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► **To cite this version:**

Pierre Mollo. Multi-Fidelity Ensemble Kalman Method with Dynamic Mode Decomposition Surrogate. MORTech 2023, Nov 2023, Saclay, France, France. hal-04771818

**HAL Id: hal-04771818**

**<https://hal.science/hal-04771818v1>**

Submitted on 7 Nov 2024

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# Multi-Fidelity Ensemble Kalman Method with Dynamic Mode Decomposition Surrogate

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## Introduction

An accurate description of the blood flow dynamic in local areas of interest is a key tool to explain emergence of certain abnormalities like stenosis. However, extraction of the entire vascular network is in general inaccessible and the truncated part is encoded via Windkessel models. These models rely on many parameters which are estimated by comparing the model with observations. This poster presents a systematic approach for this parameter estimation task using the Ensemble Kalman Method (EnKM) with a surrogate model based on Dynamic Mode Decomposition (DMD).

## Vascular model

In large scale vessels ( $\sim$  [mm]), the blood is considered as an incompressible homogeneous Newtonian fluid and the Navier-Stokes equations describe the dynamic.

$$\begin{cases} \frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} - \nu \Delta \mathbf{u} = -\nabla p & [0, T] \times \Omega \\ \nabla \cdot \mathbf{u} = 0 & [0, T] \times \Omega \\ \mathbf{u} = 0 & [0, T] \times \Gamma_{\text{wall}} \\ \mathbf{u} = \mathbf{g} & [0, T] \times \Gamma_{\text{inlet}} \\ \nabla \mathbf{u} \cdot \mathbf{n} - p \mathbf{n} = -P_{\text{ext},i} \mathbf{n} & [0, T] \times \Gamma_{\text{outlet},i} \end{cases}$$

**Variables:**

- $\mathbf{u}$  velocity field
- $p$  pressure
- $\nu$  dyn. viscosity
- $\mathbf{g}$  input velocity

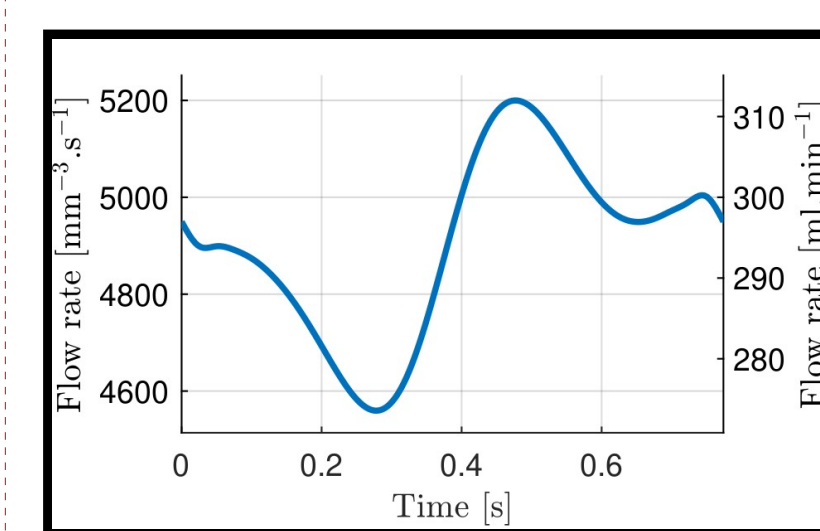
**Windkessel model:** A generic 4-components model (RCL) is used at each outlet, coupled to the 2D model by the external pressure terms  $P_{\text{ext},i}$  at the outlets.

$$P_{\text{ext},i}^{(k)} = Q_i^{(0)} e^{-\frac{\delta t}{R_{d,i} C_i}} \left( \frac{\delta t}{2C_i} + R_{d,i} \right) + \frac{\delta t}{C_i} \sum_{j=1}^{k-1} Q_i^{(j)} e^{-\frac{(j-k)\delta t}{R_{d,i} C_i}} - Q_i^{(k-1)} \frac{L_i}{\delta t} + Q_i^{(k)} \left( \frac{\delta t}{2C_i} + R_{p,i} + \frac{L_i}{\delta t} \right)$$

where  $k$  denotes the time step and  $Q_i$  the flow rate at outlet  $i$ .

Flow rate measurements are provided at inlet and outlets of the model:

- inlet meas.  $\rightarrow$  input
- outlets meas.  $\rightarrow$  observation for control



Input

$R_d$  = distal resistance  
 $R_p$  = proximal resistance  
 $C$  = compliance  
 $L$  = inductance

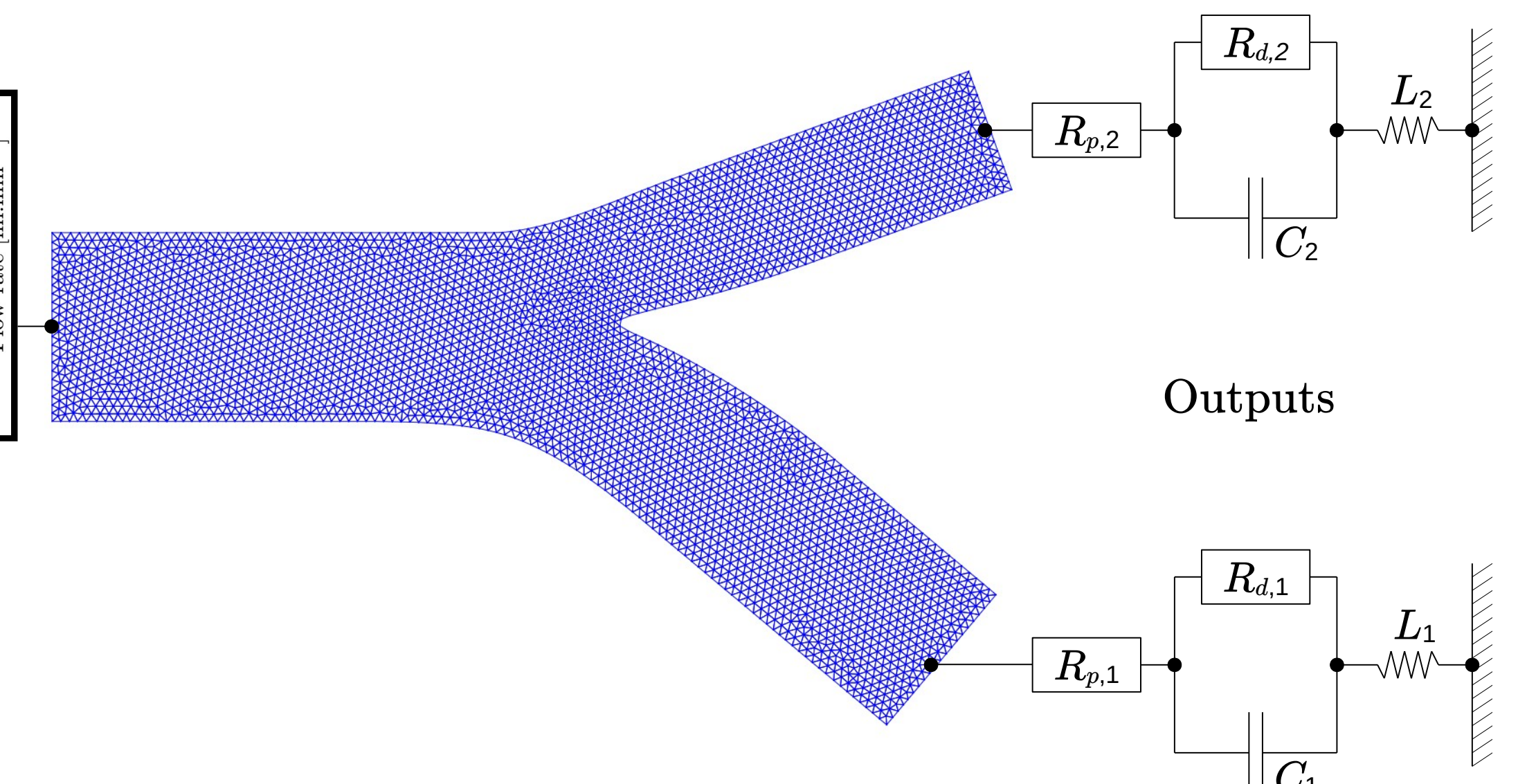


Figure 1: Vascular model.

## Parameter estimation

**Parameter:**

- $\mu = \{R_{d,1}, R_{d,2}, R_{p,1}, R_{p,2}, C_1, C_2, L_1, L_2\} \in \mathbb{R}_+^8$
- $\mathbf{u}(\mu) = \{\mathbf{u}^{(1)}(\mu), \dots, \mathbf{u}^{(K)}(\mu)\}$  the associated velocity fields at the  $K$  time steps of interest.

**Observations operator:** Given a velocity field  $\mathbf{v}$ , the flow rate at the outlet  $i$  defines a linear observation operator:

$$L_i(\mathbf{v}) = \int_{\Gamma_{\text{outlet},i}} \mathbf{v} \cdot \mathbf{n} \, ds, \quad i = 1, 2$$

**Problem formulation:** Infer the parameter  $\mu$  to fit the flow rate at the output  $i$  according to a targeted flow rate  $y \in \mathbb{R}^K$ . The estimation parameter problem reads:

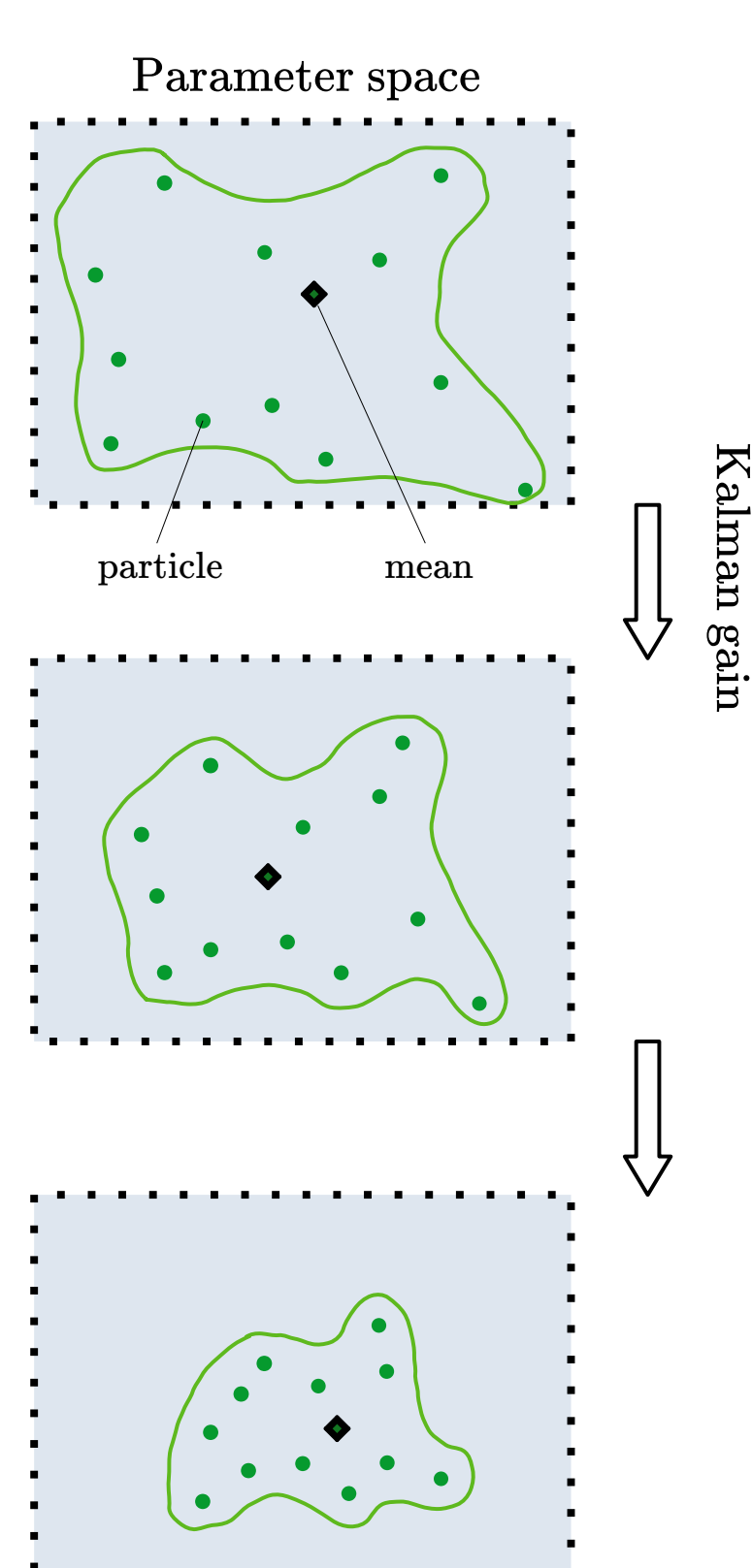
$$\hat{\mu} = \arg \inf_{\mu} \|L_i(\mathbf{u}(\mu)) - y\|_2, \quad i = 1 \text{ or } 2$$

**EnKM:** It is a robust way to perform parameter estimations based on distribution functions generated using  $J$  particles, subject to artificial Gaussian noise  $\mathcal{N}(0, \Gamma)$ . Each particle requires an evaluation of the Full Order Model (FOM) at each iteration, leading to a computationally intensive task.

```

Data:  $\mu = \{\mu^1, \dots, \mu^J\}, L, \Gamma$ 
while  $\|\hat{\mu} - \bar{\mu}\|_2 / \|\hat{\mu}\|_2 > \varepsilon$  do
  for  $j = 1 : J$  do
     $\mathbf{u}(\mu^{(j)}) \leftarrow \text{SOLVEFOM}(\mu^{(j)})$ 
     $\mathcal{G}(\mu^{(j)}) \leftarrow L(\mathbf{u}(\mu^{(j)}))$ 
  end for
   $\bar{\mu}, \bar{\mathcal{G}} \leftarrow \frac{1}{J} \sum_{j=1}^J \mu^{(j)}, \frac{1}{J} \sum_{j=1}^J \mathcal{G}(\mu^{(j)})$ 
   $\mathbb{C} \leftarrow \frac{1}{J} \sum_{j=1}^J \mathcal{G}(\mu^{(j)}) \mathcal{G}(\mu^{(j)})^t - \bar{\mathcal{G}} \bar{\mathcal{G}}^t$ 
   $\mathbb{D} \leftarrow \frac{1}{J} \sum_{j=1}^J \mu^{(j)} \mathcal{G}(\mu^{(j)})^t - \bar{\mu} \bar{\mathcal{G}}^t$ 
  for  $j = 1 : J$  do
     $\xi \leftarrow \mathcal{N}(0, \Gamma)$ 
     $\mu^{(j)} \leftarrow \mu^{(j)} + \mathbb{D} \mathbb{C}^{-1} (\mathcal{G}(\mu^{(j)}) - y^{(j)} + \xi)$ 
  end for
   $\hat{\mu} \leftarrow \frac{1}{J} \sum_{j=1}^J \mu^{(j)}$ 
end while
    
```

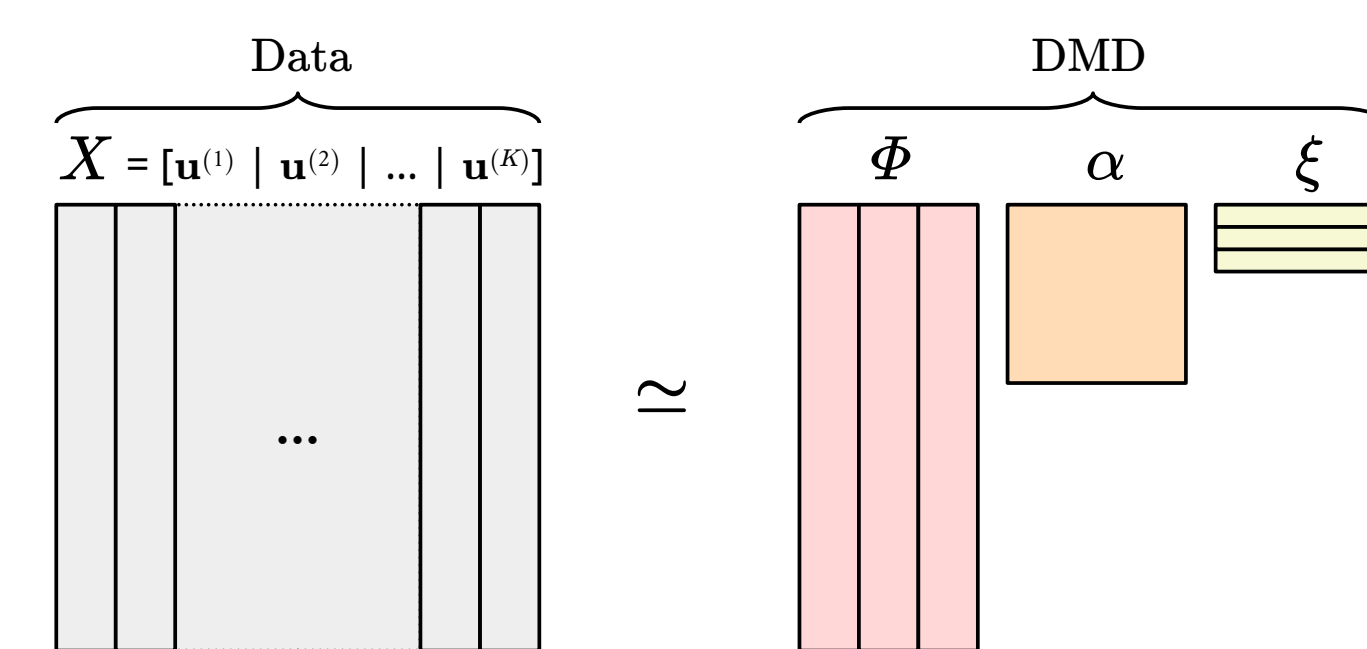
Algorithm 1: Ensemble Kalman Method



The function `SolveFOM` can be replaced by a reduced model `SolveROM`.

## Reduced order model

**DMD:** It is an equation-free algorithm which provides an efficient way to express dynamic data as a combination of spatial modes  $\Phi$ , amplitudes  $\alpha$  and time dependent dynamics  $\xi$ .



**Data set:**

$$X_n^m = [u^{(n)} | \dots | u^{(m)}]$$

**Linear mapping assumption:**

$$X_2^K = A X_1^{K-1}$$

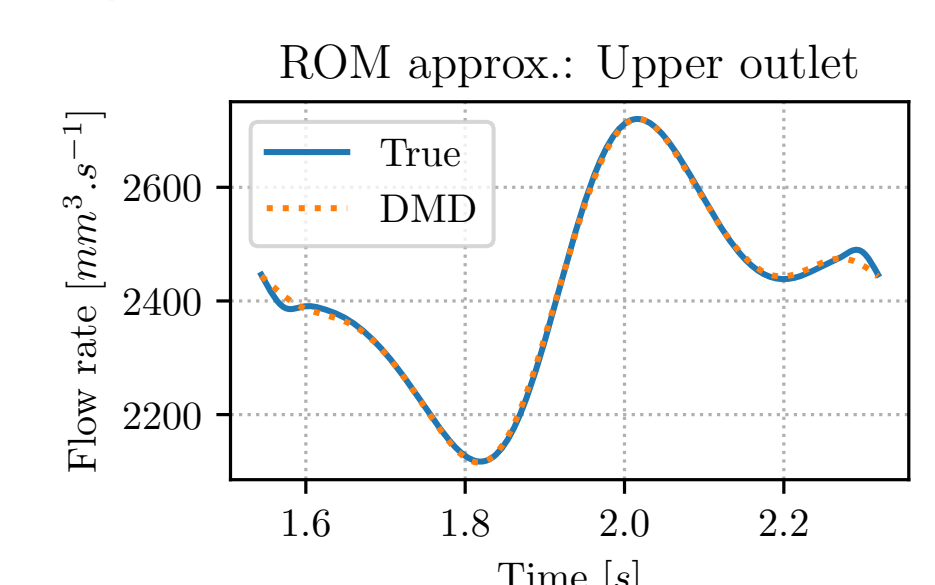
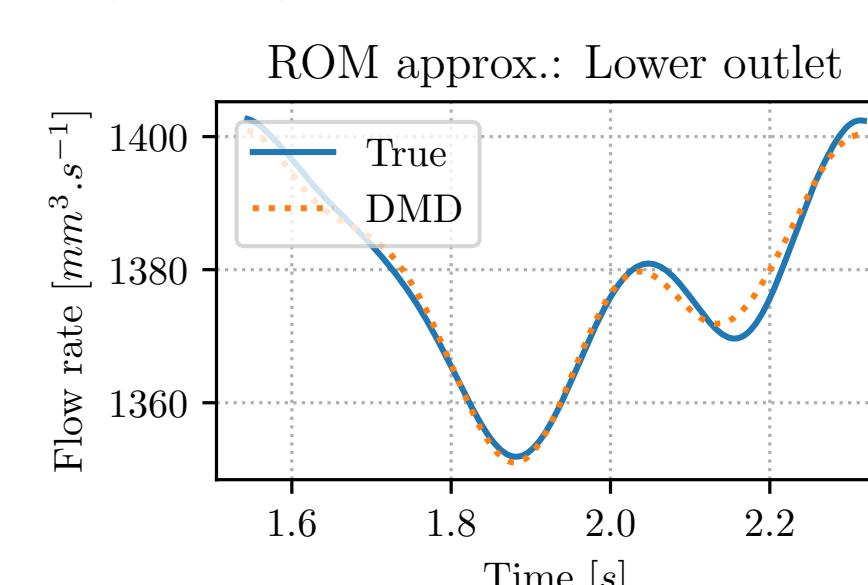
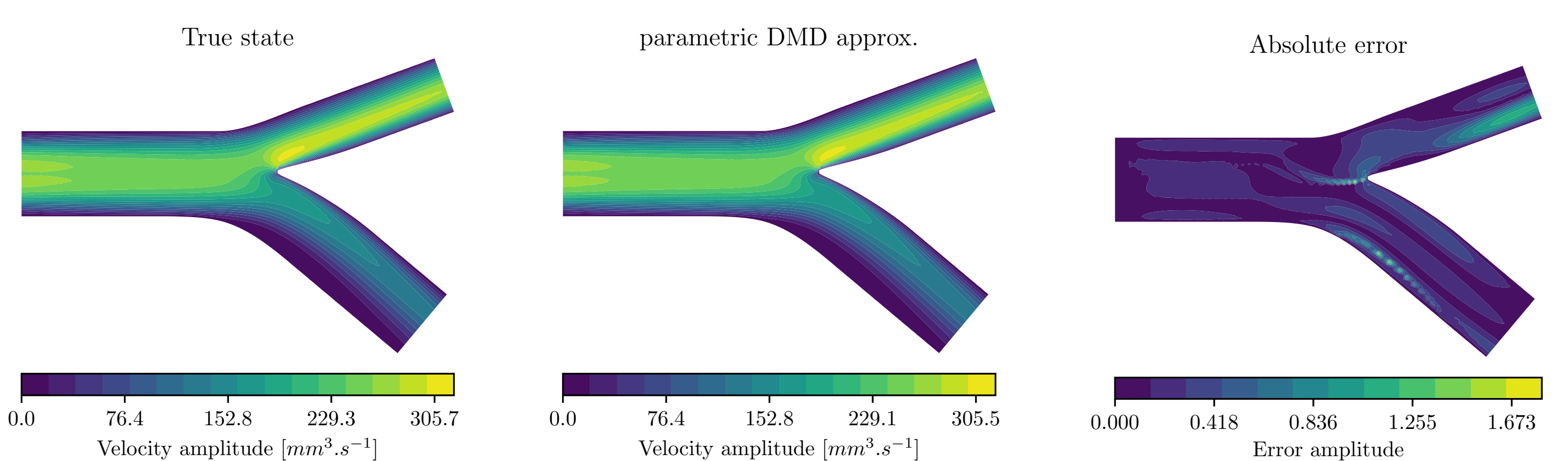
Then  $\Phi$  and  $\xi$  are given by spectral properties of  $A$ . Instead, DMD uses a reduced matrix  $\tilde{A}$  such that:

$$X_2^K \simeq X_1^{K-1} \tilde{A}$$

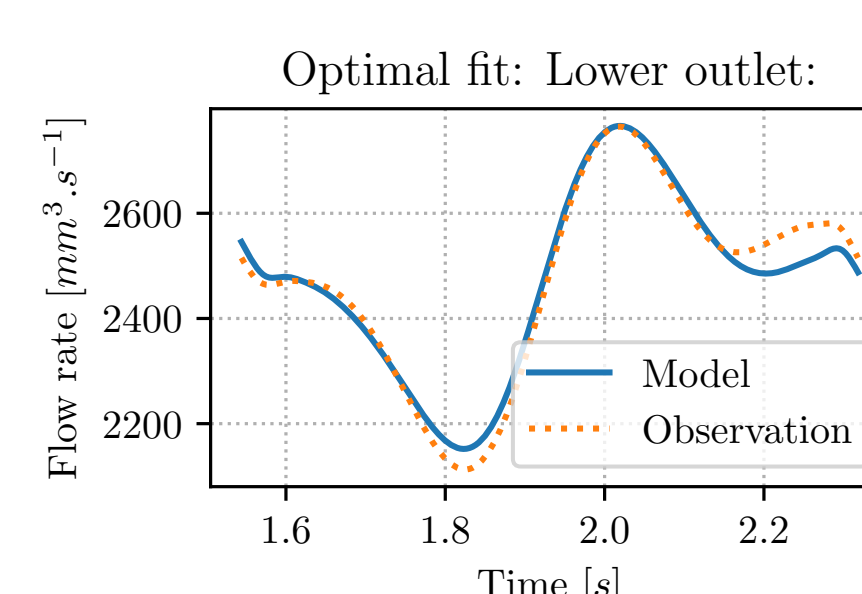
The DMD algorithm can be adapted to deal with parameterized problems to provide a map between parameter inputs and output of interest.

## Numerical results: single parameter variation

**Surrogate model:** Reduction using 24 time dependent snapshots, varying only  $L_1$ .



**Parameter estimation:** In EnKM+FOM, particles simulation run in parallel.



	EnKM + FOM	EnKM + ROM
Error		$3.4 \times 10^{-3}$
Time	10 × 40min	10 × 0.9s

Table 1: Results after 10 EnKM iterations.

## References

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- [3] J. N. KUTZ, ed., *Dynamic mode decomposition: data-driven modeling of complex systems*, Society for Industrial and Applied Mathematics, Philadelphia, 2016.

## Conclusion

- Promising preliminary results with single parameter variation.
- Need to take the model reduction bias in the EnKM algorithm.
- Possible extensions to filtering methods.