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Analytical Expression for Bistatic RCS of Rectangular Loop Resonator

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Abstract—This paper presents an analytical formulation for scattering from a rectangular loop resonator based on a circuit/transmission line model which provides the expression of the bistatic RCS. The presented closed-form RCS expression perfectly describes the resonance frequencies and the corresponding peak RCS values at all the resonant modes of the loop scatterer, which can be applied to design chipless RFID tags.

I. INTRODUCTION

The loop resonators have been widely used to design passive guided structures like microwave filters and also as scatterers to realize metasurfaces and chipless Radio Frequency IDentification (RFID) tags. In chipless technology, C-shape resonators which can be considered as symmetrically halved loops, have been employed in more designs to miniaturize the tag size [1]–[4]. While C-shape resonators have almost the same resonating behavior as rectangular loop structures, they provide smaller Radar Cross Section (RCS) levels compared to loops. The rectangular loop resonator has been applied for chipless tag design in [5], [6] whereas the RCS of the loop resonator has been analytically modeled in none of these works and all the designs have been done based on EM numerical simulations. The scattering from loop resonators has been analytically formulated in [7] respectively using integral equation and variational methods, while the presented expression in [7] is quite complex to use. Accordingly, providing a circuit model to obtain a simple but accurate RCS expression can be useful for chipless tag design using loop resonators which are attractive due to their high quality factor and RCS level.

This paper presents an analytical approach to derive the bistatic RCS of the rectangular loop resonator using a circuit/transmission line model which yields a straightforward and accurate RCS expression.

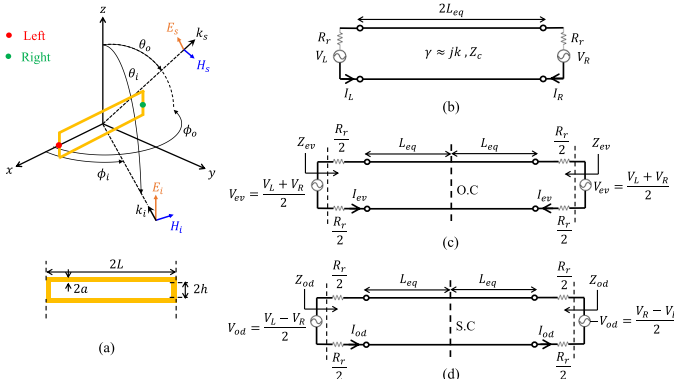


Fig. 1. (a) Scattering from wire rectangular loop. (b) Circuit/transmission line model for an excited loop. (c) Even (d) Odd mode analysis of the model.

II. ANALYTICAL MODEL

The bistatic scattering from a wire rectangular loop with the length of $2L$ (along x -axis), width of $2h$ (along z -axis), and wire radius of a is shown in Fig. 1 (a). The loop is impinged by a z -polarized plane wave at the direction of $(\theta_i = \pi/2, \phi_i)$ written as $\vec{E}_i = E_0 e^{-jk_i r} \hat{z}$ with $k = 2\pi/\lambda$, and the scattered field is observed at the direction of (θ_o, ϕ_o) . For $h \ll \lambda$, as it is shown in Fig. 1 (b), the impinged loop can be modeled as a transmission line section with the equivalent length of L_{eq} , characteristic impedance of Z_c , and propagation constant of $\gamma = jk$, which is excited by two voltage sources at the left V_L and right V_R sides followed by a resistance R_r corresponding to the radiation of the two small sides as infinitesimal dipoles. The circuit/transmission line model in Fig. 1 (b) is solved using even and odd mode analysis as it is illustrated respectively in Fig. 1 (c) and (d), which yields radiating currents in even and odd resonant modes respectively as

$$I_{ev} = \frac{V_{ev}}{Z_{ev}} = \frac{V_{ev}}{R_r - jZ_c \cot(kL_{eq})} \quad (1)$$

and

$$I_{od} = \frac{V_{od}}{Z_{od}} = \frac{V_{od}}{R_r + jZ_c \tan(kL_{eq})} \quad (2)$$

where $R_r = 80\pi^2(2h/\lambda)^2$ is the radiation resistance of infinitesimal dipoles at two sides, $Z_c = \sqrt{\mu_0/\pi^2\epsilon_0} \cosh^{-1}[(h+a)/a]$ is the characteristic impedance of the two-wire transmission line, and $L_{eq} = L + \alpha h$ is the equivalent length of the transmission line section where the coefficient α is considered to model the effect of corner bent [7]. The induced voltage sources at the left V_L and right V_R sides, and consequently $V_{ev} = (V_L + V_R)/2$ and $V_{od} = (V_L - V_R)/2$, are calculated by defining two vector effective lengths associated to each radiating dipoles respectively as \vec{l}_e^L and \vec{l}_e^R using the method presented in [8]. Using (1) and (2), by applying $V_L = \vec{E}_i(\vec{r}_L) \cdot \vec{l}_e^L$ and $V_R = \vec{E}_i(\vec{r}_R) \cdot \vec{l}_e^R$, the induced radiating currents I_{od} and I_{ev} are calculated in terms of incident field, and consequently, the scattered field can be obtained based on the currents using this general approach for all the resonant modes. Without lacking generality, we will consider the results for the dominant mode (first even mode where $2L = \lambda/2$) to discuss. Accordingly, for $\theta_o = \pi/2$, the analytical expression for the bistatic RCS of the rectangular loop is obtained as

$$\sigma(\phi_i, \phi_o) = \frac{16\eta^2 k^2 h^4}{\pi(Z_{ev})^2} \cos^2(kL \cos \phi_i) \cos^2(kL \cos \phi_o) \quad (3)$$

where $\eta = \sqrt{\mu_0/\epsilon_0}$ is the free space wave impedance.

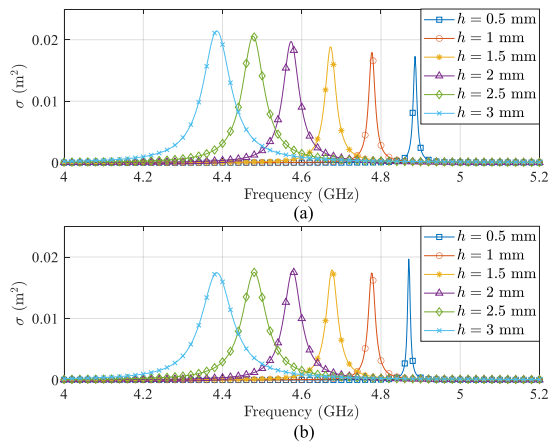


Fig. 2. (a) Calculated monostatic RCS of the rectangular loop for different widths based on the proposed model. (b) Obtained monostatic RCS of the rectangular loop using CST simulations for different widths.

III. RESULTS AND DISCUSSIONS

The proposed RCS expression in (3) is verified using full-wave simulation by CST Microwave Studio with the results shown in Fig. 2. For a rectangular loop with $L = 15$ mm and $a = 0.2$ mm, the monostatic RCS of the loop is calculated using (3) with $\phi_i = \phi_o = \pi/2$ and $\alpha = 0.7$ while the width of the loop is changed between $h = 0.5$ mm - 3 mm and the results are illustrated as a function of frequency in Fig. 2 (a). On the other hand, with the same parameters and configuration, the RCS of the considered loop is obtained using CST simulation, and the results are shown in Fig. 2 (b). Comparing the results shown in Fig. 2 (a) and (b) verifies the validity of the presented model in terms of resonance frequency, peak RCS level, and the resonance quality factor. Quantitatively expressed, the average relative error in terms of resonance frequency is less than 0.1% and in terms of RCS peak level is less than 6% for $h \leq 2$ mm while for $h > 2$ mm it grows up to 18%. These larger errors for larger h can be justified by the infinitesimal dipole approximation which directly affects the assumed value for R_r . In fact, by getting h larger, the current distribution on the small sides of the loop cannot be assumed constant anymore which cause a larger error in RCS at this situation.

In addition to resonance characteristics verified in Fig. 2, the reradiation properties of the rectangular loop at the dominant mode are studied in terms of bistatic RCS in Fig. 3 for different incident angles. For a rectangular loop with $L = 15$ mm, $h = 1$ mm, and $a = 0.2$ mm, the bistatic RCS for $\phi_i = 10^\circ - 90^\circ$ and $\phi_o = 90^\circ$ is calculated using (3) and shown in Fig. 3 (a) as a function of frequency. The simulated bistatic RCS of the considered loop is also shown in Fig. 3 (b) at the same incidence angles. The results calculated by the model are in good agreement with what is obtained by CST simulation which verifies the validity of (3) in terms of radiation properties.

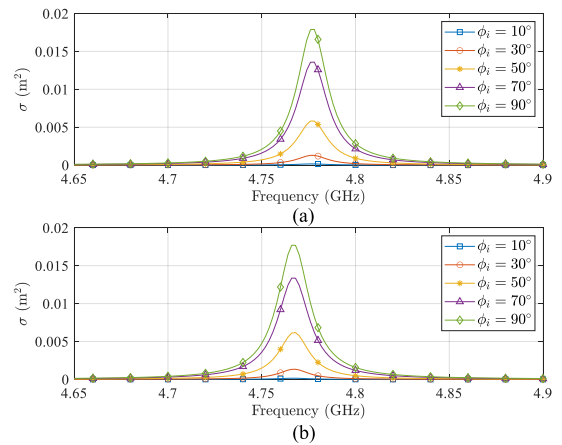


Fig. 3. (a) Calculated bistatic RCS of the rectangular loop for different incidence angles based on the model. (b) Obtained bistatic RCS of the rectangular loop using CST simulations for incidence angles.

IV. CONCLUSION

The accurate bistatic RCS expression for rectangular loop resonator has been derived based on a simple circuit/transmission line model. The presented analytical expression has been verified by simulations in terms of resonance frequency, peak RCS level, quality factor, and reradiation properties. Although the results have been discussed only for the dominant mode, the proposed general approach can be applied to all the resonant modes in this structure. In addition, the method can be employed to derive an analytical expression for RCS of the C-shape resonators since they can be considered as symmetrically halved loops.

ACKNOWLEDGEMENT

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