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Quantification of GNSS signals accuracy: an image segmentation method for estimating the percentage of sky

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Abstract—This paper is focused on the characterisation of the GNSS reception signals environment by estimating the percentage of visible sky. The estimation is based on a new segmentation technique that uses color and texture gradients with an adaptive and non-parametric combination strategy. The structural gradient, resulting from the combination, is processed with the watershed algorithm to get image segmentation. The classification process used to extract the sky region is performed using the k-means algorithm. Experimental segmentation and classification results, using real data and an evaluation methodology, are presented to demonstrate the effectiveness and the reliability of the proposed approach.

I. INTRODUCTION

Most of ITS applications rely on positioning system. Most of them are based on GNSS (Global Navigation Satellite System) such as GPS. As other radio-navigation solutions, satellite-based systems use propagation time measurements for positioning. Indeed, each satellite of the constellation broadcasts continuously its own signal and the role of the receiver is to estimate the propagation time in order to translate it into a distance, called “pseudorange”. By triangulation, three simultaneous pseudoranges allow the receiver to compute its position in a 3D absolute referential. However in constrained environment, such as urban area, signals can be reflected or blocked by obstacles (building, trees, ...). Reflections add a delay to the propagation time estimation, in particular when the direct path of the signal is not available. In order to ensure a safe and accurate position to the user, mitigation techniques can be applied [1]. The simplest one consists in excluding corrupted signals when detected but this policy enhances unavailability of the service. A more efficient approach would be, if possible, to use all available signals, weighting or estimating noises to reduce the error induced [2]. At the receiver level, the detection of disturbed signal is not always easy. Some criteria can be used such as signal to noise ratio (SNR) but a threshold to distinguish direct signal or not has to be chosen [3]. The method developed in this paper uses a video record of the environment surrounding the GPS antenna. This system has been developed for the first version of the PREDISSAT

tool [4] and has inspired other developments with infra-red cameras [5], [6]. The goal of this paper is to benefit from new image processing developments in order to enhance satellite state reception and masking elements.

The aim consists in determining the percentage of visible sky. For that, the process starts by segmenting the image into regions of interest according to a given criteria. After that, the obtained regions have to be classified into at least two classes, which correspond to the sky and the rest of the image (vegetation, buildings, ...). Many segmentation methods are proposed in the literature. Most of them can be grouped into two categories: edges and/or regions based segmentation. These methods are generally developed considering specific applications. Therefore, there is no method that can be successfully applied for any application. In the context of our application, images present two important informations that are color and texture. It is hence useful to use a segmentation method based on these informations. ANGULO [7] proposes a segmentation method combining color and texture informations. However, this method involves many parameters, which are difficult to adjust according to the considered application. The paper proposes to define a new segmentation method based on a non-parametric and adaptive combination of color and texture gradients. The proposed method uses an original technique to take into account the color and texture informations, with an adaptive combination considering local image content.

The paper is organized as follows: Section 2 presents the morphological texture and color gradients definition. Section 3 describes the structural gradient process and the proposed strategy to combine texture and color gradients. The segmentation by color watershed algorithm is explained in section 4. Before concluding, experimental results with real images are presented in section 5.

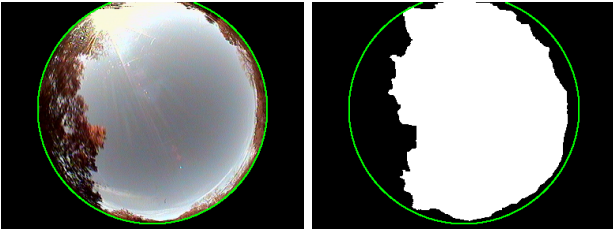


Fig. 1. Testing image (initial image and ground truth reference)

II. MORPHOLOGICAL TEXTURE AND COLOR GRADIENTS DEFINITION

The process to obtain a texture gradient can be divided into the four following stages: 1/ Image filtering; 2/ Extraction of a texture layer; 3/ Granulometric analysis; 4/ Morphological texture gradient computation. The whole process of obtaining a texture gradient is based on mathematical morphology concepts and described in literature (see [7] for details). For a better visualisation, all these stages are illustrated on a real image presented in figure 1.

A. Image Filtering

The first stage of the process consists in image filtering. The aim is to simplify textures and delete structures/regions that are not significant for the final segmentation without losing the original objects outlines. The chosen filter is a connected leveling filter. This kind of filter preserves homogeneous zones and removes noise. Let f be an image, g a generic image called “marker” (usually defined as a simplification of the first image), δ the morphological dilation and ϵ the morphological erosion. The leveling operation can then be defined, for each pixel p of the image f , as the iteration until idem-potency of the following operations:

$$g' = \lambda(f, g) \quad (1)$$

$$g'_p = \begin{cases} f_p \wedge \delta(g)_p = \min(f_p, \delta(g)_p) & \text{if } g(p) < f(p) \\ f_p \wedge \epsilon(g)_p = \max(f_p, \epsilon(g)_p) & \text{if } g(p) > f(p) \end{cases} \quad (2)$$

The used marker is a simplification of the original image. It is obtained by means of an Alternating Sequential Filter (ASF), which is defined as a series of morphological openings γ_n and closings φ_n of incremental size. The filtered image f' is obtained by the following relation:

$$f' = \lambda(ASF_n(f), f) \quad (3)$$

$$ASF_n(f) = \varphi_n \gamma_n \dots \varphi_2 \gamma_2 \varphi_1 \gamma_1(f) \quad (4)$$

B. Texture Layer

For the definition of a texture layer, only the luminance component of the HSL color space is taken into account, since texturing is a phenomenon associated to light intensity variation and not to wavelength (color). The texture layer,

referred to as f_{tex} , is obtained as the difference between the luminance component of the original image f_l and the luminance component of the filtered image f'_l as $f_{tex} = f_l - f'_l$. Indeed, the filtered image can be interpreted as a non-textured image, while the original image contains textures with other information. By applying subtraction between these images, it is expected to remove all non-pertinent information and keep the textures. The first four images of figure 2 illustrate the process of the texture layer.

C. Granulometric analysis

A granulometry is the study of the size distribution of the objects of an image based on the principle of sieving. It can be defined as a family of transformations $(\Psi_n)_{n \geq 0}$ that depends only on a parameter n and has the following properties:

- $\Psi_0(f) = f$
- $\forall n \geq 0, \Psi_n$ increasing: $f \leq g \Rightarrow \Psi_n(f) \leq \Psi_n(g)$
- $\forall n \geq 0, \Psi_n$ anti-extensive: $\Psi_n < Id$
- Absorption law: $\forall n \geq 0, \forall m \geq 0, \Psi_n \Psi_m = \Psi_m \Psi_n = \Psi_{\max(n,m)}$

For a connected structuring element, the family of increasing openings $(\gamma_n)_{n \geq 0}$ is a granulometry. The family of increasing closings $(\varphi_n)_{n \geq 0}$ is considered as an anti-granulometry, or a granulometry but for dark structures. Granulometric analysis of an image f consists on the evaluation of every opening of size n with a measure M defined as the summation of the values of the pixels of the opening:

$$M(\gamma_n(f)) = \sum_{p \in \gamma_n(f)} v(p) \quad (5)$$

Since an opening of size n is composed of all bright structures from f whose size is bigger than n , M can be considered as the number of bright structures in f with a size bigger than n . By analogy, the same concept can be applied when M is computed for a closing. In this case, the measure refers to dark structures. At this stage, a Pattern Spectrum PS can be defined as:

$$PS(f, n) = \begin{cases} M(\gamma_n(f)) - M(\gamma_{n+1}(f)) & \text{if } n \geq 0 \\ M(\varphi_n(f)) - M(\varphi_{n+1}(f)) & \text{if } n \leq -1 \end{cases} \quad (6)$$

$PS(f, n)$ measures the number of bright structures of size n in the image f if $n > 0$ (resp. $n < 0$). The pattern spectrum $PS(f, n)$ (illustrated in figure 2) indicates the distribution of structures associated to each scale n . It can be seen as a histogram. It can be used as a descriptor for texture classification. Nevertheless, it's computed globally for the entire image, while the expectation is to obtain more than one texture per image. Indeed, the classification should be applied for each pixel of the image f . To solve this problem, the granulometric analysis is computed locally for each pixel. This can be done by calculating $PS(f_{Wx}, n)$ on a window centred on each pixel x . It is defined as the local energy image of size k the image that groups all the local pattern spectrums of size k of each pixel of the image. In a k -sized local energy

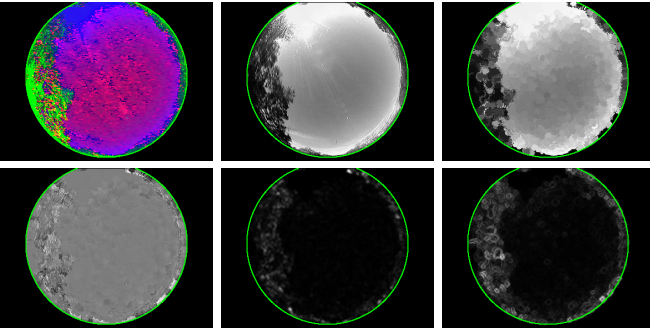


Fig. 2. Illustration of texture gradient (top to bottom and left to right: image in HSL color space, original Luminance; filtered Luminance; texture layer; pattern spectrum with $n=1$; texture gradient with $K = \{-6, -4, -2, 2, 4, 6\}$).

image, pixels associated to a structure of size k are affected with a high value. The images obtained by this procedure are then normalized.

D. Morphological texture gradient definition

A morphological texture gradient is defined using references to the concept of local energy image, presented previously. A morphological gradient is defined, for an image g , as the residue of dilatation and erosion (computed usually with a structuring element of size 1).

$$Q(g(x)) = \delta_\beta(g(x)) - \epsilon_\beta(g(x)) \quad (7)$$

The k -sized local energy image computed for the texture layer f_{tex} will be referred to as t_k . The texture gradient of the image, illustrated on the figure 2, is finally defined in reference to the morphological gradient of each t_k , and where K is a set of numbers varying from $[-n; n]$.

$$Q_{tex}(f_{tex}(x)) = \bigvee_{k \in K} [Q(t_k(x))] \quad (8)$$

E. Morphological color gradient definition

The classical definition of the morphological gradient for a gray scale image f is given by: $\nabla f = \delta(f) - \epsilon(f)$. The extension of gray scale image algorithms to color or vector valued images is not simple since there is no natural ordering on a set of color vectors, and more generally of multivariate data. According to Barnett [8], there are several possible types of multidimensional vector ordering: marginal ordering, reduced ordering, partial ordering and conditional ordering. In this paper, the conditional ordering is considered.

Let x_1, x_2, \dots, x_n denote a set of n p -dimensional vectors: $x_i = \{x_{1(i)}, x_{2(i)}, \dots, x_{p(i)}\}, x_i \in R^p$. In the conditional (also called lexicographic) ordering, the vectors are ordered according to a hierarchical order of their components. For two vectors x_i and x_j , one has:

$$x_i \leq x_j \begin{cases} x_{1(i)} < x_{1(j)} \text{ or} \\ x_{1(i)} = x_{1(j)} \text{ and } x_{2(i)} < x_{2(j)} \text{ or } \dots \\ x_{1(i)} = x_{1(j)} \text{ and } x_{2(i)} = x_{2(j)} \dots x_{p(i)} < x_{p(j)} \end{cases}$$

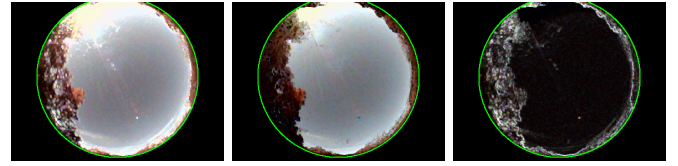


Fig. 3. Illustration of color gradient (left to right: morphological dilatation; morphological erosion; morphological color gradient).

If f is a color image, $\delta(f)$ and $\epsilon(f)$ are color vectors and the classical morphological color gradient f is given by: $\nabla f = \delta(f) - \epsilon(f)$. The different stages of the process are illustrated in figure 3.

III. STRUCTURAL GRADIENT DEFINITION

In order to achieve a robust and reliable segmentation, it is very useful to use both texture and color information. The main idea is to produce a structural gradient by combining the texture and color gradients. The problem is the fact that the color gradient is a color image while the texture gradient is a gray level image. To solve this problem, the proposed method starts by decomposing the color gradient image Q_{col} to obtain the three components which are Q_{col}^R, Q_{col}^G and Q_{col}^B . In the next step, each component of the color gradient image is combined with the texture gradient image Q_{tex} . This operation produces three gray levels images Q^R, Q^G and Q^B :

$$\begin{cases} Q^R = Q_{col}^R \otimes Q_{tex} \\ Q^G = Q_{col}^G \otimes Q_{tex} \\ Q^B = Q_{col}^B \otimes Q_{tex} \end{cases} \quad (9)$$

Q^R, Q^G and Q^B can be interpreted as the color components of a new color image, which is proposed to define the needed structural gradient. In other words, the structural gradient Q is defined as a color image with Q^R as the red component, Q^G as the green component, and Q^B as the blue component. This combination approach is suitable because, not only color information is preserved, but also the texture information is added to each color component. Indeed, texture, which is not a color phenomenon, is supposed to affect all colors equally. To combine a component of the color gradient image f and the texture gradient image g , two techniques are used: fixed combination and adaptive combination. An illustration of these combinations is given in figure 4. Let h be the output of the procedure which is applied for each pixel.

A. Fixed combination

$$h(p) = \alpha f(p) + (1 - \alpha) \times g(p) \quad (10)$$

with α , a constant coefficient taking its value in $[0; 1]$. The combination technique is not generally suitable, due to the coefficient α , which is constant for the entire image. Indeed, one may need to give priority to color or texture according to their importance in the different zones of the image. This technique (needs to adjust the coefficient α to adapt it manually to the content of the image) requires manual adjustment of a α according to the content of the image.

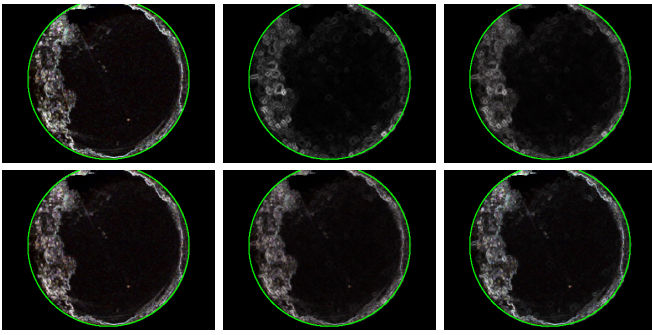


Fig. 4. Illustration of the different gradient combinations (top to bottom and left to right: with $C=1/T=0$; with $C=0/T=1$; with $C=0,2/T=0,8$; with $C=0,8/T=0,2$; with $C=0,5/T=0,5$; with adaptive combination).

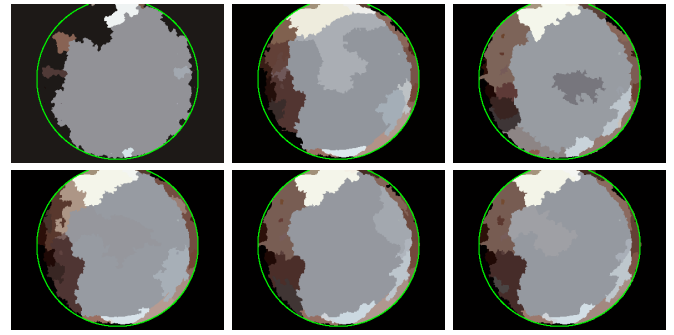


Fig. 5. Segmentation results with different combination (top to bottom and left to right: segmentation with $C=0/T=1$; with $C=1/T=0$; with $C=0,2/T=0,8$; with $C=0,8/T=0,2$; with $C=0,5/T=0,5$; with adaptive combination).

B. Adaptive combination

$$h(p) = \alpha_p f(p) + (1 - \alpha_p) \times g(p) \quad (11)$$

α_p is a coefficient taking its value in $[0; 1]$. α_p is calculated for each pixel p in order to give a high weight to the image that provides the important information for the pixel. In other words, α_p is high if the information is more important for f than for g (and vice-versa). It is computed as follows:

$$\alpha_p = \frac{f(p)}{f(p) + g(p)} \quad (12)$$

IV. SEGMENTATION BY THE COLOR WATERSHED ALGORITHM

The watershed algorithm is one of the main mathematical morphology image processing operations [9], [10]. It allows to segment an image into homogeneous regions from a seeds image (markers) and a potential image (gradient). Image segmentation based on the watershed algorithm has proved to be a powerful segmentation tool but, unfortunately, when directly applied to an image, this algorithm presents a strong over segmentation. One way to suppress this over segmentation is to use a non-parametric hierarchy of watershed, also known as the waterfall algorithm [11]. Several authors propose different types of gradients including several ordering of color vectors [12]–[15]. But in the context of our application, the processing time this approach would be too important. This is why we prefer to define a specific image of seeds positioned experimentally and adapted to our application. The watershed algorithm makes regions growing from the initial seeds using the priority given by the potential image (structural gradient based on color and texture informations).

Figure 5 presents several segmentation results with different combinations of the proposed structural gradient using the image of figure 1. We can see that when using only the texture gradient, the segmentation method extracts correctly the sky, but the results are not satisfactory for the other regions of the image. Contrariwise, using only color gradient, the vegetation is correctly segmented, while the sky is divided into several regions. This shows the interest to combine the

color and texture information. Combining the two gradients with 20% of color and 80% of texture allows improving a little the results but the segmentation remains unsatisfactory. Indeed, the sun and the sky regions are extracted (with an error in the middle of the sky region), but the vegetation is not well extracted. A combination with 80% of color and 20% of texture leads to a correct global segmentation with major errors in the sky region. A good compromise for this image test is to use a combination with 50% of color and texture. Indeed, the segmentation result is globally improved but the sky region is still not well segmented. These experimental tests show that it is difficult to adjust the percentages of color and texture to product an acceptable segmentation result. The fixed combination shows hence its limitation. As we can see, the proposed adaptive combination provides good segmentation result and does not require parameter adjustment, taking into account local image content.

V. EXPERIMENTAL RESULTS

A. Evaluation method

In this section, the discussion is focused on the classification results, which are based on the segmentation results presented in the previous section. The classification process is performed using the k-means algorithm. The goal is to classify the image regions into two classes: sky region and not-sky region. For the evaluation, a classification reference image is created. For each result, five measurements are computed: 1/ the percentage of pixels that actually make part of the sky region in the classified image and in the reference image; 2/ the percentage of pixels classified as sky but do not make part of the sky region in the reference image; 3/ the percentage of pixels classified as not-sky in both images; 4/ the percentage of pixels classified as not-sky but do not make part of the not-sky region in the reference image; 5/ The five measurement, corresponding to the sum of the first and third measurements, can be viewed as the percentage of pixels that are correctly classified. This measurement is illustrated in figure 6 for all the combinations of the figure 4, considering eight different images including sun, cloud, building, vegetation, etc. Compared to the other combination techniques, the proposed method provides good

classification rates, with the segmentation advantages cited in the previous section.

B. Observations

In matters of segmentation, the results obtained when taking into account only the color information are quite satisfying for urban images. This is not surprising since color information is more important for this sort of images than texture information. For the other kind of pictures (cloud and vegetation), sky regions are usually over-segmented and the sunny zones are mostly dilated when using only color information. This problem is also present (although in a smaller amount) with a combination of 80% color and 20% texture. On the other hand, the segmentation results computed using mostly texture information (combination of 20% color and 80% texture) are never satisfying enough. Some improvements can be seen for the most textured images compared to the segmentation results presented previously, specially in terms of sky segmentation (first, forth and sixth pictures).

The best segmentation results are those computed using a 50/50 combination or the adaptive method presented on this paper (lines fifth and sixth of figure 7). The results are, indeed, very satisfying. Nevertheless, some inconsistencies are still present on certain images segmented using the fixed combination. For example, in the second image, a small region appears on the bottom because of the excessive amount of sun. Another example is the seventh image in which the sky is divided into two regions instead of one. As for the sixth image, results obtained with a combination of 80% color and 20% texture beat those obtained with a 50/50 combination which clearly demonstrates that the best parameters for the fixed method vary from image to image. These inconsistencies are not present when applying the adaptive method, which works very well for all kinds of images, specially those featuring great amount of sky and vegetation. Moreover, this method does not require any parameters to be set manually.

As for classification, figure 6 and the last row of figure 7 illustrate the results according to the combination technique. Figure 6 presents the results in several environment types and for all methods of combination. For the sun environment which presents high difficulty ($x = 0$ in figure 6), the classification that uses only the color gradient provides very bad results and cannot be accepted for our application. The classification obtained with the texture gradient remains unsatisfactory.

For the fixed combination, the weight of each gradient must be set manually. When the weighting parameters are set as 80% of color and 20% of texture or as 20% of color and 80% of texture, the classification rate increases, when compared to the rates obtained with the classification without combination. The classification rate reaches good results when the weighting parameters are set equally. One can notice the difficulty to adjust the weighting parameters in order to take into account properly the color and texture informations considering the different environment types (axis x in figure 6).

With the proposed adaptive combination technique, the classification rates are globally equivalent, compared with those

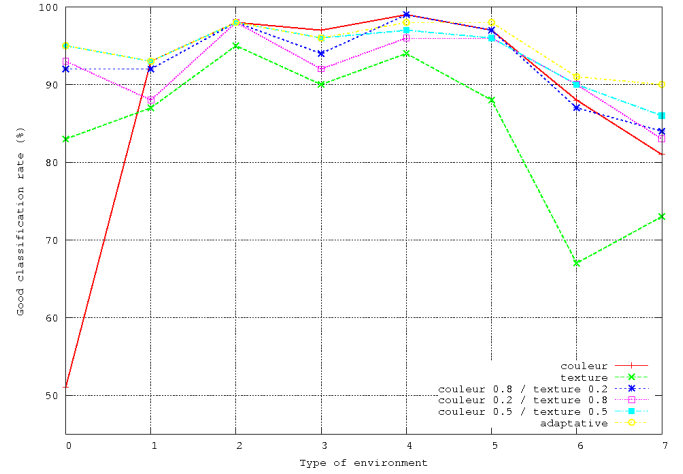


Fig. 6. Good classification rate

provided by the fixed combination. The major advantage of the proposed method is that it allows taking into account automatically the local content of the image and it requires no parameters settings. This advantage is more appreciated in terms of segmentation as shown previously. The last row of the figure 7 illustrates the classification results (according to the classification of reference) into two classes (sky or not-sky regions) obtained by k-means algorithm with the best combination. To conclude, in terms of classification results, the proposed approach performs very well for all image types with an adaptive strategy. If we consider the average percentage of well classified pixels over all images, the adaptive method takes the first place with an average of 95.3 %. Next is the 50/50 fixed combination with an average of 94.6 %. Further behind is the classification which only uses color information with an average of 88.4%.

VI. CONCLUSION

A segmentation method for characterising the environment of GNSS signals reception is presented. The main problem is to estimate the percentage of visible sky. For that, a new segmentation technique combining color and texture gradients is proposed. The originality of the method is that it takes into account the local image content by computing automatically the weighting coefficients for color and texture. The proposed method provides very good results, compared with a method based on a fixed combination of color and texture information. The drawback of this method is that it necessitates a manual adjustment of the weighting coefficients that are global for all the pixels of the image, while the proposed method does not require any parameters setting. The very good segmentation results are exploited to estimate the percentage of visible sky using the k-means algorithm. The classification results of the proposed method are very satisfactory considering those obtained by the other method with a parameter adjustment difficulty. Future works are in progress and concern the analysis in terms of different color spaces and invariant transformation to take into account the illumination changes.

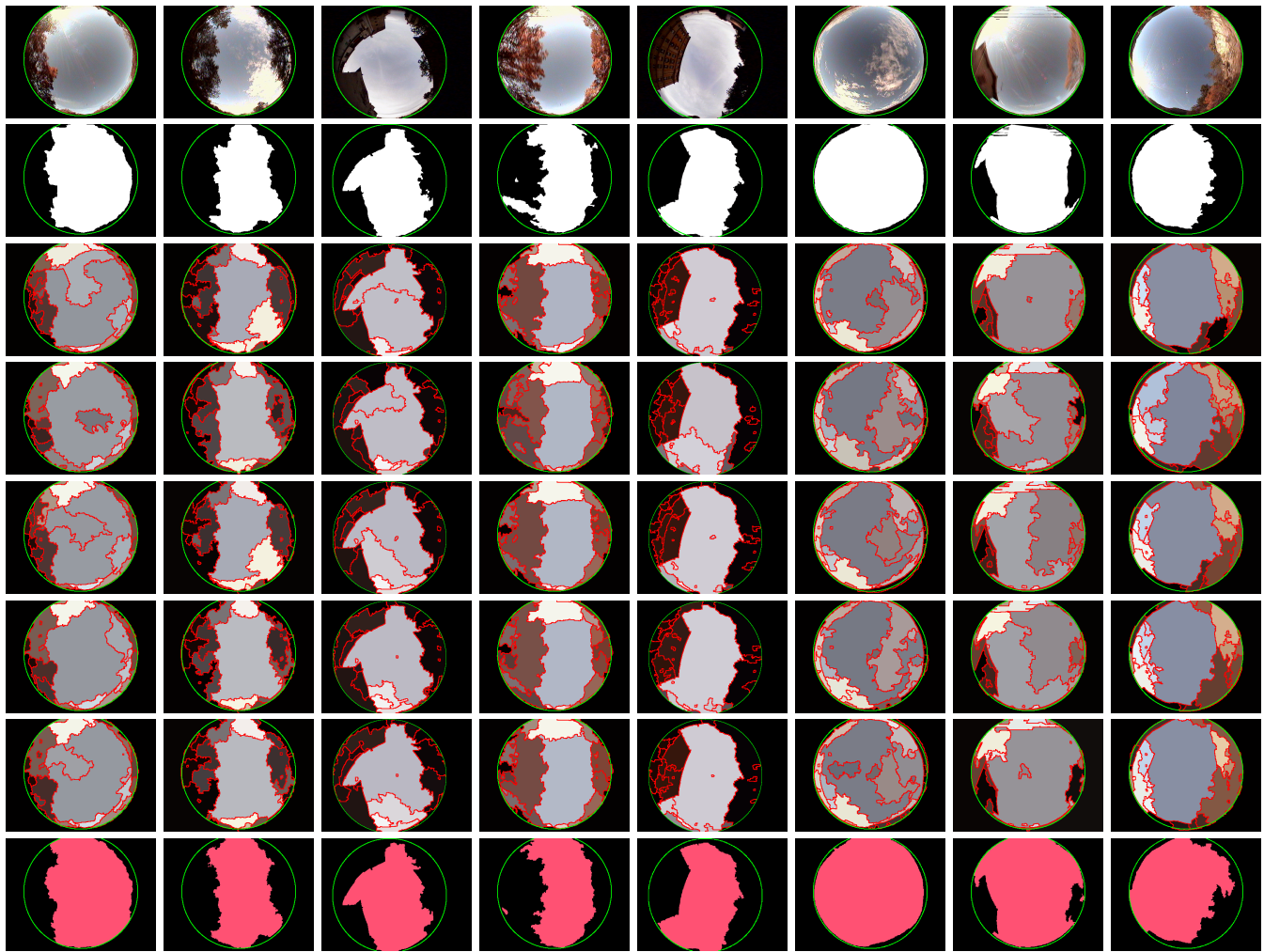


Fig. 7. Segmented images with different combinations (top to bottom : initial image, reference classified image, segmented images (with $C=1/T=0$; with $C=0.2/T=0.8$; with $C=0.8/T=0.2$; with $C=0.5/T=0.5$; with adaptive combination), classified images with the best combination).

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