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# A GRAPH APPROACH TO COLOR MATHEMATICAL MORPHOLOGY

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## ABSTRACT

The processing of color images has become a major field of interest, however the direct extension of their gray scale counterparts is not always possible since there is no natural ordering of color vectors. Mathematical morphology has to face with this problem since it needs a complete lattice which is generally based on a conditional ordering. We propose another approach based on a graph decimation of a given structuring element to find the supremum and the infimum of a color vector set. The effects of the proposed graph approach are studied on several morphological operators (erosion, dilatation, watershed) and compared with the conditional ordering.

## 1. INTRODUCTION

Color image processing is a very important area of research which is a particular case of the processing of multichannel images defined by a vector of scalar values. The extension of gray scale image algorithms to color or vector valued images is not simple since there is no natural ordering on a set of color vectors and more generally of multivariate data. According to Barnett [1], there are several possible types of multidimensional vector ordering: marginal ordering, reduced ordering, partial ordering and conditional ordering. Let  $x_1, x_2, \dots, x_n$  denote a set of  $n$   $p$ -dimensional vectors:  $x_i = \{x_{1(i)}, x_{2(i)}, \dots, x_{p(i)}\}, x_i \in \mathbb{R}^p$ . In the marginal ordering, vectors are ordered according to the scalar ordering of each component, for two vectors  $x_i$  and  $x_j$ , one has:  $x_i \leq x_j \Leftrightarrow x_{k(i)} \leq x_{k(j)}, \forall k \in \{1, 2, \dots, p\}$ . In the reduced ordering, a scalar function  $d : \mathbb{R}^p \rightarrow \mathbb{R}$  is used to order the vectors, for two vectors  $x_i$  and  $x_j$ , one has:  $x_i \leq x_j \Leftrightarrow d(x_i) \leq d(x_j)$ . In the partial ordering, convex hull like sets are used. In the conditional (or lexicographic) ordering, the vectors are ordered according to a hierarchical order of the components, for two vectors  $x_i$  and  $x_j$ , one has:

$$x_i \leq x_j \begin{cases} x_{1(i)} < x_{1(j)} \text{ or} \\ x_{1(i)} = x_{1(j)} \text{ and } x_{2(i)} < x_{2(j)} \text{ or } \dots \\ x_{1(i)} = x_{1(j)} \text{ and } x_{2(i)} = x_{2(j)} \dots x_{p(i)} < x_{p(j)} \end{cases}$$

The extension of mathematical morphology to color images is highly related to the definition of an appropriate ordering of the colors which retains the properties of their gray scale

counterparts. Basically there are two morphological operations dilatation and erosion which are used for the definition of more complex operations. This paper deals with their definition for color images. To this aim we propose a new graph approach which enables to find the supremum and the infimum of a set of colors by analyzing the hierarchical structure of the Minimum Spanning Tree (MST) within a given structuring element. The key features of the proposed approach are studied and compared to the conditional ordering. Finally hierarchies of partitions based on a non parametric pyramid of watersheds are studied with different definitions of morphological color gradients relying on our graph approach.

## 2. COLOR MATHEMATICAL MORPHOLOGY

Mathematical morphology is a non linear image processing approach which is based on a fundamental structure, the complete lattice  $\mathcal{L}$  such that [2] an ordering relation  $\leq (x \leq x, x \leq y, y \leq x \Rightarrow x = y, x \leq y, y \leq z \Rightarrow x \leq z, x, y, z \in \mathcal{L})$  is defined over  $\mathcal{L}$  and such that for every finite subset  $\mathcal{K}$  of  $\mathcal{L}$  there exists a supremum  $\vee \mathcal{K}$  and an infimum  $\wedge \mathcal{K}$ . The direct application of mathematical morphology to color images is difficult because of the vectorial nature of color, since a total ordering of the colors is needed. Therefore to perform color mathematical morphology, it is necessary to order colors, to verify the existence of a supremum and infimum and finally to avoid false colors by imposing that the sup and the inf belong to the lattice  $\mathcal{L}$ . The classical way to perform mathematical morphology operations is to use the lexicographic ordering [3]. However with this type of ordering most of the decisions are taken at the level of the first component which implies the attribution of a priority to the components. It obviously is possible to define several lexicographic orderings by imposing a dominant role to another color component. This is a severe drawback since in order to perform a morphologic operation on a color image one has to define which lexicographic ordering of the components to use [4]. Of course this definition of the ordering depends on the color space used and different results will be produced with different lexicographic orderings. Approaches based on reduced ordering were also proposed to define median filtering [5, 6]

or morphological operators [7, 8]. Conditional ordering has been studied by several authors for morphological filtering [9, 10, 11, 12, 13, 14, 15]. Partial ordering for color fuzzy pareto morphology has also been proposed by Köppen [16]. Another ordering based on space filling curves has been proposed by Chanussot [17]. Whatever the ordering methods, if one looks at the main morphological operations, the color erosion  $\epsilon$  and dilatation  $\delta$  by a structuring element  $B$  of an image  $f$  are given by

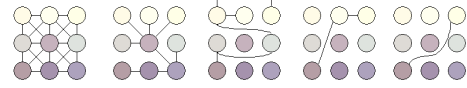
$$\begin{aligned}\epsilon_B f(x) &= \{f(y) : f(y) = \wedge(f(z)), z \in B_z\} \\ \delta_B f(x) &= \{f(y) : f(y) = \vee(f(z)), z \in B_z\}\end{aligned}$$

where  $\vee$  and  $\wedge$  are respectively the supremum and the infimum. These two operations do not really need to know the ordering of all the colors of the lattice: what is essential is to determine the two extreme values corresponding to the  $\vee$  and the  $\wedge$  which are respectively higher and lower than all the other colors of the lattice. We propose to investigate a new approach based on a graph approach to extract the  $\vee$  and the  $\wedge$  of a color lattice.

### 3. GRAPH VECTOR EROSION AND DILATATION

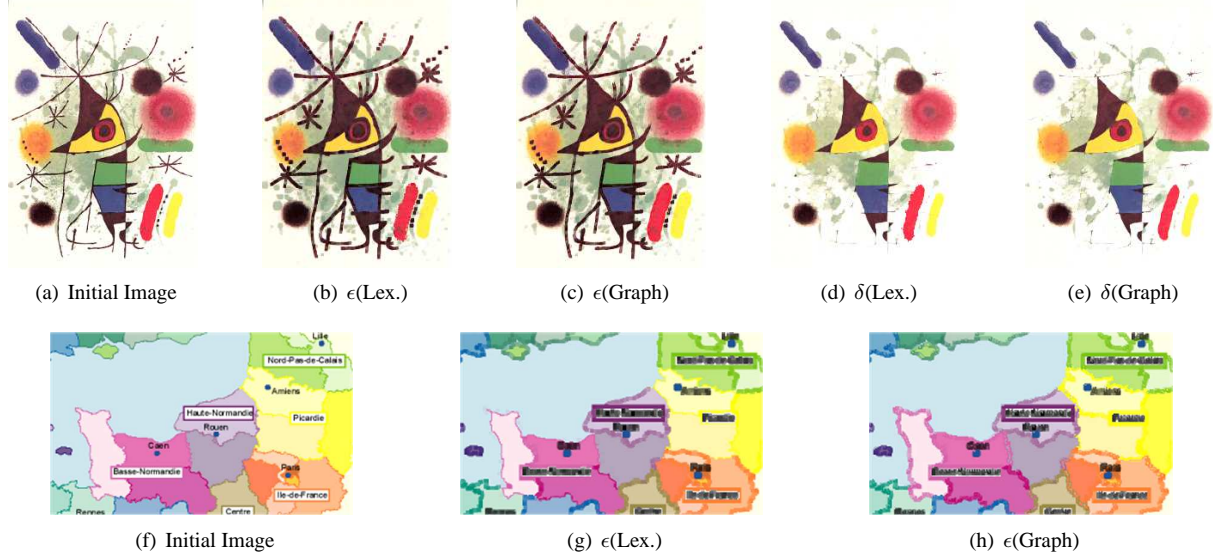
Every morphological operation is defined for a pixel over a structuring element  $B$ . Using the erosion and the dilation the aim is to find the  $\wedge$  and  $\vee$ . Ideally, one can use a total ordering such as the lexicographic which fulfills the requirements of a complete color lattice. In this section we propose an alternative to this classic way of performing color mathematical morphology. The main idea is the following: a total order makes a projection of all the 3D color vector pixels on a line which defines a path along all the pixels of the structuring element. One way to realize this is to use a 1D kohonen map but since it has to be performed for each pixel over a given structuring element, this is too computationally expensive. However can we find a faster approximative method to find the  $\wedge$  and  $\vee$  of a set of color vectors? To do this we consider the structuring element  $B$  as a Region Adjacency Graph  $\mathcal{G}_0$  where each pixel of  $B$  is a node of the graph and where nodes are linked according to 8-connectivity. To obtain a total ordering of all the color vectors, one needs to prune the RAG to obtain a path starting from the lower ( $\wedge$ ) to the upper ( $\vee$ ) bounds of the color set. This path must have two properties: it goes through all the nodes and each node is traversed only once. Such a path defines an ordering of the color vectors which is a complete lattice: a total ordering and the definition of the  $\vee$  and  $\wedge$ . It is difficult to easily find such a path over all the possible ones in  $\mathcal{G}_0$ , so we propose to make an approximation of this path by computing the Minimum Spanning Tree (MST) of the RAG  $\mathcal{G}_0$  where all the links are valued by the  $L_2$  norm between two nodes (i.e. colors). The MST is close to the solution we aim at but not exactly [18]. Figure 1 illustrates this point, the

MST of the RAG  $\mathcal{G}_0$  is computed and the nodes can be classified either as internal nodes and as leaves (final nodes). If



**Fig. 1.** Decimation process of the MST of the neighborhood RAG. From left to right: the RAG  $\mathcal{G}_0$  and the successive  $\text{MST}(\mathcal{G}_i)$  with  $i \in [0, 3]$ .

the MST is an ordering of the color vectors, it defines a path along the nodes (i.e. the colors): this path having only two leaves and each internal node is connected to exactly two nodes (this defines a projection of the set of color vectors on a line). This rarely is the case, however we can make an assumption on the properties of the nodes of the MST since it is a generalization to higher dimensions of a one dimension sorted list [18]. If a node of the MST is an internal node it cannot be the  $\vee$  or the  $\wedge$  of the set of colors, since it would have been considered as a leave following the path ordering the color vectors. To find the  $\vee$  and the  $\wedge$  of a set of color vectors, one can use the latter property by building a new RAG  $\mathcal{G}_1$  constituted of only the possible colors: the leaves of the MST of  $\mathcal{G}_0$ . Each node of  $\mathcal{G}_1$  is linked to all its other nodes (leaves of  $\text{MST}(\mathcal{G}_0)$ ) and the MST of  $\mathcal{G}_1$  is computed. This process can be iterated until the MST obtained at the iteration  $i$  is reduced to two leaves defining the  $\vee$  and the  $\wedge$ . However it remains to define which leave is the  $\vee$  (respectively the  $\wedge$ ). The  $\wedge$  is considered as the leave being closer to a reference color (usually the black). The use of a reference color has also been used in several works on color mathematical morphology [3, 19]. The complete Algorithm is summed up in the Algorithm 1. Using this algorithm, one can always define the  $\vee$  and the  $\wedge$  for a set of color vectors using an iterative decimation of the Minimum Spanning Tree built on the color vector candidates RAG. Figure 1 provides a complete illustration of the determination of the  $\vee$  and the  $\wedge$ : The first step is RAG  $\mathcal{G}_0$ , the other steps present  $\text{MST}(\mathcal{G}_{i-1})$  for  $i \in [0, 3]$  until the termination criterion is reached. In this case the reference color is black, therefore the  $\vee$  is the top right pixel and the  $\wedge$  the bottom left pixel. Once these two bounds have been determined, erosion and dilatation can be performed on the considered neighborhood of pixels.



**Fig. 2.** Comparison of Lexicographic and Graph erosion and dilatation on two sample images.

```

λ : integer;
n : integer;
end : bool;
Cref : a reference color;
end ← false
λ ← 0
Build  $\mathcal{G}_\lambda$  on the structuring element  $B$ 
Repeat
  Compute MST( $\mathcal{G}_\lambda$ )
  n = numberOfLeaves(MST( $\mathcal{G}_\lambda$ ))
  If (n=2) then
    end ← true;
  else
    λ ← λ + 1
     $\mathcal{G}_\lambda = \text{LeavesOf}(\text{MST}(\mathcal{G}_{\lambda-1}))$ 
    Link pairwise all the nodes of  $\mathcal{G}_\lambda$ 
  end If
until (end= true)
 $\mathcal{G}_\lambda = (\{V_1, V_2\}, \{E_1\})$ 
 $\vee = \text{argmax } d(V_i, C_{ref})$  and  $\wedge = \text{argmin } d(V_i, C_{ref})$ 

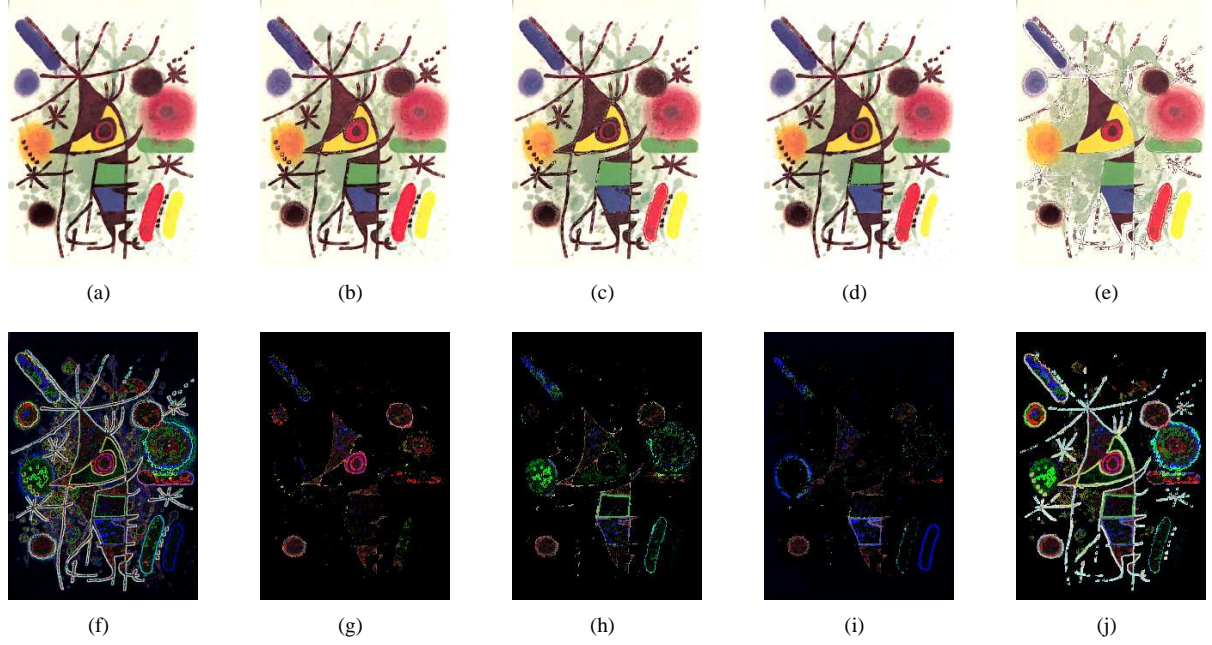
```

**Algorithm 1:** Determining the  $\vee$  and the  $\wedge$  for a set of color vectors.

Figure 2 presents a comparison between the classical lexicographic order and the proposed graph approach. The figure 2(a) presents an image of the painting "The Singer" by Joan Mirò (used in HANBURY's works [3]). Figures 2(b) and 2(d) present the erosion and dilatation results with the lexicographic order. The figures 2(c) and 2(e) present

the erosion and dilatation results with our graph approach. Everything is performed in the  $RGB$  color space with a square  $3 \times 3$  structuring element. The main advantage of our approach is that it gets rid of the disadvantage of the lexicographic order which gives a dominant role to the first component (the red here). This leads to obtaining bad visual results in some red and yellow zones of the image when the lexicographic order is used whereas our approach gives good results for these colored zones. This is all the more evident when applying operations on the image of figure 2(f). An erosion operation will accentuate the borders of the objects, however with the lexicographic order the results are not homogeneous and some colors play a dominant role. This leads to bad visual results for the red and yellow boundaries (figure 2(g)) since one would have expected the erosion to perform in a similar way everywhere in the image. Figure 2(h) presents the result of our graph approach and one can see that the defects of the classical lexicographic order are avoided.

The reference color used at the last level of the decimation of the MST has an influence on the determination of the  $\vee$  and the  $\wedge$ . Usually one will take black as a reference color, however for some special considerations, it might be interesting to use another color. We have performed an erosion with different reference colors: black, red, green, blue and yellow. All the results are presented in figure 3(a)-(e), each column of the first row corresponds to a reference color. The first row gives an erosion of figure 2(a) with the five different reference colors. To illustrate the differences between the obtained results, we first computed the difference between the original image and black erosion (figure



**Fig. 3.** Examples of the influence of the reference color (black, red, green, blue and yellow : from left to right) on the graph erosion.

3(f)). This gives an idea of the modifications made by the erosion in the image. To see the influence of the reference color, the differences between black erosion and the other reference color erosions are computed. The differences between the obtained erosions are given in figure 3(g)-(j). As expected, changing the reference color changes the way the erosion acts on some colors since it favors the colors close to the reference: look at the eye of the singer for the red erosion and the yellow elongated pattern (bottom right) for the blue erosion. This becomes quite clear for the result of yellow erosion: the majority of the colors of the image are closer to yellow than to black and this last color tends to disappear in yellow erosion.

## 4. HIERARCHY OF PARTITIONS

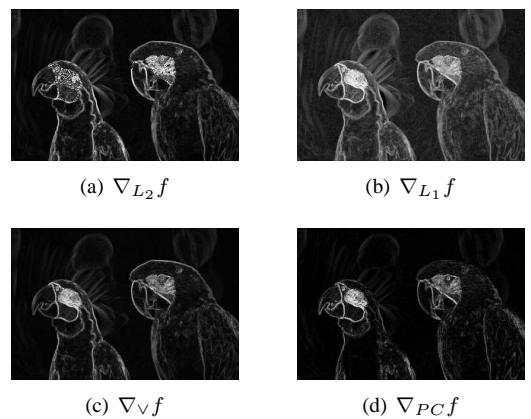
### 4.1. Morphological Gradient

The color gradient can have different formulations when color is considered [20]. Here we propose different color gradients with respect to our graph approach. The classical definition of the morphological gradient for a gray scale image  $f$  is given by:  $\nabla f = \delta(f) - \epsilon(f)$ . When  $f$  is a color image,  $\delta(f)$  and  $\epsilon(f)$  are color vectors and several definitions of the color morphological gradient can be proposed:

- $L_p$ -norm gradient: it evaluates a distance between the bounds of the colors,  $\nabla_{L_p} f(x) = \|\delta f(x) - \epsilon f(x)\|_p$

- Supremum gradient: it takes the supremum of marginal gradients,  $\nabla_{\vee} f(x) = \vee[\nabla f_1(x), \nabla f_2(x), \nabla f_3(x)]$
- Perceptual gradient: it is used in perceptual color spaces such as  $L^*a^*b^*$ :  $\nabla_P f(x) = \nabla_{L_2} f(x)$ , this gradient is perceptual chromatic ( $\nabla_{PC} f(x)$ ) when  $x$  is described by only chromatic color components.

Figure 4 presents different types of color gradients on a color image (figure 5(a)) computed in  $RGB$  color space and in  $L^*a^*b^*$  color space for the perceptual chromatic gradient.



**Fig. 4.** Examples of color morphological gradients with the proposed graph approach.

## 4.2. Non parametric watershed hierarchy

The watershed transformation is one of the principal mathematical morphology image processing operations. Image segmentation based on the watershed has proved to be a powerful segmentation tool but, unfortunately, when directly applied to an image, this transformation presents a strong over segmentation. One way to suppress this over segmentation is to use a non parametric hierarchy of watershed also known as the waterfall algorithm [21]. A watershed being a way to produce a partition of an image, the waterfall algorithm produces a hierarchy of partitions which forms a complete lattice [22]. The waterfall algorithm for a color image  $f$  defined on a given color gradient  $\nabla(f)$  can be computed in an efficient way and one can refer to [20] for more details. Since we have defined several color gradients from the erosion and dilatation graph based operations, we can build a pyramid of segmentations by the waterfall algorithm. As there are several color gradient formulations, there are of course several possible hierarchies of partitions. Some results of segmentation are presented in figure 5 for a given level (5 or 6) of the pyramid. We have compared the results for different color gradients. The first is a supremum based color gradient computed with the lexicographic order. All the others are based on gradient computed with our graph approach: a supremum based gradient, a  $L_2$  gradient, a  $L_1$  gradient, a perceptual gradient. Everything is performed in the  $RGB$  color space and in the  $L^*a^*b^*$  color space for the perceptual gradient. One has to notice that during the waterfall algorithm, everything is performed in the same color space. The first thing to point out is that the hierarchy is smoother when our graph approach is used and it is necessary to go to higher levels of the pyramids than with the lexicographic order. Considering the supremum based gradient, one obtains 235 regions at the 5th level with the lexicographic order and 300 regions at the level 6 with the graph approach. As regards all the segmentations obtained with different color gradients, it appears that the  $L_1$  and the perceptual color gradient produce the best visual results while better preserving the regions borders. The  $L_1$  norm is all the more interesting since it produces few regions within a lower level of the pyramid.

## 5. CONCLUSION

A graph approach to color mathematical morphology has been proposed based on the decimation of the MST of the RAG for a given structuring element. This approach enables to find the supremum and the infimum of a set of color vectors, therefore the main morphologic operations can be performed: erosion and dilatation. Moreover the approach can be used whatever the color space and the defects of the classical lexicographic order are avoided. Further developments will concern the definition of a graph partial ordering



**Fig. 5.** Several segmentations for different graph color gradient with the waterfall algorithm.

based on the proposed graph definition of the infimum and supremum.

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