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1 **Data-Driven Computation of the Accessibility Provided by Demand-Responsive Transport**

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27

1 **ABSTRACT**

2

3 Conventional Public Transport (PT) cannot support the mobility needs in weak demand areas. Such areas
4 could be better served by integrating, within PT, Demand-Responsive Transport (DRT), in which bus routes
5 dynamically adapt to user demand. While the literature has focused on the level of service of DRT, it has
6 overlooked its contribution to *accessibility*, which measures the ease of accessing opportunities (e.g,
7 schools, jobs, other residents). Therefore, the following simple question remains unanswered: *How many*
8 *additional opportunities per hour can be reached when DRT is deployed?*

9 However, no method exists to quantify the accessibility resulting from the integration of conventional PT
10 and DRT. We propose a novel method to compute isochrone-based accessibility. The main challenge is
11 that, while accessibility isochrones are computed on top of a graph-model of the transport system, no graph
12 can model DRT, since its routes are dynamic and stochastic. To overcome this issue, we propose a data-
13 driven method, based on the analysis of multiple days of DRT operation.

14 The methodology is tested on a case study in Acireale (Italy) simulated in Visum, where a many-to-many
15 DRT service is integrated with a metropolitan mass transit. We show that, regarding the currently deployed
16 conventional bus lines, the accessibility provided by DRT is much higher and its geographical distribution
17 more equal. While current DRT planning is based exclusively on level of service and cost, our approach
18 allows DRT planners and operators to shift their focus on accessibility.

19

20 **Keywords:** Accessibility; Multimodal Transport Network; Demand-Responsive Transport (DRT); Equity

1 INTRODUCTION

2
3 One main objective of transport planning is to ensure *accessibility*, i.e., the ability of people to
4 reach opportunities (e.g., services, activities, jobs, other people) via the available transport options (1), (2),
5 (3), (4). However, applying this concept *no matter* the transport options has resulted in car-oriented cities.
6 Recognizing the related environmental, social and economic sustainability issues and in the effort to reduce
7 externalities (5), planning strategies must now foster a modal shift from private vehicles to more sustainable
8 modes, such as active mobility and Public Transport. Since active mobility can only cover a limited fraction
9 of required accessibility (due to limits in trip lengths, city morphology, etc.), the role of Public Transport
10 (PT) is of paramount importance. This motivates why we focus here on the *accessibility provided by PT*,
11 i.e., the ability of people to reach opportunities *via PT*.

12 PT accessibility is unequally distributed in metropolitan areas (6) and is insufficient in the suburbs,
13 which therefore suffer from a chronic car-dependency (7). Indeed, to prevent the PT cost per passenger
14 from exploding in weak and scattered demand areas, PT operators can only provide low stop density and
15 line frequencies, which results in a poor level of service and insufficient PT accessibility. This issue can be
16 tackled via Demand Responsive Transport (DRT), where a fleet of buses is dynamically routed to better
17 adapt to a weak demand (8). Integrating DRT with conventional PT has the potential to improve level of
18 service (9), (10) and reduce the accessibility inequality commented above (11). In some limited areas, it
19 must also be beneficial to replace conventional PT with DRT (12).

20 Generally, DRT requires additional cost of investment and operation (10). The reluctance of
21 operators to bear this cost can be partially explained by the fact that, up to now, it has not been possible to
22 quantify its benefits in convincing terms. Indeed, DRT has mainly been studied in terms of operational
23 parameters and metrics (e.g., travel and waiting times, fleet dimensioning, etc.) that, by themselves, cannot
24 justify the high additional cost. A much more convincing argument would be to show the improvement of
25 accessibility, i.e., to understand “how many additional opportunities per hour can be reached thanks to
26 DRT”. However, no methods are available today to make this type of quantification. Assessing the
27 accessibility provided by DRT is indeed challenging: accessibility measures are generally calculated on a
28 graph. However, a DRT service cannot be represented as a graph, due to its stochasticity, as DRT vehicle
29 trips change every day, depending on user requests.

30 In this paper, we remove this barrier by proposing a new method to assess the accessibility of DRT.
31 The method is *data-driven*, as it is solely based on a simple statistical analysis of trips observed in previous
32 days of DRT operations and does not require the development of complex and hard to validate econometric
33 or simulation models. The key idea of the method is the description of DRT operation in terms of time-
34 dependent graphs and the treatment of accessibility as a random variable, to account for the stochasticity of
35 DRT. We showcase our method in a case study in Acireale (Italy), simulated in Visum, where we show the
36 improvement of accessibility when replacing conventional PT with DRT, as well as a more equal
37 geographical distribution of accessibility.

38 By allowing to assess the improvement of accessibility provided by DRT, we believe our work can
39 contribute to a better understanding of this system, an increased acceptance by planners, operators and
40 users, and a better willingness to undertake the economic investment needed for its deployment.

41 RELATED WORK

42
43
44 We focus in this section on the literature focused on the measurement of the impact of flexible
45 mobility (shared taxis, DRT, etc.) on the accessibility. Accessibility measures based on random utility have
46 been applied to on-demand mobility (13,14,15). The limit of such approach is that it requires the
47 development of agent-based simulation models to measure the utilities perceived by individual users. The
48 high complexity of developing the models, as well as calibrating and validating them, limits their
49 scalability, replicability, ease of interpretation and trust that deciders can have in their results. These
50 difficulties limit their application in practical cases.

1 Accessibility measures based on isochrones (16) are a valuable alternative to random utility
2 accessibility measures, as they provide an easy-to-understand indicator, which basically counts the number
3 of opportunities that can be reached in a given time interval. There is some relevant recent work computing
4 isochrone-based accessibility for shared or flexible mobility. Considering the work proposed by Abouelela
5 et al. [2024] (17), the authors compute an isochrone-based accessibility measure for a shared e-scooter
6 service. The limitation is that (i) they cannot capture the fact that multiple trips are shared among users,
7 which however determines the economic and environmental efficiency of DRT, (ii) they cannot capture the
8 accessibility of flexible modes *combined with* conventional PT.

9 Le Hasif et al. [2022] (18), Wang et al. [2024] (19) adopt analytical models based on continuous
10 approximation to roughly estimate DRT travel times, and compute accessibility based on estimations. While
11 such approaches can be a useful starting point in strategic planning, to reason about where DRT deployment
12 is more appropriate, their accessibility do not capture the real accessibility experienced by users, as
13 analytical models are unsuited to do so.

14 Diepolder et al., 2023 (20) are the first to propose a method to compute isochrone-based
15 accessibility based on empirical observations of DRT trips. The DRT considered in their work is *many-to-*
16 *one*: DRT buses bring multiple passengers at a time to/from major rail hubs, providing a feeder service
17 around them. Many-to-one DRT is appropriate in areas with a sufficient presence of conventional PT stops.
18 This might not be the case of weak demand areas, where door-to-door many-to-many DRT services,
19 providing a direct connection from users' origins to destinations, is more appropriate. The method of
20 Diepolder et al. 2023 is based on modeling travel and waiting times as 2-dimensional random fields around
21 each conventional PT stop. Their method relies on geostatistical analyses that exploit geographical locality:
22 (i) the similarity of travel and waiting times of user trips within close-by origins, going to the same
23 conventional PT stop and (ii) the similarity of travel times of user trips starting from the same conventional
24 PT stop and going to close-by destinations. Therefore, their method cannot generalize to a many-to-many
25 DRT service. Indeed, the user trips served in such a service cannot be positioned on a 2-dimensional surface
26 (as we have to consider the (x,y) of the origin and the (x,y) of the destination – 4 dimensions in total), which
27 prevents from using the 2-dimensional random field formalism and subsequent analysis. In this paper we
28 overcome this limitation by a more generic approach, which can handle trips *whatever they are*, requiring
29 neither any many-to-one structure nor any geographical locality.

30 To the best of our knowledge, we are the first to provide a method to quantify isochrone-based
31 accessibility of a generic DRT service (many-to-one or many-to-many), which captures trip sharing and
32 multimodality (i.e., combination with conventional PT), and which is solely based on previous observed
33 DRT trips. In terms of implementation and data format, we point out the availability of the GTFS-Flex
34 format. GTFS-Flex extends the General Transit Feed Specification (GTFS), used to describe conventional
35 PT schedules, with new tables for the description of DRT services (21). In our implementation, we describe
36 DRT in the original GTFS format, but we may consider using GTFS-Flex in the future.

37 38 **METHODOLOGY**

39 40 **Preliminaries**

41 42 *Accessibility*

43
44 We aim to compute the accessibility metrics by Biazzo et al. (22). We now explain their calculation in the
45 case of conventional PT. First, the study area is zoned with a regular tessellation in hexagons. We remove
46 the hexagons whose distance between the respective centroid and the closest PT stop is larger than
47 *max_walk_distance*. We compute accessibility scores for the remaining hexagons. Two metrics are
48 proposed, i.e. velocity score and the sociality score.

49 For a generic hexagon i we firstly set a departure time t_0 then calculate area $A(t, i)$ that can potentially be
50 visited when starting at the centroid of hexagon i at time t_0 , within time t , using walk and PT lines.

1 Transfers from line to line are possible and waiting times are considered whenever a line is boarded. We
 2 then make the simplifying assumption that area $A(t, i)$ is a circle and compute its radius:

$$3 \quad 4 \quad r(t, i) = \sqrt{\frac{A(t, i)}{\pi}} \quad (1)$$

5 We then calculate a corresponding velocity $V(t, i) = \frac{r(t, i)}{t}$. To compute a velocity score of hexagon i at
 6 time t_0 , we need to get rid, in the previous formula, from the dependence on user trip duration t . To this
 7 aim, let $f(t)$ be the probability distribution of a user trip duration, which can be computed based on trip
 8 times usually observed in cities. The **velocity score** of hexagon i at time t_0 is:

$$9 \quad V(i) = \int_0^\infty V(t, i) f(t) dt \quad (2)$$

10 The calculated velocity score provides a measure of the average speed each individual experiences while
 11 traveling on public transport services, starting from a chosen hexagon going in any direction.

12 To compute an isochrone-based accessibility score, we first compute number $S(t, i)$ of opportunities
 13 reachable within time t when starting at instant t_0 from hexagon i . $S(t, i)$ corresponds to the opportunities
 14 located inside area $A(t, i)$. Similarly, as before, we compute the **accessibility score** as:

$$15 \quad 16 \quad S(i) = \int_0^\infty S(t, i) f(t) dt \quad (3)$$

17
 18 The accessibility score can be interpreted as how many opportunities can be reached in a typical user trip
 19 thanks to PT. For simplicity and availability of data, we consider, as in (22), people as opportunities, and
 20 call $S(i)$ *sociality score*. In what follows, we will generically refer to $Acc(i)$ to refer to one of the two
 21 metrics presented above, when it will not be relevant to distinguish them.

22 *Time-expanded graph PT*

23
 24 We need to compute travel times between any pair of centroids, to calculate areas $A(i, t_0)$, on which all the
 25 aforementioned accessibility metrics are based. To this aim, as in (18) and (23), we model PT as a *time-*
 26 *expanded graph*, which is the format underlying the General Transit Feed Specification (GTFS), the
 27 standard format describing PT scheduling. Each node of such a time-expanded graph is called *stoptime* and
 28 is characterized by a pair (s, t) , where $s \in \mathbb{R}^2$ is a physical PT stop (represented as a point in the plane)
 29 and $t \in \mathbb{R}$ is a time instant in which a PT vehicle stops. So, for each PT line, a vehicle trip is modeled as a
 30 sequence of *stoptimes*. Let ST_l be the set of *stoptimes* of line l . Edge $(s, t) \rightarrow (s', t')$, where
 31 $(s, t), (s', t') \in ST_l$, models a vehicle trip within line l that starts at stop s at time t and arrives at stop s'
 32 at time t' . An edge $(s, t) \rightarrow (s', t')$ between stoptimes of two different lines $(s, t) \in ST_l$ and $(s', t') \in$
 33 $ST_{l'}$ is added to indicate a transfer from a line to another if and only if (i) the distance between s and s' is
 34 no more than *max_walk_distance* and (ii) if it is possible to walk from stop s at time t and arrive at stop s'
 35 a time $t'' \leq t'$. In this case, $t' - t''$ is the waiting time for the line l' . Let us denote by G^{PT} the time-
 36 dependent graph constructed, for any line and all vehicle trips within each line, as explained.

37 Let us consider a user who with origin $x' \in \mathbb{R}^2$ and destination $x \in \mathbb{R}^2$, with a departure time t_0 . There
 38 can be different ways in which the user journey can be performed. They can just walk, if x and x' are within
 39 D_{max} . Or they can walk a short distance, reaching stop s , then board a PT vehicle at time t , i.e. *stoptime*
 40 (s, t) , and alight from the vehicle at stop s' , i.e. *stoptime* (s', t') : finally, they can walk from stop s' reaching
 41 x' . The arrival time at his destination x' is t' plus the additional walking time. Transfers are also possible.

42 We assume that users always choose, among the possible trip combinations, that one with the earliest arrival
 43 time. We perform user trip computations using CityChroner (22), without considering capacity constraints
 44 for PT vehicles.

1 Our contribution: modelling DRT as a time-expanded graph and accessibility as a random variable

2
3 To use the same methods and tools to compute accessibility used for conventional PT, our effort is to
4 describe DRT with the same model, i.e., to extend the time-expanded graph of PT explained before with
5 additional stoptimes and edges representing DRT. The main difficulty is that trips of DRT are not
6 deterministic as described before for conventional PT. They can change from one day to another, to adapt
7 to the different user requests. We therefore need to provide here a stochastic formulation.

8 Let us assume DRT operational characteristics have been fixed a-priori: there is a certain fleet of DRT
9 vehicles, that all start from pre-fixed positions and a certain algorithm is applied to route DRT vehicles. Let
10 d be the stochastic process representing the user demand from 0:00 to 24:00. Let d_ω be one realization of
11 such stochastic process, i.e., a sequence of user requests over 24h, each characterized by the time of request,
12 the requested pick-up time, the origin and the destination. Observe that realizations d_ω are indexed by
13 sample ω in some sample space Ω . Sample ω can be interpreted as a day of DRT operation. We assume
14 requests are generated from a same underlying stochastic process d , in the sense that realizations d_ω have
15 all the same statistical properties, in terms of expected request rate and probability distribution of origins
16 and destinations. Despite this statistical consistence, it is important to note that sequence d_ω of requests
17 might be different from day to day.

18 Any sequence d_ω of user requests induces a certain set of vehicle trips performed by the DRT fleet over 24
19 hours. Let $P(d_\omega)$ be the function that returns the set of DRT vehicle trips over 24h to serve the requests in
20 d_ω . Each vehicle can perform multiple trips, one after another, with some inactivity time in between. We
21 can represent vehicle trips in $P(d_\omega)$ as stoppoints and edges and add them to time-dependent graph G^{PT} of
22 conventional PT, in the following day. For each DRT vehicle, if it stops at location x at time t to pick up or
23 drop off passengers, we add stoptime (x,t) . We then add edges between consecutive stoptimes covered by
24 each vehicle. We add all these stoptimes and edges to G^{PT} and we obtain new graph G_ω . On this graph, we
25 can compute, for any hexagon i , its accessibility $Acc_\omega(i)$. Note that, if we observe DRT operation on
26 another day ω' , we can see a different sequence of requests $d_{\omega'}$, and thus a different set of DRT vehicle
27 trips $P(d_{\omega'})$ and thus a different resulting time-dependent graph $G_{\omega'}$. As a consequence, the value of
28 accessibility $Acc_{\omega'}(i)$ in hexagon i can also be different. In other words, for any hexagon i , its accessibility
29 $Acc(i)$ is a random variable whose realizations are $Acc_\omega(i)$, $\omega \in \Omega$.

30 Let $\mathbb{E}Acc(i)$ and $\sigma_{Acc(i)}$ be the expected value and the standard deviation, respectively. Quantity $\mathbb{E}Acc(i)$
31 indicates the accessibility that a user *can expect* at hexagon i from the multimodal system composed of
32 conventional PT plus DRT. Quantity $\sigma_{Acc(i)}$ expresses instead how *variable* such accessibility is. If $\sigma_{Acc(i)}$
33 is very small, it means that every day one can expect the same accessibility at hexagon i , i.e., everyday one
34 can reach the same number of opportunities per hour.

35 Note that to compute $\mathbb{E}Acc(i)$ and $\sigma_{Acc(i)}$ one would need to observe an infinite number of days of
36 operations, which is obviously impossible. We must thus be content with their estimations: if DRT
37 operation has been observed a certain finite set $\tilde{\Omega} \subset \Omega$ of days, we compute the empirical average $\tilde{Acc}(i)$
38 and the empirical standard deviation $\tilde{\sigma}_{Acc(i)}$ just on realizations $\{Acc_\omega(i) | \omega \in \tilde{\Omega}\}$ to approximate $\mathbb{E}Acc(i)$
39 and $\sigma_{Acc(i)}$, respectively.

40 As we specify in the ‘‘Accessibility’’ section, we exclude from the analysis hexagons whose centroid is far
41 away from any stoptime (further than *max_walk_distance*). It might thus happen that a certain hexagon i is
42 excluded from accessibility computation in some realizations ω . This happens when neither conventional
43 PT stops, nor any pickups/drop-offs of DRT are close to i (it may be that no drop-offs and pick-ups in d_ω
44 are inside i or close enough, or it may also be that such requests might have existed, but the DRT service
45 was unable to serve them). To make our analysis robust, we only calculate accessibilities of the hexagons
46 that are never excluded in the observed days $\omega \in \tilde{\Omega}$. Furthermore, we provide values of accessibilities only
47 for the hexagons with small enough standard deviation, $\tilde{\sigma}_{Acc(i)} \leq \sigma_{max}$, as otherwise the measure would
48 be unreliable.

49

1 **IMPLEMENTATION**

- 2
- 3 The implementation of the method is composed of five steps, as in Figure 1:
- 4 1) Generation of the demand for multiple days of observation, i.e., set $\{d_\omega | \omega \in \Omega'\}$, where d_ω is the
- 5 sequence of user trip requests over one day of observation
- 6 2) Simulation, in Visum (24), of the operation of DRT to serve d_ω and store the resulting DRT vehicle
- 7 trips $P(d_\omega)$, for $\omega \in \Omega'$.
- 8 3) Generation, via appropriate Python scripts, of time-expanded graph G_ω modeling DRT and
- 9 conventional PT and store this graph in GTFS format, for each day of observation $\omega \in \Omega'$;
- 10 4) Computation of $Acc_\omega(i)$ using CityChrono (22), for all hexagons i and for all days of observation $\omega \in$
- 11 Ω' .
- 12 5) Computation of average accessibility $\widetilde{Acc}(i)$ and standard deviation $\tilde{\sigma}_\omega(i)$ for all hexagons i , using
- 13 Python and Excel.

14 Note that steps 1 and 2 are not needed when DRT trip observations come from a real system.

16

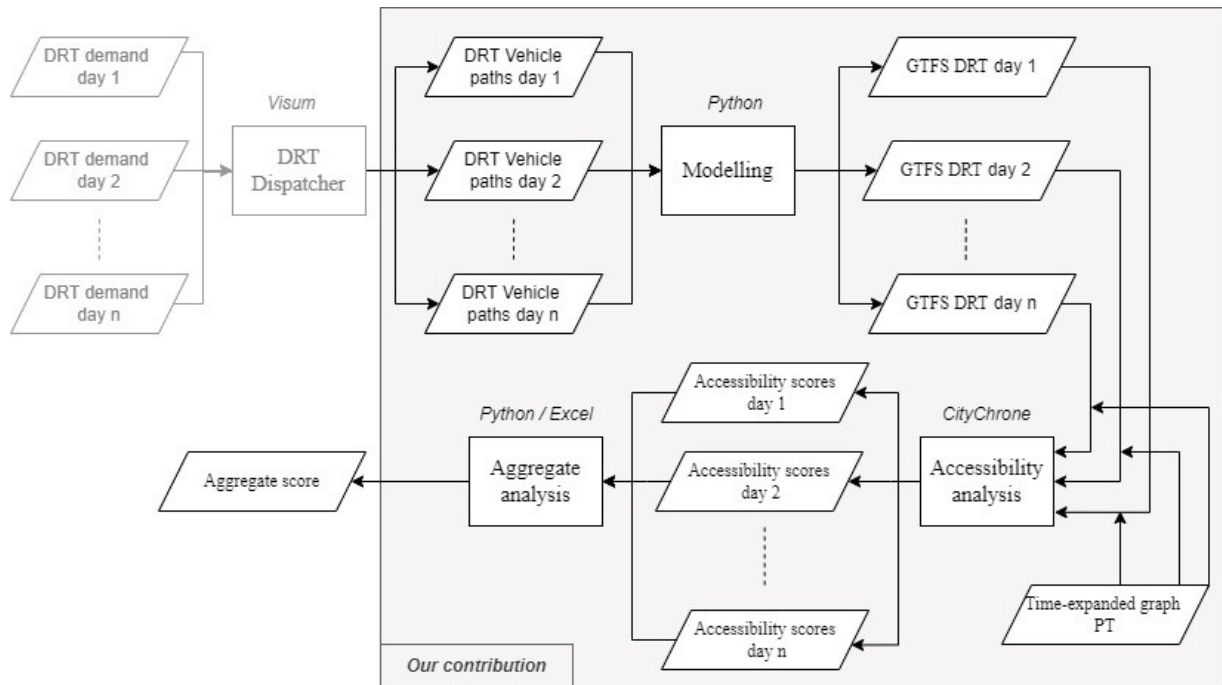


Figure 1 Implementation procedure

17
18

19 **Generation of demand d_ω and vehicle trips $P(d_\omega)$ in Visum**

20

21 First, we identify the sub-network where DRT service can be available, i.e. selected links where DRT

22 vehicles are authorized to run; then we identified physical or virtual stop points for passengers Pick-Up and

23 Drop-Off from the vehicle, named PUDOs.

24

25 To generate a demand d_ω for different observation days $\omega \in \Omega'$, we start from an hourly Origin-Destination

26 (OD) matrix, containing the number of users willing to go from an origin zone to a destination zone, via

27 DRT, at each hour. Each user is converted into a request in d_ω , characterised by an origin point and

28 destination point and a departure time. These two points are uniformly distributed at random within the

29 respective origin and destination zones. Similarly, the departure time is also uniformly distributed at random

1 d_ω , using the dispatching algorithm of Visum (25) (but the procedure is not impacted by the dispatching
2 algorithm employed).
3 Note that our procedure is oblivious to the operational rules and constraints of the DRT system under study.
4 It can indeed be applied to many-to-one or many-to-many systems, stop-based or door-to-door, with
5 prebooking or not. As long as a set of observations $P(d_\omega), \omega \in \Omega$ are available, our analysis can be
6 performed, no matter the characteristics of the DRT service that generated such observations.

7

8 **Generation of time-dependent graph G_ω and GTFS representation**

9

10 From the node-level information contained in output $P(d_\omega)$ of the simulation, we create a list in which
11 each row represents the position of a DRT vehicle, with the following attributes:

- 12 • *Vehicle id*
- 13 • *Node_ID*,
- 14 • *Node_coordinates*,
- 15 • *Is_it_stop*: indicating whether in this node the DRT has done a stop for a drop-off or pick-up
- 16 • *Arrival_time*,
- 17 • *Departure_time*,
- 18 • *Dwell time* (if provided) of a vehicle when the node is a stop;
- 19 • *Pass_on_board*, *Pass_boarded*, *Pass_alighted*: number of passengers on board, boarding and
20 alighting at this node.
- 21 • *Prev_section_length*: travelled distance along the network between the current node and the
22 previous one indicated within the previous row

23 We perform the following operations:

- 24 1. We remove the rows with *Pass_on_board* = 0, as in those moments the corresponding bus is not
25 providing a service to anyone;
- 26 2. We group the consecutive subsequence of nodes where *Pass_on_board* > 0 in a single vehicle
27 *trip*, and assign a *Trip_ID*.

28 The set of such vehicle trips constitutes $P(d_\omega)$, which is stored in the form of two files: *stop_times.txt*, with
29 the list of stops with their associated transit times and *trips.txt* with all performed vehicle trips.

30 In addition, we also consider a third file with the list of service-enabled stops (PUDO). All this information
31 is converted in the GTFS format and merged with the GTFS files of conventional PT. The result the time-
32 dependent graph G_ω .

33 We repeat the procedure above for all the considered days of DRT operation.

34

35 **Computation of accessibility $Acc_\omega(i)$ in CityChrono**

36

37 To calculate $Acc_\omega(i)$ for each day ω of DRT operation, we use the open-source platform CityChrono. The
38 input data needed for the accessibility analysis are a shape file with the number of opportunities in each
39 hexagon and the GTFS describing time-dependent graph G_ω . Within CityChrono, the study area is
40 tessellated in hexagons, and the number of opportunities contained in the aforementioned shape file is
41 distributed in the hexagons in the corresponding locations. Hexagons can be arbitrarily small, however
42 increasing the computation time.

43 The calculation of pedestrian routes between centroids of hexagons and stops is performed via the open-
44 source routing engine, i.e. Open Source Routing Machine (OSRM). At this stage, we can identify the
45 number of hexagons "served by public transport", which represent the service coverage. As output, for each
46 day of operation $\omega \in \Omega'$, CityChrono provides accessibility scores $Acc_\omega(i)$ of all centroids i , except those
47 whose centroid is not within distance *max_walk_distance* from a stop in G_ω (either of conventional PT
48 or DRT).

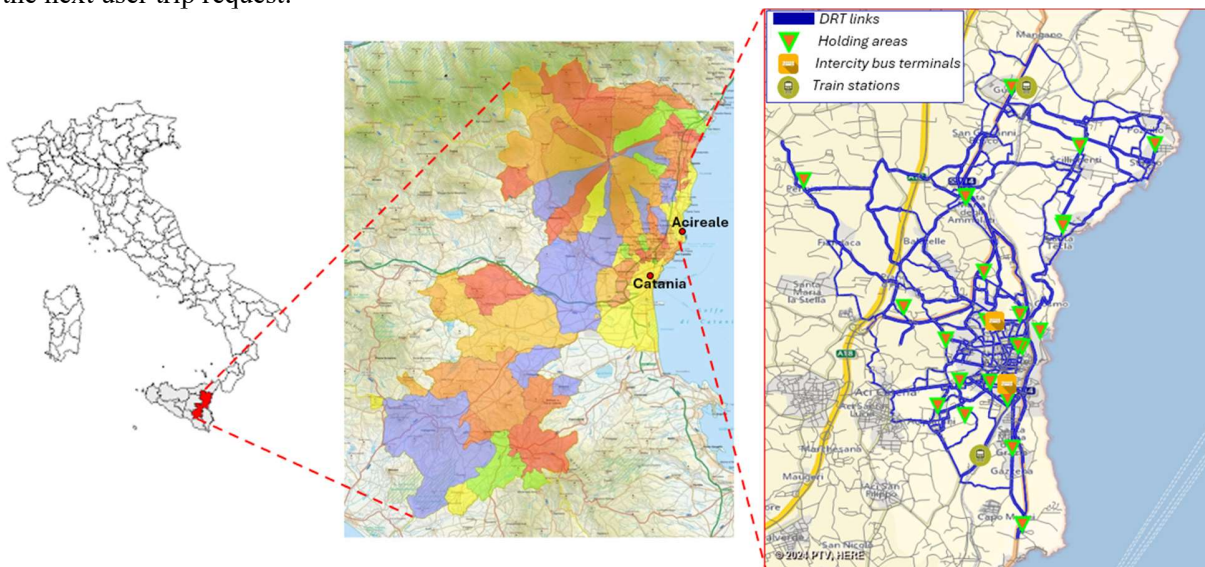
49 Finally, we aggregate the results of the different days of operation $\omega \in \Omega'$, in Python, to compute, in each
50 hexagon i , the empirical average $\widehat{Acc}(i)$ and the empirical standard deviation $\tilde{\sigma}_{Acc(i)}$.

1 **EVALUATION**

3 **Description of the use case**

5 We test the methodology in a case study to evaluate the accessibility that would result from a hypothetical
 6 implementation of a many-to-many DRT service, integrated with conventional PT, within the municipality
 7 of Acireale (Italy) (Figure 2).

8 Acireale is part of the metropolitan city of Catania, located in Sicily (Italy). It is characterized by a low
 9 frequency and poor coverage bus service, unable to provide a good connection with the main intermodal
 10 hubs (i.e. intercity bus terminals located in Piazza Livatino and Piazza Pennisi and train stations of Acireale
 11 and Guardia Mangano). The demand for PT is weak and scattered, making the implementation of DRT
 12 potentially appropriate. It is important to note that Acireale has a strong interaction with Catania, the main
 13 city in the metropolitan area (Figure 2-center), where most of the opportunities are. Figure 2-right depicts
 14 the sub-network of Acireale where the DRT service is allowed to operate, highlighting the location of main
 15 intermodal hubs and the *holding areas*, i.e., the locations where empty DRT vehicles can park, waiting for
 16 the next user trip request.



17
 18 *Figure 2 DRT area and sub-network of the simulated DRT service within the municipality of Acireale*

19 Regarding the DRT demand, we consider the systematic movements provided by the Italian National
 20 Institute of Statistics (26); specifically, we start by considering the total number of users currently using
 21 PT. We further increase such number by adding the total percentage of users who travelled by car as
 22 passengers, as we assume that those passengers can benefit the most from improved PT. We then add to
 23 our demand 5% of users who currently drive their private vehicle, assuming they may shift to PT due to the
 24 improved performance brought by integrating DRT. Table 1 indicates the numbers of users served by DRT
 25 during an hourly simulation.

26 The simulated DRT service operates under the following time constraints, according to the dispatching
 27 policy implemented in VISUM (25) with the aim of minimizing the number of vehicles. For a specific user
 28 trip request, let us define the *Ideal Travel Time (ITT)* as the travel time of a vehicle hypothetically travelling
 29 along the direct distance from the origin to the destination. Let *DRTtime* be the actual in-vehicle travel time
 30 experienced by the user considering the detours. The difference between the *DRTtime* and *ITT* is the *Detour*
 31 *Time (DT)*. The *Detour Factor* of a user trip is $DF = DT/ITT$.

32 We also define the following parameters *Maximum detour factor (MaxDF)*

- 33 • *Maximum detour time (MaxDT)*

- *Detour time always accepted (AllAccDT)*

When a user issues a request, the dispatcher computes a feasible travel plan, trying first to use the vehicles already serving other users. The plan is proposed to the user, and we assume that:

- (i) if $DT < AllAccDT$, the user accepts the plan;
- (ii) if $DT > MaxDT$, the dispatcher assigns the user to a new empty vehicle if available;
- (iii) if $AllAccDT < DT < MaxDT$ then.
 - if $DF < MaxDF$ the user accepts the plan; otherwise,
 - if $DF > MaxDF$, the dispatcher assigns the user to a new empty vehicle if available;

If there are no vehicles available for conditions (ii) and (iii), the request is rejected. Additionally, we also impose a maximum waiting time at the DRT stop ($MaxWait$) and a maximum walking time ($MaxWalk$) and the request will be rejected if these additional time constraints are not respected. We also fixed a maximum pre-booking time to make the request in advance, which could reduce the waiting time. We dimension the DRT fleet to obtain a service that is *predictable*, i.e., of which it is possible to have a good a-priori estimation of travel times. This property is important for user acceptance, in particular for DRT with no (or short) prebooking. Indeed, a user may need to know, with sufficient precision, how much time their trip is going to take, even if the trip will be requested hours or days later. With this information a user can determine if DRT is a convenient option for their trip. If it were not possible for a user to have a (at least approximate) idea of the travel time, they may perceive the DRT not reliable and may not choose it. We developed a *predictability indicator* in a previous paper (27), where we applied it in the same case study of this paper. Here, we dimension the fleet size so that the *predictability indicator* is at most 87%, which means that it is possible to provide to the user a sufficiently small range of values that contains the actual travel time 87% of times (27).

Table 1 summarizes all the DRT service parameters, as well as the parameters used for calculating accessibility metrics in CityChrone.

DRT operational parameters	
<i>Sub-network links</i>	2744 links
<i>Stops</i>	899 stops
<i>MaxDF</i>	3.5
<i>MaxDT</i>	15 minutes
<i>AllAccDT</i>	8 minutes
<i>MaxWait</i>	15 minutes
<i>MaxWalk</i>	15 minutes
<i>Max pre-booking time</i>	15 minutes
<i>Time interval of service simulation</i>	7:00 a.m. – 8:00 a.m.
<i>Number of vehicles</i>	16 vehicles
<i>Vehicle capacity</i>	8 passengers
<i>Percentage of satisfied users</i>	88%
<i>Users served</i>	166 users
Parameters related to Accessibility used in CityChrone	
<i>Walk speed</i>	3.9 km/h (28, 29)
<i>Max_walk_distance</i>	1300 m (28, 29)
<i>Hexagon_side</i>	0.5 km ²
<i>Number of Hexagons</i>	4987
<i>Hexagon_number_service areas</i>	295
<i>Geographical distribution of population</i>	EUROSTAT (30)
<i>Departure time t_0</i>	7:00 a.m.
<i>Maximum tolerated variability in accessibility σ_{max}</i>	35%
Characteristics of conventional PT within the metropolitan area of Catania	
<i>Lines (extraurban and urban)</i>	150 lines
<i>Vehicle Trips</i>	469
Characteristics of conventional PT exclusively within the municipality of Acireale	
<i>Lines (only urban)</i>	4
<i>Vehicle Trips</i>	26
<i>Removed lines in Scenario 3 Reduced PT+DRT</i>	4

Table 1 Case study parameters

1 We compare the following three scenarios, where we vary the service in Acireale while leaving the PT of
 2 the rest of the metropolitan area:

- 3 • Scenario 1 *PTonly*: PT bus lines currently operated in Acireale (no DRT);
- 4 • Scenario 2 *PT+DRT*: PT bus lines currently operated plus the DRT service in Acireale;
- 5 • Scenario 3 *ReducedPT+DRT*: Current PT supply with lines partially replaced in the areas where
 6 the DRT is implemented, removing all lines operating exclusively operating within the municipality
 7 of Acireale (Table 1).

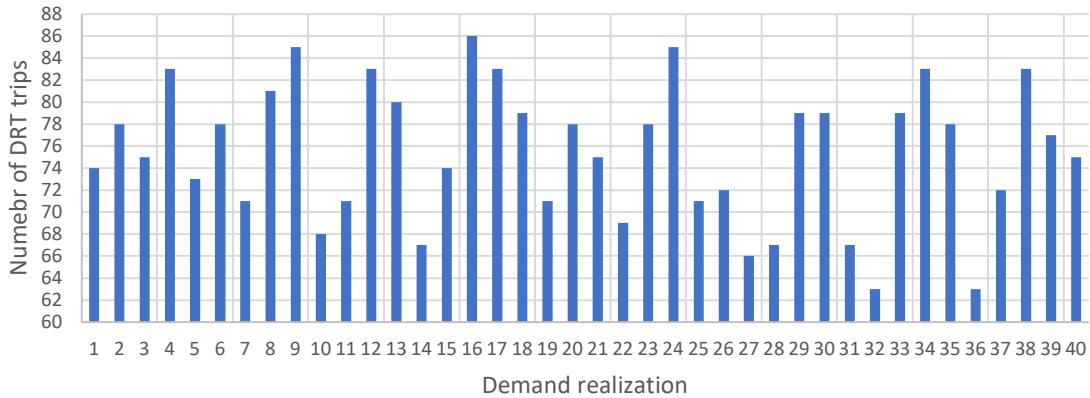
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9 **DRT service variability**

10

11 Multiple simulations have been performed, by varying the random seed, to generate the demand for multiple
 12 days of observation, i.e. $\{d_\omega | \omega \in \Omega' = \{1, \dots, 39\}\}$, to capture the variability that characterize DRT. A total
 13 of 39 simulations are carried out. For each demand d_ω we obtained a set of DRT vehicle trips $P(d_\omega)$ used
 14 to carry out the 39 simulations. Note that the accessibility computation of scenario *PT only* does not depend
 15 on the demand, as the corresponding time-dependent graph just models the GTFS data of the conventional
 16 PT, which is oblivious to the demand.

17 Figure 3 shows the number of vehicle trips performed by the entire DRT fleet in each simulation. Each
 18 realization $P(d_\omega)$ of the set of vehicle trips in a day is used to enrich the time-dependent graph of
 19 conventional PT (as we explained in the Methodology section) in both the *PT+DRT* and *ReducedPT+DRT*
 20 scenarios.



21
 22

Figure 3 Variability of simulated DRT service

23 We calculate accessibility scores of the metropolitan city of Catania to consider the entire multimodal
 24 transport system and its interactions with the municipality of Acireale. However, we show the results of
 25 only the hexagons in Acireale, where the DRT service is implemented. Note that the opportunities (people)
 26 that are reachable from the hexagons in Acireale also include those in the rest of the metropolitan area.

27 We created the GTFS file (GTFS_base) of the metropolitan area by correcting the official GTFS of the PT
 28 operator in the area, affected by errors in the location of stops and scheduling. This GTFS is the
 29 implementation of graph G^{PT} . We compute accessibility scores on G^{PT} for scenario *PT only*. The
 30 characteristics of conventional PT are in Table 1. We then remove the lines specified in Table 1 to obtain
 31 $G^{Reduced PT}$. Finally, we then build the GTFS representing the time-dependent graphs of *PT+DRT* scenario,
 32 by enriching G^{PT} with the DRT trip vehicles. We similarly for Reduced PT +DRT, by enriching
 33 $G^{Reduced PT}$ this time.

34 Via CityChrone we obtain:

- 35 - For Scenario 1 *PT only*: 1 velocity scores and 1 sociality scores for each hexagon;

1 - For Scenario 2 *PT+DRT*: 39 velocity scores $V_\omega(i)$, $\omega \in \Omega' = \{1, \dots, 39\}$ and 39 sociality scores
 2 $S_\omega(i)$, $\omega \in \Omega' = \{1, \dots, 239\}$ for each hexagon i .

3 - For Scenario 3 *ReducedPT+DRT*: similar values as in Scenario 2 *PT+DRT*

4 For both scenarios *PT+DRT* and *Reduced PT + DRT*, we compute per each hexagon the empirical
 5 average $\overline{Acc}(i)$ and standard deviation $\tilde{\sigma}_\omega(i)$. We then compute the coefficient of variation as Equation
 6 4:

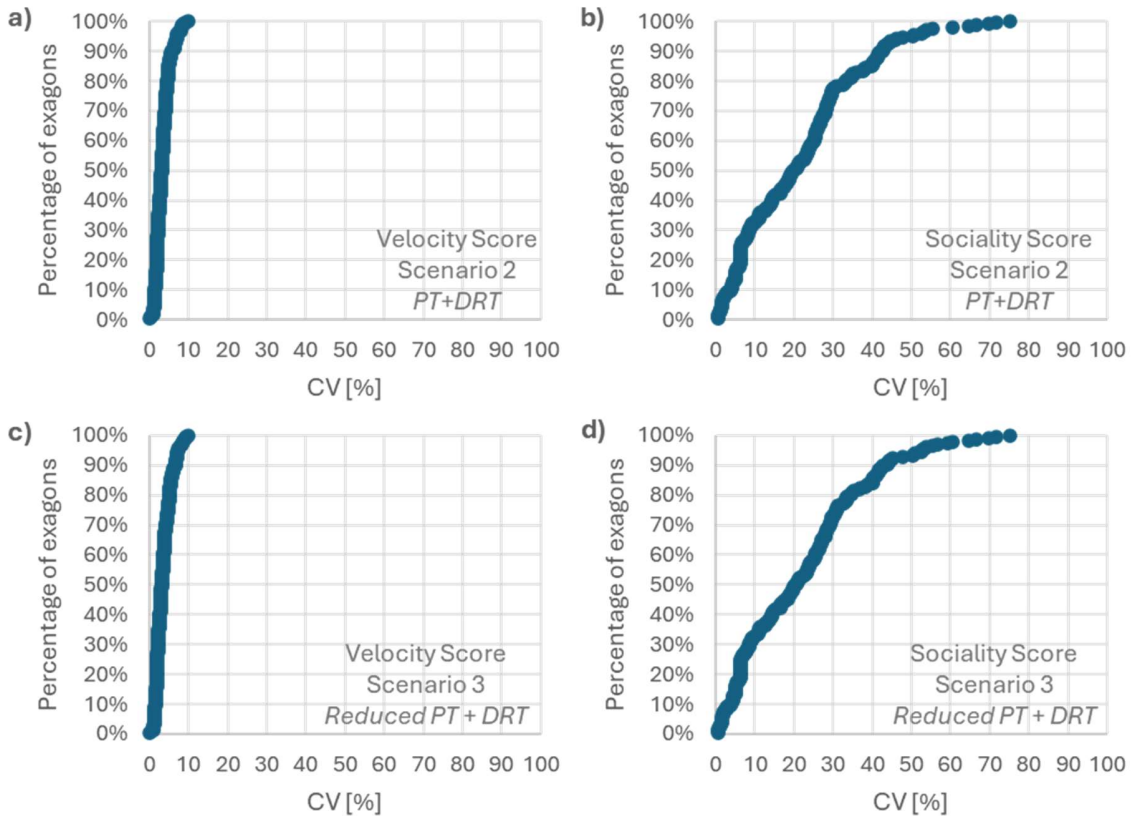
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$$8 \quad CV(i) = \frac{\tilde{\sigma}_\omega(i)}{\overline{Acc}(i)} \cdot 100 \quad (4)$$

9

10 Figure 4 shows the Empirical Cumulative Distribution Function (ECDF) of the coefficients of variation. To
 11 ensure that the accessibility metrics we will show in what follows are reliable, we exclude from the analysis
 12 all the hexagons whose coefficient of variation of the sociality score is larger than 35%- From Figure 4 one
 13 can see that this choice excludes only 20% of the hexagons in Acireale.

14



15

16

Figure 4 ECDF of the coefficients of variation

17 It is important to point out that in the 80% of the hexagons that we keep in our analysis, there are
 18 approximately 57,000 residents out of the total 60,000. In other words, we are still studying the accessibility
 19 experienced by 95% of the resident population.

20

21 Improvements of accessibility

22

23 We compare the calculated metrics in the three scenarios. ECDFs were used to correlate the accessibility
 24 scores and the percentage of population living in the analyzed hexagons.

1

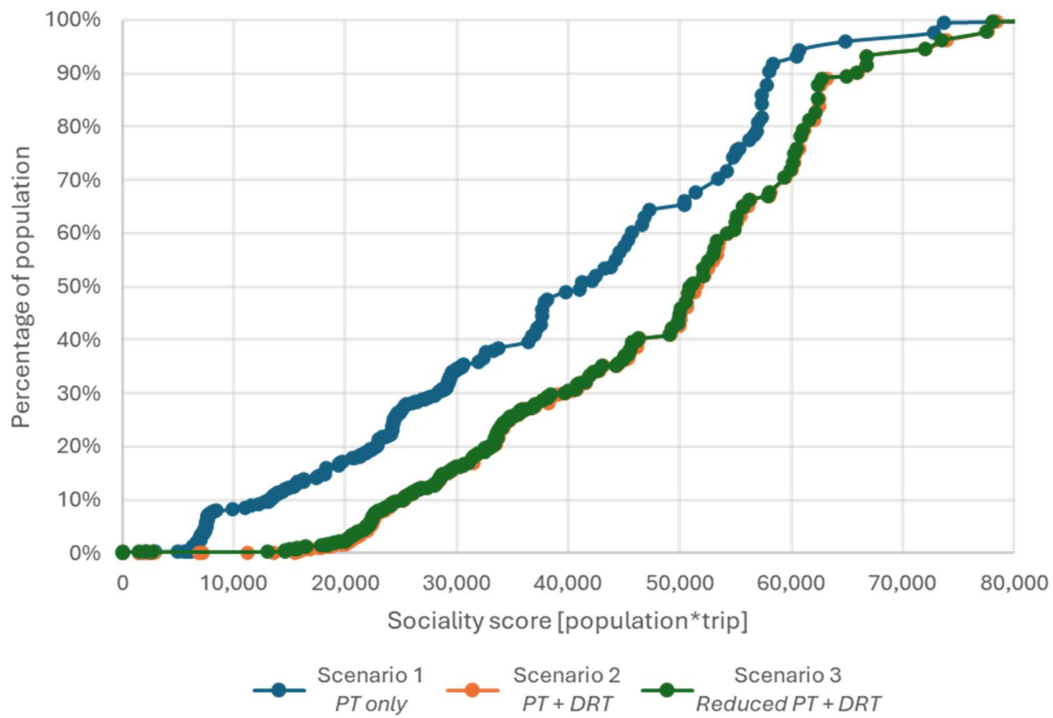


Figure 5 ECDF of the sociality scores

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4 From the ECFDs in Figure 5 it emerges that the implementation of the DRT service improves accessibility,
 5 especially in the most disadvantaged hexagons (those with the lowest scores): The accessibility of the 10%
 6 population with the worst score jumps from ~12 000 people reachable per user trip to ~24 000 (100%
 7 increase).

8 The further remarkable point is that *Reduced PT + DRT* has no lower accessibility than *PT + DRT*: after
 9 integrating DRT, certain PT lines can be removed without hurting accessibility. The scatter plot of Figure
 10 6 compares the sociality scores, hexagon by hexagon, of *PT only* and *Reduced PT + DRT*: for almost all
 11 hexagons accessibility increases, especially for the hexagons with very low values originally.

12

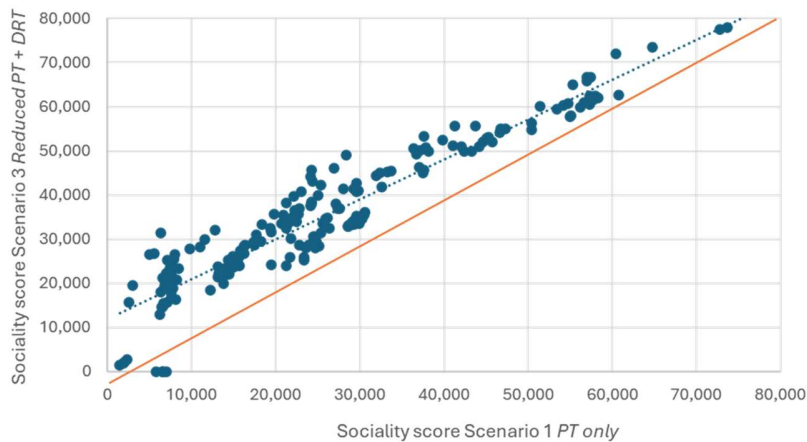
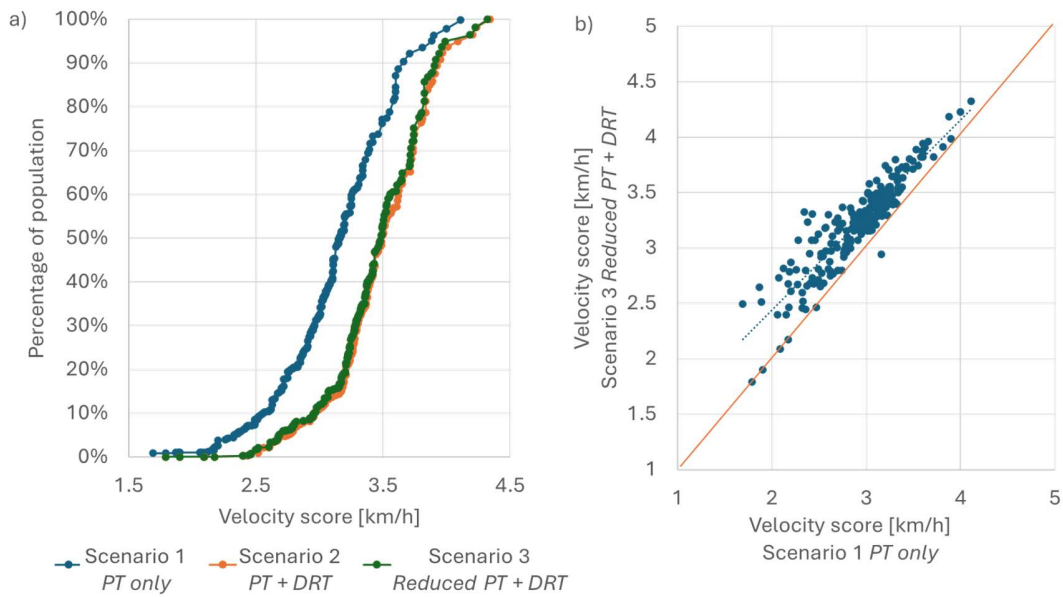


Figure 6 Scatter plot of sociality score comparing Scenario 1 PT only and Scenario 3 Reduced PT + DRT

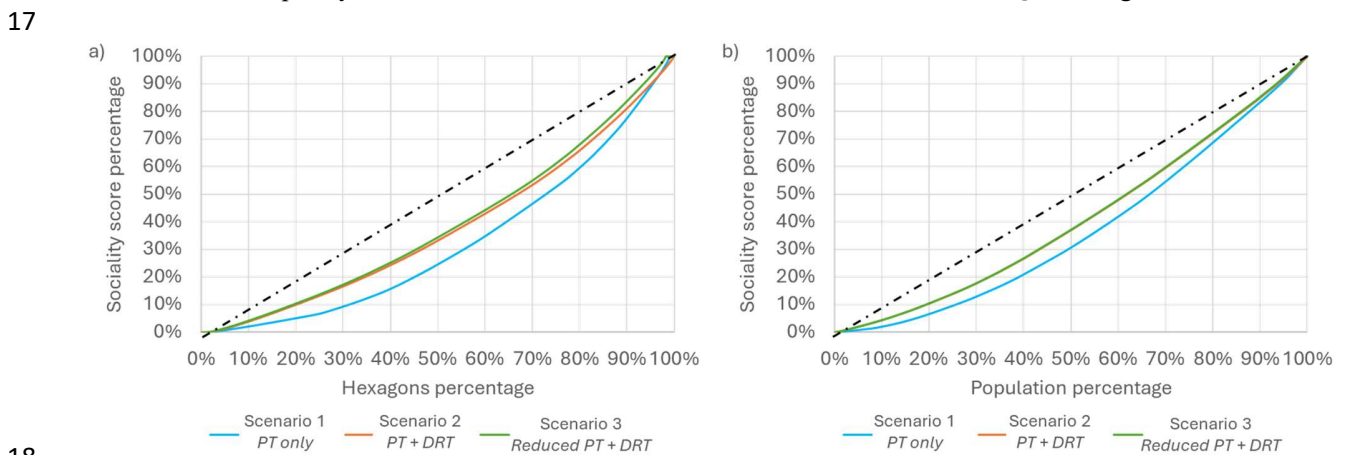
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1 Similar considerations emerge from the analyses conducted on velocity score (see Figure 7).
 2



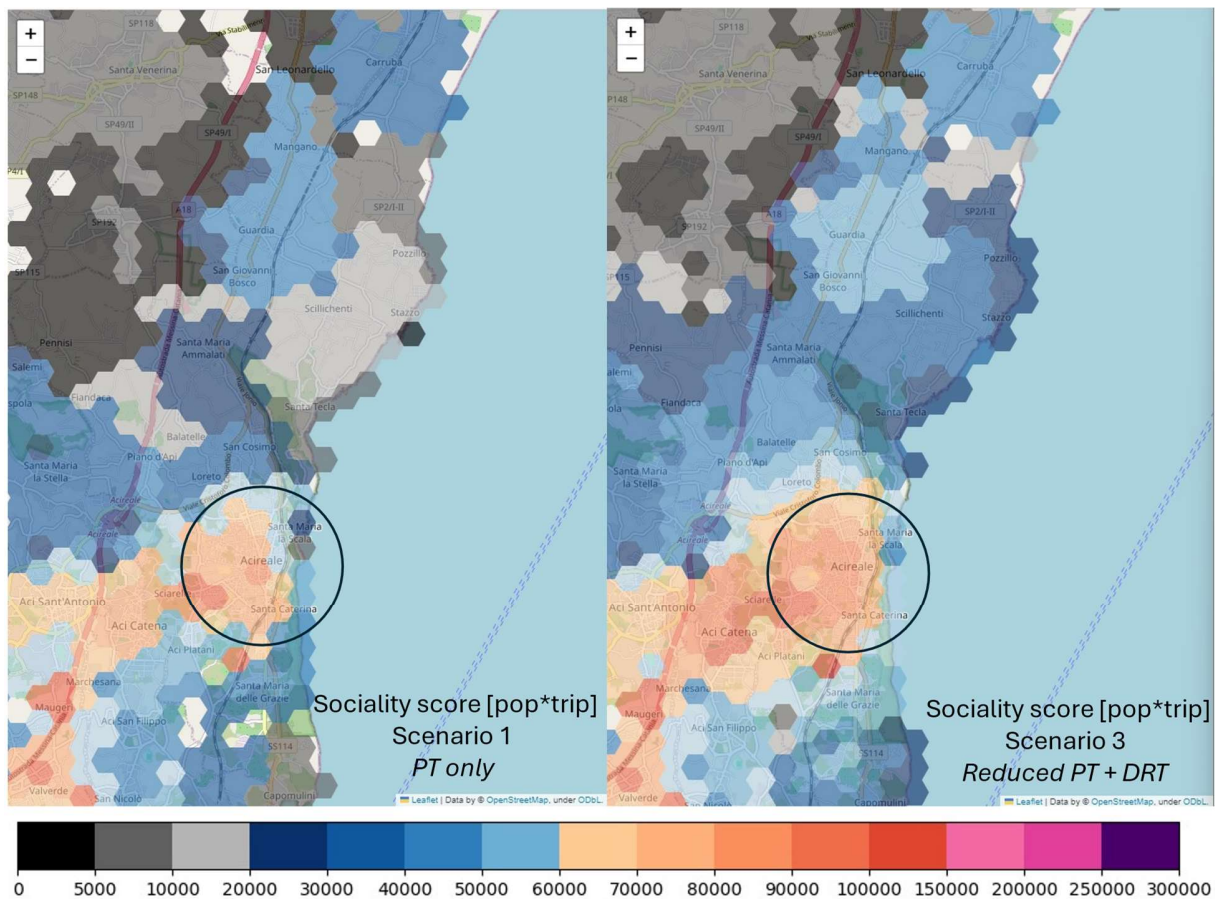
3
 4 Figure 7 a) Comparison of scenario velocity scores through ECDF, b) Scatter plot of velocity scores

5 We now analyze the equality (horizontal equity) of the distribution of accessibility in Acireale. Using
 6 Lorenz curves (31), we can visualize and quantify equality both across hexagons and across the population
 7 Following the classic approach to construct Lorenz curves, to build Figure 8-left, in the x-axis we sort
 8 hexagons in ascending order of accessibility and we report in the y-axis the cumulative sociality score. In
 9 the figure, the x and y axis are normalized to 100%. To build Figure 8-right, as in (32), we assume that all
 10 residents in a certain hexagon enjoy the sociality score of that hexagon. Then, similarly as before, we sort
 11 residents in the x -axis in ascending order of accessibility, we report in the y -axis the cumulative sociality
 12 score and we finally normalize to 100% both axes. The dotted line corresponds to the ideal "perfect
 13 equality" situation, in which $X\%$ of hexagons (or population) would enjoy $X\%$ of sociality score. We
 14 observe that DRT reduces inequality considerably. Indeed, in the *PT only* scenario, 50% of hexagons only
 15 get 25% of sociality score, while in the Reduced PT+DRT, they get 35%.
 16 Reduction of inequality across residents thanks to the DRT service is evident in Figure 8-right.



18
 19 Figure 8 Lorenz curve of sociality scores related to a) hexagons b) population

1 The maps of Figure show *where* the improvement in accessibility concentrates. Since the simulated DRT
 2 is deployed in Acireale and its suburbs (Pennisi, Santa Maria, Ammalati, Scillichenti), this is where most
 3 of the improvement is. Improvement is especially concentrated in the abovementioned suburbans, in which
 4 PT supply only is insufficient (hexagons in dark and light grey in Figure 9-top-left become even blue in
 5 Figure 9-top-right). Albeit less evident, improvements in the accessibility score are also recorded in the
 6 central of Acireale. It is interesting to notice that some improvement of accessibility is also observed in
 7 nearby areas, where the simulated DRT does not operate, such as the neighbouring municipalities (i.e. Aci
 8 Sant'Antonio, Aci Catena and Aci S. Filippo), i.e., the number of hexagons in dark orange are observed
 9 increases from Figure 10-top-left to Figure 10-top-right. This means that people living there can now
 10 connect more easily to Acireale and reach the opportunities located therein.
 11 Similarly, we realized the accessibility maps for the velocity score, shown in Figure 10. In this case,
 12 although the variations between Scenario 1 and Scenario 2 are less evident in absolute terms (i.e. increase
 13 in velocity values), the percentage variations registered between the various hexagons are in line with what
 14 was obtained for the sociality score.
 15 Since the economic component cannot be ignored, we apply the methodology of *Associazione Trasportisti*
 16 (ASS.TRA) to estimate that the cost per passenger of the simulated DRT service would be 13% smaller
 17 than the cost per passenger with the current conventional PT. A more rigorous description of the cost
 18 estimation of simulated DRT can be found in (33).
 19



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Figure 9 Maps for Sociality score

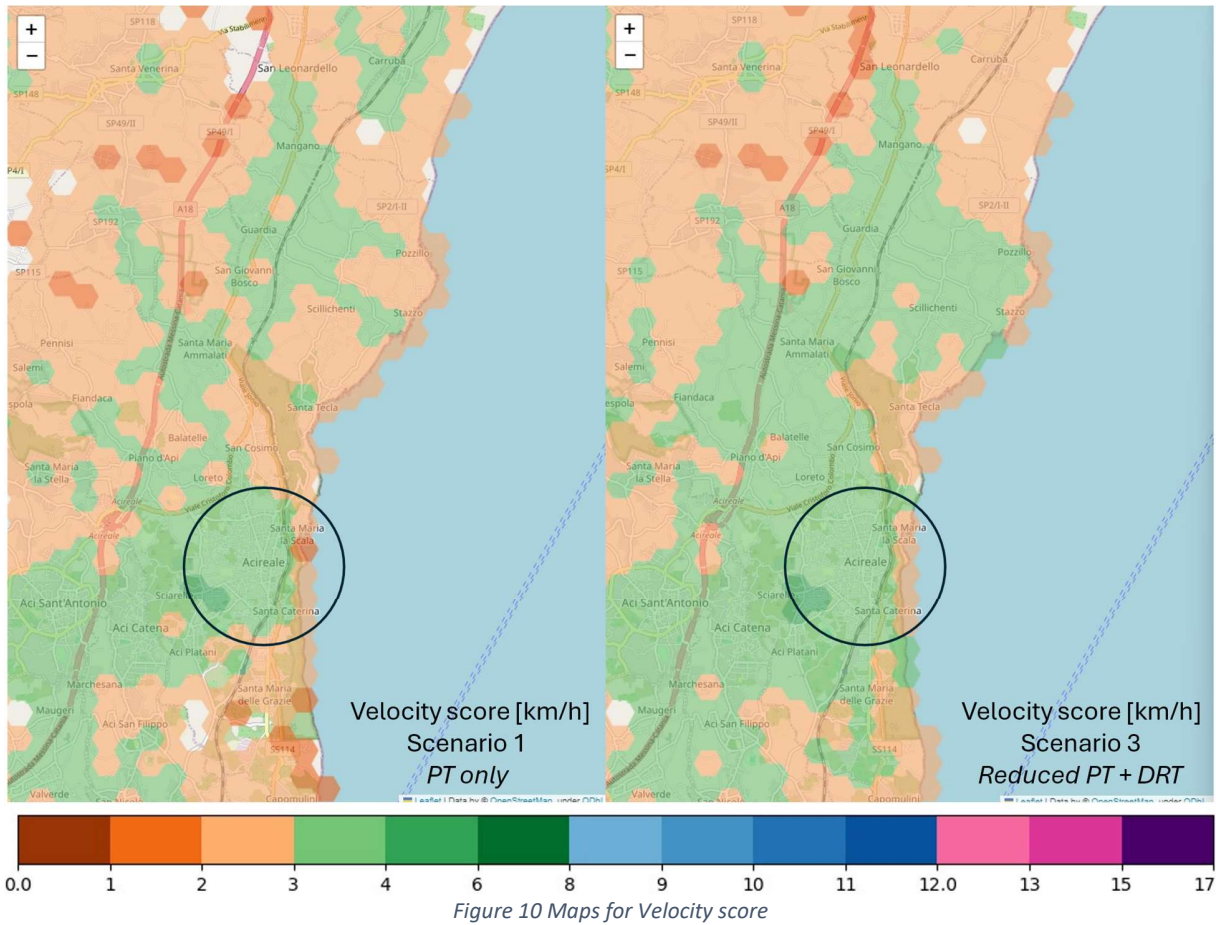


Figure 10 Maps for Velocity score

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3 CONCLUSIONS

4

5 Our study focuses on the integration of DRT systems within conventional PT and proposes a method to
 6 evaluate their impact on accessibility. The stochasticity of DRT makes classic methods for accessibility
 7 computation hard to apply. Our key idea to capture such stochasticity is to model the accessibility in each
 8 location as a random variable. To estimate such a random variable, we observe DRT over multiple days of
 9 operation, describe the operations of each day as a time-dependent graph, and compute accessibility on top
 10 of this graph. Finally, all the accessibility measures obtained over multiple days of operation are treated as
 11 the realizations of the “accessibility random variable”. Our approach can be applied on observations from
 12 real-world pilots, or from simulation models, as in our case study.

13 We evaluated two accessibility scores, namely velocity and sociality scores, performing numerical and GIS
 14 spatial analysis, testing the methodology to a case study in Acireale. Our methods allowed to quantify the
 15 increase in accessibility achieved via integrating DRT in Acireale with the conventional PT of the broader
 16 metropolitan area. In our results, accessibility increased for 95% of the resident population in Acireale. The
 17 most significant improvements were registered when the population had a very poor transport service,
 18 where sociality score increases of over 90%. Consequently, (zonal) inequalities are strongly reduced,
 19 suggesting potential for reduction of car-dependency.

20 The richness of the findings we showcased in the use case of Acireale shows that our method can shed new
 21 light on the potential of DRT: while, up to now, DRT has been evaluated in terms of provided level of
 22 service, our method allows evaluate *accessibility*, which is a deeper a much more informative measure
 23 about the impact on a territory. We believe citizens and authorities will be more willing to invest in

1 innovative services like DRT if they could quantify “*how many additional reachable opportunities per user*
2 *trip such services can bring*”. Our method enables this kind of analysis.

3 This work represents the core of a wider research effort that we are conducting, where we tackle service
4 dimensioning, predictability, cost analysis and accessibility evaluation, to build a comprehensive
5 methodology to support decisions of administrations and operators.

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11 12 **AUTHOR CONTRIBUTIONS**

13 The authors contributed to the concept. P.L., A.A. and V.T. conceptualized the research objectives and
14 developed the methodological framework. P.L. and V.T. developed the simulation model and P.L. designed
15 the algorithm to conduct analysis. A.A. and V.T. provided advice and guidance throughout the process of
16 algorithm design, analysis, and interpretation of results. A.A. and M.I. provided fundings. All authors
17 reviewed the results and approved the final version of the manuscript.

1 REFERENCES

- 2
- 3 1. Hansen W.G. 1959. "How Accessibility Shapes Land Use". *J. Am. Plan. Assoc.* 25(2).
- 4 2. Levine, J. (2020). A century of evolution of the accessibility concept. *Transportation research part D: transport and environment*, 83, 102309.
- 5 3. Banister, D. (2008). The sustainable mobility paradigm. *Transport policy*, 15(2), 73-80.
- 6 4. Litman, T. (2017). *Evaluating accessibility for transport planning*. Victoria, BC, Canada: Victoria Transport Policy Institute.
- 7 5. Sofia, D., Gioiella, F., Lotrecchiano, N., & Giuliano, A. (2020). Mitigation strategies for reducing air pollution. *Environmental Science and Pollution Research*, 27(16), 19226-19235.
- 8 6. Badeanlou, A., Araldo, A., & Diana, M. (2022). Assessing transportation accessibility equity via open data. *hEART: 10th Symposium of the European Association for Research Transportation*
- 9 7. Eppenberger, N. and Maximilian Alexander Richter. The opportunity of shared autonomous vehicles to improve spatial equity in accessibility and socio-economic developments in european urban areas. *European transport research review*, 13(1):32, 2021.
- 10 8. Kuo, Y. H., Leung, J. M., & Yan, Y. (2023). Public transport for smart cities: Recent innovations and future challenges. *European Journal of Operational Research*, 306(3), 1001-1026.
- 11 9. Machado, C. A. S., de Salles Hue, N. P. M., Berssaneti, F. T., & Quintanilha, J. A. (2018). An overview of shared mobility. *Sustainability*, 10(12), 4342.
- 12 10. Calabrò, G., Araldo, A., Oh, S., Seshadri, R., Inturri, G., & Ben-Akiva, M. (2023). Adaptive transit design: Optimizing fixed and demand responsive multi-modal transportation via continuous approximation. *Transportation Research Part A: Policy and Practice*, 171, 103643.
- 13 11. Wang, D., Araldo, A., & El Yacoubi, M. A. (2024). AccEq-DRT: Planning Demand-Responsive Transit to reduce inequality of accessibility. In *Transportation Research Board (TRB) 103rd Annual Meeting*.
- 14 12. Klumpenhower, W., Shalaby, A., & Weissling, L. (2020). The state of demand-responsive transit in Canada. *University of Toronto*, 38.
- 15 13. Nahmias-Biran, B. H., Oke, J. B., Kumar, N., Lima Azevedo, C., & Ben-Akiva, M. (2021). Evaluating the impacts of shared automated mobility on-demand services: An activity-based accessibility approach. *Transportation*, 48, 1613-1638.
- 16 14. Zhou, M., Le, D. T., Nguyen-Phuoc, D. Q., Zegras, P. C., & Ferreira Jr, J. (2021). Simulating impacts of Automated Mobility-on-Demand on accessibility and residential relocation. *Cities*, 118, 103345.
- 17 15. Ziemke, D., & Bischoff, J. (2023). Accessibilities by shared autonomous vehicles under different regulatory scenarios. *Procedia Computer Science*, 220, 747-754.
- 18 16. Miller, E. J. (2020). *Measuring accessibility: Methods and issues*. International Transport Forum Discussion Paper.
- 19 17. Abouelela, M., Durán-Rodas, D., & Antoniou, C. (2024). Do we all need shared E-scooters? An accessibility-centered spatial equity evaluation approach. *Transportation research part A: policy and practice*, 181, 103985.
- 20 18. Hasif, C. L., Araldo, A., Dumbrava, S., & Watel, D. (2022, November). A graph-database approach to assess the impact of demand-responsive services on public transit accessibility. In *Proceedings of the 15th ACM SIGSPATIAL International Workshop on Computational Transportation Science* (pp. 1-4).
- 21 19. Wang, D., Araldo, A., & Yacoubi, M. A. E. (2023). AccEq-DRT: Planning Demand-Responsive Transit to reduce inequality of accessibility. *arXiv preprint arXiv:2310.04348*.
- 22 20. Diepolder, S., Araldo, A., Chouaki, T., Maiti, S., Hörl, S., & Antoniou, C. (2023). On the Computation of accessibility provided by shared mobility. *arXiv preprint arXiv:2307.03148*.
- 23 21. Craig, T., & Shippy, W. (2020). *GTFS Flex—What Is It and How Is It Used?*. National Center for Applied Transit Technology.
- 24 22. Biazzo, I., Monechi, B., & Loreto, V. (2019). General scores for accessibility and inequality measures in urban areas. *Royal Society open science*, 6(8), 190979.
- 25 23. Fortin, P., Morency, C., & Trépanier, M. (2016). Innovative GTFS data application for transit network analysis using a graph-oriented method. *Journal of Public Transportation*, 19(4), 18-37
- 26
- 27
- 28
- 29
- 30
- 31
- 32
- 33
- 34
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- 47
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- 49
- 50
- 51

- 1 24. PTV Visum. The World's Leading Transport Planning Software. <https://www.myptv.com/en/mobility->
2 software/ptv-visum. Accessed July 31, 2024.
- 3 25. PTV Visum. Tour planning procedure. <https://cgi.ptvgroup.com/vision->
4 help/VISUM_2022_ENG/Content/1_Benutzermodell_OeV/1_6_Verfahren_Tourenplanung.htm.
5 Accessed July 31, 2024.
- 6 26. Italian National Institute of Statistics (ISTAT, 2024). Matrici di contiguità, distanza e pendolarismo.
7 <https://www.istat.it/non-categorizzato/matrici-di-contiguita-distanza-e-pendolarismo/>. Accessed July
8 31, 2024.
- 9 27. Leonardi, P., Torrisi, V., Araldo, A., Ignaccolo, M., (2024). Data-driven assessment for the predictability
10 of On-Demand Responsive Transit. Peer-review under responsibility of the scientific committee of
11 EWGT2024. [https://www.researchgate.net/publication/382756876_Data-](https://www.researchgate.net/publication/382756876_Data-driven_assessment_for_the_predictability_of_On-Demand_Responsive_Transit-NC-ND_license_httpscreativecommonsorglicensesby-nc-nd40_Peer-review_under_responsibility_of_the_scientific_committee_of_th)
12 [driven_assessment_for_the_predictability_of_On-Demand_Responsive_Transit-NC-](https://www.researchgate.net/publication/382756876_Data-driven_assessment_for_the_predictability_of_On-Demand_Responsive_Transit-NC-ND_license_httpscreativecommonsorglicensesby-nc-nd40_Peer-review_under_responsibility_of_the_scientific_committee_of_th)
13 [ND_license_httpscreativecommonsorglicensesby-nc-nd40_Peer-](https://www.researchgate.net/publication/382756876_Data-driven_assessment_for_the_predictability_of_On-Demand_Responsive_Transit-NC-ND_license_httpscreativecommonsorglicensesby-nc-nd40_Peer-review_under_responsibility_of_the_scientific_committee_of_th)
14 [review_under_responsibility_of_the_scientific_committee_of_th](https://www.researchgate.net/publication/382756876_Data-driven_assessment_for_the_predictability_of_On-Demand_Responsive_Transit-NC-ND_license_httpscreativecommonsorglicensesby-nc-nd40_Peer-review_under_responsibility_of_the_scientific_committee_of_th)
- 15 28. Bohannon, R. W., & Andrews, A. W. (2011). Normal walking speed: a descriptive meta-analysis.
16 *Physiotherapy*, 97(3),182-189.
- 17 29. Daniels, R., & Mulley, C. (2013). Explaining walking distance to public transport: The dominance of
18 public transport supply. *Journal of Transport and Land Use*, 6(2), 5-20.
- 19 30. Statistical Office of the European Union (EUROSTAT, 2024). Population grids.
20 https://ec.europa.eu/eurostat/statistics-explained/index.php?title=Population_grids. Accessed July 31,
21 2024.
- 22 31. Welch, T. F., & Mishra, S. (2013). A measure of equity for public transit connectivity. *Journal of*
23 *Transport Geography*, 33, 29-41.
- 24 32. Badeanlou, A., Araldo, A., & Diana, M. (2022). Assessing transportation accessibility equity via open
25 data. hEART: 10th Symposium of the European Association for Research Transportation
- 26 33. Leonardi, P. (2021). Integrazione modale tra trasporto pubblico condiviso a domanda (DRT), autolinee
27 urbane ed extraurbane nel comune di Acireale. Master Thesis.
28 https://www.researchgate.net/publication/382800902_INTEGRAZIONE_MODALA_TRA_TRASPO
29 [RTO PUBBLICO CONDIVISO A DOMANDA DRT AUTOLINEE URBANE ED EXTRAUR](https://www.researchgate.net/publication/382800902_INTEGRAZIONE_MODALA_TRA_TRASPO)
30 [BANE NEL COMUNE DI ACIREALE](https://www.researchgate.net/publication/382800902_INTEGRAZIONE_MODALA_TRA_TRASPO)