

Supplementary Material

eCOALIA: Neocortical Neural Mass Model for simulating electroencephalographic signals

E. Köksal-Ersöz^{a,1,2}, M. Yochum^{a,1}, P. Benquet^a, F. Wendling^a

^a*Univ Rennes, Inserm, LTSI U1099, Rennes, France.*

The system of equations of the laminar NMM representing a neocortical brain region reads:

$$\begin{aligned}
 \ddot{y}_P &= \frac{A}{\tau_A} [(-PrePost k_P stim(t)) + S(y_{P1} + C_{P,PyP} - C_{I1,PyI1} - C_{I2b,PyI2b} \\
 &\quad - C_{I2a,PyI2a} - C_{I4,PyI4} + y_{CCP} + y_{Pnoise} + PrePost k_P stim(t))] \\
 &\quad - \frac{2}{\tau_A} \dot{y}_P - \frac{1}{\tau_A^2} y_P, \\
 \ddot{y}_{P1} &= \frac{A}{\tau_A} [(-PrePost k_{P1} stim(t)) + C_{P1,P} S(C_{P,P1} y_P + PrePost k_{P1} stim(t))] \\
 &\quad - \frac{2}{\tau_A} \dot{y}_{P1} - \frac{1}{\tau_A^2} y_{P1}, \\
 \ddot{y}_{I1} &= \frac{G}{\tau_G} [(-PrePost k_{I1} stim(t)) + S(C_{P,I1} y_P - C_{I1,I1} y_{I1} - C_{I1b,I1} y_{I1b} \\
 &\quad - C_{I2,I1} y_{I2b} - C_{I4,I1} y_{I4} + y_{CCI1} + y_{I1noise} + PrePost k_{I1} stim(t))] \\
 &\quad - \frac{2}{\tau_G} \dot{y}_{I1} - \frac{1}{\tau_G^2} y_{I1}, \\
 \ddot{y}_{I1b} &= \frac{G}{\tau_G} S(C_{P,I1b} y_P - C_{I1,I1b} y_{I1}) - \frac{2}{\tau_G} \dot{y}_{I1b} - \frac{1}{\tau_G^2} y_{I1b}, \\
 \ddot{y}_{I2b} &= \frac{B_b}{\tau_{B_b}} [(-PrePost k_{I2} stim(t)) + S(C_{P,I2} y_P - C_{I3,I2} y_{I3} - C_{I4,I2} y_{I4} + y_{CCI2} \\
 &\quad + y_{TCI2} + y_{I2noise} + PrePost k_{I2} stim(t))] + \frac{2}{\tau_{B_b}} \dot{y}_{I2b} - \frac{1}{\tau_{B_b}^2} y_{I2b}, \\
 \ddot{y}_{I2a} &= \frac{B_a}{\tau_{B_a}} [(-PrePost k_{I2} stim(t)) + S(C_{P,I2} y_P - C_{I3,I2} y_{I3} - C_{I4,I2} y_{I4} + y_{CCI2}
 \end{aligned}$$

¹co-first author

²corresponding author: elif.koksal-ersoz@inserm.fr

$$\begin{aligned}
& + y_{TCI2} + y_{I2noise} PrePost k_{I2} stim(t)] - \frac{2}{\tau_{B_a}} \dot{y}_{I2a} - \frac{1}{\tau_{B_a}^2} y_{I2a}, \\
\ddot{y}_{I3} & = \frac{D}{\tau_D} [(-PrePost k_{I3} stim(t)) + S(C_{P,I3} y_P - C_{I2,I3} y_{I2b} - C_{I4,I3} y_{I4} \\
& + y_{CCI3} + y_{I3noise} + PrePost k_{I3} stim(t))] - \frac{2}{\tau_D} \dot{y}_{I3} - \frac{1}{\tau_D^2} y_{I3}, \\
\ddot{y}_{I4} & = \frac{R}{\tau_R} [(-PrePost k_{I4} stim(t)) + S(C_{I2,I4} y_{I2b} - C_{I4,I4} y_{I4} - C_{I4b,I4} y_{I4b} \\
& + y_{CCI4} + y_{I4noise} + PrePost k_{I4} stim(t))] - \frac{2}{\tau_R} \dot{y}_{I4} - \frac{1}{\tau_R^2} y_{I4}, \\
\ddot{y}_{I4b} & = \frac{R}{\tau_R} S(-C_{I4,I4b} y_{I4}) - \frac{2}{\tau_R} \dot{y}_{I4b} - \frac{1}{\tau_R^2} y_{I4b},
\end{aligned}$$

where the sigmoidal transfer function reads $S(v) = 5/(1 + \exp(0.56(5 - v)))$. Stimulation signal $stim(t)$ affects the membrane potential of a subpopulation if $PrePost = 1$ and the post-synaptic potentials otherwise.

The excitatory inputs from the region M to the subpopulations of the region K are the solution of the following system of equations:

$$\begin{aligned}
\ddot{y}_{CCP} & = \frac{A}{\tau_A} \sum_{M=1, M \neq K} C_{M;K,P} F_{M,P} - \frac{2}{\tau_A} \dot{y}_{CCP} - \frac{1}{\tau_A^2} y_{CCP}, \\
\ddot{y}_{CCI1} & = \frac{A}{\tau_A} \sum_{M=1, M \neq K} C_{M;K,I1} F_{M,P} - \frac{2}{\tau_A} \dot{y}_{CCI1} - \frac{1}{\tau_A^2} y_{CCI1}, \\
\ddot{y}_{CCI2} & = \frac{A}{\tau_A} \sum_{M=1, M \neq K} C_{M;K,I2} F_{M,P} - \frac{2}{\tau_A} \dot{y}_{CCI2} - \frac{1}{\tau_A^2} y_{CCI2}, \\
\ddot{y}_{CCI3} & = \frac{A}{\tau_A} \sum_{M=1, M \neq K} C_{M;K,I3} F_{M,P} - \frac{2}{\tau_A} \dot{y}_{CCI3} - \frac{1}{\tau_A^2} y_{CCI3}, \\
\ddot{y}_{CCI4} & = \frac{A}{\tau_A} \sum_{M=1, M \neq K} C_{M;K,I4} F_{M,P} - \frac{2}{\tau_A} \dot{y}_{CCI4} - \frac{1}{\tau_A^2} y_{CCI4},
\end{aligned}$$

where $F_{M,P}$ is the firing rate of the excitory subpopulation P in region M (TC cells in thalamus). Equations for the input noise on the subpopulations of the region K reads:

$$\begin{aligned}
\ddot{y}_{Pnoise} & = \frac{A}{\tau_A} P_{noise}(t) - \frac{2}{\tau_A} \dot{y}_{Pnoise} - \frac{1}{\tau_A^2} y_{Pnoise}, \\
\ddot{y}_{I1noise} & = \frac{A}{\tau_A} I1_{noise}(t) - \frac{2}{\tau_A} \dot{y}_{I1noise} - \frac{1}{\tau_A^2} y_{I1noise}, \\
\ddot{y}_{I2noise} & = \frac{A}{\tau_A} I2_{noise}(t) - \frac{2}{\tau_A} \dot{y}_{I2noise} - \frac{1}{\tau_A^2} y_{I2noise},
\end{aligned}$$

$$\begin{aligned}\ddot{y}_{I3noise} &= \frac{A}{\tau_A} I3_{noise}(t) - \frac{2}{\tau_A} \dot{y}_{I3noise} - \frac{1}{\tau_A^2} y_{I3noise}, \\ \ddot{y}_{I4noise} &= \frac{A}{\tau_A} I4_{noise}(t) - \frac{2}{\tau_A} \dot{y}_{I4noise} - \frac{1}{\tau_A^2} y_{I4noise},\end{aligned}$$

where $i_{noise}(t)$ are random variables following a normal distribution with mean $p_{m,i}$ and standard deviation $p_{s,i}$ with $i = \{P, I1, I2, I3, I4\}$.

Equations of the NMM representing thalamus reads:

$$\begin{aligned}\ddot{y}_{TC} &= \frac{TC}{\tau_{TC}} [(-PrePost k_{TC} stim(t)) + S(C_{TC,TC} y_{TC} - C_{RT1,TC} y_{RT1} - C_{RT2,TC} y_{RT2} \\ &+ y_{CCTC} + y_{TCnoise} + PrePost k_{TC} stim(t))] - \frac{2}{\tau_{TC}} \dot{y}_{TC} - \frac{1}{\tau_{TC}^2} y_{TC}, \\ \ddot{y}_{RT1} &= \frac{RT1}{\tau_{RT1}} [(-PrePost k_{RT1} stim(t)) + S(C_{TC,RT1} y_{TC} + y_{CCRT1} \\ &+ PrePost k_{RT1} stim(t))] - \frac{2}{\tau_{RT1}} \dot{y}_{RT1} - \frac{1}{\tau_{RT1}^2} y_{RT1}, \\ \ddot{y}_{RT2} &= \frac{RT2}{\tau_{RT2}} [(-PrePost k_{RT2} stim(t)) + S(C_{TC,RT2} y_{TC} + y_{CCRT2} \\ &+ PrePost k_{RT2} stim(t))] - \frac{2}{\tau_{RT2}} \dot{y}_{RT2} - \frac{1}{\tau_{RT2}^2} y_{RT2}, \\ \ddot{y}_{TCnoise} &= \frac{TC}{\tau_{TC}} TC_{noise}(t) - \frac{2}{\tau_{TC}} \dot{y}_{TCnoise} - \frac{1}{\tau_{TC}^2} y_{TCnoise}.\end{aligned}$$

The variable $y_{TCnoise}$ is the thalamic noise with $TC_{noise}(t)$ is a random variable following a normal distribution with mean $p_{m,TC}$ and standard variation $p_{s,TC}$. The cortico-thalamic inputs are given by

$$\begin{aligned}\ddot{y}_{CCTC} &= \frac{TC}{\tau_{TC}} \sum_{M=1, M \neq K} C_{M,TC} F_{M,P} - \frac{2}{\tau_{TC}} \dot{y}_{CCTC} - \frac{1}{\tau_{TC}^2} y_{CCTC}, \\ \ddot{y}_{CCRT1} &= \frac{TC}{\tau_{TC}} \sum_{M=1, M \neq K} C_{M,RT1} F_{M,P} - \frac{2}{\tau_{TC}} \dot{y}_{CCRT1} - \frac{1}{\tau_{TC}^2} y_{CCRT1}, \\ \ddot{y}_{CCRT2} &= \frac{TC}{\tau_{TC}} \sum_{M=1, M \neq K} C_{M,RT2} F_{M,P} - \frac{2}{\tau_{TC}} \dot{y}_{CCRT2} - \frac{1}{\tau_{TC}^2} y_{CCRT2}.\end{aligned}$$

Table 1: Parameter sets of the neocortical regions used for the illustrative examples presented in Fig 2 and 3.

Parameter	Default	Spike-wave (Fig 2)	Alpha-oscillations (Fig 3)
PSP amplitudes		NMM ₂₄	NMM _{11,44}
A	2	7	4.5
G	30	30	30
B_b	20	32	20
B_a	22	19	15
D	5	5	5
R	5	5	5
PSP time constants			
$1/\tau_A$	100	100	100
$1/\tau_G$	150	350	150
$1/\tau_{B_b}$	50	50	50
$1/\tau_{B_a}$	30	20	30
$1/\tau_D$	20	20	20
$1/\tau_R$	5	5	5
Coupling coefficients			
$C_{P,P}$	0	0	0
$C_{I1,P}$	50	50	50
$C_{I2b,P}$	25	20	0
$C_{I2a,P}$	25	35	25
$C_{I4,P}$	0	0	0
$C_{P1,P}$	100	100	100
$C_{P,P1}$	135	135	135
$C_{P,I1}$	50	50	50
$C_{I1,I1}$	200	200	200
$C_{I1b,I1}$	0	0	0
$C_{I2,I1}$	13.5	13.5	13.5
$C_{I4,I1}$	0	0	0
$C_{P,I1b}$	0	0	0
$C_{I1,I1b}$	0	0	0
$C_{P,I2}$	25	55	25
$C_{I3,I2}$	20	20	20
$C_{I4,I2}$	0	0	0
$C_{P,I3}$	0	0	0
$C_{I2,I3}$	20	20	50
$C_{I4,I3}$	0	0	0

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Table 1 – continued from previous page

Parameter	Default	Spike-wave (Fig 2)	Alpha-oscillations (Fig 3)
$C_{\{I2,I4,I4b\},I4}$	0	0	0
$C_{I4,I4b}$	0	0	0
Input noise coefficients			
$p_{m,P}$	70	80	70
$p_{m,\{P1,I1,I2,I3,I4\}}$	0	0	0
$p_{s,P}$	2	2	2
$p_{s,\{P1,I1,I2,I3,I4\}}$	0	0	0
Stimulation coefficients			
$k_{\{P,P1,I1,I2,I3,I4\}}$	0	0	0

Table 2: Parameter sets of the thalamic population used for the illustrative examples presented in Fig 2 and 3.

Parameter	Values
PSP amplitudes	
TC	2
$RT1$	20
$RT2$	50
PSP time constants	
$1/\tau_{TC}$	100
$1/\tau_{RT1}$	150
$1/\tau_{RT2}$	50
Coupling coefficients	
$C_{TC,TC}$	100
$C_{RT1,TC}$	50
$C_{RT2,TC}$	40
$C_{TC,RT1}$	50
$C_{TC,RT2}$	25
Input noise coefficients	
$p_{m,TC}$	40
$p_{s,TC}$	2
Stimulation coefficients	
$k_{\{TC,RT1,RT2\}}$	0