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# Continuous-time IRT Models: Modelling a Latent Process Trajectory with Item- level Measurement Models

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# Different Designs of PRO Studies, Same Issues

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- Intensive longitudinal data
  - time-intensive data collections over relatively short periods of time
  - primary focus on within-individual variation
- Longitudinal data
  - repeated-measures study designs with a shorter number of measurements
  - focus on longitudinal mean trajectory of an outcome and individual deviations from the mean trend
- Issues
  - The construct of interest is continuous by essence, the COA is observed at discrete time points
  - The timescale of the study may be continuous or at least it can have differing time intervals between and within patients
  - Varying number of measurements (design, missing data)
  - Account for the measurement model



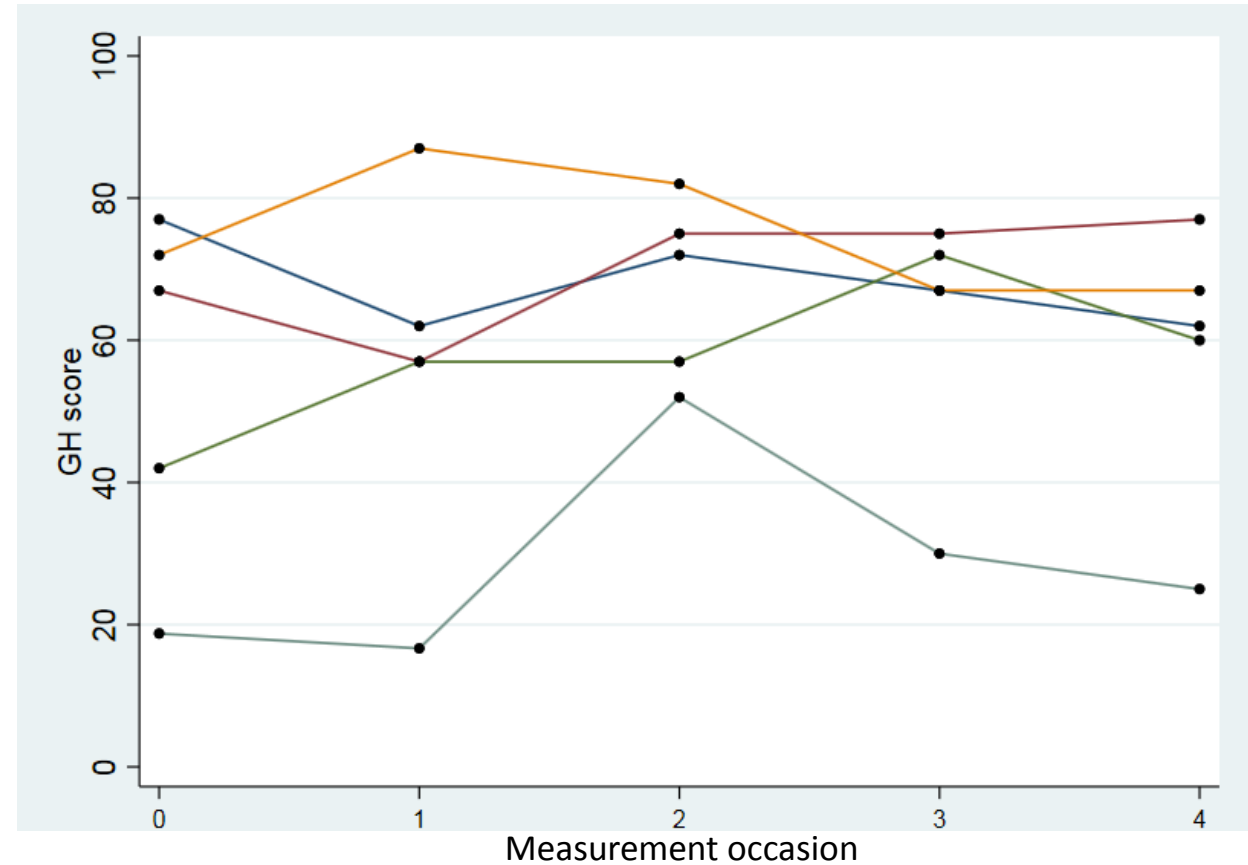
# Expectations

## Design of the study:

- Patients on waiting list for renal transplantation
- SF-36 measured every 6 months

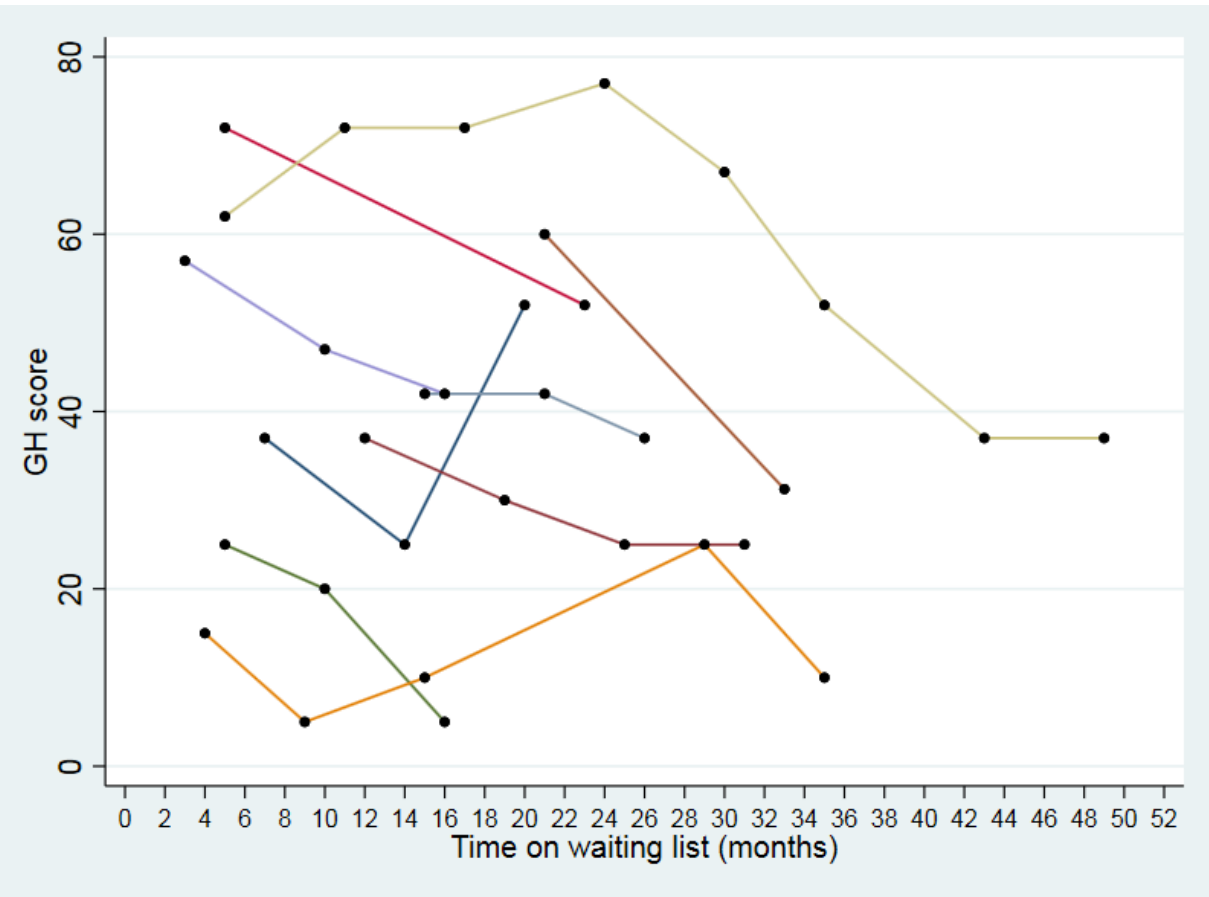
## Expectations:

- Same T0 for all patients (study entry)
- Equally spaced measurement occasions
- No missing data

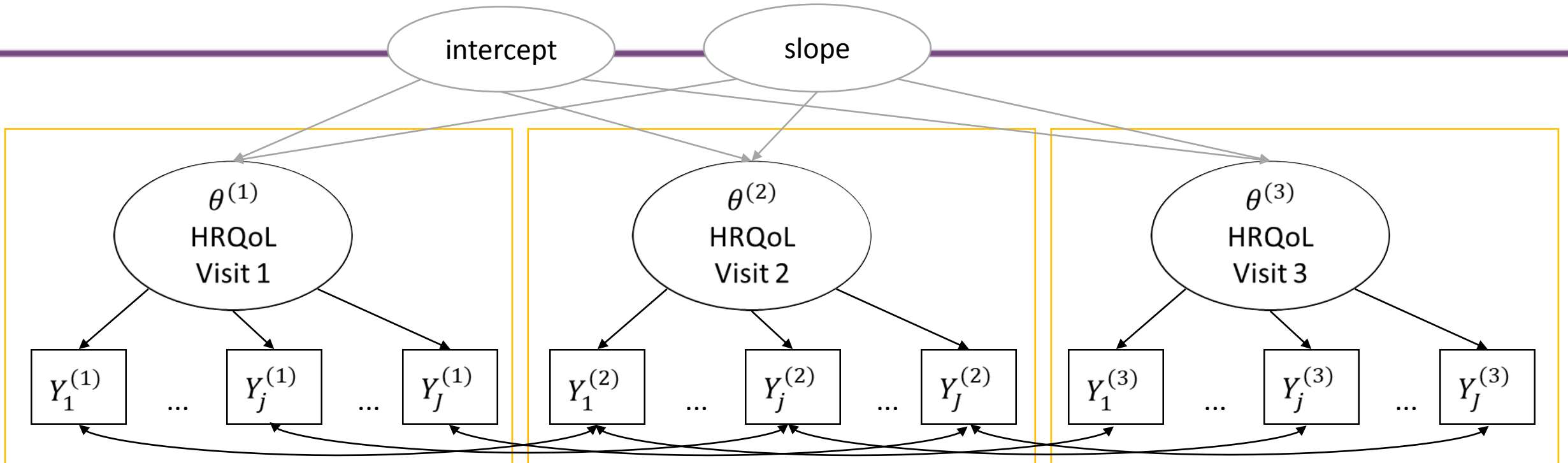


# Reality

- Time of interest: time on waiting list  
T0 is not the same for all patients
  - Unequally spaced measurement occasions
  - Missing data
  - Up to 12 measurements per patient
- Varying time intervals between and within patients



# Usual Longitudinal Latent Variable Model



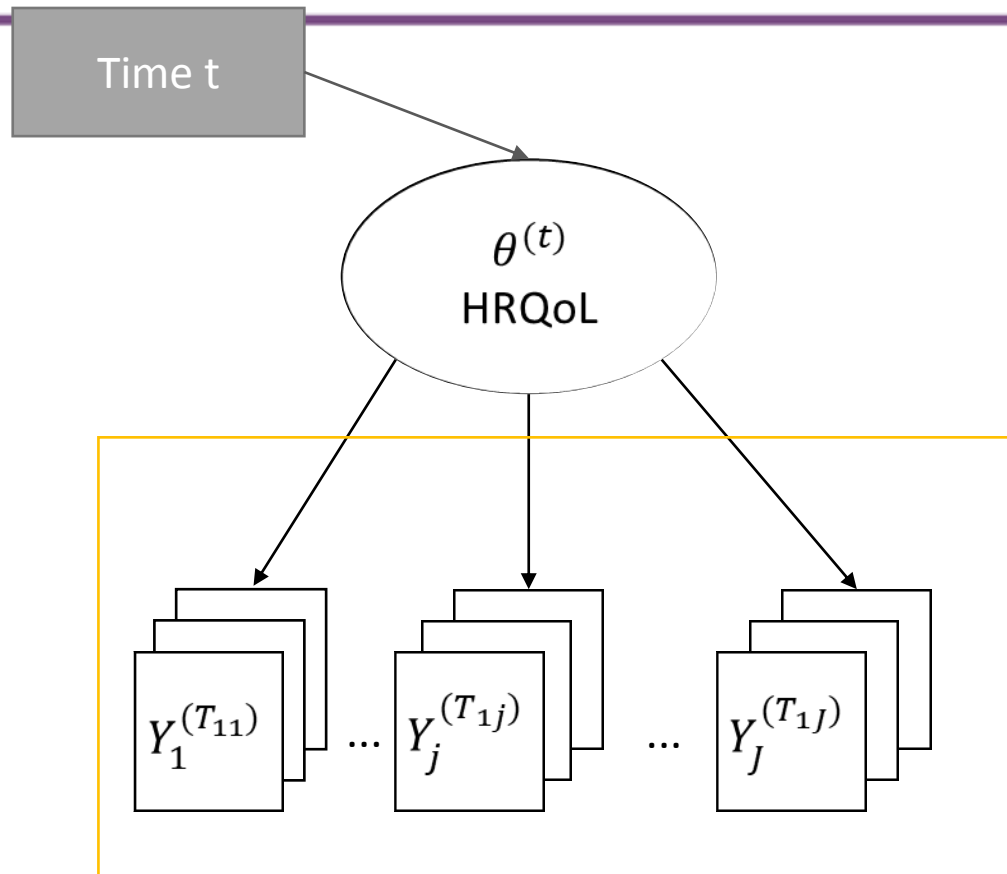
Latent variable  $\theta^{(t)}$  at time  $t$

Observed responses:  $Y_1^{(t)}, \dots, Y_J^{(t)}$

Assumption: same times of measurement for all the items and subjects  
Very long to estimate with more than 3 time points



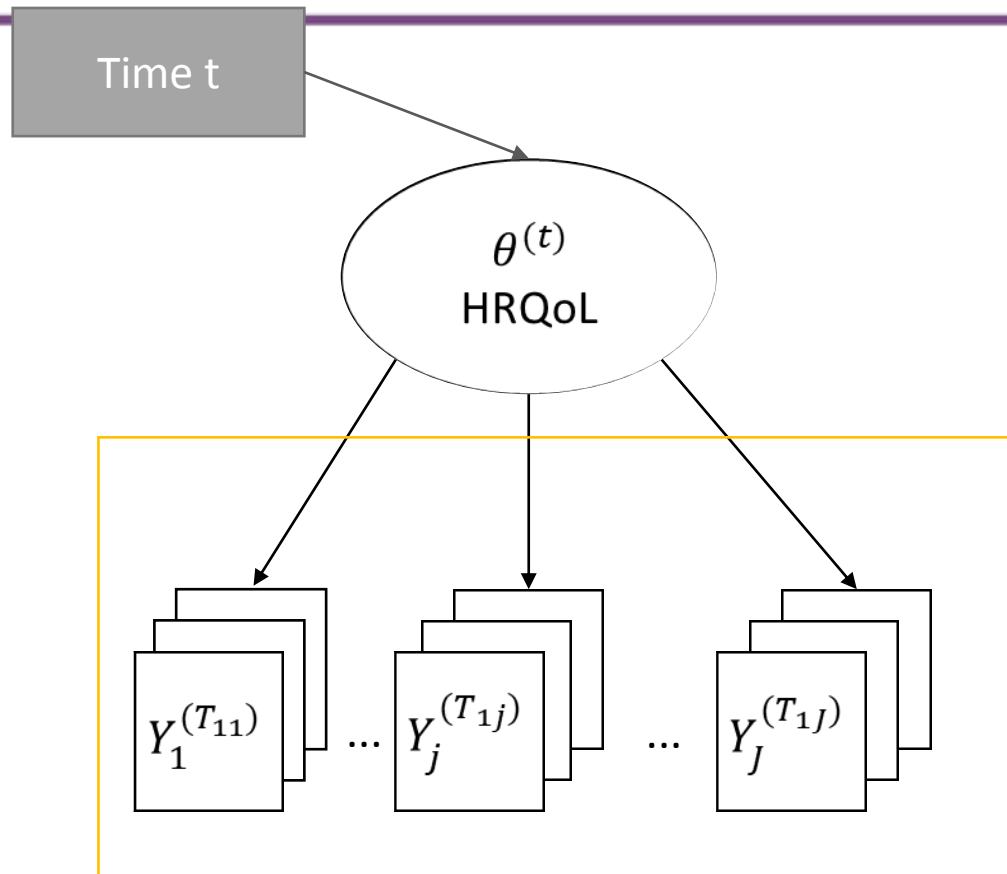
# Capturing the Dynamic Aspect of the Latent Trait



- The latent process  $\theta^{(t)}$  is defined in CONTINUOUS time ( $t \in \mathbb{R}$ )
- The item responses are collected in DISCRETE times ( $t_{ijk}$ )
- Handle varying time intervals between and within subjects
- Linear mixed model to model the trajectory of the latent process



# Capturing the Dynamic Aspect of the Latent Trait



$$\theta^{(t)} = X_i^{(t)T} \beta + Z_i^{(t)T} b_i$$

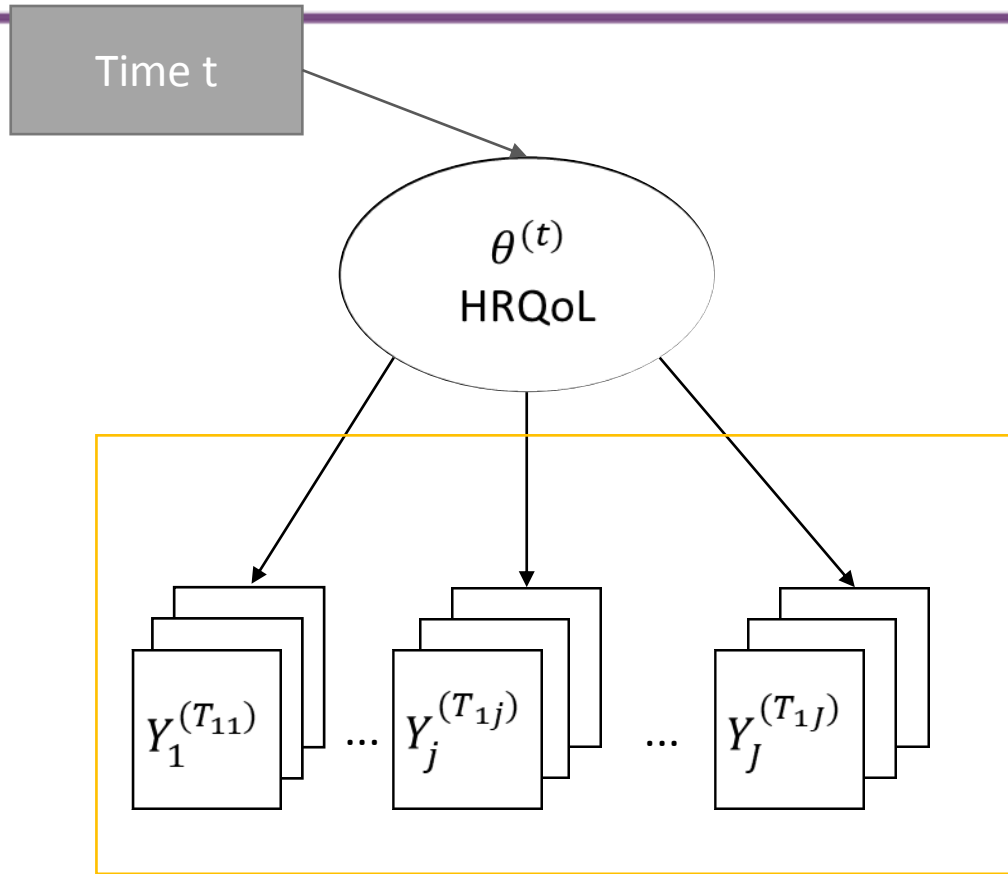
- functions of time and covariates (population level)
- shape of the trajectory of the latent process over time
- association with covariates



Proust-Lima, C., et al. (2013). Analysis of multivariate mixed longitudinal data: a flexible latent process approach. *The British journal of mathematical and statistical psychology*, 66(3), 470–487.

Proust-Lima, C., et al. (2022). Modeling repeated self-reported outcome data: A continuous-time longitudinal Item Response Theory model. *Methods*, 204, 386–395.

# Capturing the Dynamic Aspect of the Latent Trait

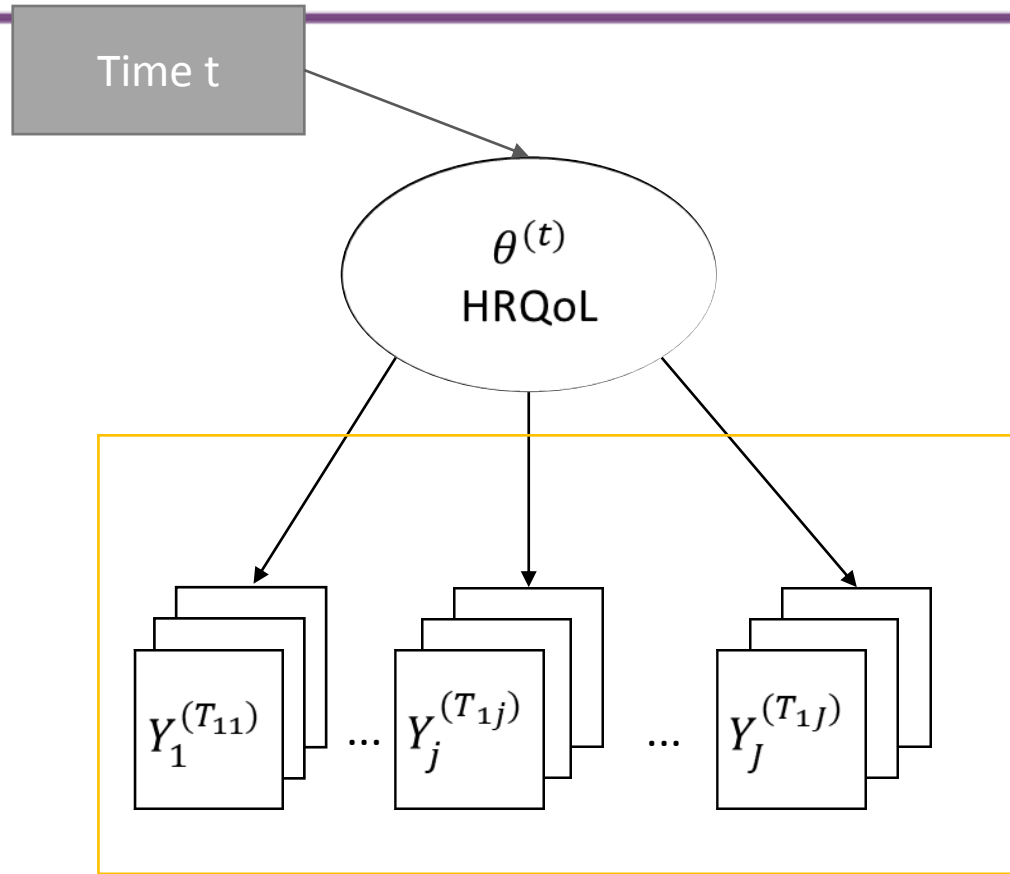


$$\theta^{(t)} = X_i^{(t)T} \beta + Z_i^{(t)T} b_i$$

- individual random-effects
- deviation from the mean trajectory



# Capturing the Dynamic Aspect of the Latent Trait



$$\theta^{(t)} = X_i^{(t)T} \beta + Z_i^{(t)T} b_i$$

- item response:  $Y_{ijk}$
- time of measurement:  $t_{ijk}$

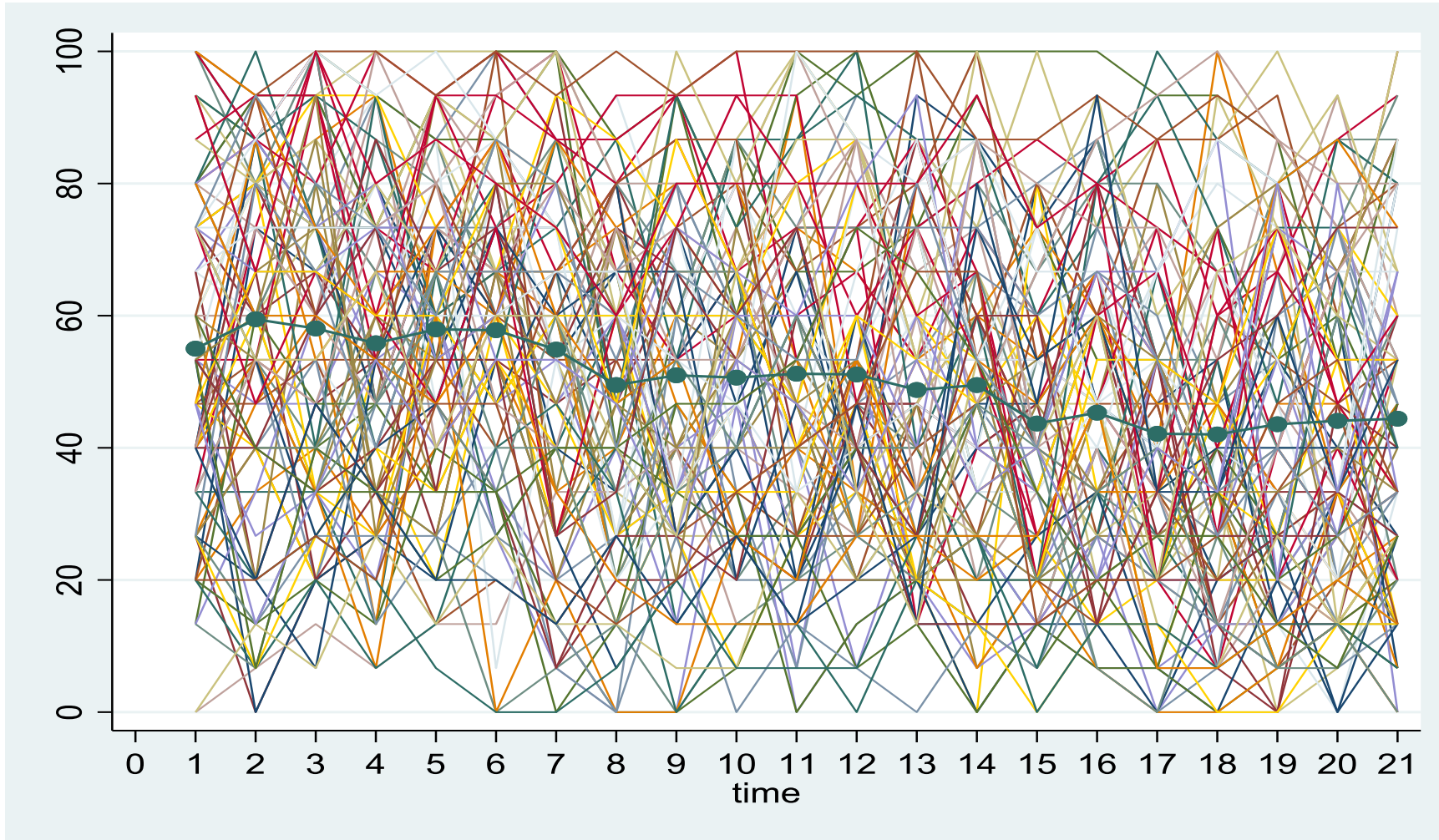
$Y_{ijk}$  = link function of the measurement model

An example for ordinal items (cumulative probit)

$$P(Y_{ijk} \leq l | \theta_i(t_{ijk})) = \Phi(\alpha_j(\eta_{jl+1} - \theta_i(t_{ijk})))$$

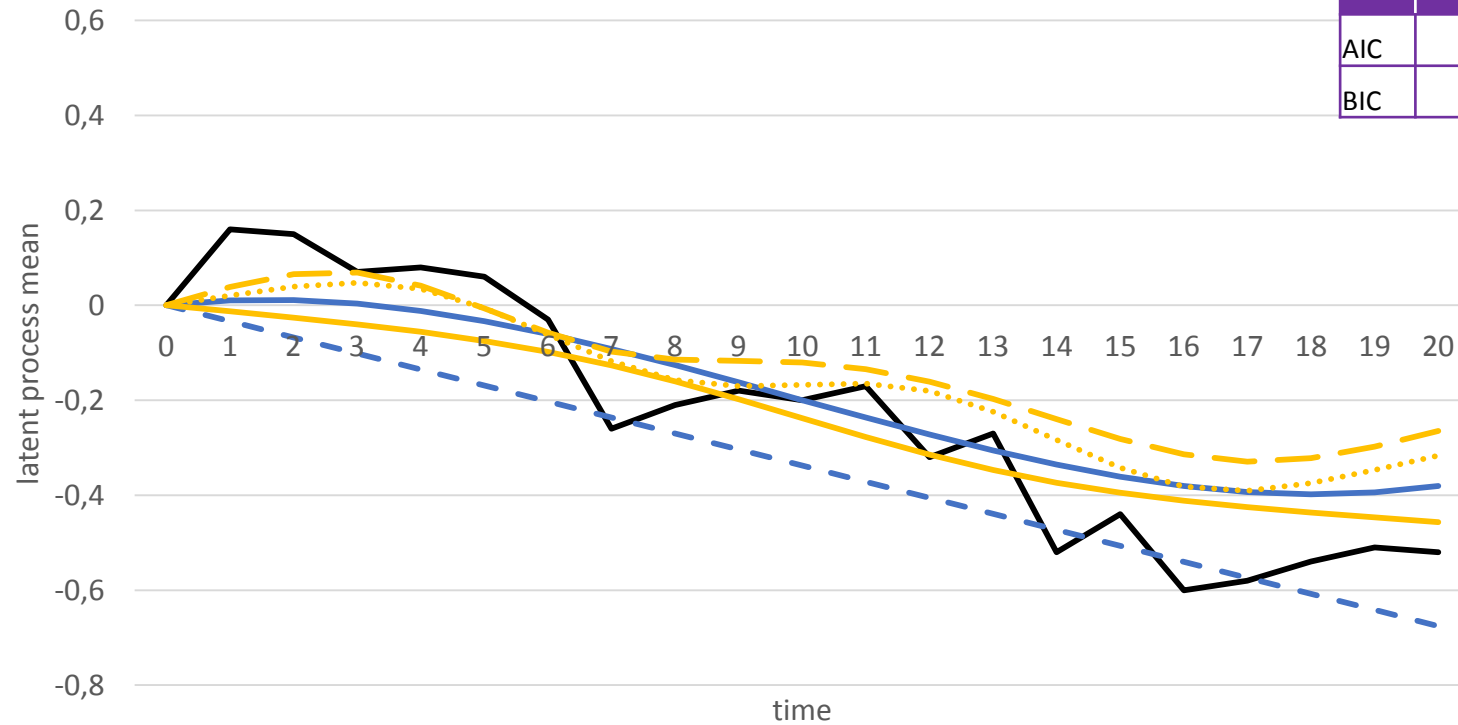


# Overall and Individual Trends of Simulated Sumscores



# Results: Estimated Trajectory

	linear	cubic polynomial	cubic splines 4	cubic splines 6	cubic splines 7
AIC	114940.1	114904.2	114921.9	114862.6	114865.7
BIC	115107.1	115085.7	115096.2	115058.5	115061.7



simulated
  linear
  polynomial cubic
  cubic splines 4 knots
  cubic splines 6 knots
  cubic splines 7 knots



# Results: Parameter Recovery

DISCRIMINATION	True value	linear	cubic polynomial	cubic splines 4	cubic splines 6	cubic splines 7
item 1	1.42	1.27	1.70	1.62	1.81	2.04
item 2	1.17	1.24	1.66	1.59	1.77	2.00
item 3	0.81	1.09	1.46	1.40	1.55	1.75
item 4	0.96	1.18	1.57	1.51	1.67	1.89
item 5	1.12	1.23	1.65	1.58	1.75	1.98

- Discrimination

Best estimated values for linear trend

- Thresholds

Best estimated values for cubic splines with 7 knots

- Bad recovery for all methods

THRESHOLDS		True value	linear	cubic polynomial	cubic splines 4	cubic splines 6	cubic splines 7
item1	cut1	-1.97	-2.86	-2.76	-2.81	-2.73	-2.70
	cut2	-0.97	-1.61	-1.51	-1.56	-1.49	-1.46
	cut3	0.03	-0.38	-0.28	-0.33	-0.25	-0.23
item2	cut1	-1.43	-2.02	-1.93	-1.97	-1.90	-1.87
	cut2	-0.43	-0.90	-0.81	-0.85	-0.78	-0.75
	cut3	0.57	0.22	0.31	0.27	0.34	0.37
item3	cut1	-1	-1.23	-1.14	-1.19	-1.12	-1.09
	cut2	0	-0.36	-0.28	-0.32	-0.26	-0.23
	cut3	1	0.50	0.58	0.54	0.61	0.63
item4	cut1	-0.57	-0.95	-0.86	-0.90	-0.83	-0.80
	cut2	0.43	0.05	0.14	0.10	0.17	0.20
	cut3	1.43	1.03	1.12	1.08	1.15	1.17
item5	cut1	-0.03	-0.46	-0.36	-0.41	-0.33	-0.31
	cut2	0.97	0.63	0.72	0.68	0.75	0.78
	cut3	1.97	1.77	1.86	1.81	1.89	1.92



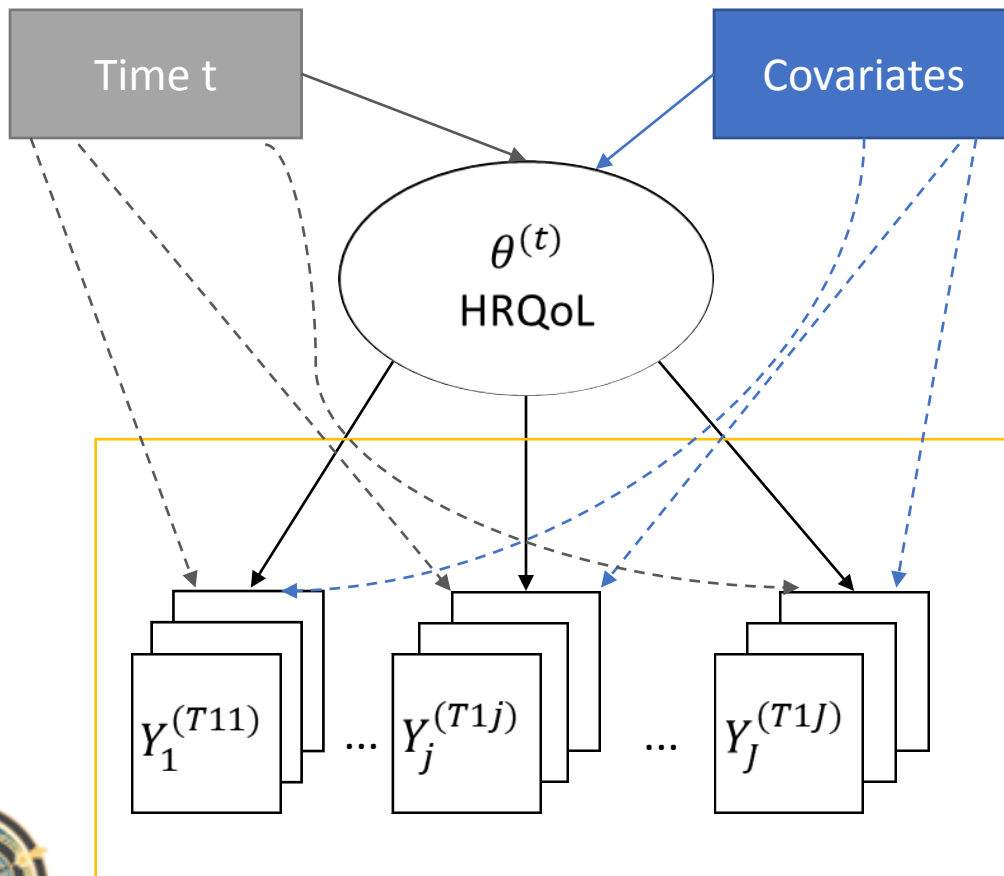
# A Promising and Powerful Method

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- Splines seem to better fit in terms of trajectory of the latent process  
But hard to interpret
- Bad recovery of item parameters: add random effects or Gaussian stochastic process to the structural mixed model
- Advantages:
  - dynamic process + measurement model
  - handle varying time intervals
- Choice of trajectory + choice of measurement model: Quality of adjustment/model fit?



# Complete Continuous-time Model



- Linear mixed model to model the trajectory of the latent process (linear, polynomial, splines...)  
+ determine the covariates associated with the latent process and its change over time
- Measurement model with influence of covariates (DIF) and time (response shift)

