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## Review of “A review of the coupled criterion”

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### Review of version 1

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**Authors** We thank both reviewers for their constructive comments that improved the quality of the paper. Our answers are provided below, and the corresponding changes are highlighted in red in the new version of the manuscript.

#### Reviewer 1 (Alberto SAPORA)

**Reviewer** The work presents a review of the most relevant applications of the coupled criterion in the last decade. As such, it can be considered the prosecution of the previous review paper by Weissgraeber et al. (2016). The study covers the recent developments in the framework of 3D, nonlinearities, fatigue and dynamic scenarios, as well as the comparison between the CC and other well-known methods (including the cohesive zone model and the phase-field approach) to fracture mechanics. According to my opinion, the review is exhaustive and well-written. The Authors are experts in the field. After reading carefully the manuscript, I would suggest the following minor improvements:

**Reviewer** 1. Some other keywords (such as 3D formulation, nonlinearities, dynamic loading, fatigue failure etc.) can be added to the list after the Abstract.

**Authors** The proposed keywords were added.

**Reviewer** 2. Section 1, please note that the references “Griffith’s theory Griffith (1921) or Irwin’s approach Irwin (1958)” can be rewritten as “Griffith’s theory (1921) or Irwin’s approach (1958)”.

**Authors** Changes were made accordingly, see page 1.

**Reviewer** 3. I would suggest to rename Section 2 as “Coupled Criterion theory” and Section 2.3 as “Coupled system”.

**Authors** Section 2 was renamed as “Coupled Criterion theory” and Section 2.3 was renamed as “Coupling stress and energy criteria”.

**Reviewer** 4. I would add a brief (trivial) explanation that  $\lambda$  is a parameter ranging from 0 to 1 after eq. 19.

**Authors** We added the explanation for lambda, see changes page 5.

**Reviewer** 6. Section 2.4.4: some acronyms (PMTE-SC, LEBIM, BEM) are introduced. If not used elsewhere in the text, they may be deleted.

- Authors** We checked that the defined acronyms are further used in the text or removed the acronym otherwise.
- Reviewer** 7. Section 2.5: I would replace "CC FFE implementation" with "FFE implementation of the CC", for the sake of clarity. Also "CC MA" (page 10) sounds a little redundant and can be replaced by "MA approach of the CC".
- Authors** Change was made, see pages 10 and 11.
- Reviewer** 8. Section 3.1 can be renamed as "3D formulation" instead of simply "3D"
- Authors** Change was made, see page 13.
- Reviewer** 9. Section 3.2: I would delete "of the CC" (which is obvious) from the title.
- Authors** Change was made, see page 15.
- Reviewer** 10. Section 3.3, the conclusion "If material nonlinearities such as, e.g., plasticity or diffuse damage are considered, the energy change due to plasticity or damage increase must be considered in the energy balance" seems a little misleading. Nonlinearities should also influence the stress field, as outlined by the Authors themselves in the previous sentence. Please, think about modifying the conclusion.
- Authors** This sentence is not a conclusion but the sequel of the last sentence. The first sentence states that nonlinearities influence the stress field, the second one that they influence the energy balance. We added a sentence just before to highlight the fact that nonlinearities influence both the stress and the energy conditions (see page 16).
- Reviewer** 11. Section 3.3.1: the observation "It was shown that the fracture stresses decreased with increasing hole diameters" is not significant in my opinion. It is a classical result in the framework of FFM (or TCD). As far as I know, Leite et al 2021 showed that the fracture stress decreases, but less than under linear elastic conditions. In other words, the nonlinear stress concentration factor is lower than the linear one. If the Authors agree, they can modify the sentence accordingly.
- Authors** Changes were made accordingly, see page 17.
- Reviewer** 12. If the Authors think that it is relevant, they can add the very recent paper to the section on fatigue failure: <https://doi.org/10.1016/j.ijfatigue.2023.107659>. It deals with lifetime of notched components.
- Authors** A reference to the proposed paper was added in the section dedicated to Fatigue, see page 18.
- Reviewer** 13. Is the acronym CZM defined? Although trivial, please check it.
- Authors** We added its definition, see page 16
- Reviewer** 14. Section 5.1: does the reference Doitrand et al. 2017 refer either to a), b) or to both works?
- Authors** It refers to b), see change page 24.
- Reviewer** 15. Section 5.1: for the sake of curiosity, what does the distance 1 mm (defining crack interaction) depend on? (e.g. material and/or geometry)
- Authors** The answer to this question is not straightforward, the analysis was done for given material and geometry as well as for fixed crack geometry. It can however be expected that this distance depends on the yarn thickness, which were approximately 0.3mm in this analysis.
- Reviewer** 16. What about changing the title to be more appealing? Like "A review of the Coupled Criterion: What is left after 20 years?". It is only a suggestion.
- Authors** As stated by the reviewer, since the review paper focuses on applications of the CC over the last decade, it almost does not include works before 2016, already present in a precedent review. Therefore, we keep the original title.

### Reviewer 2 (Anonymous)

- Reviewer** The manuscript is an excellent review of the state of the art concerning the coupled criterion of finite fracture mechanics. The review is exhaustive and very rigorous. The contribution to the study of the state of the art is very useful compared to the previous review on the topic, published in 2016. My recommendation is acceptance after some minor recommendations:
- Reviewer** Section 1, line 2: The sequence of references and Griffith seems not to be very adequate.
- Authors** Changes were made, see page 1.
- Reviewer** Section 1, line 8: According to Prof. Taylor, and his book about TCD, the TCD includes the coupled criterion, is it not? I would suggest making some notes about the notation. This would be very useful for beginners.
- Authors** The main difference between the TCD and the coupled criterion (CC) is that in the TCD, the critical length is an input of the method whereas in the CC, it is an output obtained by combining the stress criterion and the energy criterion. We added a precision regarding the TCD, see page 1.
- Reviewer** Page 3, line 1: I would mention the names of the two conditions here to improve readability.
- Authors** Changes were made, see page 2.
- Reviewer** Page 3, paragraph after equation (1):  $f$  cannot be a surface in the principal stress space.  $f = 0$  is a surface in this space,  $f \geq 0$  (Eq (1)) is a subspace.  $f$  is simply a functional because it depends on the shape of some functions, particularly the stresses.
- Authors** Changes were made according to the reviewer comment, see page 2.
- Reviewer** Equation (3): Does this criterion depend on shear strength?
- Authors** This criterion is a maximum principal stress criterion that involves the shear stress but not the shear strength.
- Reviewer** Page 2, line 4 after Equation (5): After “loading,” I would add “at crack onset.”
- Authors** Change was made, see page 3.
- Reviewer** Page 3, line 3: In my opinion, “upper bound” has a not clear sense when referring to surfaces. What is the definition of larger or smaller when referring to surfaces, the area? I guess it’s not so clear when it concerns the coupled criterion. It seems to be an extension from 2D to 3D that is not so rigorous and can be very confusing for beginners in the coupled criterion, who are some of the target readers for this review.
- Authors** The reviewer is right that the term “upper bound” can only be used provided that there is a single crack configuration corresponding to one value of crack surface, such as for instance, in 2D or in 3D for crack described by stress isocontours. We added this precision in the text, see page 3.
- Reviewer** Page 3, line 9 after Figure 2 caption: In my opinion, both classical stress criteria, pointwise and average, are non-local conditions because both are based on the values of stresses over a whole surface. The name “pointwise” can be confusing in this aspect, but the condition in this criterion is that all the values of the stresses at a certain surface exceed a certain value, so the condition is nonlocal.
- Authors** The term “pointwise” was not used in the manuscript.
- Reviewer** Page 3, last line: ERR tends to 0 in weak singular points, but not in strong or crack singular points, where it is infinite and finite, respectively. I would add “weak” before “singular.”
- Authors** The reviewer is right, we made the change, see page 4.
- Reviewer** Page 4, line 3: I would add “between the states” after “principle.”
- Authors** We made the change, see page 4.
- Reviewer** Page 4, line 6: I would use a symbol for the crack surface creation energy, instead of  $\mathcal{G}_c S$ , because in some cases,  $\mathcal{G}_c$  is not homogeneous, as discussed later in the manuscript. Thus, to be coherent,

this general expression should be compatible with this fact.

**Authors** We modified the text according to the reviewer’s comment, see page 4.

**Reviewer** Page 4, Equation (9): The arguments for  $\mathcal{G}_{\text{inc}}$  should include  $\Gamma$  instead of  $S$ , similar to the stress criterion. The confusion comes from the fact that the arguments of  $\Delta W$  were not explicitly stated in this equation, and they depend on  $\Gamma$ , not on  $S$ . Although  $S$  appears explicitly in the divider, it is actually a function of  $\Gamma$ . The same idea could be applied to the rest of the equations, including  $\mathcal{G}_{\text{inc}}$ .

**Authors** We modified the dependency of  $\mathcal{G}_{\text{inc}}$  to  $\Gamma$  instead of  $S$  for the general definition of the IERR according to the reviewer’s comment, see page 4.

**Reviewer** Page 4, lines 3-7 after equation (11): The sentence starting with “The energy condition...” seems to repeat previous ideas from the lines before.

**Authors** This is to emphasize that LEFM is included in the CC, no changes were made.

**Reviewer** Page 5, lines before and after Equation (16): Mantic (2009) and the other articles cited used the stress mixity at the crack tip, not along the presupposed crack before the crack onset, as mentioned in the lines after Eq. (16). I would recommend reading Section 4.2 that discusses it in the already cited manuscript Garcia and Leguillon (2012).

**Authors** We agree with the reviewer. Actually, we highlighted one line after that the approaches proposed by Mantic (2009), is based on the stress mixity at the crack tip. With this definition of the mixity, it is calculated along the presupposed crack path but for considering different crack lengths between 0 and the initiation crack length.

**Reviewer** Page 6, line 12: The initiation surface should be  $\Gamma_c$ , not  $S_c$ , which is reserved here for area, is it not? The same in line 3 of subsection 2.3. I suggest revising carefully the notation about surface and area.

**Authors**  $\Gamma_c$  corresponds to the set of admissible initiation cracks.  $S_c$  corresponds to the crack area or surface, which denotes the same quantity.

**Reviewer** Page 9, line 12 after Figure 5: I would add crack tip or singularity. If we are studying the initiation from an already existing crack, we should be close to the crack tip.

**Authors** Change was made, see page 9.

**Reviewer** Page 10, Section 2.4.3: I suggest reserving the name of Full Finite Element approach for the formulation implementing the coupled criterion directly in FEM, such as the Li formulation. I guess this is called “full” in contrast with the use of FEM for applying the MA.

**Authors** The name FFE was already used previously to designate a finite element implementation of the CC when the full structure is modeled, in contrast to the MA approach where a focus is made around the singularity.

**Reviewer** Page 11, Figure 6: I suggest improving the colors of this figure because in the printable version, they are not well printed.

**Authors** We increased the line width of the curves for a better visibility.

**Reviewer** Page 18, Section 4.1, Lines 4-5: “No separation of the surfaces occurs...”. This is not true in CZM in general, and it depends on the law. For example, if the exponential law is used, the separation is not zero except for zero traction.

**Authors** We made a change according to the reviewer’s comment, see page 19.

**Reviewer** Conclusions: I would add the need for a computational implementation of the coupled criterion in an efficient way and able to be used by researchers who are not experts in the Coupled Criterion. This computational implementation would allow extending the application of the coupled criterion to many complex problems and widen the user community.

**Authors** We mentioned it in the conclusion according to the reviewer’s comment, see Page 25.

**Authors** The following references, relevant to the application of the CC, were added to the manuscript: (Aranda and Leguillon 2023; Jiménez-Alfaro and Leguillon 2022; Ferrian et al. 2024; Girard et al. 2023; Girard et al. 2024; Mirzaei et al. 2024; Methfessel et al. 2024; Sakha et al. 2023; Sapora et al. 2023; Rheinschmidt et al. 2024; Duminy et al. 2024)

## Editor’s assessments (Julien RÉTHORÉ)

The utilisation and advancement of the coupled criterion undoubtedly falls within the purview of JTCAM. This document presents a review of the latest improvements and applications of this method. The previous review paper on this subject dates back to 2016. This already constitutes compelling arguments for the acceptance of this document. Furthermore, both reviewers recognised the scientific value of the document and suggested only minor comments that enabled the authors to enhance the document’s completeness and quality. Consequently, I have decided to accept the article in its revised version.

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