

The Parameterized Complexity of Local Search for MOTSP

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This research aims to connect two important areas: parameterized complexity and multi-objective optimization. Parameterized complexity is a way of studying how efficiently an algorithm can solve a problem by focusing on given parameters, while multi-objective optimization focuses on real-world problems that have several goals to achieve at once. The Multi-Objective Traveling Salesman Problem (MOTSP) is a variation of the Traveling Salesman Problem (TSP) where a salesman must visit a set of cities and return to the starting point, but instead of optimizing just one objective (like minimizing distance), there are multiple objectives to consider simultaneously. These objectives could be things like minimizing total cost, distance, or travel time, while also considering factors like fuel consumption or safety. We aim to solve MOTSP using a local search method. To enhance the effectiveness of this approach, we have proposed a new neighborhood operator, which is defined as follows:

Local MOTSP(r -swap)

Input: An H-ordered graph $G = (V, E)$, two edge weight functions w_1, w_2 with maximum weight \mathcal{W} , and a positive integer k .

Question: Is there an ordered sequence of r -swaps S of size at most k such that $\text{perm}(S)$ is an improved Hamiltonian cycle?

Parameters: r , k and \mathcal{W} .

This neighborhood operator is inspired by single objective one [1]. We show that even with multiple objectives, the problem can still be solved in FPT time, specifically in time $O(k^2 \cdot k! \cdot r^{2k} \cdot n + k^3 \cdot r \cdot \mathcal{W}^2 \cdot n^2)$.

References

- [1] J. Guo, S. Hartung, R. Niedermeier, and O. Suchý, *The Parameterized Complexity of Local Search for TSP, More Refined*, *Algorithmica* **67(1)** (2012), 89–110.