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► **To cite this version:**

Guibril Zerbo. Arbitration between insurance and self-insurance against natural risks. 2024. hal-04755582

HAL Id: hal-04755582

<https://hal.science/hal-04755582v1>

Preprint submitted on 28 Oct 2024

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Arbitration between insurance and self-insurance against natural risks *

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Abstract

This article studies the effect of preferences on an individual's optimal choice between insurance demand and self-insurance in a risk context and then in an ambiguous context. The innovative idea in this paper is to introduce ambiguity about the effectiveness of self-insurance to understand the nature of the relationship between insurance demand and self-insurance demand. We show that an increase in risk aversion increases the demand for insurance and decreases the demand for self-insurance. However, when risk is introduced on the effectiveness of self-insurance, we show that the individual still prefers self-insurance to market insurance. We also show that when ambiguity is introduced on the effectiveness of self-insurance, the individual always prefers market insurance to self-insurance. Finally, we determine the conditions under which the individual's self-insurance effort is higher or lower under ambiguity than risk.

Key words : Natural risks, arbitration, insurance, self-insurance, efficacy, risk, ambiguity

JEL Classification: D81. G22. Q54

*I want to thank all the participants in the University of Paris Nanterre doctoral student seminars and the International Risk Day in Niort for their comments and suggestions on my presentation. I want to thank Meglena Jeleva, Olivier Renault, Ludovic Julien, Saïd Souam, and Eric Langlais for their constructive comments, massive support, and helpful advice, which helped to improve the document's quality considerably.

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1 Introduction

Natural disasters such as floods and droughts are events whose probability of occurrence and intensity are difficult to estimate. Because these estimates are based on few observations and climate change affects the frequency and severity of these natural hazards, making an accurate estimate remains a difficult task. These natural disasters caused by global warming result in significant human, material, and financial losses. Between 2001-2015, some 1,391 natural disasters occurred in France and its overseas departments, causing the direct or indirect death of 25,193 people and costing more than €27 billion in damage. (CATNAT 2001-2015) ¹. Between 2001 and 2020, 18,142 damaging natural events were recorded worldwide. These events resulted in the direct or indirect death of 1,354,792 people and cost more than 3,445 billion dollars in damage.(Catnat 2001-2020) ². However, while property losses worldwide are in the billions of dollars, the percentage of insured losses remains relatively modest despite technological developments in the insurance industry and the increased involvement of governments in compensation. The same report indicated that insurance and reinsurance companies covered only 30% of losses. This large proportion of uninsured losses results from poor management of catastrophic risks and underdeveloped, sub-optimal prevention systems. Implementing prevention and insurance activities to reduce devastating risk losses is essential in global warming and constitutes a significant public policy challenge. Unlike insurance, which compensates victims once a disaster has occurred, prevention seeks to reduce the probability of occurrence or to reduce the consequences. As these two services are both costly, the relationship between insurance and the type of prevention arises. That is to say, self-insurance (action aimed at reducing the extent of the damage) or self-protection (action aimed at reducing the probability of the damage). Ehrlich and Becker (1972) have shown in their study on market insurance, self-insurance, and self-protection that self-insurance is always a substitute for insurance, whereas self-protection and insurance can be complementary. A little further on, Dionne and Eeckhoudt (1985) have shown that an increase in risk aversion leads to a rise in the demand for self-insurance, whereas the same is not necessarily true for the demand for self-protection. In a study on ambiguity aversion and the propensity to self-insure and self-protect, Snow (2011) showed that the optimal levels of self-insurance and self-protection for a decision-maker adverse to risk and ambiguity are higher in the presence of ambiguity and increase with greater ambiguity aversion. In this study, the author considered that the decision-maker is supposed to be uncertain about the probability of the loss occurring and is, therefore, in a situation of ambiguity. For Chiu (2000), an individual more averse to risk has a higher propensity for self-protection if the probability of initial loss is below a certain threshold. Brunette et al. (2009) showed in their

¹©Ubyrisk Consultants – CATNAT.net "2001-2015 : 15 ans de catastrophes naturelles en France"

²©Ubyrisk Consultants – CATNAT.net "Catnat 2001-2020 bilan statistique de 20 ans de catastrophes naturelles dans le monde"

experimental study on insurance, public intervention, and ambiguity with private forest owners that a forest owner averse to ambiguity chooses a higher demand for insurance for an ambiguous risk than an unambiguous risk. Similarly, work by [Alary et al. \(2013\)](#) has shown that ambiguity aversion always increases the demand for self-insurance and insurance but decreases the demand for self-protection. The intuition of the result of the positive effect of ambiguity aversion on the market for self-insurance is that when the insurable state has a low level of wealth, self-insurance reduces the dispersion of conditional expected utility levels. Ambiguity-averse individuals invest more in self-insurance. [Brunette et al. \(2020\)](#) have also shown that in a context of ambiguity, insurance, and self-insurance are substitutable, whether or not the self-insurance effort is observable by the insurer. However, to our knowledge, previous literature needs to pay more attention to the impact of ambiguity on the effectiveness of self-insurance and the relationship between insurance and self-insurance. Yet it is essential to know how an individual behaves when moving from an ambiguous choice to a risky choice or from an ambiguous choice to a particular choice, the results of which are not noticeable. Moreover, self-insurance activities are necessary in climate change where agricultural yields are unstable. However, their effectiveness remains unclear, so studying the impact of the effectiveness of these self-insurance actions on reducing the amount of losses is decisive for the choice made by individuals. The effect of drought on crop yields depends on the variety, the species and its genetic potential, and cultivation techniques, which are often associated with local growing conditions and climatic scenarios. Another example concerns the uncertainty surrounding the environmental benefits of consuming green products due to their complexity and time horizon. As pointed out by [Montoya-Villalobos \(2023\)](#), the current state of knowledge needs to allow us to estimate with certainty the effectiveness of green goods on environmental quality.

Our study differs from the [Brunette et al. \(2009\)](#) study, which introduced ambiguity not on the effectiveness of prevention but on the risk of loss linked to the natural disaster. The article by [Alary et al. \(2013\)](#) also focused on the effect of ambiguity aversion on assurance and self-protection. In this paper, the authors mainly considered specific ambiguous contexts where ambiguity is concentrated in a state of nature. In other words, the final wealth distribution is not vague, subject to the fact that this state does not occur. In the case of self-insurance, they examined the impact of ambiguity aversion on optimal self-insurance in favor of the state where the probability distribution is ambiguous to increase wealth in that state. In the case of self-protection, they studied the impact of the level of effort on the probability of the state where ambiguity is concentrated. Similarly, the article by [Brunette et al. \(2020\)](#) studied the effect of the unobservability by the insurer of the insured's self-insurance effort on the substitution relationship between insurance and self-insurance in a context of risk, then in a context where the insured only has ambiguous knowledge of his accident risk. In other

words, the ambiguity studied relates to the probability of loss, and the individual now only has ambiguous understanding of the risk he is running. In this paper, we introduce ambiguity not on the probability of loss as was the case in these previous studies, but on the effectiveness of self-insurance in order to examine the optimal behavior of a rational individual when he does not know to what extent his self-insurance action will reduce the level of potential damage. Thus, the research question in this paper is as follows: What is the optimal behavior of an individual between the demand for insurance and the demand for self-insurance when ambiguity concerns the effectiveness of self-insurance? The aim of this paper is twofold. Firstly, the paper uses the [Von Neumann et Morgenstern \(1944\)](#) utility expectation model to study the impact of preferences in characterizing the optimal choice of a rational individual between insurance demand and prevention demand when the probability of the loss is known. Risk is defined as a perfectly-known and measurable hazard, i.e., one whose probability distributions are assumed to be known by the individual. The results are that an increase in risk aversion reduces the demand for self-insurance and increases the demand for insurance. The positive effect of an increase in risk aversion on the demand for insurance results from the efficiency of the self-insurance technology. Moreover, when risk is introduced, the individual always increases the demand for self-insurance, contrary to the demand for insurance. Secondly, the article uses the alpha maxmin utility expectation model of [Ghirardato et al. \(2004\)](#) to examine the impact of preferences on the optimal choice of a rational individual between requesting insurance and requesting self-insurance when the ambiguity concerns the effectiveness of self-insurance. We speak of ambiguity or non-probabilistic uncertainty when the individual does not precisely know the probability distribution of an event. The results show that an increase in the ambiguity aversion index always increases the demand for insurance and decreases the demand for self-insurance. Finally, we determine the conditions under which the level of self-insurance effort is higher under ambiguity than risk. When the individual behaves optimistically (or pessimistically), the effort under ambiguity is higher (or lower) than under risk.

The remainder of this paper will be structured as follows: Section 2 presents the basic theoretical model. Section 3 studies the trade-off between insurance and self-insurance in the presence of risk. Section 4 deals with the impact of the introduction of risk on the effectiveness of self-insurance and the relationship between demand for insurance and demand for self-insurance. Section 5 deals with the impact of the introduction of ambiguity on the effectiveness of self-insurance and the relationship between the demand for insurance and the demand for self-insurance. Section 6 concludes.

2 Theoretical basic model

An individual's choice to take out an insurance policy involves transferring a share of the risk he faces to an insurance company in return for paying an insurance premium. Therefore, this choice is made in a risky universe and can be studied using the Von Neumann Morgenstern (VNM) utility expectation criterion. Individual preferences are represented by a strictly increasing ($U' > 0$) and concave ($U'' < 0$) VNM utility function (U). We consider an insurance model in which individuals can choose between insurance and self-insurance. The individual has initial wealth W and faces a risk of damage of amount L with probability p . A level of prevention "e" of monetary cost $C(e)$ with $C'(e) > 0$, $C''(e) \geq 0$ affects the amount of the loss L which depends on the self-insurance variable $L(e)$. We consider a self-insurance technology $L'(e) < 0$ and assume that $L''(e) \geq 0$. If the individual takes out an insurance contract, he receives compensation or reimbursement I in the event of a claim in return for payment of an insurance premium $\Pi = (1 + \lambda)pI$. Where $0 \leq I \leq L(e)$ allows over-insurance to be excluded; $\lambda \geq 0$ is the loading rate (management and marketing costs, etc.), and pI is the pure premium (expectation of loss). Insurance companies are assumed to be risk-neutral, offering the same menus of insurance contracts $C(\Pi, I)$ to individuals.

The condition $0 \leq I \leq L(e)$ implies that the level of wealth of the individual without a claim is greater than the level of wealth with a claim footnote $X_B \geq X_A \Rightarrow U(X_B) \geq U(X_A)$ and $U'(X_B) \leq U'(X_A)$. We assume that there are two states of the world: a state with a loss and a state without a loss.

The final wealth, when a loss occurs, is written as :

$$X_A = W - (1 + \lambda)pI - C(e) - L(e) + I \quad (1)$$

The final wealth if no claim has been made is written as follows :

$$X_B = W - (1 + \lambda)pI - C(e) \quad (2)$$

The objective function is given by :

$$EU = pU(X_A) + (1 - p)U(X_B) \quad (3)$$

3 Trade-off between insurance and self-insurance in the presence of risk

The self-insurance effort only affects the extent of the damage (the risk of consequence). We are interested in the optimal choice of an individual between insurance demand and self-insurance demand. The utility expectation of final wealth is written as :

$$EU = pU(W - (1 + \lambda)pI - C(e) - L(e) + I) + (1 - p)U(W - (1 + \lambda)pI - C(e)) \quad (4)$$

3.1 Case of risk neutrality

We begin by analysing self-insurance and insurance behaviour in the risk-neutral case. We consider a utility function of the CRRA (Constant Relative Risk Aversion) type.

$$U(x) = \frac{1}{1 - \gamma} x^{1 - \gamma}$$

Risk neutrality implies that $\gamma = 0$, so the utility function is linear in wealth ($U' > 0$, $U'' = 0$).

The objective function is written as :

$$\begin{aligned} EU &= p(W - (1 + \lambda)pI - C(e) - L(e) + I) + (1 - p)(W - (1 + \lambda)pI - C(e)) \\ &= W - \lambda pI - pL(e) - C(e) \end{aligned} \quad (5)$$

The levels of self-insurance and insurance are given by the first-order conditions :

$$\frac{\partial EU}{\partial I} = -\lambda p < 0 \Rightarrow I = 0 \quad (6)$$

$$\frac{\partial EU}{\partial e} = -pL'(e) - C'(e) \Rightarrow -pL'(e) = C'(e) \quad (7)$$

The program has an interior solution at e if : ³

$$\frac{\partial EU}{\partial e} \Big|_{e=0} > 0 \Rightarrow -pL'(0) - C'(0) > 0 \quad (8)$$

The equation (6) translates that the demand for insurance is zero when the individual is risk neutral. This implies that there is no market insurance since the individual has no reason to insure himself. The individual is indifferent between a decision involving risk and a decision without risk. The equation (7) translates that the individual chooses his self-insurance effort in such a way as to equalize the marginal benefit of the effort with its marginal cost. This implies a demand for self-insurance when the individual is risk-neutral.

3.2 Case of risk aversion

In the remainder of our analysis, we will assume risk-averse behavior on the part of the individual in order to study the effect of preferences on the relationship between the demand for self-insurance and the demand for market insurance. The coefficient of relative risk aversion is given by :

$$R_U(X) = -x \frac{U''(x)}{U'(x)} = -x \frac{-\gamma x^{-\gamma-1}}{x^{-\gamma}} = \gamma$$

The coefficient of relative risk aversion is given by the parameter γ . For γ between 0 and 1, the utility function is strictly increasing and concave. ($U' > 0, U'' < 0$).

L'équation (4) se réécrit:

$$EU = \frac{p}{1-\gamma} [W - (1+\lambda)pI - C(e) - L(e) + I]^{1-\gamma} + \frac{1-p}{1-\gamma} [W - (1+\lambda)pI - C(e)]^{1-\gamma} \quad (9)$$

The utility expectation varies with the level of compensation I . We check that ⁴:

$$\frac{\partial EU}{\partial I} = p[1 - (1+\lambda)p]X_A^{-\gamma} - (1-p)(1+\lambda)X_B^{-\gamma} \quad (10)$$

³Inada's conditions are satisfied

$$\lim_{e \rightarrow \infty} L'(e) = 0, \lim_{e \rightarrow \infty} C'(e) = \infty$$

⁴ X_{AC} is the wealth with accident evaluated in $I = 0$ and X_{SA} is the wealth without accident evaluated in $I = 0$

$$\begin{aligned}
\frac{\partial EU}{\partial I} \Big|_{I=0} > 0 \text{ if and only if } p[1 - (1 + \lambda)p]X_{AC}^{-\gamma} - (1 - p)(1 + \lambda)X_{SA}^{-\gamma} > 0 \\
&\Leftrightarrow \frac{(1 - (1 + \lambda)p)}{1 + \lambda} > \frac{(1 - p)X_{SA}^{-\gamma}}{pX_{AC}^{-\gamma}} \\
&\Leftrightarrow \frac{1}{(1 + \lambda)p} > \frac{(1 - p)X_{SA}^{-\gamma} + pX_{AC}^{-\gamma}}{pX_{AC}^{-\gamma}} \\
&\Leftrightarrow 1 + \lambda < \frac{X_{AC}^{-\gamma}}{(1 - p)X_{SA}^{-\gamma} + pX_{AC}^{-\gamma}} \\
&\Leftrightarrow \lambda < \frac{(1 - p)[X_{AC}^{-\gamma} - X_{SA}^{-\gamma}]}{(1 - p)X_{SA}^{-\gamma} + pX_{AC}^{-\gamma}} \\
\text{Let us note } \hat{\lambda} &= \frac{(1 - p)[X_{AC}^{-\gamma} - X_{SA}^{-\gamma}]}{(1 - p)X_{SA}^{-\gamma} + pX_{AC}^{-\gamma}}
\end{aligned}$$

This condition means the individual agrees to buy an insurance contract if λ , the loading rate is lower than the threshold loading rate $\hat{\lambda}$. This implies that, even if the individual is risk averse, his participation in the insurance market depends on the values of λ . The first-order conditions, therefore, give the inner solution to the program :

$$\begin{aligned}
\frac{\partial EU}{\partial I} = 0 &\Leftrightarrow p[1 - (1 + \lambda)p]X_A^{-\gamma} - (1 - p)(1 + \lambda)pX_B^{-\gamma} = 0 \tag{11} \\
&\Leftrightarrow \frac{pX_A^{-\gamma}}{(1 - p)X_B^{-\gamma}} = \frac{(1 + \lambda)p}{1 - (1 + \lambda)p}
\end{aligned}$$

$$\begin{aligned}
\frac{\partial EU}{\partial e}(I, e) = 0 &\Leftrightarrow -p[C'(e) + L'(e)]X_A^{-\gamma} - C'(e)(1 - p)X_B^{-\gamma} = 0 \tag{12} \\
&\Leftrightarrow -pL'(e)X_A^{-\gamma} = C'(e)[pX_A^{-\gamma} + (1 - p)X_B^{-\gamma}]
\end{aligned}$$

Equation (11) implies that the marginal substitution rate equals the implicit insurance price at the optimum. Comparing equation (11) with equation (6) where the demand for insurance is zero, we find that the individual obtains a marginal benefit by requesting insurance; the effect of risk aversion explains this. The equation (12) means that the individual determines his optimal level of effort by equalizing the expected marginal benefit (in terms of utility) from the reduction in magnitude and the expected marginal cost (in terms of utility).

The second-order conditions are satisfied (**See Appendix A**)

$$\frac{\partial^2 EU}{\partial I^2} * \frac{\partial^2 EU}{\partial e^2} - \left(\frac{\partial^2 EU}{\partial e I}\right)^2 > 0$$

The first-order conditions on e allow us to compare the level of effort in the case where the individual is risk-neutral and in the case where the individual is risk-averse.

From the equation (12):

$$\begin{aligned}
-pL'(e)X_A^{-\gamma} &= pC'(e)X_A^{-\gamma} + (1-p)C'(e)X_B^{-\gamma} \\
\Rightarrow -pL'(e)X_A^{-\gamma} - C'(e)X_A^{-\gamma} &= pC'(e)X_A^{-\gamma} + (1-p)C'(e)X_B^{-\gamma} - C'(e)X_A^{-\gamma} \\
&\Rightarrow pL'(e) + C'(e) = C'(e)(1-p)\left[1 - \frac{X_B^{-\gamma}}{X_A^{-\gamma}}\right]
\end{aligned} \tag{13}$$

We can see that the left-hand side of the equality represents the first-order condition in the case where the individual is risk-neutral. The right-hand side of the equality represents the effect of risk aversion on the level of effort (or the price of risk). In the case of full insurance, wealth in the two states of the world is identical ($X_A^{-\gamma} = X_B^{-\gamma}$), and the right-hand side of the equality cancels out (since the ratio of marginal utilities =1), which corresponds to a zero price of risk. The first-order condition is again found in the case of risk neutrality. This suggests that insurance demand behavior is the same in the presence or absence of risk aversion. If insurance is partial, the right-hand side of the equality is positive because $X_A^{-\gamma} > X_B^{-\gamma}$ since $X_A < X_B$. This corresponds to the case where the price of risk is strictly favorable. As shown in Figure 1 below, as the price of risk increases, the curve shifts to the right, suggesting an increase in the self-insurance effort, even in the presence of market insurance. The intuition is that there is a cost of risk that the individual has to bear, which leads him to make a more significant effort to prevent it.

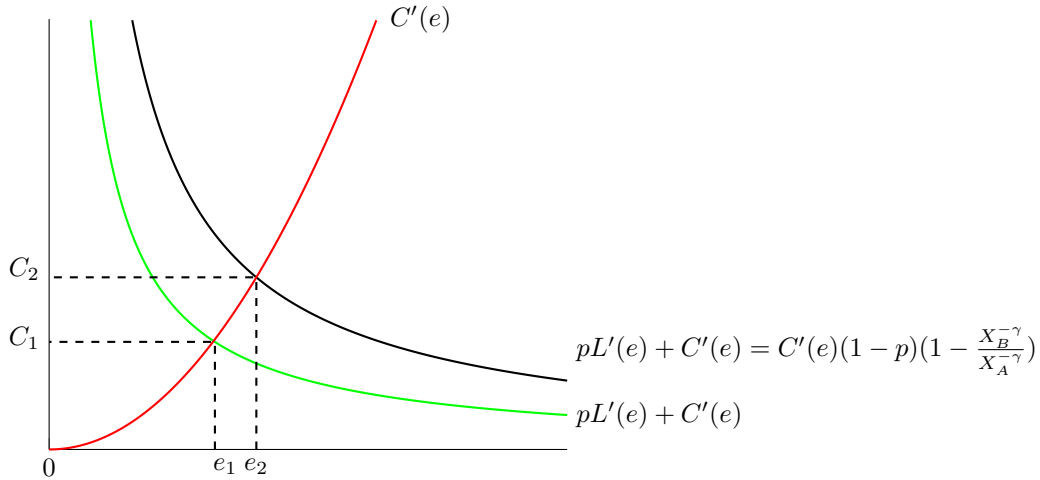


Figure 1

We now seek to determine how the optimal choices of self-insurance and insurance vary according to the increase in risk aversion. We have differentiated the two first-order conditions with respect to the parameter γ .

The results of the comparative statics yield ⁵:

$$\frac{dI}{d\gamma} > 0 \quad \text{and} \quad \frac{de}{d\gamma} < 0$$

There is a decreasing relationship between an increase in risk aversion and the demand for self-insurance and a positive relationship between an increase in risk aversion and the demand for insurance.

Proposition 1: *Increased risk aversion reduces demand for self-insurance and increases demand for insurance.*

This result shows that investment in self-insurance depends negatively on the increase in risk aversion. However, the demand for market insurance increases when risk aversion increases. This result can be explained by the effectiveness of the self-insurance technology, which the individual perceives as being unknown. This makes it riskier to invest in self-insurance activities and does not encourage people to undertake more self-insurance activities. Thus, in this case, greater risk aversion amplifies the risks of self-insurance activities, and the individual will seek to avoid this risk by reducing his self-insurance effort and increasing his demand for insurance. The result concerning the negative effect of an increase in risk aversion on the demand for self-insurance is in opposition to the results of [Dionne and Eeckhoudt \(1985\)](#); [Briys and Schlesinger \(1990\)](#), who concluded that the level of self-insurance activity always increases with an increase in the individual's risk aversion. The nuance of our results results from the fact that in the work of these authors, the effect of increasing risk aversion on self-insurance and self-protection is studied in the absence of market insurance. However, the increase in demand for insurance as risk aversion increases is consistent with the results of [Lambregts et al., 2021](#); [Belissa et al., 2019](#), which showed that an increase in risk aversion increases the demand for insurance. We find that the effect of an increase in risk aversion is opposite to the effect of an increase in price on the relationship between insurance and self-insurance. To see this, we again differentiate the two first-order conditions concerning λ . The results of the comparative statics yield ⁶:

$$\frac{dI}{d\lambda} < 0 \quad \text{and} \quad \frac{de}{d\lambda} > 0$$

The demand for insurance decreases with price, while the demand for self-insurance increases. Insurance and self-insurance are, therefore, substitutes. The increase in the price of insurance leads individuals to reduce their demand for market insurance and turn more to self-insurance. This is consistent with the results of [Ehrlich and Becker, 1972](#); [Courbage, 2001](#); [Corcos et al., 2019](#); [Brunette et al., 2020](#) who showed that insurance and self-insurance are substitutes.

⁵Evidence for the results of the comparative statics is provided in Appendix A

⁶Evidence for the results of the comparative statics is provided in Appendix A

4 Introduction of the risk on the effectiveness of self-insurance

As we pointed out earlier, investment in self-insurance depends on its effectiveness. We will look at the trade-off between the demand for insurance and the demand for self-insurance when the efficiency of self-insurance is assumed to be known. Let's consider that an individual provides an effort "e" according to the self-insurance technology : $L(e) = L_0 - \mu e$. The parameter $\mu \neq 0$, measures the effectiveness of self-insurance. We assume two levels of μ , with $\mu \in \{\underline{\mu}, \bar{\mu}\}$ and $\underline{\mu} < \bar{\mu}$. $\bar{\mu}$ being the best possible result (the best impact that the self-insurance action can have on the amount of damage). When a claim occurs, there is a probability q that the self-insurance effort reduces the level of damage with a high efficiency $\bar{\mu}$ and a probability $1 - q$ that the prevention effort reduces the level of damage with a low efficiency $\underline{\mu}$. We have the following two cases associated with the effect of the self-insurance effort on the amount of damage.

$$L'(e) = \begin{cases} \frac{\partial L(e)}{\partial e} = \underline{\mu} & (a) \\ \frac{\partial L(e)}{\partial e} = \bar{\mu} & (b) \end{cases}$$

(a) An increase of one unit of effort reduces the amount of damage by $-\underline{\mu}$;

(b) An increase of one unit of effort reduces the amount of damage by $-\bar{\mu}$

. Let $\bar{\mu}$ be associated with a damage level $\underline{L}(e)$ and $\underline{\mu}$ be associated with a damage level $\bar{L}(e)$.

The objective function is written as ⁷ :

$$EU = \frac{p}{1-\gamma} [q[W - (1+\lambda)pI - \underline{L}(e) + I - C(e)]^{1-\gamma} + \frac{1-q}{1-\gamma} [W - (1+\lambda)pI - \bar{L}(e) + I - C(e)]^{1-\gamma}] + \frac{1-p}{1-\gamma} [W - (1+\lambda)pI - C(e)]^{1-\gamma} \quad (14)$$

The individual's levels of self-insurance and insurance are given by the first conditions.

$$\frac{\partial EU}{\partial I} = 0 \Leftrightarrow p[q[1 - (1+\lambda)p]X_1^{-\gamma} + (1-q)[1 - (1+\lambda)p]X_2^{-\gamma}] - (1-p)(1+\lambda)pX_3^{-\gamma} = 0 \quad (15)$$

$$\begin{aligned} \frac{\partial EU}{\partial e} = 0 &\Leftrightarrow -pq[C'(e) + \underline{L}'(e)]X_1^{-\gamma} - p(1-q)[C'(e) + \bar{L}'(e)]X_2^{-\gamma} - C'(e)(1-p)X_3^{-\gamma} = 0 \quad (16) \\ &\Leftrightarrow -p[q\underline{L}'(e)X_1^{-\gamma} + (1-q)\bar{L}'(e)X_2^{-\gamma}] = C'(e)[pqX_1^{-\gamma} + p(1-q)X_2^{-\gamma} + (1-p)X_3^{-\gamma}] \end{aligned}$$

⁷ $X_1 = W - (1+\lambda)pI - \underline{L}(e) - C(e) + I$; $X_2 = W - (1+\lambda)pI - \bar{L}(e) - C(e) + I$;
 $X_3 = W - (1+\lambda)pI - C(e)$

The equation (16) implies that the individual chooses his level of effort by equating the marginal benefit of the effort with the marginal cost of the effort. The second-order conditions are satisfied because the determinant of the matrix is positive :

$$\frac{\partial^2 EU}{\partial I^2} * \frac{\partial^2 EU}{\partial e^2} - \left(\frac{\partial^2 EU}{\partial e I}\right)^2 > 0 \quad (\text{See appendix B})$$

We differentiate between the two first-order conditions and then study the impact of the effectiveness of self-insurance on the trade-off between demand for insurance and prevention using comparative statics. Let h be the second derivative of the utility expectation function.

$$h_{eq} = -p[C'(e) + \underline{L}'(e)][X_A^{-\gamma} + p[C'(e) + \bar{L}'(e)]X_C^{-\gamma}] > 0 \quad (17)$$

$$h_{Iq} = p[1 - (1 + \lambda)p][X_A^{-\gamma} - X_C^{-\gamma}] < 0 \quad \text{Car } X_A > X_C \quad \text{et } X_A^{-\gamma} \leq X_C^{-\gamma}$$

$$\frac{de}{dq} = \frac{h_{eI}h_{Iq} - h_{eq}h_{II}}{\tau_{ee}h_{II} - h_{eI}^2} \quad \text{et} \quad \frac{dI}{dq} = \frac{h_{eI}V_{eq} - h_{Iq}h_{ee}}{h_{ee}h_{II} - h_{eI}^2}$$

$$\frac{de}{dq} > 0 \quad \text{and} \quad \frac{dI}{dq} < 0$$

Proposition 2: *In the presence of a risk concerning the effectiveness of self-insurance, individuals always prefer prevention to market insurance.*

As the effectiveness of self-insurance increases, the demand for self-insurance increases, whereas the demand for insurance decreases. It is as if the individual knows to what extent his self-insurance action will reduce potential damage or knows perfectly well the probability distribution associated with the different levels of damage. He will always tend to invest in self-insurance rather than insurance. This result highlights the importance of taking into account the effectiveness of self-insurance in explaining the relationship between demand for insurance and demand for prevention.

5 Introducing ambiguity on the effectiveness of self-insurance

In this section, we study the impact preferences on an individual's optimal choice between insurance and self-insurance when the effectiveness of self-insurance is not precisely known. In a context where climate change forecasts are uncertain as to their amplitude and severity, the effectiveness of prevention activities is less and less

known. This is the case, for example, with the drought resistance of new crop variants and high temperatures to ensure good agricultural yields, the effectiveness of which is only sometimes satisfactory or precisely known because of the complexity of adaptation or the severity or expected duration of the drought. We also determine the conditions under which we compare the self-insurance effort level under ambiguity and risk. This is the case, for example, with the resistance of new crop varieties to drought or the size of dykes needed to contain rivers. In a context of uncertain choice, it is challenging for the individual to formulate a precise belief due to the scarce nature of the information. In this case, economic theory assumes that individuals make up for the lack of information with subjective probabilities that derive directly from their preferences. Thus, to make up for this lack of information, individuals can formulate a single distribution of subjective probabilities or several distributions of subjective probabilities or second-order beliefs. However, the literature has identified several behavioral paradoxes associated with the first approach, the best known of which is the [Ellsberg \(1961\)](#) paradox.

We use the α MaxMin utility expectation model of [Ghirardato et al. \(2004\)](#) to study the impact of ambiguity on the effectiveness of prevention. In addition to being used in numerous theoretical and experimental studies ([Ahn et al., 2014](#); [Ravanelli and Svindland, 2019](#)), The choice of this criterion for our analysis is justified by the fact that it represents a wide range of preferences, from ambiguity-seeking to ambiguity-averse. We, therefore, do not want to exclude individuals who have either extreme pessimism illustrated by the Maxmin model or extreme optimism illustrated by the maxmax model. When the individual engages in a self-insurance action, he does not know the effectiveness associated with this action on the level of damage. If the individual behaves pessimistically, he only focuses on the worst-case scenario. If the individual behaves optimistically, he only focuses on the best-case scenario. The idea is to be able to take account of this heterogeneity of preferences in the optimal choice of insurance and self-insurance demand. The α MaxMin model of utility expectation tempers the extremes of pessimism and optimism, modulating the individual's attitude as a weighted average of the maxmin and maxmax criteria. This model thus evaluates a decision as a weighted average of the minimum and maximum expected utility attainable, given the set of beliefs. The individual maximises his decision which corresponds to a linear combination of the worst and best results by weighting the worst result by the coefficient of pessimism α and the best result by the coefficient of optimism " $1 - \alpha$ ". For $\alpha = 0$ the individual is extremely optimistic and for $\alpha = 1$ the individual is extremely pessimistic. Formally:

$$\alpha \cdot \min_{q \in \Delta} E_q U(D) + (1 - \alpha) \cdot \max_{q \in \Delta} E_q U(D)$$

Where Δ represents a set of subjective additive beliefs or the perceived level of ambiguity, and D represents an act (consequence associated with the decision D). For a given level of ambiguity, the individual's aversion to

ambiguity increases with the value of α , and the lower α is, the less aversion the individual has to ambiguity. In our decision model, we introduce this ambiguity into the parameter μ , which measures the effectiveness of self-insurance with a probability q , which is no longer assumed to be known. Thus, we have a probability q of having an efficiency $\bar{\mu}$ and a probability $1-q$ of having a self-insurance effectiveness $\underline{\mu}$. We assume that q can take only two possible values: $q \in \{\underline{q}, \bar{q}\}$.

$$\alpha \cdot \max_{q \in \{\underline{q}, \bar{q}\}} \min EU(x) = \alpha \cdot \min_{q \in \{\underline{q}, \bar{q}\}} EU(e) + (1 - \alpha) \cdot \max_{q \in \{\underline{q}, \bar{q}\}} EU(e)$$

Let V be the objective function ⁸ :

$$\alpha \cdot \max_{q \in \{\underline{q}, \bar{q}\}} \min V(x) = p[\alpha \underline{q} + (1 - \alpha) \bar{q}]U(X_1) + p[\alpha(1 - \underline{q}) + (1 - \alpha)(1 - \bar{q})]U(X_2) + 2(1 - p)U(X_3) \quad (18)$$

(See appendix C)

The optimal levels of self-insurance and insurance for the individual are given by the first-order conditions ⁹ :

$$\begin{aligned} \frac{\partial V}{\partial e} = 0 &\Leftrightarrow -pr[C'(e) + \underline{L}'(e)]X_1^{-\gamma} - ps[C'(e) + \bar{L}'(e)]X_2^{-\gamma} - 2C'(e)(1 - p)X_3^{-\gamma} = 0 \\ &\Rightarrow -pr\underline{L}'(e)X_1^{-\gamma} - ps\bar{L}'(e)X_2^{-\gamma} = prC'(e)X_1^{-\gamma} + psC'(e)X_2^{-\gamma} + 2C'(e)(1 - p)X_3^{-\gamma} \end{aligned} \quad (19)$$

$$\frac{\partial V}{\partial I} = 0 \Leftrightarrow pr[1 - (1 + \lambda)p]X_1^{-\gamma} + ps[1 - (1 + \lambda)p]X_2^{-\gamma} - 2(1 - p)(1 + \lambda)pX_3^{-\gamma} = 0 \quad (20)$$

The term on the left of the equation (19) represents the expected marginal benefit (in terms of utility) of a one-unit increase in self-insurance and the term on the right represents the expected marginal cost (in terms of utility) of an increase in self-insurance effort. We find that marginal benefit and marginal cost are all affected by the presence of ambiguity.

The second-order conditions are verified (see Appendix C).

$$\frac{\partial^2 V}{\partial I^2} * \frac{\partial^2 V}{\partial e^2} - \left(\frac{\partial^2 V}{\partial e \partial I}\right)^2 > 0$$

We rewrite the equations (12) and (19) so that we can compare the prime conditions on e .

⁸ $X_1 = W - (1 + \lambda)pI - \underline{L}(e) - C(e) + I$; $X_2 = W - (1 + \lambda)pI - \bar{L}(e) - C(e) + I$; $X_3 = W - (1 + \lambda)pI - C(e)$

⁹ $r = \alpha \underline{q} + (1 - \alpha) \bar{q}$; $s = \alpha(1 - \underline{q}) + (1 - \alpha)(1 - \bar{q})$

Equation (12) is rewritten :

$$-pL'(e) - C'(e)[p + (1-p)\frac{X_B^{-\gamma}}{X_A^{-\gamma}}] = 0 \quad (21)$$

Equation (19) can be written as the sum of two first-order conditions :

$$\{-p\underline{L}'(e) - C'(e)[p + (1-p)\frac{X_3^{-\gamma}}{rX_1^{-\gamma}}]\}rX_1^{-\gamma} + \{-p\bar{L}'(e) - C'(e)[p + (1-p)\frac{X_3^{-\gamma}}{sX_2^{-\gamma}}]\}sX_2^{-\gamma} = 0 \quad (22)$$

It is not easy to systematically compare first-order conditions under risk and under ambiguity. In other words, it is not possible to state that in the presence of ambiguity the individual will increase or decrease his self-insurance effort. This suggests that the behaviour of individuals in situations of risk and ambiguity are not the same and that theoretical predictions do not immediately lead to general or unique predictions. We put forward additional hypotheses.

$$\underline{L}(e) < L(e) < \bar{L}(e); \underline{L}'(e) < L'(e) < \bar{L}'(e); 1 - \bar{q} > \frac{X_A^{-\gamma}}{X_B^{-\gamma}} * \frac{X_3^{-\gamma}}{X_2^{-\gamma}}; \bar{q} > \frac{X_A^{-\gamma}}{X_B^{-\gamma}} * \frac{X_3^{-\gamma}}{X_1^{-\gamma}}$$

This last condition means that the probability of the effectiveness of self-insurance associated with the best result is higher than the product between the marginal rate of substitution in risk and in ambiguity. If the damage level $\underline{L}(e)$ occurs. Equation (22) becomes :

$$-p\underline{L}'(e) - C'(e)[p + (1-p)\frac{X_3^{-\gamma}}{rX_1^{-\gamma}}] = 0 \quad (23)$$

We have the following two conditions, depending on whether the individual is optimistic ($\alpha = 0$) or pessimistic ($\alpha = 1$). If the individual adopts an extremely optimistic attitude, the result is :

$$-p\underline{L}'(e) - C'(e)[p + (1-p)\frac{X_3^{-\gamma}}{\bar{q}X_1^{-\gamma}}] = 0 \quad (24)$$

Let e_1^* be the solution to equation (24), and we can make the difference with equation (21) evaluated in e_1^* . We have :

$$p[L'(e_1^*) - \underline{L}'(e_1^*)] + C'(e_1^*)(1-p)[\frac{X_B^{-\gamma}}{X_A^{-\gamma}} - \frac{X_3^{-\gamma}}{\bar{q}X_1^{-\gamma}}] > 0 \quad (25)$$

Since the utility function is strictly concave, the first-order condition under the risk evaluated in e_1^* is negative.

This implies that the individual makes more effort under ambiguity than under risk. When the individual is extremely optimistic, he underestimates or underweights the probability of bad results. In self-insurance activity in situations of ambiguity, as shown in Figure 2 below. [Pulford \(2009\)](#) showed in an experimental study that highly optimistic people have much less aversion to ambiguity.

On the other hand, if the individual adopts a highly pessimistic attitude, we obtain :

$$-p\underline{L}'(e) - C'(e)[p + (1-p)\frac{X_3^{-\gamma}}{qX_1^{-\gamma}}] = 0 \quad (26)$$

Proceeding in the same way, we obtain :

$$p[L'(e_1^*) - \underline{L}'(e_1^*)] + C'(e_1^*)(1-p)\left[\frac{X_B^{-\gamma}}{X_A^{-\gamma}} - \frac{X_3^{-\gamma}}{qX_1^{-\gamma}}\right] \quad (27)$$

The sign of the equation (27) depends on the differences between the marginal benefits and marginal costs under risk and under ambiguity.

$$p[L'(e_1^*) - \underline{L}'(e_1^*)] > -C'(e_1^*)(1-p)\left[\frac{X_B^{-\gamma}}{X_A^{-\gamma}} - \frac{X_3^{-\gamma}}{qX_1^{-\gamma}}\right] \quad (28)$$

Thus, for an highly pessimistic individual, the level of effort under ambiguity may be higher than the level of effort under risk if the difference between the marginal benefits under risk and under ambiguity is higher than the difference between the marginal costs under risk and under ambiguity. If the level of damage $\bar{L}(e)$ occurs. Equation (22) becomes :

$$-p\bar{L}'(e) - C'(e)[p + (1-p)\frac{X_3^{-\gamma}}{sX_2^{-\gamma}}] = 0 \quad (29)$$

We have the following two conditions depending on whether the individual is optimistic ($\alpha = 0$) or pessimistic ($\alpha = 1$). For an optimistic individual :

$$-p\bar{L}'(e) - C'(e)[p + (1-p)\frac{X_3^{-\gamma}}{(1-\bar{q})X_2^{-\gamma}}] = 0 \quad (30)$$

Let e_2^* be the solution to equation (30), we can make the difference with equation (21) evaluated in e_2^* . We have :

$$p[L'(e_2^*) - \bar{L}'(e_2^*)] + C'(e_2^*)(1-p)\left[\frac{X_B^{-\gamma}}{X_A^{-\gamma}} - \frac{X_3^{-\gamma}}{(1-\bar{q})X_2^{-\gamma}}\right] \quad (31)$$

The sign of equation (31) depends on the difference between marginal benefits and marginal costs under risk and ambiguity.

$$C'(e_2^*)(1-p)\left[\frac{X_B^{-\gamma}}{X_A^{-\gamma}} - \frac{X_3^{-\gamma}}{(1-\bar{q})X_2^{-\gamma}}\right] < -p[L'(e_2^*) - \bar{L}'(e_2^*)] \quad (32)$$

Thus, for an highly optimistic individual, the level of effort under ambiguity may be higher than the level of effort under risk if the difference between the marginal benefits under risk and under ambiguity is greater than the difference between the marginal costs under risk and under ambiguity. For a pessimistic individual:

$$-p\bar{L}'(e) - C'(e)\left[p + (1-p)\frac{X_3^{-\gamma}}{(1-\underline{q})X_2^{-\gamma}}\right] = 0 \quad (33)$$

Proceeding in the same way, we obtain :

$$p[L'(e_2^*) - \bar{L}'(e_2^*)] + C'(e_2^*)(1-p)\left[\frac{X_B^{-\gamma}}{X_A^{-\gamma}} - \frac{X_3^{-\gamma}}{(1-\underline{q})X_2^{-\gamma}}\right] < 0 \quad (34)$$

When people behave extremely pessimistically, their level of effort under ambiguity is lower than their level under risk, as seen in Figure 2. This is logical since when people are highly pessimistic, they give greater weight to the occurrence of unfavorable events. This pessimistic attitude translates into an extreme belief that unfavorable events occur more often, leading to an unrealistic overweighting of unfavorable probabilities. Once the individual perceives the effort of self-insurance as always catastrophic, he has no reason to increase his preventive effort under the ambiguity, which is interpreted in another way as a reinforcement of risk aversion.

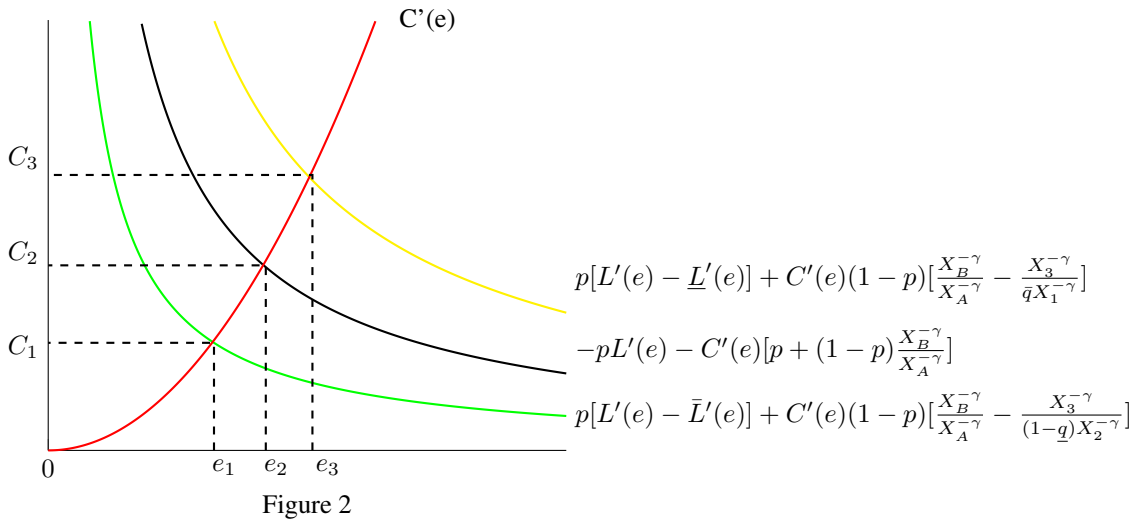


Figure 2 shows that the level of effort is higher under ambiguity than risk when the individual behaves optimisti-

cally (a shift of the black curve to the right). In contrast, the level of effort is low when the individual behaves extremely pessimistically (a shift of the black curve to the left) under ambiguity rather than under risk.

The second-order conditions on I and e are satisfied if the Hessian matrix is negative definite.

$$\frac{\partial^2 V}{\partial I^2} * \frac{\partial^2 V}{\partial e^2} - \left(\frac{\partial^2 V}{\partial e \partial I}\right)^2 > 0 \quad (\text{See appendix C})$$

As in the case of risk, we will study the impact of preferences on the trade-off between the demand for insurance and the demand for self-insurance. To analyse the impact of α , we will differentiate between the two first-order conditions.

$$V_{e\alpha} = -p[\underline{q} - \bar{q}][C'(e) + \underline{L}'(e)]X_1^{-\gamma} - p[\bar{q} - \underline{q}][C'(e) + \bar{L}'(e)]X_2^{-\gamma} < 0 \quad (35)$$

$$\begin{aligned} V_{I\alpha} &= p[\underline{q} - \bar{q}][1 - (1 + \lambda)p]X_1^{-\gamma} + p[(1 - \underline{q}) - (1 - \bar{q})][1 - (1 + \lambda)p]X_2^{-\gamma} \\ &= p(\underline{q} - \bar{q})[1 - (1 + \lambda)p]X_1^{-\gamma} + p(\bar{q} - \underline{q})[1 - (1 + \lambda)p]X_2^{-\gamma} \\ &= p(\bar{q} - \underline{q})[1 - (1 + \lambda)p][X_1^{-\gamma} - X_2^{-\gamma}] > 0 \quad \text{car } X_1^{-\gamma} \leq X_2^{-\gamma} \end{aligned} \quad (36)$$

The results of the comparative statics give ¹⁰:

$$\frac{de}{\partial \alpha} < 0 \quad \text{and} \quad \frac{dI}{\partial \alpha} > 0$$

The level of effort decreases with the ambiguity aversion index, while the level of benefit chosen increases with ambiguity aversion.

Proposition 3: *When there is ambiguity about the effectiveness of self-insurance, individuals always prefer market insurance to self-insurance.*

This result is because, as the ambiguity aversion index increases, individuals become more pessimistic and invest less in self-insurance than insurance. In our case, as the probability of the effectiveness of self-insurance is unknown, the individual overestimates the probability of unfavorable states and underestimates the probability of favorable states. This is because when the individual undertakes a self-insurance action, he no longer knows how much this will reduce the associated damage. This aversion to ambiguity translates into a form of inherent mistrust on the part of the individual, which would be all the stronger; in this case, it reduces the optimal level of self-insurance activity. This is because the individual can only make imprecise probability judgments because he has only imprecise knowledge of the probability of effectiveness of self-insurance. This result shows how

¹⁰The proofs are given in appendices C

much ambiguity impacts the individual's decision-making behavior and must be considered in the analyses. It is also important to note that the effect of ambiguity aversion on insurance and self-insurance differs from that of risk aversion. In the context of risk, an increase in risk aversion reduces the demand for insurance and increases the demand for self-insurance. However, in the context of ambiguity, the demand for insurance increases with the ambiguity aversion index, while the demand for self-insurance decreases. This difference in results is due to whether or not the probability of the effectiveness of self-insurance actions on the amount of damage is known. As pointed out by [Cecchi et al. \(2024\)](#), ambiguity aversion considerably increases the willingness to pay for insurance.

The result that the introduction of ambiguity about the effectiveness of self-insurance has a positive impact on the demand for insurance complements the results obtained in the literature from [Brunette et al. \(2009\)](#) and [Alary et al. \(2013\)](#), which, by introducing ambiguity about the risk of loss, concluded that the demand for insurance increases with ambiguity aversion. We can now generalize the results according to which when ambiguity concerns the probability of loss or the effectiveness of self-insurance, the demand for insurance always increases with the ambiguity aversion index.

6 Conclusion

This article proposes a theoretical model to study the trade-off between the demand for insurance and the demand for self-insurance. Specifically, it aims to study the effect of preferences on the relationship between the demand for insurance and self-insurance in a context of risk and then in a context of ambiguity. Several previous studies have looked at this relationship and have produced exciting results. However, we found that most of these previous works introduced ambiguity only in the probability of the claim. This article aims to study the impact of introducing ambiguity not on the probability of a claim but on the effectiveness of self-insurance on the relationship between the demand for insurance and self-insurance. This is the significant contribution of this paper to the existing literature.

To study the effect of preferences on the relationship between the demand for insurance and self-insurance in a risk context, we use the Von Neumann and Morgenstern (1944) utility expectation model to represent individuals' preferences. We show that an increase in risk aversion increases the demand for insurance and decreases the demand for self-insurance. Furthermore, we show that when risk is introduced on the effectiveness of self-insurance, individuals always prefer self-insurance to market insurance. To study the effect of preferences on the relationship between demand for insurance and demand for self-insurance in a context of ambiguity, we use

the $\alpha MaxMin$ utility expectation model of [Ghirardato et al. \(2004\)](#) to represent individuals' preferences. We show that when the ambiguity concerns the effectiveness of self-insurance, the individual prefers market insurance to self-insurance. Finally, we show that comparing the individual's level of effort under risk and ambiguity depends on the individual's beliefs. When the individual behaves extremely optimistically (or pessimistically), the level of effort under ambiguity is higher (lower) than the level under risk.

This theoretical analysis of the trade-off between demand for insurance and self-insurance against natural risks adds a new element to the existing literature by introducing ambiguity into the effectiveness of self-insurance. Moreover, it completes the results on the impact of ambiguity aversion on the relationship between demand for insurance and prevention. By understanding the optimal choices or preferences of individuals between demand for self-insurance and demand for insurance in the presence of risk and ambiguity, this study makes it possible to improve the functioning of the insurance market and, therefore, to be able to offer individuals more suitable insurance policies. These results are also helpful for implementing public policies to increase the demand for insurance or self-insurance while targeting individuals according to their pessimistic and optimistic attitudes.

However, it is essential to note that this study has several limitations. Firstly, this study does not consider the effect of an increase in the probability of loss on analyzing the relationship between the demand for insurance and self-insurance. Secondly, it does not consider the impact of prices on the relationship between the demand for insurance and self-insurance when the ambiguity concerns the effectiveness of self-insurance. Thirdly, it only considers two levels of effectiveness (high or low), yet there can be several levels of effectiveness of self-insurance. For example, in the case where the self-insurance measure does not reduce the level of damage. Future research may extend these critical points.

Appendix

A. Insurance versus self-insurance

Proof of the second-order conditions of the maximisation programme.

The second-order conditions are given by the determinant of the second derivative matrix (determinant of the Hessian matrix).

The second-order conditions on I and e are:

$$\frac{\partial^2 EU}{\partial I^2} = -\gamma p [1 - (1 + \lambda)p]^2 X_A^{-\gamma-1} - \gamma(1-p)[(1 + \lambda)p]^2 X_B^{-\gamma-1} < 0 \quad (37)$$

$$\frac{\partial^2 EU}{\partial e^2} = -p\gamma[C'(e) + L'(e)]^2 X_A^{-\gamma-1} - \gamma C'(e)^2 (1-p) X_B^{-\gamma-1} < 0 \quad (38)$$

The cross derivative gives :

$$\frac{\partial^2 EU}{\partial e I} = \frac{\partial^2 EU}{\partial I e} = \gamma p[C'(e) + L'(e)][1 - (1 + \lambda)p] X_A^{-\gamma-1} - \gamma C'(e)(1-p)(1 + \lambda)p X_B^{-\gamma-1} < 0 \quad (39)$$

The second derivative product gives :

$$\begin{aligned} \frac{\partial^2 EU}{\partial I^2} * \frac{\partial^2 EU}{\partial e^2} = & \gamma^2 p^2 [C'(e) + L'(e)]^2 [1 - (1 + \lambda)p]^2 X_A^{-\gamma-1} * X_A^{-\gamma-1} \\ & + \gamma^2 p(1-p)(1 + \lambda)^2 p^2 [C'(e) + L'(e)]^2 X_A^{-\gamma-1} X_B^{-\gamma-1} \\ & + \gamma^2 p(1-p) C'(e)^2 [1 - (1 + \lambda)p]^2 X_B^{-\gamma-1} X_A^{-\gamma-1} \\ & + \gamma^2 (1-p)^2 C'(e)^2 [(1 + \lambda)p]^2 X_B^{-\gamma-1} X_B^{-\gamma-1} \end{aligned} \quad (40)$$

The product of cross-derivatives gives :

$$\begin{aligned} \left(\frac{\partial^2 EU}{\partial e I}\right)^2 = & \gamma^2 p^2 [C'(e) + L'(e)]^2 [1 - (1 + \lambda)p]^2 X_A^{-\gamma-1} * X_A^{-\gamma-1} \\ & + \gamma^2 (1-p)^2 C'(e)^2 (1 + \lambda)^2 p^2 X_B^{-\gamma-1} * X_B^{-\gamma-1} \\ & - 2\gamma^2 p(1-p) C'(e)(1 + \lambda)p [C'(e) + L'(e)][1 - (1 + \lambda)p] X_A^{-\gamma-1} * X_B^{-\gamma-1} \end{aligned} \quad (41)$$

Thus :

$$\begin{aligned} \frac{\partial^2 EU}{\partial I^2} * \frac{\partial^2 EU}{\partial e^2} - \left(\frac{\partial^2 EU}{\partial e I}\right)^2 = & \gamma^2 p(1-p) X_A^{-\gamma-1} * X_B^{-\gamma-1} [C'(e)^2 + 2C'(e)L'(e)(1 + \lambda)p \\ & + L'(e)^2 (1 + \lambda)^2 p^2] > 0 \end{aligned} \quad (42)$$

Because the term in square brackets is a remarkable identity

The second-order conditions on I and e are therefore satisfied because the determinant of the matrix is strictly Hessian positive (Δ). The maximisation program therefore has a global maximum.

Proof of the comparative statics results

We denote f the second derivatives of the utility expectation function.

$$\begin{cases} f_{ee} * \frac{de}{d\gamma} + e_I * \frac{dI}{d\gamma} + f_{e\lambda} = 0 \\ f_{Ie} * \frac{de}{d\gamma} + f_{II} * \frac{dI}{d\gamma} + f_{e\lambda} = 0 \end{cases}$$

We obtain the following system, which is solved using Cramer's method :

$$\begin{vmatrix} f_{ee} & f_{eI} \\ f_{Ie} & f_{II} \end{vmatrix} * \begin{pmatrix} \frac{de}{d\gamma} \\ \frac{dI}{d\gamma} \end{pmatrix} = \begin{vmatrix} -f_{e\gamma} \\ -f_{I\gamma} \end{vmatrix}$$

Comparative statics are given by :

$$\frac{de}{d\gamma} = \frac{f_{eI} * f_{I\gamma} - f_{e\gamma} * f_{II}}{f_{II} * f_{II} - (f_{Ie})^2} \quad \text{et} \quad \frac{dI}{d\gamma} = \frac{f_{Ie} * f_{e\gamma} - f_{I\gamma} f_{ee}}{f_{II} * f_{II} - (f_{Ie})^2}$$

Differentiating the equations (11) and (12) with respect to γ gives :

$$f_{I\gamma} = -p[1 - (1 + \lambda)p] \ln X_A * X_A^{-\gamma} + (1 - p)(1 + \lambda)p \ln X_B * X_B^{-\gamma} \quad (43)$$

$$f_{I\gamma} > 0 \Rightarrow \frac{pX_A^{-\gamma} * \ln(X_A)}{(1 - p)X_B^{-\gamma} * \ln(X_B)} < \frac{(1 + \lambda)p}{1 - (1 + \lambda)p}$$

From equation (11)

$$\frac{pX_A^{-\gamma}}{(1 - p)X_B^{-\gamma}} = \frac{(1 + \lambda)p}{1 - (1 + \lambda)p}$$

$f_{I\gamma} > 0$ Because $\ln(X_A) < (\ln X_B)$

$$f_{e\gamma} = p[C'(e) + L'(e)]X_A^{-\gamma} * \ln X_A + C'(e)(1 - p)X_B^{-\gamma} * \ln X_B \quad (44)$$

$$V_{e\gamma} < 0 \Rightarrow \frac{pX_A^{-\gamma} * \ln X_A}{(1 - p)X_B^{-\gamma} * \ln X_B} < -\frac{C'(e)}{C'(e) + L'(e)}$$

From equation (12)

$$\frac{pX_A^{-\gamma}}{(1 - p)X_B^{-\gamma}} = -\frac{C'(e)}{C'(e) + L'(e)}$$

$V_{e\gamma} < 0$ Because $(\ln X_A) < (\ln X_B)$

The result is as follows:

$$\frac{de}{d\gamma} = \frac{(-) * (+) - (-) * (-)}{+} < 0 \quad \text{and} \quad \frac{dI}{d\gamma} = \frac{(-) * (-) - (+) * (-)}{+} > 0$$

With regard to the impact of an increase in the price of insurance, the differentiation of equations (11) and (12) according to λ gives:

$$f_{e\lambda} = \gamma p^2 I X_A^{-\gamma-1} - \gamma p I (1 - p) X_B^{-\gamma-1} \quad (45)$$

$$f_{e\lambda} > 0 \quad \text{if} \quad \frac{1 - p}{p} < \frac{X_A^{-\gamma-1}}{X_B^{-\gamma-1}}$$

This condition implies that for a low value of λ ($\lambda < \hat{\lambda}$), the self-insurance effort depends positively on λ . As

soon as the value of λ is high ($\lambda > \hat{\lambda}$), the self-insurance effort no longer depends on λ , since there is no longer any market insurance.

$$\begin{aligned} f_{I\lambda} &= -p^2 X_A^{-\gamma} + p^2 I \gamma [1 - (1 + \lambda)p] X_A^{-\gamma-1} - p(1-p) X_B^{-\gamma} - p^2 I \gamma (1-p)(1+\lambda) X_B^{-\gamma-1} \\ &= p^2 X_A^{-\gamma-1} (\gamma I - X_A) - \gamma p^2 I (1+\lambda) p X_A^{-\gamma-1} - p(1-p) X_B^{-\gamma} - \gamma p^2 I (1-p)(1+\lambda) X_B^{-\gamma-1} < 0 \end{aligned} \quad (46)$$

This inequality is always verified because the compensation weighted with the risk aversion coefficient is lower than the wealth in the bad state of nature.

As $f_{I\lambda} < 0$, $f_{e\lambda} > 0$ and $\Delta > 0$, we can deduce that :

$$\frac{de}{d\lambda} > 0 \quad \text{and} \quad \frac{dI}{d\lambda} < 0$$

B. Introduction of risk on the effectiveness of self-insurance

The second-order conditions on I and e are given by :

$$\frac{\partial^2 EU}{\partial I^2} = -\gamma p q [1 - (1 + \lambda)p]^2 X_1^{-\gamma-1} - \gamma p (1 - q) [1 - (1 + \lambda)p]^2 X_2^{-\gamma-1} - \gamma (1 - p) [(1 + \lambda)p]^2 X_3^{-\gamma-1} < 0 \quad (47)$$

$$\frac{\partial^2 EU}{\partial e^2} = -\gamma p q [C'(e) + \underline{L}'(e)]^2 X_1^{-\gamma-1} - \gamma p (1 - q) [C'(e) + \bar{L}'(e)]^2 X_2^{-\gamma-1} - \gamma (1 - p) X_3^{-\gamma-1} < 0 \quad (48)$$

$$\begin{aligned} \frac{\partial^2 EU}{\partial e I} &= \gamma p q [1 - (1 + \lambda)p] [C'(e) + \underline{L}'(e)] X_1^{-\gamma-1} - \gamma C'(e) (1 - p) (1 + \lambda) p X_3^{-\gamma-1} \\ &\quad + \gamma p (1 - q) [1 - (1 + \lambda)p] [C'(e) + \bar{L}'(e)] X_2^{-\gamma-1} < 0 \quad \text{if} \quad q < -\frac{C'(e)}{\underline{L}'(e)} * \frac{X_2^{-\gamma-1}}{X_1^{-\gamma-1}} \end{aligned} \quad (49)$$

This condition means that the sign of the cross derivative is negative if the probability of the effectiveness of self-insurance is lower than the marginal return on self-insurance weighted by the marginal rate of substitution.

The product of the two second derivatives gives :

$$\begin{aligned}
\frac{\partial^2 EU}{\partial I^2} * \frac{\partial^2 EU}{\partial e^2} = & \gamma^2 p^2 q^2 [1 - (1 + \lambda)p]^2 [C'(e) + \underline{L}'(e)]^2 X_1^{-\gamma-1} X_1^{-\gamma-1} \\
& + \gamma^2 p^2 q(1 - q) [1 - (1 + \lambda)p]^2 [C'(e) + \bar{L}'(e)]^2 X_1^{-\gamma-1} X_2^{-\gamma-1} \\
& + \gamma^2 pq(1 - p) C'(e)^2 [1 - (1 + \lambda)p]^2 X_1^{-\gamma-1} X_3^{-\gamma-1} \\
& + \gamma^2 p^2 q(1 - q) [1 - (1 + \lambda)p]^2 [C'(e) + \underline{L}'(e)]^2 X_2^{-\gamma-1} X_1^{-\gamma-1} \\
& + \gamma^2 p^2 q(1 - q)^2 [1 - (1 + \lambda)p]^2 [C'(e) + \bar{L}'(e)]^2 X_2^{-\gamma-1} X_2^{-\gamma-1} \\
& + \gamma^2 p(1 - p)(1 - p) C'(e)^2 [1 - (1 + \lambda)p]^2 X_2^{-\gamma-1} X_3^{-\gamma-1} \\
& + \gamma^2 pq(1 - p)(1 + \lambda)^2 p^2 [C'(e) + \underline{L}'(e)]^2 X_3^{-\gamma-1} X_1^{-\gamma-1} \\
& + \gamma^2 p(1 - p)(1 - q)(1 + \lambda)^2 p^2 [C'(e) + \bar{L}'(e)]^2 X_3^{-\gamma-1} X_2^{-\gamma-1} \\
& + \gamma^2 (1 - p)^2 (1 + \lambda)^2 p^2 C'(e)^2 X_3^{-\gamma-1} X_3^{-\gamma-1}
\end{aligned} \tag{50}$$

The product of the two cross derivatives gives :

$$\begin{aligned}
\left(\frac{\partial^2 EU}{\partial eI}\right)^2 = & \gamma^2 p^2 q^2 [1 - (1 + \lambda)p]^2 [C'(e) + \underline{L}'(e)]^2 X_1^{-\gamma-1} X_1^{-\gamma-1} \\
& + \gamma^2 (1 - p)^2 C'(e)^2 (1 + \lambda)^2 p^2 X_3^{-\gamma-1} X_3^{-\gamma-1} \\
& + \gamma^2 p^2 (1 - q)^2 [1 - (1 + \lambda)p]^2 [C'(e) + \bar{L}'(e)]^2 X_2^{-\gamma-1} X_2^{-\gamma-1} \\
& - 2\gamma^2 pq(1 - p) C'(e)(1 + \lambda)p [1 - (1 + \lambda)p]^2 [C'(e) + \underline{L}'(e)] X_1^{-\gamma-1} X_3^{-\gamma-1} \\
& - 2\gamma^2 p(1 - q)(1 - p) C'(e)(1 + \lambda)p [1 - (1 + \lambda)p]^2 [C'(e) + \bar{L}'(e)] X_3^{-\gamma-1} X_2^{-\gamma-1} \\
& + 2\gamma^2 p^2 q(1 - q) [1 - (1 + \lambda)p]^2 [C'(e) + \underline{L}'(e)][C'(e) + \bar{L}'(e)] X_1^{-\gamma-1} X_2^{-\gamma-1}
\end{aligned} \tag{51}$$

Thus

$$\frac{\partial^2 EU}{\partial I^2} * \frac{\partial^2 EU}{\partial e^2} - \left(\frac{\partial^2 EU}{\partial eI}\right)^2 = \gamma^2 pq(1 - p) X_1^{-\gamma-1} X_3^{-\gamma-1} [C'(e) + (1 + \lambda)p \underline{L}'(e)]^2 > 0 \tag{52}$$

Introduction of ambiguity about the effectiveness of self-insurance

$$\alpha.\max_{q \in \{\underline{q}, \bar{q}\}} \min EU(x) = \alpha.\min_{q \in \{\underline{q}, \bar{q}\}} Eu(x) + (1 - \alpha).\alpha.\min_{q \in \{\underline{q}, \bar{q}\}} Eu(x) \tag{53}$$

Minimum utility is given by :

$$\alpha.min_{q \in \{q, \bar{q}\}} Eu(x) = E[u(x)/q = \underline{q}] = p[\underline{q}U(X_1) + (1 - \underline{q})U(X_2) + (1 - p)U(X_3)] \quad (54)$$

Maximum utility is given by :

$$\alpha.max_{q \in \{q, \bar{q}\}} Eu(x) = E[u(x)/q = \bar{q}] = p[\bar{q}U(X_1) + (1 - \bar{q})U(X_2) + (1 - p)U(X_3)] \quad (55)$$

The sum of the two utilities weighted by the pessimism and optimism coefficients is given by :

$$\begin{aligned} \alpha.maxmin EU(x) &= \alpha p[\underline{q}U(X_1) + (1 - \underline{q})U(X_2) + (1 - p)U(X_3)] \\ &\quad + (1 - \alpha)p[\bar{q}U(X_1) + (1 - \bar{q})U(X_2)] + (1 - p)U(X_3) \quad (56) \\ &= p[\alpha \underline{q} + (1 - \alpha)\bar{q}]U(X_1) + p[\alpha(1 - \underline{q}) + (1 - \alpha)(1 - \bar{q})]U(X_2) + 2(1 - p)U(X_3) \end{aligned}$$

Proof concerning the determinant of the Hessian matrix :

Second derivatives give :

$$\frac{\partial^2 V}{\partial I^2} = -\gamma pr[1 - (1 + \lambda)p]^2 X_1^{-\gamma-1} - \gamma ps[1 - (1 + \lambda)p]^2 X_2^{-\gamma-1} - 2\gamma(1 - p)[(1 + \lambda)p]^2 X_3^{-\gamma-1} < 0 \quad (57)$$

$$\frac{\partial^2 V}{\partial e^2} = -\gamma pr[C'(e) + \underline{L}'(e)]^2 X_1^{-\gamma-1} - \gamma ps[C'(e) + \bar{L}'(e)]X_2^{-\gamma-1} - 2\gamma C'(e)^2(1 - p)X_3^{-\gamma-1} < 0 \quad (58)$$

The cross derivative gives :

$$\begin{aligned} V_{eI} &= \gamma pr[1 - (1 + \lambda)p][C'(e) + \underline{L}'(e)]X_1^{-\gamma-1} + \gamma ps[1 - (1 + \lambda)p][C'(e) + \bar{L}'(e)]X_2^{-\gamma-1} \\ &\quad - 2\gamma C'(e)(1 - p)(1 + \lambda)pX_3^{-\gamma-1} \quad (59) \\ V_{eI} < 0 \text{ if } s < \frac{2(1 - p)(1 + \lambda)X_3^{-\gamma-1}}{X_2^{-\gamma-1}} \end{aligned}$$

The product of second derivatives gives :

$$\begin{aligned}
\frac{\partial^2 V}{\partial e^2} * \frac{\partial^2 V}{\partial I^2} &= \gamma^2 p^2 r^2 [1 - (1 + \lambda)p]^2 [C'(e) + \underline{L}'(e)]^2 X_1^{-\gamma-1} X_1^{-\gamma-1} \\
&+ \gamma^2 p^2 r s [1 - (1 + \lambda)p]^2 [C'(e) + \bar{L}'(e)]^2 X_1^{-\gamma-1} X_2^{-\gamma-1} \\
&+ 2\gamma^2 p r C'(e)^2 (1 - p) [1 - (1 + \lambda)p]^2 X_1^{-\gamma-1} X_3^{-\gamma-1} + \\
&\gamma^2 p^2 s r [1 - (1 + \lambda)p]^2 [C'(e) + \underline{L}'(e)]^2 X_2^{-\gamma-1} X_1^{-\gamma-1} \\
&+ \gamma^2 p^2 s^2 [1 - (1 + \lambda)p]^2 [C'(e) + \bar{L}'(e)]^2 X_2^{-\gamma-1} X_2^{-\gamma-1} \\
&+ 2\gamma^2 p s C'(e)^2 (1 - p) [1 - (1 + \lambda)p]^2 X_2^{-\gamma-1} X_3^{-\gamma-1} \\
&+ 2\gamma^2 p r (1 - p) (1 + \lambda)^2 p^2 [C'(e) + \underline{L}'(e)]^2 X_3^{-\gamma-1} X_1^{-\gamma-1} \\
&+ 4\gamma^2 C'(e)^2 (1 - p)^2 (1 + \lambda)^2 p^2 X_3^{-\gamma-1} X_3^{-\gamma-1} \\
&+ 2\gamma^2 p s (1 - p) (1 + \lambda)^2 p^2 [C'(e) + \bar{L}'(e)]^2 X_3^{-\gamma-1} X_2^{-\gamma-1}
\end{aligned} \tag{60}$$

The product of the cross derivatives gives :

$$\begin{aligned}
\left(\frac{\partial^2 V}{\partial e I}\right)^2 &= \gamma^2 p^2 r^2 [1 - (1 + \lambda)p]^2 [C'(e) + \underline{L}'(e)]^2 X_1^{-\gamma-1} X_1^{-\gamma-1} \\
&+ \gamma^2 p^2 s^2 [1 - (1 + \lambda)p]^2 [C'(e) + \bar{L}'(e)]^2 X_2^{-\gamma-1} X_2^{-\gamma-1} \\
&+ 4\gamma^2 C'(e)^2 (1 - p)^2 (1 + \lambda)^2 p^2 X_3^{-\gamma-1} X_3^{-\gamma-1} \\
&+ 2\gamma^2 p^2 r s [1 - (1 + \lambda)p]^2 [C'(e) + \underline{L}'(e)][C'(e) + \bar{L}'(e)] X_1^{-\gamma-1} X_2^{-\gamma-1} \\
&- 4\gamma^2 p r C'(e) (1 - p) (1 + \lambda) p [1 - (1 + \lambda)p][C'(e) + \underline{L}'(e)] X_1^{-\gamma-1} X_3^{-\gamma-1} \\
&- 4\gamma^2 p s C'(e) (1 - p) (1 + \lambda) p [1 - (1 + \lambda)p][C'(e) + \bar{L}'(e)] X_1^{-\gamma-1} X_3^{-\gamma-1}
\end{aligned} \tag{61}$$

Hence

$$\frac{\partial^2 V}{\partial I^2} * \frac{\partial^2 V}{\partial e^2} - \left(\frac{\partial^2 V}{\partial e I}\right)^2 = 2\gamma^2 p r (1 - p) X_1^{-\gamma-1} X_3^{-\gamma-1} [C'(e) + \underline{L}'(e)(1 + \lambda)p]^2 > 0 \tag{62}$$

Proof of the results of the comparative statics

We know that $V_{eI} < 0$, $V_{I\gamma} > 0$; $V_{e\gamma} < 0$, so :

$$\frac{de}{d\gamma} = \frac{(-) * (-) - (+) * (-)}{+} < 0 \quad \text{and} \quad \frac{dI}{d\gamma} = \frac{(-) * (+) - (-) * (-)}{+} > 0$$

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