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Fatiha Alabau-Boussouira. Comments on "Erratum: "Existence and asymptotic stability for the semi-linear wave equation with boundary damping and source term [J. Math. Phys. 49, 053511 (2008)]"". 2024. hal-04754495

HAL Id: hal-04754495

<https://hal.science/hal-04754495v1>

Submitted on 25 Oct 2024

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Comments on

“Erratum: ”Existence and asymptotic stability for the semi-linear wave equation with boundary damping and source term [J. Math. Phys. 49, 053511 (2008)]””

Fatiha Alabau-Boussouira*

18 October, 2024

Abstract

A paper entitled “Erratum: ”Existence and asymptotic stability for the semi-linear wave equation with boundary damping and source term” [J. Math. Phys. 49, 053511 (2008)]” was published in 2009 [2], in the *Journal of Mathematical Physics*. In the 2008 version [1], the authors assumed $\mu \in W^{2,\infty}(0,T)$ and “ $\mu(t) \geq \mu_0 \geq 1$, $\mu'(t) \leq -\mu_1\mu(t)$ and $\mu''(t) \leq 0$ a.e. in $[0,\infty)$, (2.3), where μ_0 and μ_1 are positive constants.” In the Erratum, the authors mentioned “Hypothesis 2.3 on p. 053511-3 is not suitable.” They explained that the two first inequalities in (2.3) “cannot be used together.” The Erratum further corrects the assumptions of Theorem 2.3: the original assumption (2.3) is replaced by a new hypothesis, still denoted by (2.3) stated as: $\mu \in W^{2,\infty}(0,\infty) \cap W^{2,1}(0,\infty)$ and “ $\mu(t) \geq \mu_0 > 0$, $\mu'(t) < 0$ and $\mu''(t) \leq 0$ a.e. in $[0,\infty)$, (2.3), where μ_0 is a positive constant.” The Erratum then presents the arguments to correct the proof of Theorem 2.3 thanks to this new assumption.

In the present document we prove that:

1. The new assumptions (2.3) cannot be satisfied: the two first inequalities are basically contradictory with the third inequality.
2. The claim that the Erratum corrects the proof of Theorem 2.3 is flawed: the required property is μ' is bounded away from 0 and not that μ' is bounded. This can be basically noted in the stated inequality: “ $\int_{\Gamma_1} |u|^2 d\Gamma \leq C \int_{\Omega} |\nabla u|^2 dx \leq C(-\mu'(t)) \int_{\Omega} |\nabla u|^2 dx$ ”
3. We prove that the requirement that μ' has to be assumed to be bounded away from 0 is basically contradictory with the two first assumptions on μ in the new assumption (2.3), so that keeping only the two first inequalities of (2.3) does not allow to correct the proof of Theorem 2.3.

Hence we show that the Erratum contains two major flaws, and through a deeper short analysis we establish that the proof of Theorem 2.3 cannot be corrected in this 2009 Erratum nor in the 2008 original version of the proof. As a consequence, Theorem 2.3 remains unproved as stated in the Erratum.

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5 Data on dissemination through citations of [2]

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1 Introduction

We analyze the "Erratum: "Existence and asymptotic stability for the semi-linear wave equation with boundary damping and source term" [J. Math. Phys. 49, 053511 (2008)]" published in 2009, in the *Journal of Mathematical Physics*" and written by Jong Yeoul Park and Tae Gab Ha. This Erratum presents a short correction of the hypothesis (2.3) stated in the paper entitled "Existence and asymptotic stability for the semi-linear wave equation with boundary damping and source term" published in 2008 in the *Journal of Mathematical Physics*.

In a first step, we first prove that the new hypothesis (2.3) cannot hold due to a major flaw: namely the third inequality is contradictory with the two first inequalities in this new assumption. Hence the three inequalities cannot hold together.

The two first inequalities are used at several places and cannot be dropped. We therefore study in all the sequel the mathematics presented in the Erratum only assuming the two first (non contradictory) inequalities of (2.3) of [2].

In a second step, assuming only the two first inequalities of (2.3) in [2], we prove that the mathematical arguments presented in the Erratum [2] by Jong Yeoul Park and Tae Gab Ha to assert that the proof of Theorem 2.3 is still correct, contain a major flaw. Namely these arguments state that the proof is still correct since μ' is bounded on $[0, \infty)$. We show that this is not the required property on μ' . One needs the property that μ' stays bounded away from 0, this even only assuming the two first inequalities of the new assumption (2.3).

In a third step, assuming only the two first inequalities of (2.3) in [2], we also prove that this Erratum does not allow to correct the proof of Theorem 2.3. This is due to the concerned proof itself, which generates a contradiction when the authors want to estimate a boundary term with respect to the part of the dissipation of energy involving μ' (μ (being the square of the time dependent speed of propagation of (1.1)) and the L^2 norm of the gradient (with respect to space) of the solution u , using Sobolev's embeddings. This strategy requires assumptions on μ which are contradictory with the two other hypotheses on μ , namely the two first inequalities of the new assumption (2.3). Thus, we prove that Theorem 2.3 remains unproved even with weaker assumptions than the new hypothesis (2.3) of the 2009 Erratum [2].

We first present the 2009 Erratum [2] and its goal. In this Erratum, the authors replace their original assumption (2.3) stated as: " $\mu \in W^{2,\infty}(0, T)$ "

and

"

$$\mu(t) \geq \mu_0 \geq 1, \quad \mu'(t) \leq -\mu_1 \mu(t) \text{ and } \mu''(t) \leq 0 \quad \text{a.e. in } [0, \infty), \quad (2.3)$$

where μ_0 and μ_1 are positive constants. " by a new hypothesis, still denoted by (2.3) in the Erratum [2], formulated as follows:

$$" \mu \in W^{2,\infty}(0, \infty) \cap W^{2,1}(0, \infty) "$$

and

"

$$\mu(t) \geq \mu_0 > 0, \quad \mu'(t) < 0 \text{ and } \mu''(t) \leq 0 \quad \text{a.e. in } [0, \infty), \quad (2.3)$$

where μ_0 is a positive constant. "

For the sake of clearness and to avoid confusion, we shall denote this new hypothesis (2.3) by (2.3N) in all the sequel of this paper, that we recopy here for easy reading:

$$\mu(t) \geq \mu_0 > 0, \quad \mu'(t) < 0 \text{ and } \mu''(t) \leq 0 \quad \text{a.e. in } [0, \infty), \quad (2.3N)$$

where μ_0 is a positive constant, and " $\mu \in W^{2,\infty}(0, \infty) \cap W^{2,1}(0, \infty)$ ".

In all the sequel, whenever (2.3N) is used, it means that it is the new hypothesis (2.3) of the Erratum [2] which is used.

Theorem 2.3 is stated at 053511-6 of the original 2008 version [1]. It is devoted to the asymptotic behavior of the solution as times goes to infinity. The authors state existence for all nonnegative time t and show exponential and polynomial energy decay estimates.

The proof of Theorem 2.3 starts at page 053511-20 in the original 2008 version [1] in which u is assumed to be a regular solution of (5.1). The system is an evolution equation in time and space. The space domain is assumed to be a connected open and nonempty subset Ω with a boundary Γ of class C^2 . The boundary Γ is formed by a disjoint partition $\{\Gamma_0, \Gamma_1\}$ where Γ_0 and Γ_1 are assumed to be close and such that $\Gamma_0 \cap \Gamma_1 = \emptyset$. The system (5.1) involves a wave equation with a time dependent speed propagation equal to $\sqrt{\mu}$, and a semi-linear term $h(u)$. The boundary conditions involve an homogeneous boundary condition on Γ_0 , where Γ_0 is of positive Lebesgue measure. The boundary condition on Γ_1 is of the form:

“

$$\mu(t) \frac{\partial u}{\partial \nu} + \alpha u' + |u'|^\rho u' = |u|^\gamma u.$$

”

The associated initial conditions are given by (u_0, u_1) .

In the next section, we first prove that the Erratum [2] contains a major basic flaw. We then prove that the published correction of the proof of Theorem 2.3 is flawed even when keeping only the first two inequalities of the new assumption (2.3). Then in a third step, we prove that even when dropping the last inequality of the new assumption (2.3) it is not possible to correct the proof of Theorem 2.3 as performed. For this we give some basic mathematical arguments to show an elementary contradiction both in the 2009 Erratum [2] and the original published article [1] concerning Theorem 2.3 and its proof.

2 Flaws in the Erratum [2] published to correct the proof of Theorem 2.3

Remark 2.1. *The Erratum [2] concerns the pages 053511-20 to 053511-25 of the original 2008 version [1]. The system (5.1) is presented at page 053511-20. The authors also precise in the same page that: “And in the following section, the symbol C indicates positive constants, which may be different.”*

Proposition 2.2. *The new hypothesis (2.3) (renamed (2.3N) in the present paper) of [2] contains contradictory assumptions.*

Proof. We consider the new assumption of [2], that is μ_0 is a positive constant, $\mu \in W^{2,\infty}(0, \infty) \cap W^{2,1}(0, \infty)$ and (2.3N) holds. Thanks to the assumption $\mu \in W^{2,\infty}(0, \infty) \cap W^{2,1}(0, \infty)$, we have $\mu \in C^1([0, \infty))$. We claim that under the new hypothesis in the 2009 Erratum [2], $\mu' \equiv 0$ on $[0, \infty)$. Thanks to the second inequality in (2.3N), there exists $t_0 \geq 0$ such that $\mu'(t_0) < 0$. Thanks to the third assumption $\mu'' \leq 0$ a.e. on $[0, \infty)$ in [2], we deduce that

$$\mu'(t) \leq \mu'(t_0) \quad \forall t \geq t_0,$$

which after an integration with respect to t on both sides of the above inequality implies that

$$\mu(t) \leq \mu'(t_0)(t - t_0) + \mu(t_0) \quad \forall t \geq t_0.$$

But since we assume $\mu'(t_0) < 0$, we have that

$$\mu'(t_0)(t - t_0) + \mu(t_0) < 0 \quad \forall t > t_0 + \mu(t_0)/(-\mu'(t_0)) > t_0.$$

Hence we have

$$\mu(t) < 0 \quad \forall t > t_0 + \mu(t_0)/(-\mu'(t_0)),$$

which contradicts the first inequality in (2.3N). Hence the assumption that there exists $t_0 \geq 0$ such that $\mu'(t_0) < 0$ leads to a contradiction. Therefore we prove our claim:

$$\mu'(t) = 0 \quad \forall t \geq 0.$$

But this contradicts the second inequality of (2.3N). Hence the new hypotheses assumed in the 2009 Erratum [2] are contradictory. This invalidates the Erratum and the correction of the proof of Theorem 2.3. \square

Remark 2.3. *The assumption that μ' is a non-positive function on the real half-line cannot be dropped. It is used from the very beginning of the proof of Theorem 2.3 [1] (and consequently [2]) at page 053511-20 to insure the energy E of the solution is a non-increasing function, that is that “ $E(T) - E(S) = \frac{1}{2} \int_S^T \mu'(t) \|\nabla u(t)\|_2^2 dt - \alpha \int_S^T \|u'(t)\|_{2,\Gamma_1}^2 dt - \int_S^T \|u'(t)\|_{\rho+2,\Gamma_1}^{\rho+2} dt$ ” is non-positive for all $0 \leq S \leq T < \infty$. The first inequality of (2.3N) is also used in a key way in several places of [1] (and consequently [2]) and cannot be dropped, for instance its is used to obtain the left hand side of (5.24) from (5.5).*

Hence, we shall now drop in (2.3N) the third inequality since the two first inequalities have to be kept, whereas the third inequality is contradictory with the two first ones.

From now on and in the sequel, we keep only the two first inequalities of (2.3N). Under this new assumption, we prove in our next Propositions and in the Corollary that the mathematical arguments presented in the 2009 Erratum [2] to correct the proof of Theorem 2.3 are false independently of the basic contradiction in the new assumption (2.3). We then perform a deeper analysis of the proof of Theorem 2.3 to show that it is the strategy itself that does not allow to prove Theorem 2.3, namely we prove that the authors' strategy necessarily requires contradictory assumptions, so that Theorem 2.3 remains totally unproved.

Proposition 2.4. *Assume that the initial data (u_0, u_1) in (5.1) (see page 053511-20 of [1]) is non vanishing. Then this implies necessarily that the solution u of (5.1) in [1] satisfies*

$$\int_{\Omega} |\nabla u|^2 dx \neq 0. \quad (1)$$

Proof. Assume by contradiction that (1) does not hold. Then, since Ω is assumed to be a connected bounded open subset in \mathbb{R}^n , one deduces, thanks to the boundary condition $u = 0$ (almost everywhere for $x \in \Gamma_0$ and for t in $(0, \infty)$) in (5.1), that u and its time derivative are identically vanishing in $\Omega \times (0, \infty)$, so that the initial data (u_0, u_1) in (5.1) are vanishing, which is a contradiction. \square

Hence we assume from now on that (1) holds.

Let us now show that a contradiction appears and can be analyzed both in the 2009 published Erratum [2] and the 2008 original version [1]. Let us start by the Erratum version [2]. We recall, that the new hypothesis (2.3) is denoted in the present paper by (2.3N) to avoid confusion. The authors state that thanks to their new hypothesis (2.3N), μ' is bounded on $[0, \infty)$ so that there exists a positive constant “ C_{12} ”, such that

$$|\mu'(t)| \leq C_{12} \text{ for all } t.$$

”

Then the authors expose how they correct the proof of Theorem 2.3 of the 2008 original version [1], thanks to the above argument: the boundedness of μ' over $[0, \infty)$. They proceed as follows. The authors assert that due the continuous Sobolev embedding of $H_{\Gamma_1}^1(\Omega)$ in $L^2(\Gamma_1)$ and thanks to the boundedness of μ' , the following estimate “

$$\int_{\Gamma_1} |u|^2 d\Gamma \leq C \int_{\Omega} |\nabla u|^2 dx \leq C(-\mu'(t)) \int_{\Omega} |\nabla u|^2 dx$$

”

holds true. Let us refer to this inequality as (E1).

Remark 2.5. *Note that the first constant C and the second one are not the same constants. The authors precise at page 053511-20, next after (5.2):*

“And in the following section, the symbol C indicates positive constants, which may be different.”

Note also that (E1) is supposed to hold for all $t \geq 0$.

We now prove in a second step that the mathematical arguments presented in the 2009 Erratum [2] to correct the proof of Theorem 2.3 contain a major flaw.

Proposition 2.6. *Assume only that μ satisfies the first and second inequalities of (2.3N) and the regularity assumptions $\mu \in W^{2,\infty}(0, \infty) \cap W^{2,1}(0, \infty)$ of [2]. Assume moreover that the initial data (u_0, u_1) in (5.1) (see page 053511-20 of [1]) is non vanishing. Then, for (E1) to hold as stated at page 069901-1 in [2], μ' should be bounded away from 0, that is:*

$$\exists C_1 > 0, \text{ such that } |\mu'(t)| \geq C_1 \quad \forall t \geq 0, \quad (2)$$

contrarily to the statement “ $|\mu'(t)| \leq C_{12}$ for all t ” formulated at page 069901-1 in the 2009 Erratum [2].

Proof. Let us write completely the last inequality in the above estimate (E1). Taking into account that the two constants C in (E1) are not the same, there exists a positive constant C_1 such that:

$$C_1 \int_{\Omega} |\nabla u|^2 dx \leq (-\mu'(t)) \int_{\Omega} |\nabla u|^2 dx \quad \forall t \geq 0 \quad (3)$$

Thanks to (1), we deduce that

$$0 < C_1 \leq -\mu'(t) \quad \forall t \geq 0$$

which is equivalent to

$$\mu'(s) \leq -C_1 < 0 \quad \forall s \in [0, \infty) \quad (4)$$

□

Remark 2.7. *Note that since μ' is assumed to be a non-positive function in [2, 1], the property μ' bounded on the real positive half-line is equivalent to:*

$$\exists C > 0, \text{ such that } 0 \leq -\mu'(s) \leq C \quad \forall s \in [0, \infty),$$

which is therefore not the required assumption for (E1) to hold, contrarily to the assertion in [2].

In a third step, let us now prove that assuming that μ stays positive and bounded away from 0 (to guarantee the uniform hyperbolic properties of the first equation of (5.1) (see page 053511-20 of [1])), that $\mu' \leq 0$ holds for a.e. every $t \geq 0$ (to guarantee that the dissipation of the energy E stated at page 053511-20 of [1] (see the inequalities stated just next to (5.2) and before (5.3)), together with the necessary condition that μ' should be bounded away from 0 for the arguments of the proof of Theorem 2.3 in [1] to apply, necessarily leads to a mathematical contradiction.

Proposition 2.8. *Assume that μ satisfies the first and second inequalities of (2.3N) and the regularity assumptions $\mu \in W^{2,\infty}(0, \infty) \cap W^{2,1}(0, \infty)$ of [2]. Assume moreover that the initial data (u_0, u_1) in (5.1) (see page 053511-20 of [1]) is non vanishing. Then the second inequality of (E1) as it appears at page 069901-1 in the 2009 Erratum [2] cannot hold true.*

Proof. We argue by contradiction. Let us assume that (E1) holds. Thanks to Proposition 2.6, (2) holds. Since μ' is nonnegative on $[0, \infty)$, the inequality (4) holds. Integrating (4) between 0 and t for any nonnegative t , we deduce that

$$\mu(t) \leq -C_1 t + \mu(0) \quad \forall t \in [0, \infty) \quad (5)$$

Hence, we have:

$$\mu(t) < 0 \quad \forall t > \frac{\mu(0)}{C_1}$$

which contradicts the first inequality in the new hypothesis (2.3) of the 2009 Erratum [2] referenced in the present paper as (2.3N), stated as follows in (2.3N)

“

$$\mu(t) \geq \mu_0 > 0 \text{ a.e. in } [0, \infty).$$

” Hence the inequality (E1) cannot hold. □

Corollary 2.9. *Assume that μ satisfies the first and second inequalities of (2.3N) and the regularity assumptions $\mu \in W^{2,\infty}(0, \infty) \cap W^{2,1}(0, \infty)$ of [2]. In particular, we do not assume the third inequality of (2.3N). Assume moreover that the initial data (u_0, u_1) in (5.1) (see page 053511-20 of [1]) is non vanishing. Then the proof of Theorem 2.3 as presented in the 2009 Erratum [2] cannot be corrected.*

Remark 2.10. We prove that the Erratum [2] does not allow intrinsically to correct the flaw in the proof of Theorem 2.3 by the two first inequalities of (2.3N) ((2.3) in [2]). This can be easily checked in the original proof of Theorem 2.3 at page 05311-25 of the 2008 original article [1] when the authors derive (5.25) from the inequality (5.24). This invalidates the 2009 Erratum [2] and the proof of Theorem 2.3 both in the original 2008 [1] version and in the 2009 Erratum [2] version.

Remark 2.11. One can remark that the authors Jong Yeoul Park and Tae Gab Ha do not provide any examples of functions μ satisfying the new assumption (2.3) in their 2009 Erratum [2] (see also [1]). This would have helped to detect that this assumption was basically contradictory and may have given the idea to remark that the assumption $\mu'' \leq 0$ a.e. on $[0, \infty)$ does not seem to be really necessary in the part of the proofs where it is involved. Examples of functions μ which satisfies (only) the first two assumptions of (2.3N) and such that μ'' is bounded over the positive real half-line, as presented in the present document, are for instance given by:

$$\mu(t) = \mu_0 + \frac{A}{t+1} \quad \forall t \geq 0, \quad (6)$$

where μ_0 and A are arbitrary positive constants. Then $\mu \in W^{2,\infty}(0, \infty) \cap W^{2,1}(0, \infty)$, and μ satisfies the two first inequalities of (2.3N). Moreover, μ does not satisfy the last inequality of (2.3N). Many such examples can be given.

3 Conclusion

We prove in the present paper that major flaws are still present in the Erratum [2] both in the statements and in the proof of Theorem 2.3. We moreover prove that Theorem 2.3 cannot be corrected. Hence these major flaws affect the stability part mentioned in the title of the Erratum (see also [1]).

4 Description of the presentation, open access, submission history

A presentation of the 2009 Erratum version of the article is available at the link:

<https://doi.org/10.1063/1.3155787>

The pdf of the 2009 Erratum article starts at page 069901-1 and reduces to a single page.

The history of submission is indicated at the top of page 069901-1 as follows:

“(Received 14 April 2009; accepted 28 May 2009; published online 23 June 2009)”

A presentation of the 2008 original version of the article is available at the link:

<https://doi.org/10.1063/1.2919886>

The pdf of the 2008 article starts at page 053511-1 and ends at page 053511-26.

The history of submission is indicated at the top of page 053511-1 as follows:

“(Received 24 January 2008; accepted 15 April 2008; published online 8 May 2008)”

Acknowledgments at page 053511-25 mention that: “This work was supported by Korea Research Foundation Contract/Grant No. 20060168.”

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The following presentation: “JMP welcomes original research of the highest quality in all active areas of mathematical physics, including the following:” and “From initial evaluation through peer review and final decision, your paper will be handled by experts” is available on the website of the journal.

5 Data on dissemination through citations of [2]

The 2009 Erratum version [2] does not appear as cited in Google Scholar. In the zbMATH database, in the review of the 2008 original version [1], accessible at the link <https://zbmath.org/1152.81576>, an additional text indicates that the assumption (2.3) has been corrected in an Erratum. In the MathSciNet database, a text has been added to the review of the 2008 original version and indicate that for additional information see the refers to the MathSciNet 2009 review "MR2536125".

At the time this paper was written the citations of [1] co-authored by Jong Yeoul Park and Tae Gab Ha in some of the existing databases or platforms are the following:

Google scholar citations	34
Web of Science citations	28
MathSciNet citations	23
MathSciNet citations in Reviews	1
zbMATH citations	23

References

- [1] Jong Yeoul Park and Tae Gab Ha, Existence and asymptotic stability for the semi-linear wave equation with boundary damping and source term. *Journal of Mathematical Physics* 49, 953511 (2008).
- [2] Jong Yeoul Park and Tae Gab Ha, "Erratum: Existence and asymptotic stability for the semi-linear wave equation with boundary damping and source term" [*J. Math. Phys.* 49, 053511 (2008)]. *Journal of Mathematical Physics* 50, 069901 (2009)