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# 1 Large vortices from soft vibrations

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6 Steady streaming is a peculiar flow, induced by the oscillation of an object with  
7 respect to the fluid where it is embedded. While the oscillation amplitude may  
8 be extremely small, but the non-linearity due to fluid inertia gives rise to this  
9 steady flow. Large and powerful vortices develop around the object. Usually the  
10 streaming is scrutinized in the case of homogeneous objects. Here Cui *et al* (*J.*  
11 *Fluid Mech.* vol. 979, 2024, A7) derive the streaming around a soft elastic sphere  
12 containing a solid core. This elaborate derivation opens a new description of  
13 streaming in biological soft environments and robotics.

## 14 1. Streaming from vibrating objects

15 It must have been a wonderful surprise, the discovery of the streaming wind  
16 by Faraday (1831), when he observed the streams of air above vibrating plates  
17 showing the famous Chladni patterns. These steady vortices play a key role in  
18 moving very light particles on top of the plate, gathering particles in specific  
19 nodes, different from the ones followed by the heavy particles bouncing and  
20 reaching vibration nodes. The same streaming phenomenon occurs within a  
21 Kundt tube, gathering lycopodia particles at specific nodes.

22 We should first stress that it is not that usual that vibrations induce flow.  
23 The reverse phenomenon is more common: steady flows induce vibrations by the  
24 apparition of a self-sustained vibration of a boundary. An example is the pleasant  
25 singing of vocal cords vibrating under the air exhaled out of our lungs, or the  
26 harsh noise from the turbulent wake behind a high-speed vehicle.

27 Why do tiny vibrations induce a steady flow in a fluid? The vibration may come  
28 from the solid, or equivalently from the flow around a fixed object. According  
29 to the well-know Stokes second problem, the tangential vibration of the solid  
30 boundary shears the fluid alternately back and forth, and this shear "diffuses"  
31 into the fluid because of inner viscous forces. It sets an oscillatory boundary layer  
32 in the fluid near the solid body whose thickness scales like  $\delta \sim (\nu/\omega)^{1/2}$ , with  $\nu$   
33 the kinematic viscosity and  $\omega$  the pulsation frequency. A non-dimensional number  
34 can be defined: the ratio of the typical size  $L$  of the object to this boundary layer  
35 thickness  $\delta$ , known as the Womersley number and defined as  $M = L/(\nu/\omega)^{1/2}$ .

36 Within this layer inertia plays a role, especially if there are velocity gradients  
37 involved. The non-linear term of the Navier-Stokes equations provides a steady

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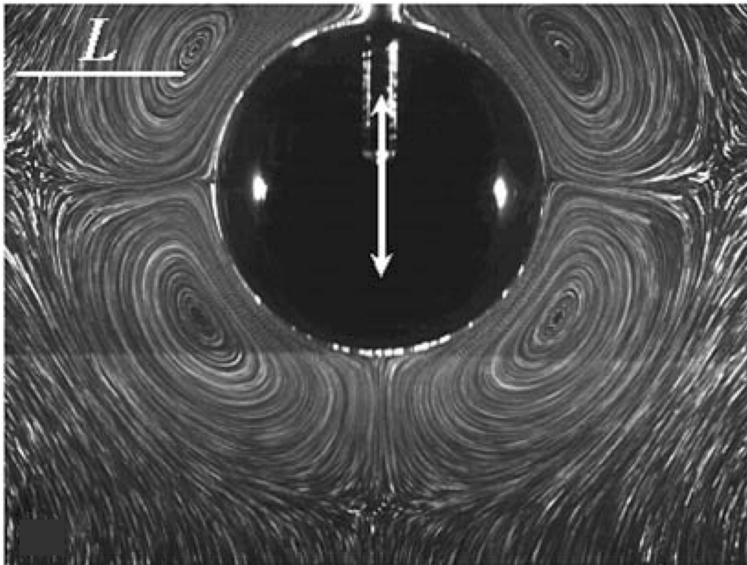


Figure 1: Streaming flow around a vibrating rigid acrylic sphere. The fluid is a solution of 90% glycerin by weight, seeded with particles. The ratio of the object size  $L$  to the oscillatory boundary layer thickness is such that  $M = 6$ , with  $L = 1.4$  cm. Reproduced with permission from Kotas *et al.* (2006).

38 forcing which is quadratic in the oscillation amplitude. As a result the sinusoidal  
 39 vibration is rectified by this non-linearity into a steady flow pattern. The steady  
 40 flow velocity is also quadratic in the oscillation amplitude, while the geometry of  
 41 the pattern depends only on the oscillation frequency. A beautiful visualisation  
 42 of a steady flow around a rigid sphere is given in Fig. 1 taken from Kotas *et al.*  
 43 (2006).

44 Note that the streaming flow may give rise to very strong velocities: it is then  
 45 called Stuart streaming if the Reynolds number associated with the streaming  
 46 velocity and object size is large, while it is called Rayleigh-Nyborg-Westervelt  
 47 streaming if this Reynolds number is small (Lighthill 1978).

## 48 2. Introducing a touch of softness

49 The derivation of the streaming flow field is a complex calculation. It has been  
 50 solved in several cases: around a rigid cylinder or sphere whose center of mass  
 51 oscillates. The geometry of the object is important, for instance it was recently  
 52 shown that in channels, sharp edges intensify the streaming at the tip of the edge  
 53 (Zhang *et al.* 2020). Conversely, some geometries can nearly suppress acoustic  
 54 streaming, as shown by Bach & Bruus (2020). In other derivations, a more  
 55 complex vibration of the object was imposed to compute the flow, such as around  
 56 solids exhibiting arbitrary shape modes (Spelman & Lauga 2017), or around  
 57 bubbles vibrating in both translation and radius (Longuet-Higgins 1998), giving  
 58 rise to specific vortices, or around liquid droplets Rednikov (2006).

59 Most prior streaming derivations considered rigid body motions or fluid objects.  
 60 The full elasticity of the object was considered recently by Doinikov *et al.* (2021).  
 61 Cui *et al.* (2024) fill a gap in the literature by considering an elastic (or visco-  
 62 elastic) solid in a particularly useful geometry. The geometry of the problem is

63 very simple: it consists of an elastic sphere containing a rigid spherical core. The  
64 vibration is imposed by a back and forth motion of the liquid around the object,  
65 the solid core being fixed. By a change of reference, this is equivalent to assuming  
66 that the solid core is driven by a given oscillation in translation. The flexibility  
67 of the sphere surface is now involved in the streaming.

### 68 **3. An elaborate analytical solution**

69 The 3D derivation is performed in spherical coordinates around the sphere. It  
70 follows the same steps as the 2D calculation around a soft cylinder made by the  
71 same team (Bhosale *et al.* 2022).

72 The analytical derivation is an elaborate and lengthy piece of work. It is a note-  
73 worthy solution of the Navier-Stokes equations for small vibration amplitudes,  
74 including an elastic solid. In practice it consists of a perturbation series of the  
75 Navier-Stokes equation, using an asymptotic expansion in powers of the vibration  
76 amplitude. It can be decomposed in the following steps:

77 • The first order of the perturbation in powers of the amplitude gives the  
78 back and forth (acoustic) vibration of the fluid. The originality is that here, the  
79 vibration within the solid displacement is also derived.

80 • The second order gives the steady streaming, forced by non-linear terms.  
81 One then obtains a steady flow which is an Eulerian flow field.

82 • The Lagrangian flow field is computed in order to obtain the trajectories  
83 of tracer particles in this flow, by adding the Stokes drift (Lighthill 1978). This  
84 effect originates from the fact that particles embedded in the field oscillate in  
85 position, and therefore experience different flow velocities, which result in a drift  
86 in addition to the Eulerian flow.

87 The final result is an analytical solution of all the vibration fields in the solid  
88 and in the fluid, and of course of the steady streaming flow. It correctly predicts  
89 typical streaming flows with an inner layer and an outer layer (see again Fig. 1). A  
90 simple observation of the flow field which confirm this theory is the measurement  
91 of thickness of the inner layer. The authors observe that with a softer elastic  
92 solid the same inner layer thickness can be obtained with a lower frequency of  
93 vibration. The elastic vibration seems to decrease the extent of the inner layer,  
94 provided the elastic shear modulus is small in front of the fluid inertial forces.

### 95 **4. Implications for outcomes in biology, microfluidics and robotics**

96 These results will be of high interest in three fields of research. First, these results  
97 will be important for biological fluids where it is common to observe a pulsation of  
98 the flow, or for oscillating boundaries which generate steady streaming patterns  
99 in soft environments. Examples of such streaming situations are flows within the  
100 alveoli of the lungs due to breathing (Kumar *et al.* 2011), rectified flows due  
101 to intestinal villi (Puthumana Melepattu & de Loubens 2022), or flow induced  
102 by the vibration of otolith stones in fish ears (Kotas *et al.* 2006). In medicine,  
103 acoustic streaming flows enhance the delivery of drugs, or accelerate the healing of  
104 injuries. The derivation by the authors might help to quantify the exact direction  
105 of circulation of the fluid around these objects, by predicting the extent of the  
106 inner boundary layer as well as the streaming velocities.

107 Second, microfluidic flows are an important possible outcome, since microfluidic  
108 channels are usually made of soft elastomeric materials, or because they contain

109 soft or biological objects circulating. Many microfluidic applications rely on sound  
 110 oscillations to set the fluid in motion, a convenient way to move fluid at small  
 111 scales without external pumps (Muller *et al.* 2012) in order to mix or to propel  
 112 fluids. There we envision that the streaming predictions will be useful for the  
 113 development of hybrid systems consisting of solid parts covered by soft elastomeric  
 114 or biological layers, while set into vibration.

115 Finally, the fields of robotics and micro-swimmers could also benefit from this  
 116 study. Swimming can be achieved from steady streaming (Dombrowski *et al.*  
 117 2019), and considering a body with an elastic part is a natural extension. 3D  
 118 printing technologies allow the manufacturing of soft micro-propellers or micro-  
 119 swimmers down to micrometric scales (Kaynak *et al.* 2020). The design of these  
 120 objects will benefit from the streaming predictions, for the choice of the elasticity  
 121 or the thickness of those materials, in order to enhance their velocities.

122 To conclude, we envision that this study will have ramifications at the funda-  
 123 mental level as well. The analytical derivations by Cui *et al.* (2024) still need to be  
 124 compared with experiments: this work calls for new fluid mechanics experiments  
 125 on vibrating soft spheres.

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