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Reconciling and Validating the Ashworth-Davies Doppler Shifts of an Arbitrarily Translating Mirror

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We simplify, to first order in $\frac{v}{c}$, the generalized, special relativistic treatment of a Doppler shift from an arbitrarily translating mirror originally derived by Ashworth and Davies [Proc. IEEE 64, 280 (1976)]. We show that it is in good agreement with a somewhat modified, but more intuitive derivation that only considers the constancy of the speed of light. We experimentally demonstrate the theoretical predictions using phase-insensitive frequency measurements in a liquid crystal light valve with mirror translations of only a few 10's of nanometers per second.

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1. INTRODUCTION

A Doppler shift is the frequency differential of a wave when a source and detector are in relative motion. Harnessing the Doppler effect has brought about great gains in scientific, engineering, and society at large. Doppler shifts are used extensively in astronomy [1–3], remote weather monitoring [4–6], non-invasive medical diagnostics [7–9] and laser velocimetry (fluid flow) [10, 11] to name a few applications.

Einstein's derivation of the Doppler shift of light from a uniformly translating mirror considered only the Doppler shift resulting from light reflected at an oblique angle [12]. While the angle of incidence of the beam relative to the surface normal was arbitrary, the direction of mirror propagation was in the same direction as the surface normal.

A derivation of the Doppler shift that considers both arbitrary mirror propagation direction and arbitrary incidence angle was derived by Ashworth and Davies [13]. Follow-on experiments demonstrated the intended prediction from the theory namely that there is no Doppler effect for transversely moving mirrors [14, 15]. However, no one has experimentally verified the results for the general case, including nonrelativistic mirror velocities, until this experiment. In this study, we measured the Doppler shift of a beam incident on a mirror moving at non-relativistic

velocities, varying the angles.

Understanding the general case of a Doppler shift from a moving mirror is important for our previous work on a Doppler-based gyroscope (see [16]). In that work, it was shown that Doppler shifts are fundamental in passive gyroscopes. However, it is impossible to derive a generalized theory about the Doppler gyroscope without an understanding of the Doppler shifts result from the movement of the mirror with respect to its surface normal.

When considering a rotating interferometer, each mirror in the interferometer can move in a nontrivial direction, depending on its position relative to the axis of rotation. To calculate the difference in frequency shift between the two optical paths in the interferometer, we need to understand what is the frequency shift that results from the reflection of each individual mirror.

Here, we show that the Ashworth-Davies result in the non-relativistic domain can be written in an intuitive form that reconciles with a modification of a Doppler shift model by Gjurchinovski [17], and we experimentally validated this derivation for the first time. Our verification employs sensitive differential frequency measurements in a liquid crystal light valve [18] for a moving mirror at a velocity of 60 nanometers per second.

2. THEORY

We consider the Doppler shift scenario as originally proposed by Ashworth and Davies as shown in Fig. 1. A mirror is translating at constant velocity along the x-axis. An incoming light beam with an angle of incidence β is specularly reflected from the surface. Using a convention by Ashworth and Davies, α is the angle between the surface normal and the propagation direction of the mirror and ϕ is the angle between the velocity vector and the angle of incidence.

In an effort to prove that transverse Doppler shifts do not exist in reflection from a mirror, Ashworth and Davies derived a generalized special relativistic Doppler shift formula

$$f_f = f_i \frac{[\tan \alpha + \frac{v}{c} \sin \phi]^2 + [1 - \frac{v}{c} \cos \phi]^2}{1 - \frac{v^2}{c^2} + \tan^2 \alpha}, \quad (1)$$

where f_f and f_i are the frequency of the light after and before the reflection from the mirror, respectively and v is the speed of

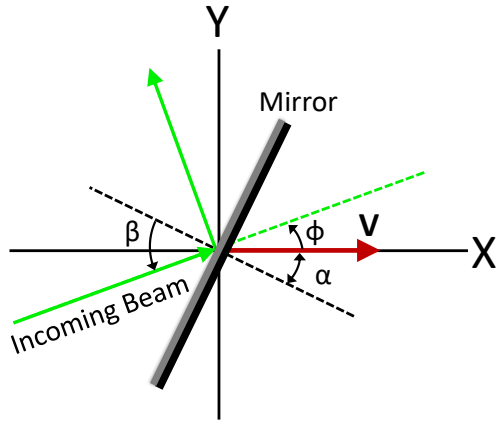


Fig. 1. Light reflecting from a mirror moving with velocity v along the x -axis. We use the convention of Ashworth and Davies [13] in which α is the angle between the velocity vector and the surface normal and ϕ is the angle between the angle of incidence (from the opposite side of the mirror) and the velocity vector. We also define β as the angle of incidence.

the mirror. In the limit $v \ll c$, we simplify the equation, namely

$$\Delta f = \frac{2v}{\lambda} \frac{(\tan \alpha \sin \phi - \cos \phi)}{1 + \tan^2 \alpha}, \quad (2)$$

where $\Delta f = f_f - f_i$ and λ is the wavelength of the light.

Our aim is to rewrite this result only in terms of the mirror's surface normal rather than with respect to the velocity vector. We first use the trigonometric identity $1 + \tan^2 \alpha = \sec^2 \alpha = 1/\cos^2 \alpha$, which allows us to write

$$\Delta f = \frac{2v}{\lambda} (\sin \alpha \cos \alpha \sin \phi - \cos^2 \alpha \cos \phi). \quad (3)$$

We now make a change of variables. Instead of using ϕ (the angular difference between the angle of incidence and the velocity vector), we directly use the angle of incidence β given by $\beta = \phi + \alpha$. This implies

$$\Delta f = \frac{2v}{\lambda} [\sin \alpha \cos \alpha \sin(\beta - \alpha) - \cos^2 \alpha \cos(\beta - \alpha)]. \quad (4)$$

Using the angle sum relations $\cos(\beta - \alpha) = \cos \beta \cos \alpha + \sin \beta \sin \alpha$ and $\sin(\beta - \alpha) = \sin \beta \cos \alpha - \cos \beta \sin \alpha$ as well as $\sin^2 \alpha + \cos^2 \alpha = 1$, we arrive at the relation

$$\Delta f = -\frac{2v}{\lambda} \cos \beta \cos \alpha. \quad (5)$$

This formula is more intuitive than the original Ashworth-Davies result and easier to use experimentally in describing a physical system as all angles are defined with respect to the mirror's surface normal. The formula contains the familiar Einstein formula [12], in the low-velocity limit, for $\alpha = 0$ (the scenario where the mirror surface normal also lies on the x -axis).

The result is also in good agreement with a simple addition to the intuitive derivation by Gjurchinovski [17]. Gjurchinovski considered sequential wavefronts reflecting with temporal separation Δt from a mirror moving at velocity v parallel to the surface's normal. The first wavefront reflected from the mirror at the first time and after the time interval Δt the mirror reflected the second wavefront. During the time interval Δt the mirror

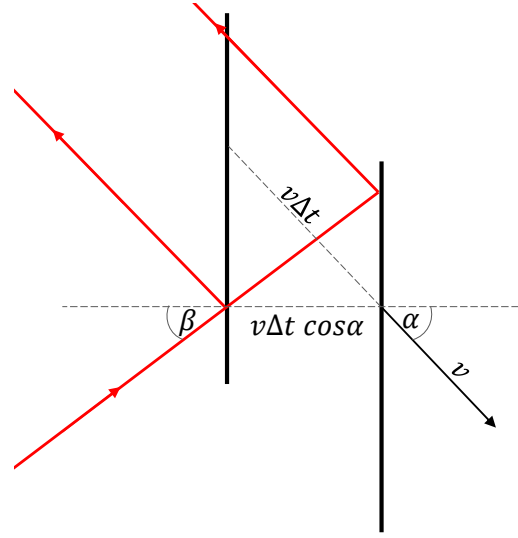


Fig. 2. Consider a mirror moving at nonrelativistic speed v in an angle α relative to its surface normal, during a time interval Δt . The mirror velocity in the direction of its normal is $v \cos \alpha$, so the distance between the mirror before and after the time interval will be $v \cos \alpha \Delta t$. Repeating Gjurchinovski derivation in [17] but replacing $v \Delta t$ with $v \cos \alpha \Delta t$ will yield Eq.(5)

propagated a distance $v \Delta t$. The distance between those two surfaces (the mirror surface at two different times) was $v \Delta t$ if the mirror propagated in the direction of the surface normal. Using only the constancy of the speed of light in a vacuum with this assumption, and without using a Lorentz transform, he reproduces Einstein's equation [12]:

$$f_f = f_i \frac{1 - 2\frac{v}{c} \cos \beta + \frac{v^2}{c^2}}{1 - \frac{v^2}{c^2}}, \quad (6)$$

However, we note that if the velocity of the mirror were not in the direction of the surface normal, but at an angle α , the mirror would have only moved $v \Delta t \cos \alpha$ between sequential wavefronts, as seen in Fig 2. If we only care about the velocity in order to calculate the distance between the mirror before and after a time interval Δt and consider only nonrelativistic velocities (and thus ignoring length contraction), we can easily replace v with $v \cos \alpha$ and ignore second order of $\frac{v}{c}$ in Eq. (6), thus reproducing Eq. (5).

3. EXPERIMENTAL SETUP

To experimentally test these results, we use a liquid crystal light valve (LCLV) [18] (see Supplementary Materials for more information). While any experimental system that can precisely measure Doppler shifts can be used, we find the LCLV to be exceptionally ideal. As shown in [18], a LCLV can measure down to $\mu\text{Hz}/\text{Hz}^{1/2}$ meaning that it is several orders of magnitude more sensitive than any other system per measurement time. This means that the movement of the mirror can be made to be very small, which has the effect of minimizing alignment issues as we rotate through the various measurement angles.

The experimental setup is shown in Fig.3. Collimated light from a laser at 532 nm is split on a 50/50 beamsplitter. One of the mirrors is mounted to an adjustable moving platform that is controlled by a piezoactuator (PZT) so that the beam that's

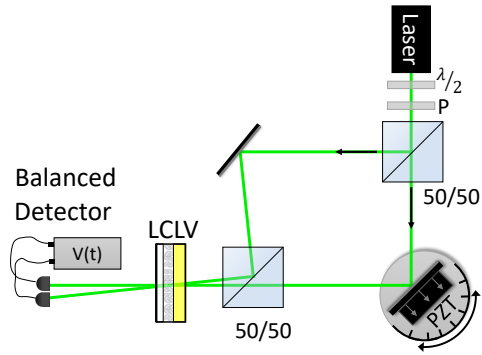


Fig. 3. The experimental setup. A displaced Mach-Zehnder interferometer, where one of the mirrors is mounted to an adjustable moving platform, controlled by a piezo-electric crystal (PZT). A slight angle between the beams is fabricated in order to create a wave mixing where the beams meet at the Liquid Crystal Light Valve (LCLV). The two primary diffracted output orders of the LCLV are focused on a balanced detector.

reflected from the mirror experiences a Doppler shift relative to the other beam. A Mach-Zehnder type setup is used to create a slight angle between the two beams on the order of 0.01 radians, and both beams are then incident on a LCLV. A small angle between the beams is needed in order to create two-wave mixing in the Raman-Nath regime of the LCLV; see Supplementary Materials for more information. The two primary diffracted output orders of the LCLV are focused on a balanced detector. The difference in the intensities of the two beams hitting the balance detector is proportional to the difference in frequency between the two beams, Δf [18].

To test the Doppler shift dependence on the direction of propagation of the mirror, the PZT direction was changed while the angle of incidence was fixed. With each iteration of the experiment, the moving platform was tuned to a different angle, and the mirror sitting on it was tuned to be with a fixed angle relative to the incoming beam at $\beta = 45^\circ$. This caused the movement of the platform to be in a different direction relative to the mirror's surface normal (which corresponds to α as defined above). The PZT was driven by an arbitrary waveform generator producing triangle waves at 20 mHz and a peak-to-peak voltage of 15V. The PZT response was measured to be approximately 100 nm/V. The experiment was repeated 36 times, with alpha ranging from -180° to 180° . The absolute value of the mirror's velocity was constant through all the iterations and equal to $V = 60$ nm/s.

The errors in the experiment are primarily from seismic and acoustic vibrations as well as from fluctuating air currents. While we had various forms of active and passive noise reduction, the light valve is particularly sensitive to noise. Further, the experiments are run at slow speeds meaning that $1/f$ noise is significant.

4. RESULTS AND DISCUSSION

The experimental results are shown in Fig. 4. The blue data points are the measured amplitude of the Doppler shift from the moving mirror as a function of the angle between the direction of the mirror's propagation to the mirror's normal (α). The orange curve is a least-squares fitted cosine function. Hence, the experimental results are in excellent agreement with Eq. (5), which is the main result of this paper. We have thus shown

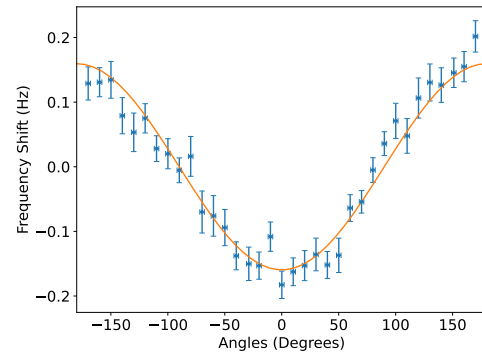


Fig. 4. Experimental results. The Doppler signal amplitude vs velocity angle α relative to the surface normal is shown in the blue points along with the error bars. We used a fixed angle of incidence of $\beta = 45^\circ$. The orange curve is the cosine curve from Eq. 5, when plugging the parameters of our experiment ($\lambda = 532$ nm, $V = 60$ nm/s).

that at the Doppler shift non-relativistic limit formula, there is not only a cosine term that arises from the angle of incidence that is measured relative to the surface normal but also from the direction of mirror propagation relative to the surface normal.

5. CONCLUSION

In conclusion, we have developed an intuitive nonrelativistic Doppler correction to the frequency of light, which includes both the angle of impact on the mirror and the mirror propagation angle relative to the surface normal. We showed that this result is a simplification of the formula proposed by Ashworth-Davies for velocities that are in the above regime. In addition, we were the first to measure the Doppler shift for a mirror moving at a speed of only tens of nanometers in different relative directions, where we measured accurate differential frequencies using LCLV. We received cosine behavior, which is in line with the intuitive formula we showed that Doppler shift, in this limit, is proportional to the cosine of the mirror propagation angle, in addition to the known cosine of the light incidence angle. These results not only provide a new abstract form for the nonrelativistic regime of the Doppler shift but also have a significant impact on practical application areas, like a Doppler-based gyroscope [16].

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6. SUPPLEMENTAL DOCUMENT

See supplemental Supplement 1 for supporting content.

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