- same transfer function form as the black box model
- parameters obtained from the wind known physical characteristics

⇒ **Generating random realistic wind speed turbulence without reference data**

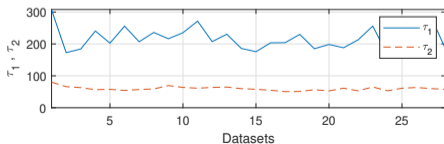




# System Identification for 28 datasets

Selected model: **Wind CC×2 model**:

$$H_{CC \times 2}(s) = \frac{\sqrt{K}}{\left(1 + \left(\frac{\tau_1}{2\pi} s\right)^\nu\right) \left(1 + \left(\frac{\tau_2}{2\pi} s\right)^{2\nu}\right)}$$



# System Identification for 28 datasets

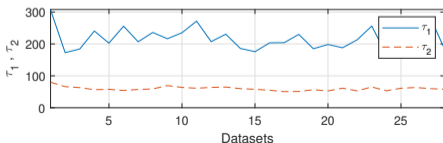
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- Same static gain expression as the Von Kármán model:

$$K = 4\sigma^2 \frac{L}{V}$$

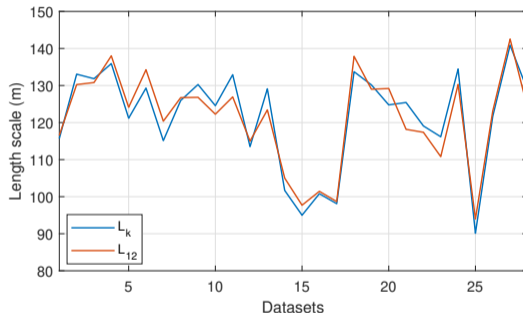
- The mean value of  $\nu = 0.516$   
( $\nu \in [0.49, 0.53]$ )
- The ratio between  $\tau_1$  and  $\tau_2$  is in [2.6, 4.6]: average value **3.6**



# Model parameter tuning

(Von Kármán model:  $K = 4\sigma^2 \frac{L}{\bar{V}}$  and  $\tau = 8.41 \frac{L}{\bar{V}} \implies L_K = \frac{K\bar{V}}{4\sigma^2}$ .)

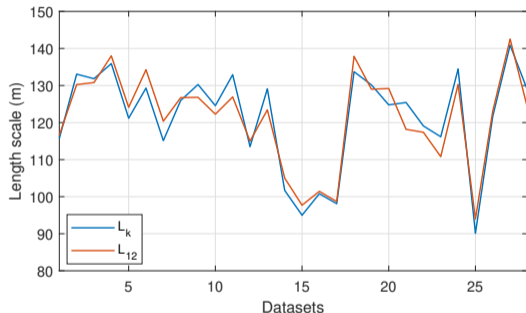
- **Validation** by comparing the length scales  $L_k$  and  $L_{12} = \frac{\bar{V}\sqrt{\tau_1\tau_2}}{4.7}$  obtained from  $\tau_1$  and  $\tau_2$  for 28 different datasets



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## Grey-box wind turbulence model

Wind CC×2 model defined by:

$$\mathbf{K} = 4\sigma^2 \frac{\mathbf{L}}{\bar{V}}, \quad \tau_1 = 8.9 \frac{\mathbf{L}}{\bar{V}}, \quad \tau_2 = \frac{\tau_1}{3.6}, \quad \nu = 0.516 \quad (14)$$

# Rational filter design: Oustaloup's approximation

## Recursive distribution of zeros and poles

The frequency band  $[\omega_A, \omega_B] = [10^{-1}, 10^1]$ . Therefore, the rational approximation will be carried out on  $[10^{-2}, 10^2]$ .  $s^\nu$  is approximated by:

$$s^\nu \rightarrow s_{[\omega_A, \omega_B]}^\nu = C_0 \left( \frac{1 + \frac{s}{\omega_b}}{1 + \frac{s}{\omega_h}} \right)^\nu \approx C_0 \prod_{k=1}^N \frac{1 + \frac{s}{\omega'_k}}{1 + \frac{s}{\omega_k}} \quad (15)$$

where:

$$\omega'_1 = \eta^{1/2} \omega_b, \quad \omega_1 = \alpha \omega'_1,$$

$$\omega'_{k+1} = \alpha \eta \omega'_k, \quad \omega_{k+1} = \alpha \eta \omega_k,$$

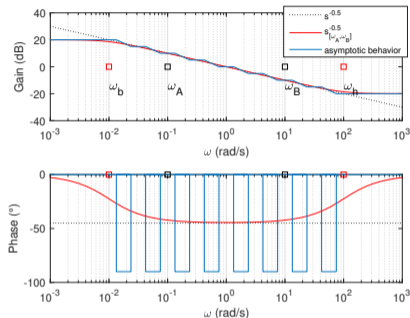
$$\nu = \frac{\log \alpha}{\log(\alpha \eta)}, \quad \alpha = \left( \frac{\omega_h}{\omega_b} \right)^{\frac{\nu}{N}}, \quad \eta = \left( \frac{\omega_h}{\omega_b} \right)^{\frac{1-\nu}{N}}$$

and

$$C_0 = \left| \frac{1 + j \frac{1}{\omega_h}}{1 + j \frac{1}{\omega_b}} \right|^\nu = \left( \frac{\omega_h}{\omega_b} \right)^\nu \left( \frac{1 + \omega_b^2}{1 + \omega_h^2} \right)^{\nu/2}.$$



**A. Oustaloup, F. Levron, B. Mathieu, and F. M. Nanot (2000)**, "Frequency-band complex noninteger differentiator: characterization and synthesis" *IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications*, 47(1):25–39, DOI:10.1109/81.817385.



**Figure:** Fractional integrator  $s^{-0.5}$  and its approximation  $s_{[0.1, 10]}^{-0.5}$  based on Oustaloup's recursive poles and zeros approximation

# Time simulation

- Measured wind characteristics:

$$\bar{V} = 6.6\text{m/s}, \sigma = 1.92, L = 120\text{m}$$

- Identified CCx2 model:

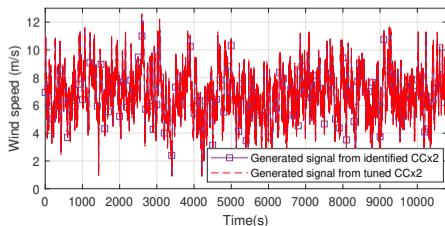
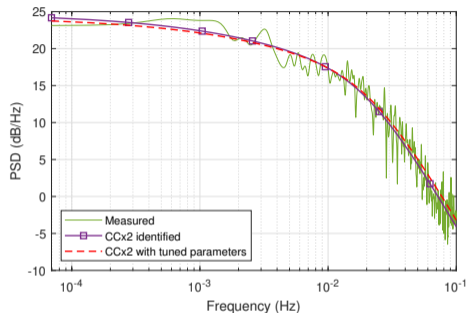
$$H_{BB}(s) = \frac{17.35}{48.72s^{1.554} + 8.59s^{1.036} + 5.67s^{0.518} + 1}$$

- Tuned grey box model:

$$K = 269.94, \tau_1 = 161.74s, \tau_2 = 44.93s$$

$$H_{GB}(s) = \frac{16.43}{40.7s^{1.548} + 7.62s^{1.032} + 5.34s^{0.516} + 1}$$

⇒ Both models provide wind speed with a PSD similar to the measured wind PSD



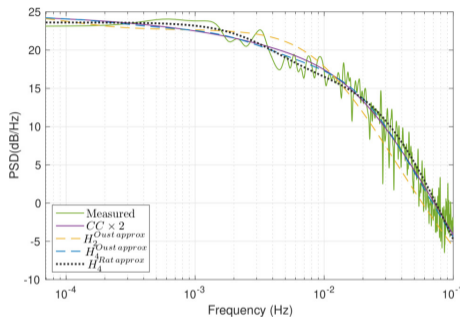
# Comparison with rational models

- Rational approximation from Oustaloup's approximation:

$$H_{N_{cells}}^{Oust approx} = H_0 \prod_{k=1}^N \frac{1 + \frac{s}{\omega'_k}}{1 + \frac{s}{\omega_k}}, \quad (16)$$

- Rational approximation:

$$H_4^{Rat approx} = H_0 \frac{1 + \frac{s}{\omega_2}}{\left(1 + \frac{s}{\omega_1}\right) \left(1 + \frac{s}{\omega_3}\right)}, \quad (17)$$



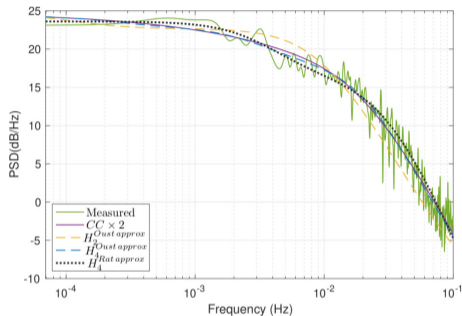
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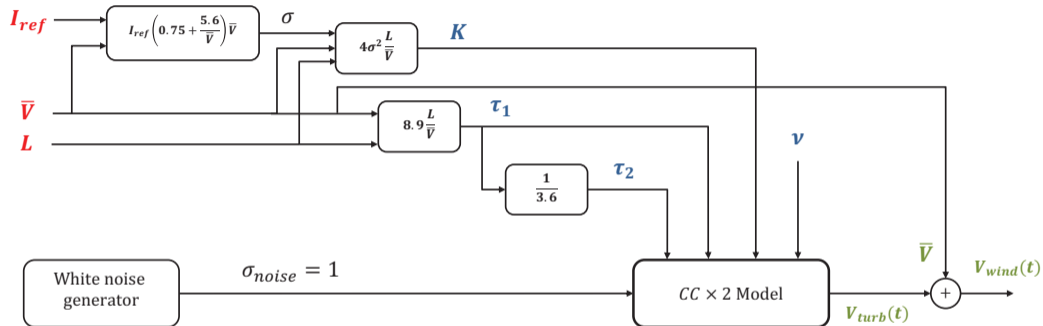
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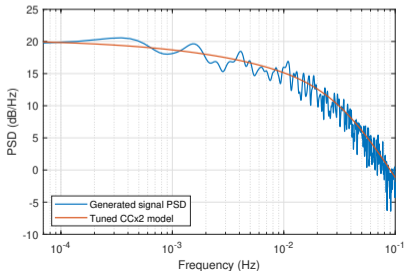
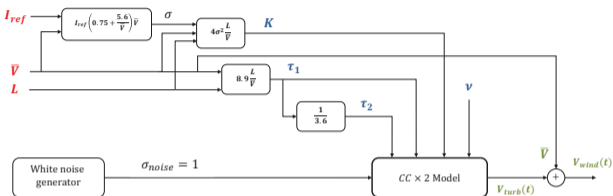
Model	Number of parameters	Fit (%)	nAIC
Cole-Cole×2	4	96.473	1.193
Oust. Approx $H_2^{Oust\ approx}$	14	91.514	2.083
Oust Approx $H_4^{Oust\ approx}$	26	96.502	1.214
Rat. Approx $H_4^{Rat\ approx}$	4	73.166	3.222



# Wind speed generator



# Wind speed generator



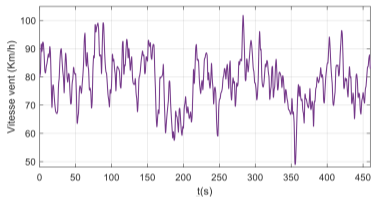
```

1 %% Model inputs
2 L = 110; %Length scale (m)
3 Vm = 80/3.6; %Mean speed (m/s)
4 I_ref = 0.12; %Turbulence intensity reference (low: 0.12, mean: 0.14,
5             % high: 0.16)
6 Ts = 1;
7 %% Tuning model parameters
8 I = I_ref.*(0.75+5.6/Vm);
9 sigma = I*Vm;
10 K = 4.*(sigma)^2.*L./Vm;
11 Taul = 8.*L./Vm;
12 Tau2 = Taul./3;
13 nu = 0.51;
14
15 %% Fractional transfer function
16 num=sqrt(K);
17 den1=frac.poly_exp([(Taul/2/pi)^nu 1],[nu 0]);
18 den2=frac.poly_exp([(Tau2/2/pi)^(2*nu) 1],[2*nu 0]);
19
20 sys = frac.tf(num,den1)*frac.tf(1,den2);
21
22 %% Approximated rational transfer function
23 set(sys,'N',10); %Cell number
24 set(sys,'band',[1e-4,10]); %Frequency range + 2 decades
25 set(sys,'sim','Oustaloup');
26
27 H_CCx2 = frac2int(sys);

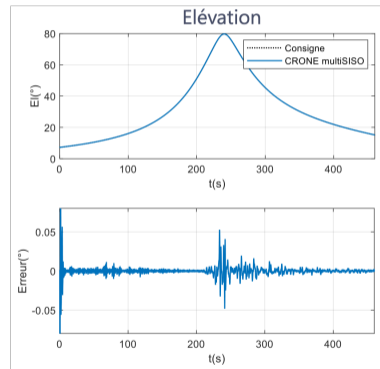
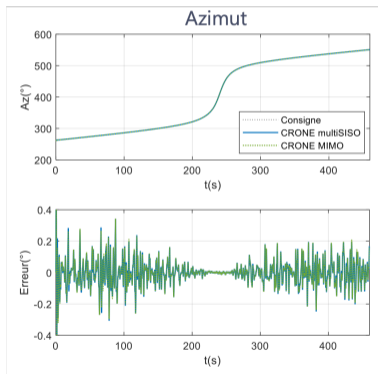
```



# Application on a 3-axis antenna




- Wind turbulence average:  
 $\bar{V} = 80\text{km/h}$
- Wind turbulence speed:  
 $[50, 100]\text{km/h}$
- MIMO CRONE control for turbulence rejection



- 1 Introduction
- 2 State of the art
  - Wind speed components
  - von Kármán model
  - Model identification
- 3 Improved fractional models
  - Davidson-Cole model
  - Cole-Cole model
  - Improved Cole-Cole model
- 4 Wind turbulence model design
  - Model parameter tuning
  - Time simulation
  - Wind speed generator
  - Application on a 3-axis antenna
- 5 Conclusion

# Conclusion

- Wind models based on the spectral characteristics and not on time series data, then shaping filters are synthesized for time simulation
- Study and identification of the Von Kármán approximation and its limits
- Improved fractional models with: Davidson-Cole, Cole-Cole and Cole-Cole $\times$ 2 models
- Tuning methodology to transform the CC $\times$ 2 black box model into a grey box model  $\Rightarrow$  not limited by access to reference data
- **A long period wind mean speed generator can be developed for more realistic scenarios**

 **M. Hajjem, S. Victor, P. Melchior, P. Lanusse, and L. Thomas (2023)**, “Wind turbulence modeling for real-time simulation.” *Fractional Calculus & Applied Analysis* vol. 26, pp.1632–1662 doi: 10.1007/s13540-023-00165-0.