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# COMPUTATION OF ELECTROMAGNETIC RADIATION BY HEAVY CURVED INTERFACES USING CONFORMAL GAUSSIAN BEAMS

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## ABSTRACT

An Asymptotic technique have been successfully applied to compute electromagnetic wave radiation in various high-frequency engineering domains. Recent approaches based on Gaussian beams for tracking fields may overcome some problems inherent to the ray methods such as caustics. However, some difficulties may arise with curved surface on some complex situations. In this paper, we propose a new efficient way to treat curved surfaces. For this purpose, a new beam formulation called Conformal Gaussian Beam (CGB) is used. The CGBs have been developed to overcome some limitation in Gaussian Beams Tracking methods. The analytical Plane-Wave Spectrum and far-field of a CGB are derived and compared with numerical calculations. The goal of this technique is to realise fast radome radiation computation in an optimisation process.

Key words: Gaussian Beams (GBs), Conformal Gaussian Beams (CGBs), curved interfaces.

## 1. INTRODUCTION

For several years, high-frequency techniques have been successfully applied to describe electromagnetic wave radiation in various domains: antenna analysis, propagation, Radar Cross-Section computation, compatibility issues, etc. As the size of the objects under consideration tends to be large according to the wavelength, rigorous approaches such as the finite-element method (FEM) or the method of moments (MoM) become computer time and resource prohibitive. In contrast, at sufficiently high-frequency, the electromagnetic phenomena such as radiation, propagation and diffraction, exhibit localized behavior properties. As the asymptotic methods use a local description of EM fields, such as the ray techniques, their efficiency increases with the frequency.

Ray techniques like the shooting or bouncing ray methods represent the fields by a set of Geometrical Optics (GO) ray tubes for which the propagation, reflection and transmission obey the generalized Fermat's principle and

the power conservation. However, some difficulties may arise with some complex situations, such as caustics or heavy curved surfaces. Because of the necessary discretization used on the objects under consideration, the spatial extent of grazing ray tubes on a curved surface may be quite large according to the wavelength. Local assumptions are not valid any more in such a case and one may need to increase the spatial resolution on the enlightened surfaces. Therefore, the number of rays and the computation time may increase too. These problems are of great importance in many practical situations such as radome computation.

Current based approaches such as Physical Optics (PO) and Physical Theories of Diffraction (PTD) do not rely on the concept of ray and then can circumvent ray-based problems like caustics. However, the asymptotic evaluation of radiating fields by the PO/PTD currents needs numerical integration which can be also time and resources consuming on large surface. Many directions and techniques such as hybrid methods exist in order to deal with these theoretical and practical challenges.

Some approaches may use different basis functions involving Gaussian Beams (GBs) for tracking fields in complex environments [1]. The Gaussian Beams set enables one to reduce the number of required ray tubes and avoids caustic problems. Moreover, asymptotic methods allow to express analytical formulations of the electromagnetic fields. Gaussian Beams Tracking (GBT) methods are based on the ability to expand any source fields into a summation of Gaussian beams [2]. As the ray based techniques, this method is able to deal with curved dielectric surfaces, under some curvature and incidence restrictions. During the last years, GBT has been successfully applied to the computation of the fields scattered by mono and multilayer dielectric radomes [3]. Recently, it has been applied to the Airbus A380 front antenna radome, reducing computation time by a factor 70 in contrast with a classic Plane-Wave Spectrum (PWS) based method [4].

Just like the ray methods in the special case of the heavy curved interface combined with an oblique incidence, the size of the Gaussian beams may be too large on a surface. In order to obtain some localized beams instead, one may think of a Gabor decomposition of the fields. However,

this has been performed rigorously only over a plane [5] or a cylindrical surface [6]. Then, such an expansion on a curved surface may be wrong evaluated. In order to treat a more general class of interfaces such as sharp surfaces, [7] proposed to use a Gabor expansion with respect to the curvilinear coordinate of a regular interface in a two-dimensional case. This expansion requires a new kind of Gaussian beams, named Conformal Gaussian Beams (CGB). The basic idea of the 2D-CGB is still to expand non-local fields into a summation of localized fields. Hence, a Gabor expansion of equivalent currents is used on a curved surface into some localized Gaussian amplitude currents. The field radiated by each of these localized currents can be expressed through a closed-form by applying standard asymptotic techniques.

In section 2, the principal characteristics of the CGBs extended to the 3D case are presented. We derive in section 3, the analytical Plane Wave Spectrum (PWS) and in section 4 the far-field of a CGB. Comparisons with numerical evaluations are shown. An  $\exp(+j\omega t)$  time dependence for the electromagnetic fields is assumed and suppressed throughout in this paper.

## 2. CONFORMAL GAUSSIAN BEAM

Under oblique incidence, Gaussian beams spots may be large on a curved surface. In this case, the local assumption according to the wavelength is not valid and the radiated fields may be wrong evaluated. In order to circumvent this problem, one needs to keep a suitable size beam on the surface and to have the possibility to specify a main radiation direction. In this paper, we present a new type of beam combining both properties. We called them Conformal Gaussian Beams (CGB). In [7], CGBs were defined in two-dimension on curvilinear coordinates. In this section, we briefly present an extension of the CGB to the three-dimensional case. In this extension, one should note that amplitude and phase do not depend of the curvilinear coordinates. However, the beams keep the same properties as in the 2D-version. By expanding fields using a combination of these CGB, one should be able to compute the field scattered by heavy curved surface as fast as standard GB tracking methods.

We first assume that the incident electric and magnetic fields are known on a curved surface  $S$ . Equivalent electric  $\mathbf{J}$  and magnetic  $\mathbf{M}$  currents are then given by the Equivalence Theorem on  $S$ . As presented in [3, 7], the basic idea is to expand the surface-currents  $\mathbf{J}, \mathbf{M}$  into  $n$  elementary Gaussian amplitude currents:

$$\mathbf{J} = \sum_n a_n^J \mathbf{J}_n^{CGB} \quad (1)$$

$$\mathbf{M} = \sum_n a_n^M \mathbf{M}_n^{CGB} \quad (2)$$

where  $\mathbf{J}_n^{CGB}$  and  $\mathbf{M}_n^{CGB}$  are the electric and magnetic elementary gaussian currents and  $a_n^{J/M}$  the expansion coefficients. Unprimed characters will denote

the observation points and primed characters the radiating points on the surface. Each elementary current  $\mathbf{J}_n^{CGB}, \mathbf{M}_n^{CGB}$  is expressed in its own local coordinate system  $(O_n, x'_n, y'_n, z'_n)$ , which is translated along the surface according to the order  $n$ , by:

$$\mathbf{J}_n^{CGB}(\mathbf{x}') = \hat{\mathbf{e}}_n^J(\mathbf{x}') u_n(\mathbf{x}') \quad (3)$$

$$\mathbf{M}_n^{CGB}(\mathbf{x}') = \hat{\mathbf{e}}_n^M(\mathbf{x}') u_n(\mathbf{x}') \quad (4)$$

where  $\hat{\mathbf{e}}_n^{J/M}$  are local unit-vector directions related to each elementary current and  $u$  a Gaussian function defined by:

$$u_n(\mathbf{x}') = \exp\left(-j\frac{k}{2} {}^t \mathbf{x}'_n \mathbb{Q}_f \mathbf{x}'_n - j\mathbf{B}_n \cdot \mathbf{x}'_n\right) \quad (5)$$

where  $k$  is the wave number,  $\mathbf{x}' = {}^t[x'_n, y'_n]$ ,  $\mathbf{B}_n = {}^t[\beta_{x,n}, \beta_{y,n}]$  and  $\mathbb{Q}_f$  the complex curvature matrix of the elementary currents whose eigenvalues depend of the beam waist  $(W_{0,x}, W_{0,y})$ . This formulation is analogue to that of conventional Gaussian Beam field formalism [8].  $(\beta_{x,n}, \beta_{y,n})$  stands for the linear phase terms with respect to  $x$  and  $y$  coordinates. They define the main propagation direction of the radiated fields. The fields radiated by those elementary Gaussian currents are named Conformal Gaussian Beam (CGB).

The general geometry for the problem is shown on Fig. (1). Let  $O$  be the center of the direct local system  $\{\hat{\mathbf{e}}_x, \hat{\mathbf{e}}_y, \hat{\mathbf{e}}_z\}$ . As in the following we consider only an elementary CGB for the sake of clarity, subscripts  $n$  have been suppressed. The  $z$ -axis is normal to the curved surface  $S$  on  $O$ . Let  $\mathbf{r} = x\hat{\mathbf{e}}_x + y\hat{\mathbf{e}}_y + z\hat{\mathbf{e}}_z$  be an observation point off  $S$  and  $\mathbf{r}' = x'\hat{\mathbf{e}}_x + y'\hat{\mathbf{e}}_y + z'\hat{\mathbf{e}}_z$  a point on  $S$ .

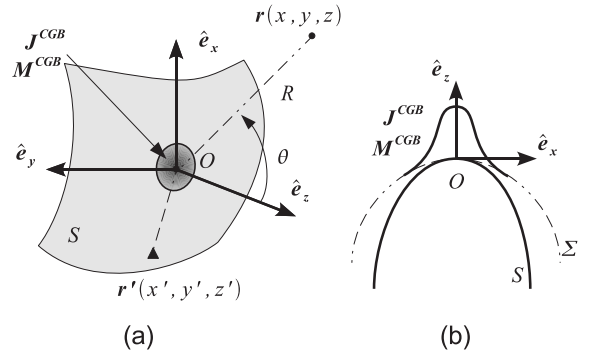


Figure 1. (a) Geometry of the problem. (b)  $\{\hat{\mathbf{e}}_x, \hat{\mathbf{e}}_z\}$  plane cut. The surface  $S$  is approximated at  $O$  by a paraboloid  $\Sigma$ . An elementary current  $\mathbf{J}_n^{CGB}$  or  $\mathbf{M}_n^{CGB}$  is centered on  $O$ .

It is assumed that the surface  $S$  at  $O$  can be approximated locally by a quadric  $\Sigma$  defined by its principal radii of curvature  $R_a, R_b$ . The curvature matrix  $\mathbb{Q}^\Sigma$  is deduced in the local coordinates system from  $R_a, R_b$ . Therefore,  $z'$  can be approximated by:

$$z' \approx -\frac{1}{2} {}^t \mathbf{x}' \mathbb{Q}^\Sigma \mathbf{x}' \quad (6)$$

Using the FRANZ's integrals [9], the radiated fields from electric  $\mathbf{J}^{CGB}$  and magnetic  $\mathbf{M}^{CGB}$  elementary currents can be expressed as electric  $\mathbf{E}_J, \mathbf{H}_J$  and magnetic  $\mathbf{E}_M, \mathbf{H}_M$  CGBs by:

$$\mathbf{E}_J(\mathbf{r}) = \frac{Z_0}{jk} \nabla \times \nabla \times \mathbf{A}_J(\mathbf{r}) \quad (7a)$$

$$\mathbf{E}_M(\mathbf{r}) = -\nabla \times \mathbf{A}_M(\mathbf{r}) \quad (7b)$$

$$\mathbf{H}_J(\mathbf{r}) = \nabla \times \mathbf{A}_J(\mathbf{r}) \quad (8a)$$

$$\mathbf{H}_M(\mathbf{r}) = \frac{1}{jkZ_0} \nabla \times \nabla \times \mathbf{A}_M(\mathbf{r}) \quad (8b)$$

where the differential operators act only on the observation point dependant terms  $\mathbf{r}$ .  $Z_0 = \sqrt{\mu_0/\varepsilon_0}$  is the free-space impedance and:

$$\mathbf{A}_M(\mathbf{r}) = \iint_{\Sigma} \mathbf{M}^{CGB}(\mathbf{x}') g(\mathbf{r}, \mathbf{r}') dS' \quad (9)$$

$$\mathbf{A}_J(\mathbf{r}) = \iint_{\Sigma} \mathbf{J}^{CGB}(\mathbf{x}') g(\mathbf{r}, \mathbf{r}') dS' \quad (10)$$

$$g(\mathbf{r}, \mathbf{r}') = \frac{e^{-jk\|\mathbf{r}-\mathbf{r}'\|}}{4\pi\|\mathbf{r}-\mathbf{r}'\|} \quad (11)$$

To illustrate the aspect of a CGB, we represent on Fig. (2) the numerical computation of the radiation of an elementary two-dimensional electric current  $\mathbf{J}^{CGB} = \hat{\mathbf{e}}^J(x')u(x')$  centered on  $O$  by a KOTTLER current integral [7]. As any conventional current on a surface, elementary currents radiate toward both sides of the surface  $\Sigma$ . However, the radiated fields on each side are not the same because of the curvature of the surface. Some of the far-field properties of a CGB will be exhibited in part 4.

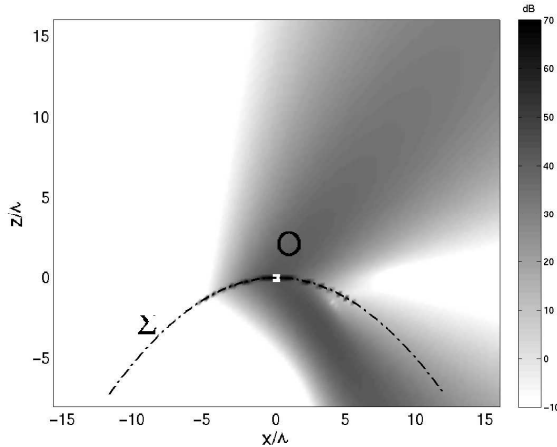


Figure 2. Example of electric field radiated by a single two-dimensional elementary electric current  $\mathbf{J}^{CGB}$  centered on  $O$  with  $W_{0x} = 2\lambda$ ,  $R_x = 10\lambda$  and  $\beta_x = k \sin(45^\circ)$ . The elementary current radiates in two main directions which are given by the value of the  $\beta_x$  parameter.

### 3. CGB PLANE-WAVE SPECTRUM

#### 3.1. Plane-Wave Spectrum derivation

In this section, we derive the asymptotic expression of the Plane-Wave Spectrum (PWS) of a CGB. The spatial integrals (9) and (10) issued from FRANZ formulations can be transformed into their spectral integrals by expanding the free-space Green function  $g(\mathbf{r}, \mathbf{r}')$  into its Plane-Wave Spectrum using the WEYL expansion [10] :

$$g(\mathbf{r}, \mathbf{r}') = \frac{1}{8j\pi^2} \iint_{-\infty}^{+\infty} \frac{e^{-j\mathbf{k} \cdot (\mathbf{r}-\mathbf{r}')}}{k_z} dk_x dk_y \quad (12)$$

where  $\mathbf{k} = {}^t[k_x, k_y, k_z]$ ,  $|z - z'| > 0$  and

$$k_z = \begin{cases} \sqrt{k^2 - k_x^2 - k_y^2}, & \text{if } k^2 > k_x^2 + k_y^2 \\ -j\sqrt{k_x^2 + k_y^2 - k^2}, & \text{if } k^2 < k_x^2 + k_y^2 \end{cases} \quad (13)$$

Substituting (12) into (9) and (10), and interchanging the order of spatial and spectral integration, one can put the spatial integral into the general reduced form:

$$\mathbf{A}_{J,M} = \frac{1}{4\pi^2} \iint_{-\infty}^{+\infty} \mathbf{I}(k_x, k_y) e^{-j\mathbf{k} \cdot \mathbf{r}} dk_x dk_y \quad (14)$$

where

$$\mathbf{I}(k_x, k_y) = \iint_{-\infty}^{+\infty} \mathbf{f}(\mathbf{x}') e^{k\Phi(\mathbf{x}')} dx' dy' \quad (15)$$

with  $\mathbf{f}$  and  $\Phi$  complex functions. As  $k \gg 1$ ,  $\mathbf{I}$  has an appropriate form to be evaluated with the saddle-point method [10]. Finally, the radiated fields  $\mathbf{E}_{J,M}$  and  $\mathbf{H}_{J,M}$  can be expressed as:

$$\mathbf{E}_{J,M}(\mathbf{r}) = \frac{1}{4\pi^2} \iint_{-\infty}^{+\infty} \tilde{\mathbf{E}}_{J,M}(k_x, k_y) e^{-j\mathbf{k} \cdot \mathbf{r}} dk_x dk_y \quad (16)$$

$$\mathbf{H}_{J,M}(\mathbf{r}) = \frac{1}{4\pi^2} \iint_{-\infty}^{+\infty} \tilde{\mathbf{H}}_{J,M}(k_x, k_y) e^{-j\mathbf{k} \cdot \mathbf{r}} dk_x dk_y \quad (17)$$

where one can deduce by identification the analytical Plane-Wave Spectrum of a Conformal Gaussian Beam:

$$\tilde{\mathbf{E}}_J(k_x, k_y) = \frac{Z_0}{k} (\mathbf{k} \times \mathbf{k} \times \hat{\mathbf{e}}^J) \frac{\gamma(k_x, k_y)}{2k_z} \quad (18a)$$

$$\tilde{\mathbf{E}}_M(k_x, k_y) = (\mathbf{k} \times \hat{\mathbf{e}}^M) \frac{\gamma(k_x, k_y)}{2k_z} \quad (18b)$$

$$\tilde{\mathbf{H}}_J(k_x, k_y) = (-\mathbf{k} \times \hat{\mathbf{e}}^J) \frac{\gamma(k_x, k_y)}{2k_z} \quad (19a)$$

$$\tilde{\mathbf{H}}_M(k_x, k_y) = \frac{1}{Z_0 k} (\mathbf{k} \times \mathbf{k} \times \hat{\mathbf{e}}^M) \frac{\gamma(k_x, k_y)}{2k_z} \quad (19b)$$

with

$$\gamma(k_x, k_y) = \frac{2\pi}{k} N_s (\det \mathbb{Q})^{-\frac{1}{2}} \cdot \exp \left[ \frac{j}{2k} {}^t(\mathbf{B} - \mathbf{k}) \mathbb{Q}^{-1} (\mathbf{B} - \mathbf{k}) \right] \quad (20)$$

$$\mathbb{Q} = \mathbb{Q}^f + \frac{k_z}{k} \mathbb{Q}^\Sigma \quad (21)$$

where  $N_s$  is the norm of the normal vector to the surface  $\Sigma$  evaluated on the saddle-point.

One can note that the derivation does not depend on a paraxial approximation. Therefore, the analytical Plane-Wave Spectrum of a CGB can be used beyond the paraxial limitation which relates to some conventional GBs formulations.

### 3.2. Numerical results

In order to validate our preceding results, a comparison between analytical and numerical computation of the PWS has been realised. First, the fields radiated by an elementary electric current  $\mathbf{J}^{CGB}$  are evaluated on a plane surface  $\Lambda$  near the radiating surface  $\Sigma$  by a numerical current integration. Then, the PWS of the field on this plane surface is computed and compared with our analytical formulation (18). The geometry of the test case is depicted on Fig. (3). Results are presented on Fig. (4) where we represent the two components  $\tilde{\mathbf{E}}_x, \tilde{\mathbf{E}}_y$  of the PWS. The  $\tilde{\mathbf{E}}_z$  component can be deduced from the two others. We used on this comparison the elementary currents  $\mathbf{J}^{CGB} = \hat{\mathbf{e}}^J(x', y')u(x', y')$ ,  $\mathbf{M}^{CGB} = \mathbf{0}$ ,  $W_0 = 2\lambda$ ,  $R_x = R_y = 10\lambda$ ,  $\delta z = 1\lambda$ ,  $\beta_x = \beta_y = 0$ . Unit-vector  $\hat{\mathbf{e}}^J$  is defined in the  $(y, z)$ -plane. A 0.05 dB gap error is found between the maximum values of the numerical and analytical formulations.

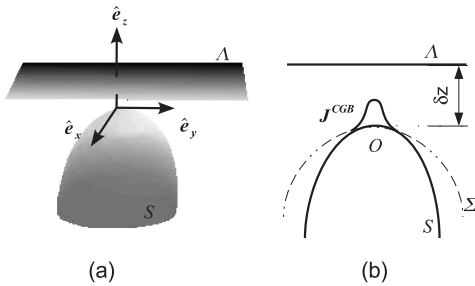


Figure 3. Geometry for the numerical validation of the PWS of a CGB. (a) General view of the geometry. (b)  $\hat{\mathbf{e}}_x, \hat{\mathbf{e}}_z$  plane cut.

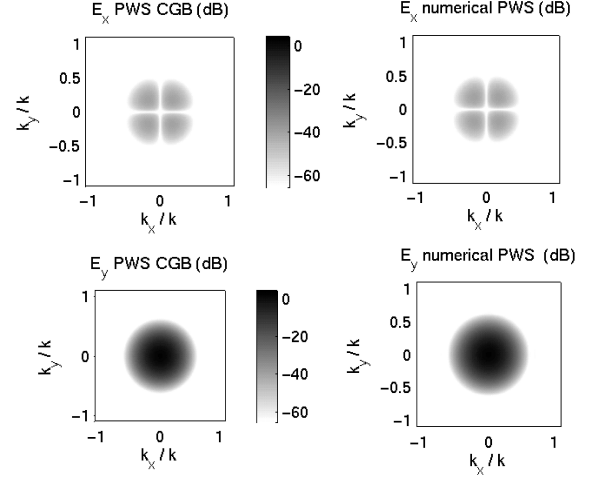


Figure 4. Comparisons between analytical Plane Wave Spectrum (PWS) of a Conformal Gaussian Beam with a numerical PWS computation of the field calculated on the plane  $\Lambda$ .

## 4. CGB FAR-FIELD

### 4.1. Far-field derivation

We derive here the asymptotic approximation of a CGB for large value of  $kr$  ( $r = \|\mathbf{r}\|$ ). For the sake of brevity, the development is only performed for the  $\mathbf{E}_J$  field. The same results can be obtained for  $\mathbf{E}_M, \mathbf{H}_J$  and  $\mathbf{H}_M$  fields. We rewrite equations (16) into the following general compact form

$$\mathbf{E}_J(\mathbf{r}) = \frac{1}{4\pi^2} \iint_{-\infty}^{+\infty} \tilde{\mathbf{E}}_J(k_x, k_y) e^{jkr\Psi(\mathbf{k})} dk_x dk_y \quad (22)$$

with

$$\Psi(\mathbf{k}) = -\frac{1}{kr} \mathbf{k} \cdot \mathbf{r} = -\frac{1}{kr} (k_x x + k_y y + k_z z) \quad (23)$$

The approximation may be obtained by the application of stationary phase method. According to this well-known principle, the asymptotic approximation to (22) for large  $kr$  is

$$\mathbf{E}_J(r, \theta, \phi) \approx \frac{jk}{2\pi} \cos \theta \frac{e^{-jkr}}{r} \tilde{\mathbf{E}}_J(k \sin \theta \cos \phi, k \sin \theta \sin \phi) \quad (24)$$

where  $(r, \theta, \phi)$  are the spherical coordinates with origin  $O$ .

### 4.2. Numerical results

The far-field expression (24) is compared to a numerical computation based on the Kottler integrals for the



electric field radiated by an elementary current  $\mathbf{J}^{CGB} = \hat{\mathbf{e}}^J(x', y')u(x', y')$ . The unit vector  $\hat{\mathbf{e}}^J(x', y')$  is defined in the  $(y, z)$ -plane.  $\mathbf{J}^{CGB}$  is defined on the local system  $(O, x, y, z)$ , as represented on Fig. (1). The fields are evaluated at large distance  $R$ . Results are shown on Fig. (5). A 0.06 dB gap error is found between the maximum values of the numerical and analytical formulations.

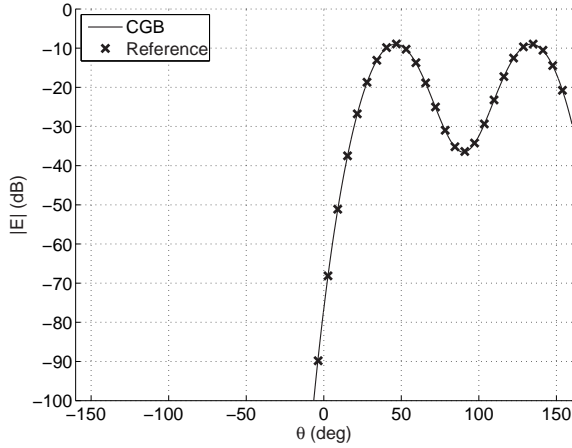


Figure 5. Electric far-field of a CGB compared to a Kotler numerical radiation integral (Reference). The geometry of the problem is depicted on fig. (1).  $\mathbf{J}^{CGB} = \hat{\mathbf{e}}^J(x', y')u(x', y')$ ,  $W_0 = 2\lambda$ ,  $R_x = R_y = 10\lambda$  and  $\beta_x = k \sin(45^\circ)$ ,  $\beta_y = 0$ .

## 5. CONCLUSION

The Conformal Gaussian Beams (CGB) have been presented. These new beams are radiated by elementary Gaussian currents on a curved surface. Using the FRANZ field integral and the WEYL expansion of the free-space Green function, a closed form of the plane-wave spectrum (PWS) of a CGB has been obtained. A comparison between a numerical evaluation of the PWS and the analytical form has been made. The far-field of a CGB has been expressed and compared with a classical numerical current integration method. A very good agreement is observed.

A multi-beams summation can be applied in three-dimensional problems to expand electric and magnetic currents on a curved interface into elementary Gaussian currents as done in the two-dimensional case [7]. With the analytical plane-wave spectrum of a CGB and using the Fresnel coefficients, a closed-form calculation of the reflected and transmitted fields by a mono or multi dielectric layers curved interface is possible as realised in [3]. This technique permit one to compute the fields scattered by heavy curved radome faster than conventionnal methods based on plane-wave spectrum. These will be presented in future communications.

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