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# Growth of surface wind-waves in water of finite depth: a laboratory experiment

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## Abstract

This paper reports on laboratory experiment results on wind-driven surface waves in finite depth and their comparison with theoretical predictions and experimental in-situ studies. We introduce the Miles theory's extension to the case of finite depth, as well as the rules transforming theoretical expressions to formulae commonly used in experimental routines, in particular the important rules transforming theoretical growth rates to experimental ones. Wind waves depend strongly on boundary marine layer parameters, namely, the aerodynamic roughness length, the Charnock constant as well as, the drag coefficient. Consequently, this work gives detailed measurements of these parameters in finite depths. Experiments conducted in the IRPHE/Pythéas wind-wave tank (Marseille, France), reveal that for a given wind speed, these values are higher in finite depth than in deep water. In the limit case of fully developed surface, due to depth, theoretical and empirical formulas relating the critical values of wave age to the non-dimensional depth, have been experimentally verified. Plots of non-dimensional peak frequency, non-dimensional energy, and the inverse of wave age, against non-dimensional depth are presented. The plots clearly show that these quantities admit a limit due to the depth influence. All data obey the range of empirical values established in field experiments. Experimental data, obtained in the facility, agree with the theoretical family of depth-dependent wave growth rate as a function of wave age in finite depth. The non-dimensional growth rate data obtained in our laboratory, as a function of the inverse of wave age, are consistent with the theoretical predictions of Miles theory as well as measurements of other laboratories.

*Keywords:* Wind-generated waves; wind-wave growth rates; Miles' theory; finite depth; wind-wave facility;

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## 1. Introduction

Understanding the coupling between the atmosphere and the ocean surface is important for many scientific, environmental and engineering problems. The wind driven momentum exchange and amplification factor of the water waves by the wind at the air-water interface are key drivers of hydrodynamic and ecological processes in open seas. Surface water waves and their generation by wind is a fascinating issue. The task of coupling atmosphere and oceans requires rather detailed knowledge of the wave-field and, in particular, the momentum transfer between boundary layer winds and surface waves. While coastal zones are limited, compared to the global ocean, their impacts on and response to human activity are profound and should be better understood. The vast majority of air-sea interaction study involved measurements made across the open ocean and, excepting relatively few studies, the role transition in the momentum flux from open ocean to nearshore has not been fully characterized: amplification of water waves by the wind in finite depth environment is still an unsolved problem. Despite differentiated observed physical air-sea behaviours between deep waters and shallow waters, atmospheric models apply the same wind stress formulation regardless of the depth of the waters. Wave growth parametrizations for deep water are commonly applied to shallow water, however the impact that depth-limited processes have on the growth of the waves by the wind remains under-explored. Additional well controlled experiments are essential to gaining a better understanding of the growth of waves over water of finite depth. Any theory needs ultimate validation by experimental means.

A few experiments have been conducted to study the influence of the water depth on air-sea interactions at the water surface. Most of them were dedicated to the role of the finite depth  $h$  on the modification of prediction laws of non-dimensional energy ( $g^2\sigma^2/U_{ref}^4$ ), non-dimensional frequency ( $fU_{ref}/g$ ), wave age ( $c/U_{ref}$ ), non dimensional fetch ( $gX/U_{ref}^2$ ) and non-dimensional depth ( $gh/U_{ref}$ ), with  $\sigma$  the standard deviation of the water elevation,  $U_{ref}$  the wind speed at a reference level,  $g$  the gravity,  $f$  the dominant wave frequency,  $c$  the wave celerity and  $X$  the fetch [Thijsse, 1949; Bretschneider, 1954, 1958; Ijima and Tang, 1966; CERC, 1977, 1984; Young and Verhagen, 1996 a,b].

Other works were done on the influence of water depth on momentum flux ( $\tau = \rho_a u_*^2$ ), with  $\rho_a$  the air density,  $u_*$  the friction velocity [Geernaert, 1987; Donelan et al., 2005, 2006; Zhao et al., 2015; Ortiz-Suslow et al., 2018; Chen et al., 2019], on the drag coefficient ( $C_d = u_*^2 / U_{ref}^2$ ) [Anctil and Donelan, 1986; Donelan et al., 2012; Bi et al., 2015; Oost et al., 2002; Toffoli et al., 2012; Shabani et al., 2014], or on the roughness length  $z_0$  [Anctil and Donelan, 1986; Smith, 1980; Gao et al., 2009; MacMahan, 2017; Jimenez and Dudhia, 2018]. These papers show the increase of both  $u_*$ ,  $C_d$  and  $z_0$  in shallow water environment. Authors explain this increase by the modification of wave dynamics and kinematics by the shallow bottom. Finite depth decreases the wave celerity  $c$ , thus the wave age  $c/U_{ref}$ , and increases wave height, wave steepness by shoaling effect. This modification of celerity and wave geometric profile seems to explain the increase of drag and roughness. Very close to the shore, the slower wave celerity and the sawtooth wave shape in the surf zone are an important contributor to the increased wind stress. For example, [Donelan et al., 2006] argued that shallow water measurements exhibit higher drag coefficients for same wind speed, because the waves of the same frequency are slower and shorter and consequently steeper, thus producing higher stress. Based on their air-flow separation model, [Makin and Kudryavtsev, 2002] have reproduced the increase in  $C_d$  with decreasing water depth: at moderate and high winds, decreasing water depth induces a steepening of dominant waves and thus more air-flow separation at the crest of the waves, and then more drag. The constant flux model of [Chen et al., 2019] has shown how the waves, modified by the bottom in finite depth conditions, influence the wind profile over the waves and by the way the stress budget between the water and the air. Other writers like MacMahan [2017] related the increase of roughness near the coasts to surf-zone foam coverage due to wave breaking. Recently [Ortiz-Suslow et al., 2018] clearly demonstrated that the mechanisms that characterize air-sea interaction in deep water, may not apply near shore in finite depth waters.

Very few papers [Young and Verhagen, 1996 a,b; Donelan et al., 2005, 2006; Young, 1997; Young et al., 2005] were dedicated specifically on the growth rate modification by the water depth. The experiment described in [Young and Verhagen, 1996 a] was the first really comprehensive attempt to

measure the evolution of fetch limited waves in water of finite depth. The Lake George finite-depth field experiment is well-documented in literature and we refer to [Young et al., 2005] for the most complete and detailed summary on this experiment. A large set of atmospheric and wave data were collected in finite water depth regimes at the Lake George measurement site. The aim of the Lake George project was to simultaneously measure source and sink functions for wave prediction models, as well as fetch-limited wave evolution. In particular, the effects of water depth and wave steepness were addressed. The lake George measurements have confirmed the water depth dependence of the asymptotic limits to wave growth. Young [1997], also reproduced in Young [1999], derived an empirical relation with appropriate non-dimensional parameters able to reproduce the experimental data of Young and Verhagen [1996 a]. In particular the relationship between the variation of fractional energy and the inverse wave age, found by Donelan et al. [1992] for deep water, was extended to the finite depth domain. These open field results together with plots of the empirical laws have shown that, contrary to the deep water case, the wave age at which the growth rate becomes zero is wind-dependent and depth-dependent. In Young [1997] the curves of the wave growth versus wave age have been presented for ranges of a non-dimensional water depth parameter  $\delta$ . Beyond a critical wave age, the growth rate vanishes.

Pioneering theories on surface wind-wave growth have been developed for deep water waves since [Jeffreys, 1924, 1925; Phillips, 1957] and [Miles, 1957], and until more recently, for example by [Belcher and Hunt, 1993] or other authors reviewed in [Janssen, 2004]. These theories are thus restrictive with regard to wind generated near-shore waves, bays, rivers or shallow lake waves. The well known Miles' theory has been extended to the finite depth under breeze to moderate winds conditions by several authors [Montalvo et al., 2013 a,b; Latifi et al., 2017]. They derived a surface wind-wave growth theory in finite depth based on a) the Euler equations in the water domain for finite depth, and b) the linearised governing equation of steady air flow, with a prescribed mean horizontal velocity  $U(z)$  depending on the vertical coordinate  $z$  together with perturbations to the mean. Developments leads to the well known Rayleigh equation [Rayleigh, 1880]. Then a non-dimensional water depth parameter depending to the depth and to a characteristic wind speed, induced a family of curves representing the wave growth as a function of the wave phase velocity and the wind speed. Their model provides an agreement with the data and empirical relationships obtained from the Lake George experiment and the Australian Shallow Water Experiment. For small wave age the wave growth rates are comparable to those of deep water, but for larger wave age a finite-depth limited growth is reached, with wave growth rates going to zero. The [Montalvo et al., 2013 a] study was focused on the wave growth of monochromatic waves, the work of [Montalvo et al., 2013 b] was extended to the evolution of wave packets growing with the wind, and [Latifi et al., 2017] added the effect of shelter mechanism in the theory [Jeffreys, 1925], and they also added non linearity effects and studied the evolution of non-linear breathers and solitons in shallow depth environments and windy conditions.

To date, there is no referenced laboratory direct experimental work that confirms or denies the theoretical study of [Montalvo et al., 2013 a,b; Latifi et al., 2017], more particularly the fact that the wave growth rate induced by the wind could drop to zero amplification for some configuration of water depth, wave dominant frequency, wave age and wind speed. Therefore a well control experiment on wave growth in finite depth environment is lacking. The goal of our study is to conduct a specific water tank experiment with wind blowing over the water surface, and with different combinations of water depth, dominant waves, and wind speeds. Marine air boundary layer characteristics (friction velocities, drag coefficients, surface roughness lengths) will be estimated, and water elevation will be measured at different locations to measure the wave growth in relatively smooth wind and steepnesses conditions: we avoided wave breaking dissipation, because in [Montalvo et al., 2013 a,b; Latifi et al., 2017] theories breaking dissipation is disregarded. Wave growth will be compared to theoretical developments. The asymptotic depth limited growth of non-dimensional energy, non-dimensional peak frequency and inverse wave age will be tested and validated.

Notations, parameters and theoretical developments are presented in section 2. Material and

methods are described in section 3, with the presentation of the wind-wave facility, the experimental arrangement, data processing, experimental conditions, air and water measured values and their precision. Section 4 describes the effects of water depth on air flow, on air-water transfer parameters, on wave growth rate, and this section is ended by the comparison with theory developments. The last section provides a short summary and outlook.

## 2. The air-water interface in finite depth. Theoretical wave growth rates $\gamma$ and $\beta$

### 2.1. Wave growth rates $\gamma$ and $\beta$ : analytical developments

The classical Miles' theory in deep water is based on the dispersion relation of the air-sea interface and the related Rayleigh equation [Rayleigh, 1880; Conte and Miles, 1959; Drazin and Reid, 1982]. We consider the water and air particles to be located relative to a two-dimensional Cartesian coordinate system with axes  $x, z$ , origin  $x = z = 0$ ,  $x \in [-\infty, +\infty]$ ,  $z$  being the upward vertical direction and  $z \in [-h, +\infty]$ . Before perturbations the water lies between the impermeable bottom at  $z = -h$  and the still water surface at  $z = 0$ . The undisturbed water steady state is incompressible, inviscid and with zero surface tension. It is assumed that the air is also inviscid and incompressible. In absence of surface waves the air can be described by a prescribed mean shear flow  $U(z)$  in the  $x$ -direction which varies with the height  $z$  above the surface i.e.:

$$\vec{U} = U(z)\vec{e}_x \quad (1)$$

with  $\vec{e}_x$  the unitary vector in the  $x$  direction. Let's consider a perturbation to the free surface which propagates like a progressive plane wave of wave number  $k$ , phase velocity  $c$  and constant amplitude  $\eta_0$  i.e.  $\eta(x, t) = \eta_0 e^{ik(x-ct)}$ . Miles's mechanism of waves generation by wind assumes that  $\eta(x, t)$  induces small perturbation of the air pressure and velocities. From reference [Montalvo et al., 2013 a] we have that the perturbed air pressure  $P_a(x, z, t)$  is given by

$$P_a(x, z, t) = P_0 - \rho_a g z + ik\rho_a \int_z^\infty [U(z) - c]w_a(x, z, t)dz \quad (2)$$

where  $P_0$  is the atmospheric pressure,  $\rho_a$  the air density,  $g$  gravity,  $w_a(x, z, t) = \mathcal{W}_a(z)e^{ik(x-ct)}$  the  $z$ -component of the perturbed air velocity. The expression for  $P_a$  is of primary importance in the Miles mechanism of wave-generation by wind since it is supposed that the momentum transfer from the air into the surface waves is due to the air pressure oscillations in quadrature with the surface slope [Deardorff, 1967].

In (2) neither  $w_a(x, z, t)$  nor  $c$  are known. The perturbed velocity  $w_a(x, z, t)$  is calculated using the Rayleigh equation [Montalvo et al., 2013 a] and  $c$  is determined by the relation dispersion which reads

$$\rho_w \eta(x, t) (c^2 k \coth kh - g) + P_0 = P_a(x, \eta, t). \quad (3)$$

In the single-domain problem  $P_a(x, \eta, t) = P_0$  and equation (3) brings to the usual expression  $c$  of the phase velocity governing surface waves in finite depth:  $c = \sqrt{(g/k) \tanh(kh)}$ . In this case in order to calculate  $c$  from (3) we use  $P_a$  evaluated at  $z = \eta$  from (2). We obtain [Montalvo et al., 2013 a]

$$g(1 - s) + c \frac{sk^2}{W_0} I_1 - c^2 \left( \frac{sk^2}{W_0} I_2 + k \coth(kh) \right) = 0, \quad (4)$$

with

$$I_1 = \int_{z_0}^\infty U(z)\mathcal{W}_a(z)dz, \quad I_2 = \int_{z_0}^\infty \mathcal{W}_a(z)dz, \quad (5)$$

with  $z_0$  the aerodynamic sea surface roughness located just above the interface,  $W_0 = \lim_{z \rightarrow z_0} \mathcal{W}_a(z)$ , and  $s$  the ratio of air density to water density:  $s = (\rho_a/\rho_w) \sim 10^{-3}$ .

The function  $\mathcal{W}_a(z)$  is complex and consequently  $c$  too, so  $c = \text{Re}\{c\} + i \text{Im}\{c\}$  where  $\text{Im}\{c\}$  ( $\text{Re}\{c\}$ ) is the imaginary (real) part of  $c$ .

The imaginary part of  $c$  gives the growth rate of  $\eta(x, t)$  defined by

$$\gamma = k \operatorname{Im}\{c\}, \quad (6)$$

The theoretical and numerical results concerning the growth rate  $\gamma$  are studied and computed with three non-dimensional parameters  $\delta$ ,  $\theta_{dw}$  and  $\theta_{fd}$  defined by

$$\delta = \frac{gh}{U_1^2}, \quad \theta_{dw} = \frac{1}{U_1} \sqrt{\frac{g}{k}}, \quad \theta_{fd} = \theta_{dw} \sqrt{T}, \quad \text{with,} \quad T = \tanh(kh) = \tanh\left(\frac{\delta}{\theta_{dw}^2}\right) \quad (7)$$

The non-dimensional parameter  $\delta$ , for constant  $U_1$ , measures the influence of the finite fluid depth on the rate of growth of  $\eta(x, t)$ . The parameter  $\theta_{dw}$  is a *theoretical analogous of the deep water wave age*. It measures the relative value of the deep water phase speed in relation to the characteristic wind velocity  $U_1$ . Finally  $\theta_{fd}$  is a *finite depth wave age*  $\theta_{fd}$ . We introduce the following non-dimensional variables and scaling, hats meaning non-dimensional quantities

$$U = U_1 \hat{U}, \quad W_a = W_0 \hat{W}_a, \quad z = \frac{\hat{z}}{k}, \quad c = U_1 \hat{c}, \quad t = \frac{U_1}{g} \hat{t}. \quad (8)$$

The non-dimensional growth rate  $\hat{\gamma} = \frac{U_1}{g} \gamma$  is given by

$$\hat{\gamma} = \frac{s}{2} \left( \frac{T \operatorname{Im}\{\hat{I}_1\}}{\theta_{dw}^2} - \frac{T^{3/2} \operatorname{Im}\{\hat{I}_2\}}{\theta_{dw}} \right). \quad (9)$$

We have the following transformation rule between non-dimensional parameters  $\beta$  and  $\hat{\gamma}$

$$\beta = \frac{2\hat{\gamma}}{s} \theta_{dw}^3 T^{1/2}, \quad (10)$$

The deep water limit  $\delta \rightarrow \infty$  ( $T \rightarrow 1$ ) gives back  $\hat{\gamma}$  and  $\beta$  for Miles' theory. The existence of a finite depth  $h$  transforms the unique curve of wave growth rate  $\hat{\gamma}$  or  $\beta$  in deep water in a *family of curves* indexed by  $\delta = gh/U_1^2$ , i.e., a curve for each value of  $\delta$  [Montalvo et al., 2013 a,b; Latifi et al., 2017].

## 2.2. Wave growth rates $\gamma$ and $\beta$ : wind-wave tank environment

Experimental results are obtained using appropriate parameters and formulae commonly employed in the laboratory routine. In this section we are looking for the transformation rules between theoretical expressions and their laboratory counterpart.

Let's consider a monochromatic progressive wave moving in a wave tank. The wave frequency is  $\omega$ , the wavenumber  $k$  and the amplitude  $a(x, t)$  with  $x$  the distance from the wavemaker which is termed the *fetch*  $x$  and  $t$  is the time

$$\eta(x, t) = a(x, t) \sin(kx - \omega t) \quad (11)$$

If we consider the wind action, with wind  $U_{ref}$  (the wind speed at the altitude  $z = z_{ref}$ ), as the unique process causing the wave growth, and we consider the viscosity and bottom friction the only two existing mechanisms of wave dissipation, (i.e. no wave breaking because we did not see wave breaking during our experiments, and the present theory excludes wave breaking), we may write :

$$\left( \frac{1}{a} \frac{\partial a}{\partial x} \right)_{U_{ref}} = \gamma_{es} - \nu \quad (12)$$

where  $\gamma_{es}$  is the *experimental spatial wave growth rate*, and  $\nu$  is the total rate of dissipation by viscosity and friction along the walls and on the bottom of the tank.

For each experiment we did two runs: the first one was done with mono-frequency paddle waves with amplitude  $a$ , frequency  $\omega$  and wind speed  $U_{ref}$  and the second one was done with the same mono-frequency waves (amplitude  $a$ , and frequency  $\omega$ ) but without wind speed, i.e. with  $U_{ref} = 0m/s$ . When there is no wind speed, the wind notation is  $U_0$ .

We made the strong hypothesis that the dissipation process was the same in presence of wind and without wind. This hypothesis is questionable, but in our experiments, we never saw specific additional dissipation process, or wave breaking events due to the presence of the wind. We just noticed a slight enhancement of the waves when we turned the wind on. So without wind we may write:

$$\left(\frac{1}{a} \frac{\partial a}{\partial x}\right)_{U_0} = -\nu \quad (13)$$

Finally, the spatial growth rate may be inferred from (12) and (13)

$$\left(\frac{1}{a} \frac{\partial a}{\partial x}\right)_{U_{ref}} - \left(\frac{1}{a} \frac{\partial a}{\partial x}\right)_{U_0} = \gamma_{es} \quad (14)$$

In many papers, model theories and wave prediction models, the temporal growth rate  $\gamma$  [Janssen, 2004; Montalvo et al., 2013 a,b] or the temporal growth rate  $\beta_{Miles}$  [Miles, 1957] are used instead of the spatial growth rate. By analogy we can define an experimental temporal wave growth rate  $\gamma_{et}$ , related to the experimental spatial growth rate  $\gamma_{es}$  by the group velocity  $C_g$  [Gaster, 1962] :

$$\gamma_{et} = C_g \gamma_{es} \quad (15)$$

The non-dimensional temporal wave growth  $\hat{\gamma}_{et}$  may be defined as [Latifi et al., 2017] :

$$\hat{\gamma}_{et} = \left(\frac{u^*}{g\kappa}\right)\gamma_{et} \quad (16)$$

with  $u^*$  the friction velocity and  $\kappa = 0.41$  the Von Karman constant.

[Miles, 1957] used the other non-dimensional temporal wave growth  $\beta$  :

$$\beta = 2 \left(\frac{c}{u^*}\right)^2 \kappa^2 \frac{1}{\omega} \left(\frac{\rho_a}{\rho_w}\right) \gamma_{et} \quad (17)$$

Another widely used non-dimensional temporal growth rate [Plant, 1982; Belcher et al., 1994; Young, 1997] is  $\frac{\gamma_{et}}{f} = \frac{\gamma_{et} 2\pi}{\omega}$

Experimentally, if the wave height is measured at two different fetches (i.e. different distances from the wind inlet and the wavemaker), namely  $a_0$  and  $a_1$  at fetch  $x_0$  (close to the beginning of the tank) and  $x_1$  (far away from  $x_0$ ), the dimensional spatial growth rate may be estimated by (c.f. equation (14)):

$$\gamma_{es} = \left(\frac{1}{a_1} \frac{a_1 - a_0}{x_1 - x_0}\right)_{U_{ref}} - \left(\frac{1}{a_1} \frac{a_1 - a_0}{x_1 - x_0}\right)_{U_0} \quad (18)$$

In this relation, subscripts  $U_{ref}$  and  $U_0$  refer to experiments with wind speed  $U_{ref}$  and experiments with wind speed  $U_0 = 0$

The non-dimensional temporal growth rates  $\hat{\gamma}_{et}$ ,  $\beta$  and  $\frac{\gamma_{et}}{f}$  could be estimated by (c.f. equations (15), (16) and (17)):

$$\hat{\gamma}_{et} = \left(\frac{u^*}{g\kappa}\right) C_g \left(\left(\frac{1}{a_1} \frac{a_1 - a_0}{x_1 - x_0}\right)_{U_{ref}} - \left(\frac{1}{a_1} \frac{a_1 - a_0}{x_1 - x_0}\right)_{U_0}\right) \quad (19)$$

$$\beta = 2 \left(\frac{c}{u^*}\right)^2 \kappa^2 \frac{1}{\omega} \left(\frac{\rho_a}{\rho_w}\right) C_g \left(\left(\frac{1}{a_1} \frac{a_1 - a_0}{x_1 - x_0}\right)_{U_{ref}} - \left(\frac{1}{a_1} \frac{a_1 - a_0}{x_1 - x_0}\right)_{U_0}\right) \quad (20)$$

and

$$\gamma_{et}/f = \frac{C_g}{f} \left(\left(\frac{1}{a_1} \frac{a_1 - a_0}{x_1 - x_0}\right)_{U_{ref}} - \left(\frac{1}{a_1} \frac{a_1 - a_0}{x_1 - x_0}\right)_{U_0}\right) \quad (21)$$

Usually only  $\omega$  is measured.  $k$ ,  $c$  and  $C_g$  are not directly measured. Wavenumber  $k$  is estimated by solving the dispersion relationship between  $k$  and pulsation  $w$  :

$$\omega^2 = gk \tanh(kh) \quad (22)$$

$c$  and  $C_g$  are respectively:

$$c = \sqrt{\frac{g}{k} \tanh(kh)} \quad (23)$$

$$C_g = \frac{c}{2} \left(1 + \frac{2kh}{\sinh(2kh)}\right) \quad (24)$$

### 3. Material and methods

#### 3.1. Wind-wave facility and experimental arrangement

Experiments were conducted in the IRPHÉ/Pythéas wind-wave tank, in order to estimate the wave-growth of regular monochromatic mechanical waves under the action of the wind for different water depth configurations. The IRPHÉ/Pythéas wind-wave tank, sketched in Figure 1, consists of a closed-loop tunnel for air-circulation with a 1.5-m-high air cavity above a basin, with a test section 40-m-long and 2.60-m-wide. A complete description of the facility can be found in [Coatic et al., 1981], and at the following address: <https://www.osupytheas.fr/?-Uniqueness->. The wind is generated by an air blower in the upper part of the tunnel, and guided through a settling chamber with a honeycomb and a converging nozzle. This system produces a uniform airflow at the entrance of the water basin. The bottom is flat, and the water depth may be adjusted from 0.1m up to 0.80 m depending upon the water quantity put in the tank. An adjustable 6m-long divergent section was added recently between the entrance of the air section and the water section when the tank is not full of water, thus giving a smooth and non-perturbing air transition between the two air and water sections. The height of this air-divergent can be adjusted to any water depth (see Figure (2)). Consequently for each run, the wind was always tangent to the mean water surface at the entrance of the water section. There is a slightly divergence of the air section along the wind-wave tank, allowing a zero pressure longitudinal gradient, and a constant momentum transfer flux layer from the air to the water along the tank. The air blower is controlled by a graduated potentiometer.

At the upwind-end of the basin, an immersed paddle wave-maker (piston-type) is used for generating monochromatic waves propagating along the wind direction. The wave maker is controlled by a Labview program. These paddle waves do not hit the air divergent located juste above them. At the downwind-end, a permeable absorbing beach, with a 6° slope, prevents wave reflection. The reflection rate ranges between 2% to 5% depending upon the wavelength of the incoming waves. The basin side walls are punctuated with windows to observe the water surface. An inside view of the facility showing the experimental arrangement is shown in Figure 3. A sketch of the experimental arrangement is drawn in Figure 4 showing the location of the measuring devices.

The instantaneous horizontal wind velocity was measured with a Constant Temperature Anemometer (E+E Elektronik, Langwiesen, Austria). This sensor was calibrated against a reference sonic GILL 2D anemometer on several occasions during the experiments. The anemometer was supported by a vertical telescopic pole allowed to move it at different altitudes above the water level. Measurements were done from a point slightly above the highest wave, up to about several vertically distributed points and a duration of 300 s to obtain the vertical profile of the mean horizontal velocity  $U(z)$ . The precision of the anemometer was about  $\pm 4cm/s$

For these experiments, we used two DISA capacitance-type wave gauges in order to measure the wave growth between two different locations. One wave gauge was located at fetch  $x_0 = 2.40m$ , the other one at fetch  $x_1 = 20.10m$ . The origin of the fetch ( $x = 0m$ ), is by definition, the location where the wind hits the water, i.e. at the downwind end of the air-divergent. For each run, wave gauges



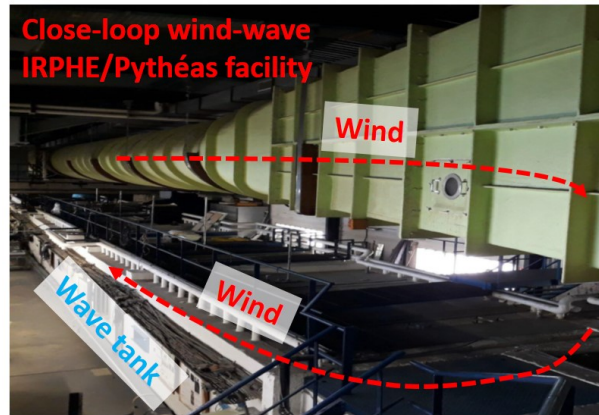


Figure 1: External view of the IRPHÉ/Pythéas wind-wave facility

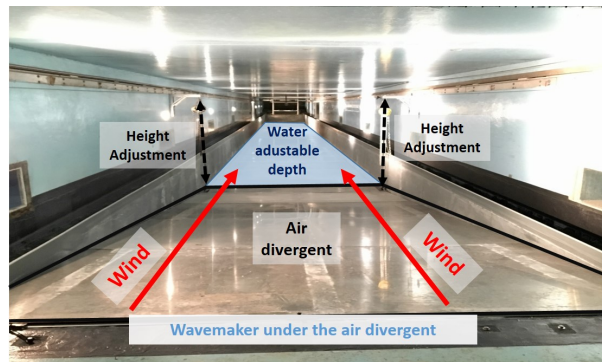


Figure 2: Internal view of the IRPHÉ/Pythéas wind-wave facility: the height of the downwind end of the air-divergent is adjustable, depending upon the water depth. The wavemaker is under the air-divergent. Mechanical waves do not hit the air divergent.

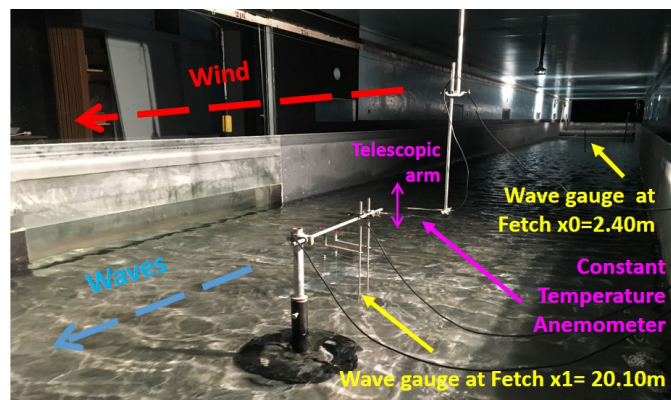


Figure 3: View of the experimental arrangement inside the IRPHÉ/Pythéas wind-wave facility. We used one DISA capacitance wave gauge at fetch  $x_0 = 2.40m$ , another one at fetch  $x_1 = 21.10m$ , a DANTEC Constant Temperature Anemometer (CTA) located on a telescopic arm than can move vertically from the water surface up to 1m above the waves, thus allowing us to have vertical profiles of horizontal wind speed  $U(z)$ .

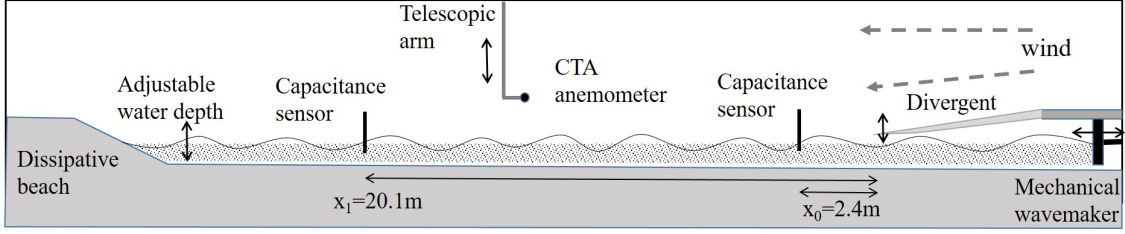


Figure 4: Sketch of the IRPHÉ/Pythéas wind-wave tank facility showing the location of the measuring devices

signals were acquired during 20 minutes at a sampling rate of 256 Hz. Before every acquisition, it was verified that the waves in the wave tank were in stationary fetch-limited conditions by waiting a sufficient long time before acquiring data. Calibrations of the wave gauges were required every day.

### 3.2. Measurements

#### 3.2.1. Wind parameters

In the facility, air and water temperature were roughly at the same temperature, which means that the marine atmospheric conditions were "neutral". The friction velocity  $u^*$  is considered constant along the test section because of the slightly divergence of the air section along the wind-wave tank allowing a zero pressure longitudinal gradient [Coantic et al., 1981]. Assuming a logarithmic wind profile and neutral conditions, Monin and Obukhov [1954] scaling allows evaluation of friction velocity  $u^*$ , roughness length  $z_0$ , and  $U_{10}$ , from the measured  $U(z)$  profiles values with:

$$U(z) = \frac{u^*}{\kappa} \ln \frac{z}{z_0} \quad (25)$$

where  $\kappa = 0.41$  is the von Kármán constant,  $z$  is the measurement height above mean water level. The logarithmic profile is the solution of the horizontal momentum equations for the near-surface boundary sublayer dominated by friction forces, both viscous and turbulent [Komen, 1994]  $\partial\tau/\partial z = 0$ , with  $\tau$  the momentum flux from air to water. A linear fit between  $\ln z$  and  $U(z)$  gives easily an estimation of  $u^*$  and  $z_0$ . Once  $u^*$  and  $z_0$  are evaluated,  $U_{10}$  is taken as :

$$U_{10} = \frac{u^*}{\kappa} \ln \frac{10}{z_0} \quad (26)$$

This method, called the "profile method", is routinely used for wind parameter determination in this facility (see for example recent papers of [Villefer et al., 2021; Bruch et al., 2021] where vertical wind profiles are displayed with  $u^*$ ,  $z_0$ , and  $U_{10}$  determinations). Two examples of vertical wind profiles are plotted in Figure (5). We did several times the same runs to estimate the precision on  $u^*$  and  $z_0$ . The relative precision was estimated to be 15% for  $u^*$  and  $U_{10}$  and 30% for  $z_0$ :  $\frac{du^*}{u^*} = 15\%$ ,  $\frac{dU_{10}}{U_{10}} = 15\%$ ,  $\frac{dz_0}{z_0} = 30\%$ . The estimated values of  $u^*$ ,  $U_{10}$ , and  $z_0$  for the 15 different runs are listed in Table 1.

The drag coefficient  $Cd_{10} = \frac{\rho_a u^{*2}}{\rho_a U_{10}^2} = \left(\frac{u^*}{U_{10}}\right)^2$ , with  $\rho_a$  the air density, is a commonly used parameter to characterise the shear air-flow above the waves and the non-dimensional momentum transfer from the air to the water.  $\tau = \rho_a u^{*2}$  is the wind stress on the water surface. Values of  $Cd_{10}$  for the 15 different runs are listed in Table 1.

In open seas, it is very difficult to measure  $z_0$ . A simple parametrization like :  $z_0 = \alpha_{Ch} u^{*2} / g$ , with  $g$  the gravity, and  $\alpha_{Ch}$  the Charnock parameter could be used to estimate  $z_0$  from  $u^*$  [Charnock, 1955]. Values of  $\alpha_{Ch}$  for the 15 runs are listed in Table 1

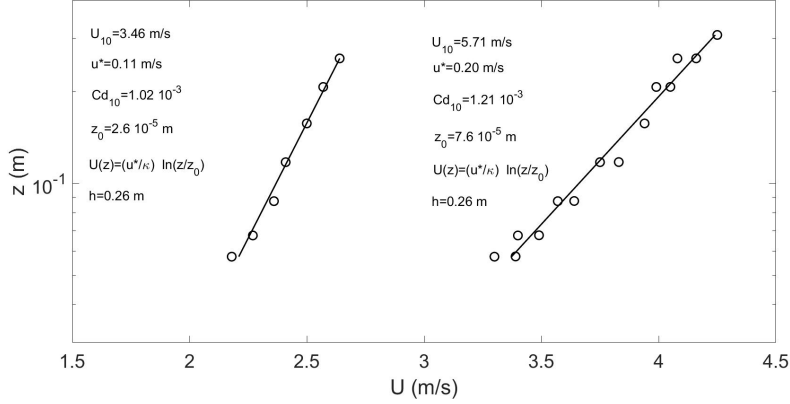


Figure 5: Two examples of logarithmic vertical wind profiles, with the estimated  $u^*$  and  $z_0$  values (runs 6 and 9).  $z$  is the altitude above the mean water level.

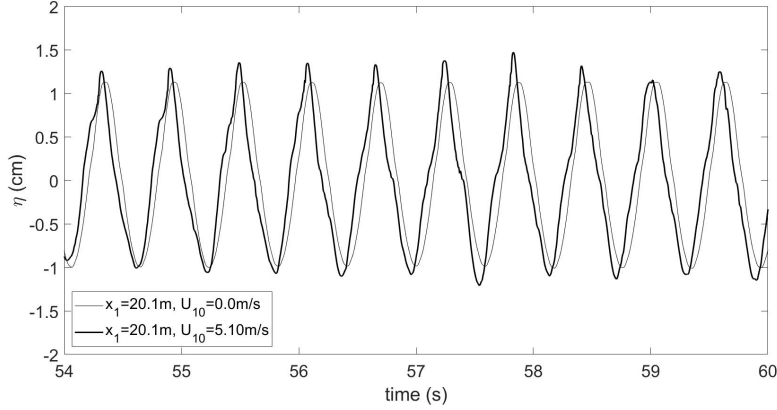


Figure 6: Example of time series of water elevation recorded by the wave gauges during run 5 at fetch  $x_1 = 20.1m$ , without wind and with wind on.

### 3.2.2. Wave parameters

The goal of the experiments was to measure the spatial and temporal growth rates of mechanical monochromatic waves forced by wind for different water depth conditions and to compare these growth rates with the theoretical values of [Montalvo et al., 2013 a]. An example of time series of the water elevation recorded by wave gauges during run 5 at fetch  $x_0 = 20.1m$ , without wind and with wind on ( $U_{10} = 5.10m/s$  is shown on Figure (6). The amplification by the wind of the dominant waves appears clearly on the plot. An example of the energy density spectra of the waves recorded during run 5 is displayed in Figure (7). The spectra are plotted at fetches  $x_0 = 2.4m$  and  $x_1 = 20.1m$ , when there is no wind and when the wind is on. Wind waves appear clearly on spectra (d) at high frequencies, and only the second harmonics of the dominant waves are visible at fetch  $x_1$  when wind is blowing.

In equations (19), (20) and (21),  $a_0$  and  $a_1$  are not the amplitude of each individual waves, but they must be considered as the average amplitude of waves with frequency close to the frequency  $f_0$ , the initial frequency of the wavemaker.

The determination of the amplitude  $a$  of the paddle waves, at the location of the wave gauges, was computed from the recorded time series of the wave gauges. We did a standard up-zero crossing analysis to separate each individual wave. Only the waves with a frequency  $f$  ranging between  $0.8f_0 < f < 1.2f_0$

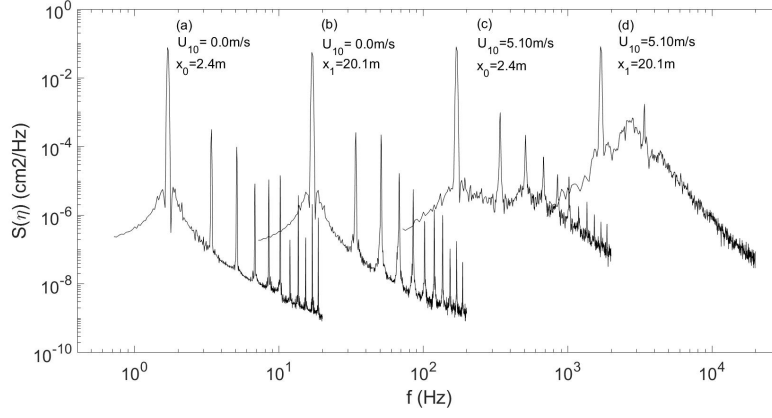


Figure 7: Density energy spectra of recorded waves during run 5. (a) fetch  $x_0 = 2.4m$  no wind; (b)  $x_1 = 20.1m$  no wind; (c)  $x_0 = 2.4m$   $U_{10} = 5.10m/s$ ; (d)  $x_1 = 20.1m$   $U_{10} = 5.10m/s$ .

were considered to avoid noisy short wave or long wave perturbations and to focus on the mono-frequency generated waves. For each wave number  $i$ , the amplitude  $a_i$  was determined as half the crest-to-trough height of the wave number  $i$  :

$$a_i = \frac{(H_{max} - H_{min})_i}{2} \quad (27)$$

The mean amplitude  $\bar{a}$  was computed as :

$$\bar{a} = \frac{1}{N} \sum_{i=1}^N a_i \quad (28)$$

with  $N$  the total number of waves of the run.

The experimental non-dimensional temporal wave growths (equations (19), (20) and (21) were finally computed with :

$$\hat{\gamma}_{et} = \left(\frac{u^*}{g\kappa}\right) C_g \left( \left(\frac{1}{\bar{a}_1} \frac{\bar{a}_1 - \bar{a}_0}{x_1 - x_0}\right)_{U_{ref}} - \left(\frac{1}{\bar{a}_1} \frac{\bar{a}_1 - \bar{a}_0}{x_1 - x_0}\right)_{U_0} \right) \quad (29)$$

$$\beta = 2 \left(\frac{c}{u^*}\right)^2 \kappa^2 \frac{1}{\omega} \left(\frac{\rho_a}{\rho_w}\right) C_g \left( \left(\frac{1}{\bar{a}_1} \frac{\bar{a}_1 - \bar{a}_0}{x_1 - x_0}\right)_{U_{ref}} - \left(\frac{1}{\bar{a}_1} \frac{\bar{a}_1 - \bar{a}_0}{x_1 - x_0}\right)_{U_0} \right) \quad (30)$$

and

$$\gamma_{et}/f_0 = \frac{C_g}{f_0} \left( \left(\frac{1}{\bar{a}_1} \frac{\bar{a}_1 - \bar{a}_0}{x_1 - x_0}\right)_{U_{ref}} - \left(\frac{1}{\bar{a}_1} \frac{\bar{a}_1 - \bar{a}_0}{x_1 - x_0}\right)_{U_0} \right) \quad (31)$$

with  $\bar{a}_0$  and  $\bar{a}_1$  the mean wave amplitude of the mechanical waves at fetches  $x_0$  and  $x_1$

### 3.3. Experimental conditions and measured values

Due to: a) the intrinsic mechanical performance of the piston wavemaker in low depth conditions (this wavemaker was designed long time ago for deep waters) , b) the fact that the paddle waves should not hit the rigid air divergent close to the water surface, c) the requirement to have not to steep waves to avoid wave amplitude modulation by modulational instabilities and wave breaking, d) the requirement to have low wind speeds to avoid air-flow separation and wave breaking too, the possible combinations of depths, wave frequencies, wave steepnesses, and wind speeds was quite low. We finally conducted 15 different runs. Main physical parameters characterizing waves and wind are

| Run | $h$<br>$m$ | $f_0$<br>$Hz$ | $L_0$<br>$m$ | $k_0h$ | $(ak)_0$ | $U_{10}$<br>$m/s$ | $u^*$<br>$m/s$ | $z_0$<br>$mm$ | $Cd_{10}$            | $\alpha_{Ch}$        |
|-----|------------|---------------|--------------|--------|----------|-------------------|----------------|---------------|----------------------|----------------------|
| 1   | 0.14       | 1.7           | 0.51         | 1.78   | 0.120    | 3.18              | 0.117          | 0.150         | $1.36 \cdot 10^{-3}$ | $10.7 \cdot 10^{-2}$ |
| 2   | 0.14       | 1.7           | 0.51         | 1.78   | 0.121    | 5.58              | 0.226          | 0.402         | $1.64 \cdot 10^{-3}$ | $7.68 \cdot 10^{-2}$ |
| 3   | 0.14       | 1.2           | 0.85         | 1.07   | 0.100    | 5.70              | 0.231          | 0.400         | $1.64 \cdot 10^{-3}$ | $7.36 \cdot 10^{-2}$ |
| 4   | 0.26       | 1.7           | 0.54         | 3.03   | 0.124    | 3.39              | 0.108          | 0.026         | $1.02 \cdot 10^{-3}$ | $2.18 \cdot 10^{-2}$ |
| 5   | 0.26       | 1.7           | 0.54         | 3.03   | 0.124    | 5.10              | 0.164          | 0.030         | $1.04 \cdot 10^{-3}$ | $1.09 \cdot 10^{-2}$ |
| 6   | 0.26       | 1.4           | 0.77         | 2.10   | 0.166    | 3.46              | 0.110          | 0.026         | $1.02 \cdot 10^{-3}$ | $2.09 \cdot 10^{-2}$ |
| 7   | 0.26       | 1.4           | 0.77         | 2.10   | 0.168    | 5.43              | 0.175          | 0.030         | $1.04 \cdot 10^{-3}$ | $0.96 \cdot 10^{-2}$ |
| 8   | 0.48       | 1.7           | 0.54         | 5.58   | 0.091    | 5.70              | 0.198          | 0.076         | $1.21 \cdot 10^{-3}$ | $1.90 \cdot 10^{-2}$ |
| 9   | 0.48       | 1.4           | 0.80         | 3.79   | 0.126    | 5.71              | 0.199          | 0.076         | $1.21 \cdot 10^{-3}$ | $1.89 \cdot 10^{-2}$ |
| 10  | 0.48       | 1.1           | 1.27         | 2.38   | 0.119    | 3.60              | 0.117          | 0.035         | $1.07 \cdot 10^{-3}$ | $2.49 \cdot 10^{-2}$ |
| 11  | 0.74       | 1.7           | 0.54         | 8.59   | 0.026    | 3.96              | 0.127          | 0.030         | $1.04 \cdot 10^{-3}$ | $1.81 \cdot 10^{-2}$ |
| 12  | 0.74       | 1.7           | 0.54         | 8.59   | 0.026    | 5.79              | 0.198          | 0.064         | $1.18 \cdot 10^{-3}$ | $1.60 \cdot 10^{-2}$ |
| 13  | 0.74       | 1.7           | 0.54         | 8.59   | 0.034    | 3.92              | 0.126          | 0.030         | $1.04 \cdot 10^{-3}$ | $1.84 \cdot 10^{-2}$ |
| 14  | 0.74       | 1.7           | 0.54         | 8.59   | 0.034    | 5.79              | 0.198          | 0.064         | $1.18 \cdot 10^{-3}$ | $1.60 \cdot 10^{-2}$ |
| 15  | 0.74       | 1.4           | 0.80         | 5.83   | 0.051    | 5.81              | 0.199          | 0.064         | $1.18 \cdot 10^{-3}$ | $1.58 \cdot 10^{-2}$ |

Table 1: Experimental conditions.  $h$  is the water depth,  $f_0$  is the paddle wave frequency,  $L_0$  is the wavelength of the paddle waves,  $(ak)_0$  is the steepness of the paddle waves measured without wind at fetch  $x_0$ ,  $U_{10}$  is the wind speed extrapolated to the standard altitude of 10m,  $u^*$  and  $z_0$  are the friction velocity and roughness length estimated by the wind profile method,  $Cd_{10}$  is the drag coefficient  $Cd_{10} = (u^*/U_{10})^2$ ,  $\alpha_{Ch}$  is the Charnock constant  $\alpha_{Ch} = gz_0/u^{*2}$ .

listed in Table 1 . Water depth  $h$  ranged from  $0.14m$  to  $0.74m$ , mono frequency waves  $f_0$  from  $1.1Hz$  to  $1.7Hz$ ,  $k_0h$  from 1.07 to 8.59. Finite depth is usually reached when the ratio of water depth over wavelength:  $h/L_0$  is lower than  $1/2$ , which corresponds, in term of  $kh$  to:  $kh < \pi = 3.14$ . According to the traditional classification of waves with respect to depth [Young, 1999] runs number 1 to seven correspond to intermediate depths:  $\pi/10 < kh < \pi$ , and run 8 to 15 to deep water conditions.

Initial steepness  $(ak)_0$  ranged from 0.026 to 0.168, wind speed  $U_{10}$  from  $3.18m/s$  to  $5.81m/s$ , friction velocity  $u^*$  from  $0.108m/s$  to  $0.231m/s$ , roughness length  $z_0$  from  $0.030mm$  to  $0.402mm$ . It was not possible to use higher wind speeds and/or higher initial wave steepness because we wanted to avoid natural wave breaking. Wave breaking induces additional wave dissipation which is not included in the theory here. With higher wind speeds or higher initial wave steepnesses we effectively obtained wave breaking inside the wave tank, either by wave amplification by the wind, or by natural sub-harmonic Benjamin-Feir instability leading to strong wave groupiness with the highest wave breaking inside the group [Bliven et al., 1986; Benjamin and Feir, 1967; Hwung et al., 2011]. The drag coefficient  $Cd_{10}$  ranged from  $1.0210^{-3}$  to  $1.6410^{-3}$ , and the Charnock constant  $\alpha_{Ch}$  from 0.0096 to 0.107.

By making several times the same experiment, the relative precision on  $(\bar{a}_0)_{U_0}$ ,  $(\bar{a}_0)_{U_{ref}}$ ,  $(\bar{a}_1)_{U_0}$ , and  $(\bar{a}_1)_{U_{ref}}$  was estimated to be 8%:  $\frac{d\bar{a}}{\bar{a}} = 8\%$ . The precision on  $u^*$  and on  $U_{10}$  is 15%, the precision on  $z_0$  is 30%. (c.f. paragraph 3.2.1)  $\frac{du^*}{u^*} = 15\%$ ,  $\frac{dU_{10}}{U_{10}} = 15\%$ ,  $\frac{dz_0}{z_0} = 30\%$ .

Since  $Cd_{10} = (\frac{u^*}{U_{10}})^2$ , the relative precision on  $Cd_{10}$  is:  $\frac{dCd_{10}}{Cd_{10}} = 2(\frac{du^*}{u^*} + \frac{dU_{10}}{U_{10}}) = 2(15\% + 15\%) = 60\%$ .

Since  $\alpha_{Ch} = \frac{gz_0}{u^{*2}}$ , the relative precision on  $\alpha_{Ch}$  is:  $\frac{d\alpha_{Ch}}{\alpha_{Ch}} = \frac{dz_0}{z_0} + 2\frac{du^*}{u^*} = 30\% + 2(15\%) = 60\%$ .

The precision on  $f_0$  is 2%:  $\frac{df_0}{f_0} = 2\%$ . The precision on  $h$  is 1%:  $\frac{dh}{h} = 1\%$ , the precision on  $x$  is 0.5%:  $\frac{dx}{x} = 0.5\%$ .

Since  $L_0 \approx \frac{g}{2\pi f_0^2}$ , we have  $\frac{dL_0}{L_0} = 2\frac{df_0}{f_0} = 4\%$ .

$\frac{d(k_0h)}{k_0h} = \frac{dk_0}{k_0} + \frac{dh}{h} = \frac{dL_0}{L_0} + \frac{dh}{h} = 4\% + 1\% = 5\%$ .

$\frac{d(ak)_0}{(ak)_0} = \frac{da}{a} + \frac{dk_0}{k_0} = 8\% + 4\% = 12\%$ .

Since  $c \approx \frac{g}{\omega}$ ,  $\frac{dc}{c} = \frac{d\omega}{\omega} = \frac{df_0}{f_0} = 2\%$ .

$C_g = \frac{d\omega}{dk}$ . We estimated the precision on  $C_g$  to be :  $\frac{d(C_g)}{C_g} \approx \frac{d\omega}{\omega} + \frac{dk}{k} = 2\% + 4\% = 6\%$  .

The reference wind speed  $U_1$  was taken as  $U_1 = u^* / \kappa$  [Montalvo et al., 2013 a,b; Latifi et al., 2017].

$$\delta = \frac{gh}{U_1^2} = \frac{gh\kappa^2}{u^{*2}}, \text{ so } \frac{d\delta}{\delta} = \frac{dh}{h} + 2\frac{du^*}{u^*} = 1\% + 30\% = 36\% .$$

$$\theta_{dw} = \frac{1}{U_1} \sqrt{\frac{g}{k}} = \frac{\kappa}{u^*} \sqrt{\frac{g}{k}}, \text{ so } \frac{d\theta_{dw}}{\theta_{dw}} = \frac{du^*}{u^*} + \frac{1}{2} \frac{dk}{k} = 15\% + 2\% = 17\% .$$

$$\theta_{fd} = \theta_{dw} \sqrt{\tanh kh}, \text{ so } \frac{d\theta_{fd}}{\theta_{fd}} \approx \frac{d\theta_{dw}}{\theta_{dw}} + \frac{1}{2} \frac{2kh}{\sinh(2kh)} \left( \frac{dk}{k} + \frac{dh}{h} \right) < 17\% + \frac{1}{2}(4\% + 1\%) = 19.5\% .$$

Equation (29) reads :  $\hat{\gamma}_{et} = \left( \frac{u^*}{g\kappa} \right) C_g \left( \left( \frac{1}{\bar{a}_1} \frac{\bar{a}_1 - \bar{a}_0}{x_1 - x_0} \right) U_{ref} - \left( \frac{1}{\bar{a}_1} \frac{\bar{a}_1 - \bar{a}_0}{x_1 - x_0} \right) U_0 \right)$ , so the precision on  $\hat{\gamma}_{et}$  is:  
 $\frac{d\hat{\gamma}_{et}}{\hat{\gamma}_{et}} \approx \frac{du^*}{u^*} + \frac{d(C_g)}{C_g} + 4 \frac{d\bar{a}}{\bar{a}} + 2 \frac{dx}{x} = 15\% + 6\% + 32\% + 1\% = 54\%$ .

Equation (30) reads :  $\beta = 2 \left( \frac{c}{u^*} \right)^2 \kappa^2 \frac{1}{\omega} \left( \frac{1}{\bar{a}} \right) C_g \left( \left( \frac{1}{\bar{a}_1} \frac{\bar{a}_1 - \bar{a}_0}{x_1 - x_0} \right) U_{ref} - \left( \frac{1}{\bar{a}_1} \frac{\bar{a}_1 - \bar{a}_0}{x_1 - x_0} \right) U_0 \right)$ , so the precision on  $\beta$  is:

$$\frac{d\beta}{\beta} \approx 2 \frac{du^*}{u^*} + 2 \frac{dc}{c} + \frac{d\omega}{\omega} + \frac{d(C_g)}{C_g} + 4 \frac{d\bar{a}}{\bar{a}} + 2 \frac{dx}{x} = 30\% + 4\% + 2\% + 6\% + 32\% + 1\% = 75\% .$$

Equation (31) reads :  $\gamma_{et}/f_0 = \frac{C_g}{f_0} \left( \left( \frac{1}{\bar{a}_1} \frac{\bar{a}_1 - \bar{a}_0}{x_1 - x_0} \right) U_{ref} - \left( \frac{1}{\bar{a}_1} \frac{\bar{a}_1 - \bar{a}_0}{x_1 - x_0} \right) U_0 \right)$ , so the precision on  $\gamma_{et}/f_0$  is:  
 $\frac{d(\gamma_{et}/f_0)}{\gamma_{et}/f_0} \approx \frac{d(C_g)}{C_g} + \frac{df_0}{f_0} + 4 \frac{d\bar{a}}{\bar{a}} + 2 \frac{dx}{x} = 6\% + 2\% + 32\% + 1\% = 41\%$ .

In the following paragraphs, we will consider non-dimensional variables which may be active in fetch or duration limited growth. Like [Sverdrup and Munk, 1947; Kitaigorodskii, 1970; Young, 1999] the following quantities to be of importance are the non-dimensional energy  $\frac{g^2 \sigma^2}{U_{10}^4}$ , with  $\sigma$  the standard deviation of the water elevation, the non-dimensional frequency  $\frac{f_0 U_{10}}{g}$ , the non-dimensional depth  $\frac{gh}{U_{10}}$ , the inverse wave age  $\frac{U_{10}}{c}$ . By making several times the same experiment, the relative precision on  $\sigma$  was found to be the same as the precision on the mean amplitude  $\bar{a}$  :  $\frac{d\sigma}{\sigma} = \frac{d\bar{a}}{\bar{a}} = 8\%$ .

The precision on  $U_{10}/c$  is :  $\frac{d(U_{10}/c)}{U_{10}/c} = \frac{dU_{10}}{U_{10}} + \frac{dc}{c} = 15\% + 2\% = 17\%$

The precision on  $gh/U_{10}^2$  is :  $\frac{d(gh/U_{10}^2)}{gh/U_{10}^2} = \frac{dh}{h} + 2\frac{dU_{10}}{U_{10}} = 1\% + 2(15)\% = 31\%$

The precision on  $f_0 U_{10}/g$  is :  $\frac{d(f_0 U_{10}/g)}{f_0 U_{10}/g} = \frac{df_0}{f_0} + \frac{dU_{10}}{U_{10}} = 2\% + 15\% = 17\%$

The precision on  $g^2 \sigma^2 / U_{10}^4$  is :  $\frac{d(g^2 \sigma^2 / U_{10}^4)}{g^2 \sigma^2 / U_{10}^4} = 2\frac{d\sigma}{\sigma} + 4\frac{dU_{10}}{U_{10}} = 2(8)\% + 4(15)\% = 76\%$ .

Relative errors are summarized in Table 2

|            |               |                |                           |               |               |                     |           |                   |          |       |       |
|------------|---------------|----------------|---------------------------|---------------|---------------|---------------------|-----------|-------------------|----------|-------|-------|
| $x$        | $h$           | $f_0$          | $c$                       | $C_g$         | $L_0$         | $k_0 h$             | $\bar{a}$ | $\sigma$          | $(ak)_0$ | $u^*$ | $z_0$ |
| 0.5%       | 1%            | 2%             | 2%                        | 6%            | 4%            | 5%                  | 8%        | 8%                | 12%      | 15%   | 30%   |
| $U_{10}$   | $Cd_{10}$     | $\alpha_{Ch}$  | $\delta$                  | $\theta_{dw}$ | $\theta_{fd}$ | $\hat{\gamma}_{et}$ | $\beta$   | $\gamma_{et}/f_0$ |          |       |       |
| 15%        | 60%           | 60%            | 36%                       | 17%           | 19.5%         | 54%                 | 75%       | 41%               |          |       |       |
| $U_{10}/c$ | $gh/U_{10}^2$ | $f_0 U_{10}/g$ | $g^2 \sigma^2 / U_{10}^4$ |               |               |                     |           |                   |          |       |       |
| 17%        | 31%           | 17%            | 76%                       |               |               |                     |           |                   |          |       |       |

Table 2: Relative errors on estimated values.

The measured values of  $(\bar{a}_0)_{U_0}$  ,  $(\bar{a}_0)_{U_{ref}}$  ,  $(\bar{a}_1)_{U_0}$  ,  $(\bar{a}_1)_{U_{ref}}$  are displayed in Table 3, as well as the computed wave phase  $c$  (relation (23)), wave group  $C_g$  (relation (24)), wave age  $c/u^*$ , and non-dimensional parameters  $\delta$ ,  $\theta_{dw}$ ,  $\theta_{fd}$  (relation (7)),  $\hat{\gamma}_{et}$  (relation (29)),  $\beta$  (relation (30)) and  $\gamma_{et}/f_0$  (relation (31)).

| Run | $(\bar{a}_0)_{U_0}$ | $(\bar{a}_0)_{U_{ref}}$ | $(\bar{a}_1)_{U_0}$ | $(\bar{a}_1)_{U_{ref}}$ | $c$  | $Cg$ | $c/u^*$ | $\delta$ | $\theta_{dw}$ | $\theta_{fd}$ | $\hat{\gamma}_{et}$ | $\beta$ | $\gamma_{et}/f_0$ |
|-----|---------------------|-------------------------|---------------------|-------------------------|------|------|---------|----------|---------------|---------------|---------------------|---------|-------------------|
|     | cm                  | cm                      | cm                  | cm                      | m/s  | m/s  |         |          |               |               | $\times 10^{-5}$    |         | $\times 10^{-4}$  |
| 1   | 0.97                | 0.95                    | 0.78                | 0.80                    | 0.87 | 0.52 | 7.4     | 17       | 3.1           | 3.0           | 0.71                | 0.34    | 1.4               |
| 2   | 0.99                | 1.03                    | 0.79                | 1.00                    | 0.87 | 0.52 | 3.8     | 4.7      | 1.6           | 1.6           | 36                  | 2.5     | 38                |
| 3   | 1.36                | 1.55                    | 0.80                | 0.81                    | 1.03 | 0.78 | 4.4     | 4.5      | 2.0           | 1.8           | 0.20                | 0.10    | 0.3               |
| 4   | 1.06                | 1.07                    | 0.87                | 0.96                    | 0.91 | 0.47 | 8.5     | 36       | 3.5           | 3.5           | 8.1                 | 5.5     | 18                |
| 5   | 1.06                | 1.13                    | 0.87                | 1.19                    | 0.91 | 0.47 | 5.6     | 16       | 2.3           | 2.3           | 30                  | 5.9     | 44                |
| 6   | 2.04                | 2.29                    | 1.60                | 1.83                    | 1.08 | 0.61 | 9.8     | 35       | 4.1           | 4.0           | 2.1                 | 2.3     | 5.4               |
| 7   | 2.07                | 2.12                    | 1.63                | 1.84                    | 1.08 | 0.61 | 6.2     | 14       | 2.6           | 2.5           | 18                  | 4.9     | 30                |
| 8   | 0.79                | 0.63                    | 0.51                | 0.89                    | 0.91 | 0.46 | 4.6     | 20       | 1.9           | 1.9           | 10                  | 12      | 128               |
| 9   | 1.59                | 1.74                    | 1.35                | 1.49                    | 1.11 | 0.56 | 5.6     | 20       | 2.3           | 2.3           | 2.0                 | 0.39    | 2.9               |
| 10  | 2.41                | 2.42                    | 2.30                | 2.39                    | 1.40 | 0.75 | 11.9    | 57       | 4.9           | 4.9           | 4.2                 | 7.9     | 13                |
| 11  | 0.22                | 0.25                    | 0.25                | 0.65                    | 0.92 | 0.46 | 7.2     | 75       | 3.0           | 3.0           | 40                  | 17      | 75                |
| 12  | 0.22                | 0.24                    | 0.25                | 0.92                    | 0.92 | 0.46 | 4.6     | 31       | 1.9           | 1.9           | 79                  | 8.8     | 94                |
| 13  | 0.30                | 0.37                    | 0.39                | 0.83                    | 0.92 | 0.46 | 7.3     | 76       | 3.0           | 3.0           | 25                  | 11      | 47                |
| 14  | 0.30                | 0.30                    | 0.39                | 0.88                    | 0.92 | 0.46 | 4.6     | 31       | 1.9           | 1.9           | 52                  | 5.8     | 62                |
| 15  | 0.65                | 1.04                    | 0.48                | 1.08                    | 1.12 | 0.56 | 5.6     | 31       | 2.3           | 2.3           | 63                  | 12      | 91                |

Table 3: Experimental measured values: mean wave amplitudes at fetch  $x_0$  without wind  $(\bar{a}_0)_{U_0}$  and with wind  $(\bar{a}_0)_{U_{ref}}$ , and at fetch  $x_1$  without wind  $(\bar{a}_1)_{U_0}$  and with wind  $(\bar{a}_1)_{U_{ref}}$ . Estimated values: celerity  $c$ , group velocity  $Cg$ , wave age  $c/u^*$ , non-dimensional parameters  $\delta$ ,  $\theta_{dw}$ ,  $\theta_{fd}$ ,  $\hat{\gamma}_{et}$ ,  $\beta$  and  $\gamma_{et}/f_0$ .

## 4. Discussion

### 4.1. Depth influence on air-to-water transfers

#### 4.1.1. Depth influence on drag coefficient $C_d$

Figure (8) shows the evolution of measured  $Cd_{10}$  with estimated  $U_{10}$ . Drag coefficients in low water depth environment ( $kh < 1.8$ ) are clearly higher than those in large depth configurations. In open seas, when  $U_{10}$  is known from satellite or models, a model of the drag like :  $Cd_{10} = a + bU_{10}$ ,  $a$  and  $b$  being constant, is widely used to estimate  $u^*$  from  $U_{10}$  :  $u^* = U_{10} (Cd_{10})^{0.5}$ . A quite large range of  $a$  and  $b$  values have been reported from measurements like, for example :  $a = 0.75 \cdot 10^{-3}$ ,  $b = 0.067 \cdot 10^{-3}$  [Garratt, 1977];  $a = 0.61 \cdot 10^{-3}$ ,  $b = 0.063 \cdot 10^{-3}$  [Smith, 1980];  $a = 0.80 \cdot 10^{-3}$ ,  $b = 0.065 \cdot 10^{-3}$  [Wu, 1982];  $a = 0.96 \cdot 10^{-3}$ ,  $b = 0.041 \cdot 10^{-3}$  [Donelan, 1982];  $a = 0.58 \cdot 10^{-3}$ ,  $b = 0.084 \cdot 10^{-3}$  [Geernaert, 1987];  $a = 0.49 \cdot 10^{-3}$ ,  $b = 0.071 \cdot 10^{-3}$  [Anderson, 1993];  $a = 0.50 \cdot 10^{-3}$ ,  $b = 0.071 \cdot 10^{-3}$  [Yelland et al., 1998];  $a = 0.96 \cdot 10^{-3}$ ,  $b = 0.060 \cdot 10^{-3}$  [Toffoli et al., 2012]. Our measured trend of  $Cd_{10}$  with  $U_{10}$  is equivalent to the trend observed in literature (i.e. the same as [Smith, 1980]:  $b = 0.063 \cdot 10^{-3}$ ) for large depth configuration. But for low depth,  $Cd_{10}$  values are significantly higher and the trend is much higher :  $b = 0.112 \cdot 10^{-3}$ .

[Bi et al., 2015] investigated the drag coefficient  $Cd$  in the South China Sea from 2008 to 2014, and one of their tower to collect data was located in 15-m of water. Some of their data were considered to be in finite depth environment (i.e. depth lower than half the dominant wavelength, i.e.  $kh < \pi$ ). [Bi et al., 2015] found that the drag coefficient was higher in shallow water by 40% for  $U_{10} < 10m/s$ . Jimenez and Duthia [2018] showed that the drag at the location of FINO1 offshore platform in shallow waters was about 30% higher than the drag in deep waters. Very close to the shore, the drag coefficients over the surf zone during onshore winds were determined by Shabani et al. [2014] to be almost 200% the values expected at the same wind speed and open ocean conditions ! [Ancil and Donelan, 1986] found that the wind dependence of  $Cd$  derived for shallow water differs from that of deep water. sometimes, their  $Cd$  values were 40% to 80% higher for shallow water. They explain the modification of  $Cd$  by the modification of wave age  $c/U_{10}$  and modification of the root mean square wave slope by the bottom effect on waves in shallow water. The geometric shape of the wave and the kinematics are modified by shoaling effects, thus modifying air-water transfers. [Toffoli et al., 2012] argued that water depth effects together with large wind result in a steeper and more vertically

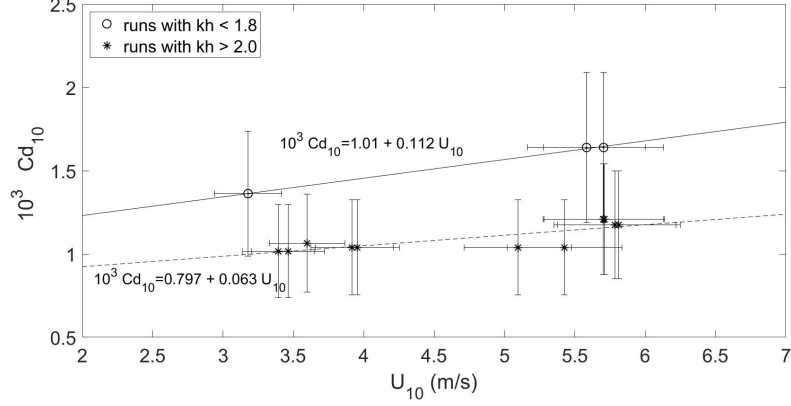


Figure 8: Drag coefficient  $Cd_{10}$  as function of  $U_{10}$ . Drag coefficients are higher for low water depth (i.e  $kh < 1.8$ )

asymmetric profile for dominant waves, which enhances breaking probability, air flow separation and sea drag. They proposed a parametrisation of  $C_d$  with an inverse quadratic function of a non linear parameter  $\mu$  depending on both wave steepness and water depth. In our figure(8) we see that our Drag coefficient is 45% higher at low water depth ( $kh < 1.8$ ) if compared to larger water depth ( $kh > 2.0$ ), and the larger wind speed, the larger relative difference between  $Cd$  values at low depth and large depth.

[Zhao et al., 2015] explored also  $Cd$  in the South China Sea from wind propeller anemometers stationed on a coastal observation tower (water depth 14m), to observe the effect of water depth on  $Cd$ . But their study was more focused on typhoon measurements. [Zhao et al., 2015] concluded that water depth unequivocally affects the drag coefficient when compared to deep open ocean results, but more particularly at high winds ( $U_{10} > 15m/s$ ), which is beyond our scope here. They found that in shallow water depth environment, the  $Cd$  maximum occurs approximately at 24 m/s, which is 5–15 m/s lower than that of the open sea. This is prospectively due to wave shoaling and wave breaking conditions pertaining to the shallow water in high wind conditions. [Zhao et al., 2015] proposed a  $Cd$  formulation as a function of wind speed and water depth, but in their formulation, the water depth plays a role only in the  $U_{10} > 15m/s$  part of the  $Cd$  curves. Before them, [Geernaert et al., 1986] have also shown that the shallow water  $Cd$  values for wind speeds above  $15m/s$  are higher than those over the open ocean and they attributed the difference to the variations in the surface wave energy spectrum. Based on their air-flow separation model, [Makin and Kudryavtsev, 2002] and their WOWC (Wind Over Waves Coupling) model have reproduced the increase in  $Cd$  with decreasing water depth: at moderate and high winds, decreasing water depth induces a steepening of dominant waves and thus more air-flow separation at the crest of the waves, and more drag. Makin et al. [2004] explained the wind speed, wave age and finite depth dependences of the sea drag in a wave tank environment with this model.

What is new in our measurements, it is the fact that we find a significant increase of  $C_d$ , even at relatively low wind speed (i.e.  $U_{10} < 6m/s$ ). [Donelan, 1982; Oost et al., 2002; Gao et al., 2009; Toffoli et al., 2012] have also shown that the  $C_d$  over shallow water is higher than that over deep water at the same wind speed, but they have related this phenomenon to wave state parameters, e.g., wave age and wave steepness. They do not include water depth  $h$  or non-dimensional parameter  $kh$  in their drag model.

#### 4.1.2. Depth influence on aerodynamic roughness length $z_0$

The evolution of  $z_0$  with the wind  $U_{10}$  is shown on figure (9).  $z_0$  values are much higher for low water depth (i.e  $kh < 1.8$ ) than for larger depths ( $kh > 2.0$ ), and the variation trend of  $z_0$  with  $U_{10}$  is six times higher for low depth than deep water. Recently [Jimenez and Dudhia, 2018] pointed out



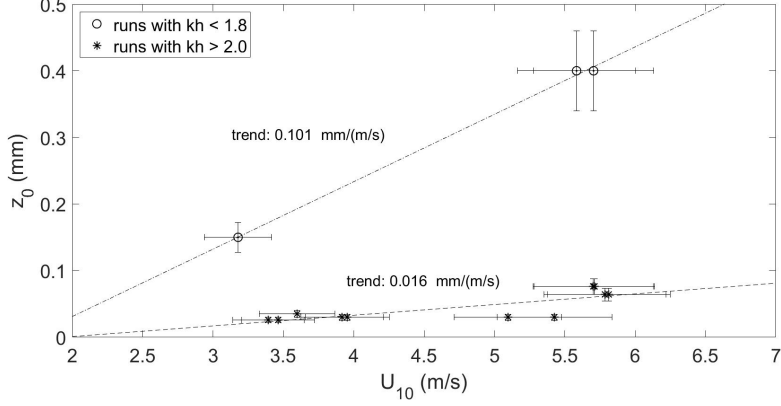


Figure 9: Roughness parameter  $z_0$  as function of  $U_{10}$ .  $z_0$  values are higher for low water depth (i.e  $kh < 1.8$ ), and the trend is six times higher.

the need to modify the sea surface roughness formulation over shallow waters taking into account the water depth. [Anctil and Donelan, 1986] explained the increase of  $z_0$  by the increase of the "inverse wave age"  $U_{10}/c$ , the increase of root mean square wave slope  $\eta_X$ , and the increase of significant wave height  $H_s$  due to shoaling effect near the coast in intermediate and shallow water conditions. They proposed a relationship like :  $z_0 = a_1 H_s (U_{10}/c)^{b_1} (\eta_X)^{c_1}$ . [Smith et al., 1992] have suggested that celerity and steepness are the wave properties more likely to influence the surface roughness  $z_0$  in finite depth environment because celerity is restricted by water depth and wave age is shorter at short fetch. [Gao et al., 2009] using three different databases, illustrated the increase of  $z_0$  with low depth, because the finite depth change the wave celerity  $c$ , and by the way the inverse wave age ( $U_{10}/c$ ). They also proposed a model of  $z_0$  with the inverse wave age  $U_{10}/c$ . [Oost et al., 2002] took also into account the wave steepness in their model of  $z_0$ , the waves being steeper when the depth is low, but they do not include parameter  $kh$  in the  $z_0$  estimation. MacMahan [2017] related the increase of roughness length  $z_0$  near the coasts in finite depth to surfzone foam coverage due to wave breaking. But in our experiment we did not observed wave breaking although roughness parameter  $z_0$  was higher in low depth conditions.

#### 4.1.3. Depth influence on Charnock parameter $\alpha_{Ch}$

In open seas, the Charnock parameter  $\alpha_{Ch}$  is generally assumed constant, although a range of values have been reported from measurements: 0.014 [Garratt, 1977], 0.018 [Wu, 1980], 0.019 [Geernaert et al., 1986], 0.035 [Kitaigorodskii and Volkov, 1965] among others. [Vickers and Mahrt, 1997] conducted measurements by fetch length and they measured  $\alpha_{Ch} = 0.018$  for onshore and  $\alpha_{Ch} = 0.073$  for offshore wind conditions during the RASEX campaign [Lüko et al., 2020], i.e. at very short fetches and intermediate water depth. Above the water surface in the intertidal area and in the inner-surf and swash zones, which means in very shallow water environment and even sometimes with no water but along the shoreline, during the Stradbroke 2012 campaign, [Shabani et al., 2014] observed Charnock coefficients an order of magnitude larger than open ocean values : the average Charnock coefficient of their data was  $\alpha_{Ch} = 0.110$  ! To our knowledge, these are the highest observed  $\alpha_{Ch}$  values so far. They explained this large increase of Charnock coefficients by the slower celerity in the surf zones, the breaking of the waves when they reach the shore and more particularly the sawtooth wave shape in the inner surf zone and swash zone, with asymmetric and very steep waves. From Lake George measurements, Toffoli et al. [2012] have proposed a parametrisation of  $\alpha_{Ch}$  using an inverse quadratic function of a complex non linear parameter  $\mu$  depending on wave steepness and water depth.

Figure (10) shows the evolution of our estimated  $\alpha_{Ch}$  data with the non-dimensional depth parameter  $kh$ . For large depth, our mean  $\alpha_{Ch}$  value is 0.018, which is the same value reported by [Wu, 1980]

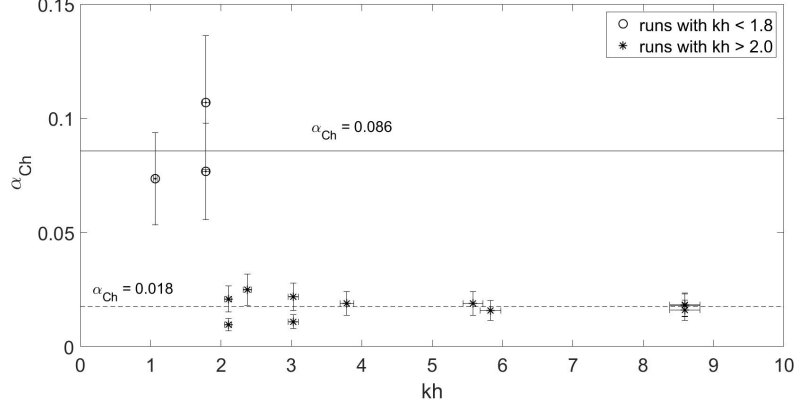


Figure 10: Charnock parameter  $\alpha_{Ch} = \frac{gz_0}{u_*^2}$  as function of non-dimensional depth parameter  $kh$ .  $\alpha_{Ch}$  values are higher for low water depth (i.e  $kh < 1.8$ )

and [Vickers and Mahrt, 1997]. This value of 0.018 was also proposed by [Charnock, 1955]. But for low depth configuration (i.e.  $kh < 1.8$ ),  $\alpha_{Ch}$  is 5 times larger: 0.086 . This value is slightly larger than the RASEX campaign Charnock value for offshore winds [Lüko et al., 2020], but slightly lower than the Stradbroke campaign value recorded along the shore line [Shabani et al., 2014]. However, one of our measurement (run 1,  $\alpha_{Ch} = 0.107$ ) is close equal to their mean recorded value:  $\alpha_{Ch} = 0.110$ . The depth effect on Charnock parameter appears clearly on our figure (10).

#### 4.2. Depth influence on wave growth

[Montalvo et al., 2013 a,b] and [Latifi et al., 2017] have shown that the limit for which the growth rate  $\hat{\gamma}_{et}$  goes to zero was defined by the equation  $\theta_{fd}^c = \sqrt{\delta}$ . Figure (11) shows the location of the 15 experiments in the  $(\theta_{fd}, \delta)$  plane, together with the [Montalvo et al., 2013 a] limit and the data of [Donelan et al., 2005, 2006], from the Australian Shallow Water Experiment. This figure shows the wide range of  $\delta$  and  $\theta_{fd}$  values covered by our experiments. Even if the relative errors are quite high, values remains below the physical limit for which sea state evolution (wave amplification) is not possible.

Figure (12) displays the evolution of the non-dimensional frequency  $f_0 U_{10}/g$  with non-dimensional depth  $gh/U_{10}^2$ . This graph shows that are data are inside the asymptotic non-dimensional limit-law proposed by [Young and Verhagen, 1996 a; Young, 1997]:  $\frac{f_0 U_{10}}{g} = 1.25(\frac{gd}{U_{10}^2})^{-0.45}$ . Our data are closer to the [Young and Verhagen, 1996 a] limit than the [Bretschneider, 1958] limit:  $\frac{f_0 U_{10}}{g} = 0.16(\frac{gd}{U_{10}^2})^{-0.375}$ .

Figure (13) displays the evolution of the inverse wave age  $U_{10}/c$  with non-dimensional depth  $gh/U_{10}^2$ . This graph shows that the low depth limits the wave age: the wave age cannot be beyond the non-dimensional depth asymptotic limit proposed by [Young, 1997]:  $\frac{U_{10}}{c} = 1.25(\frac{gd}{U_{10}^2})^{-0.45}$ .

Figure (14) displays the evolution of the non-dimensional energy  $g^2 \sigma^2/U_{10}^4$  with non-dimensional depth  $gh/U_{10}^2$ . This graph shows that our data are inside the asymptotic limit-laws proposed by [Young and Verhagen, 1996 a] :  $\frac{g^2 \sigma^2}{U_{10}^4} = 1.06 \cdot 10^{-3} (\frac{gd}{U_{10}^2})^{1.3}$  and [Bretschneider, 1958; CERC, 1977] :  $\frac{g^2 \sigma^2}{U_{10}^4} = 1.40 \cdot 10^{-3} (\frac{gd}{U_{10}^2})^{1.5}$ . The low depth limits the total energy of the sea state. Waves cannot be amplified beyond a threshold value depending on the non-dimensional depth.

The non-dimensional growth rate  $\hat{\gamma}_{et}$  is now compared in figure (15) with the theoretical curves of [Montalvo et al., 2013 a]. The existence of a finite depth  $h$  transforms the unique Miles curve of wave

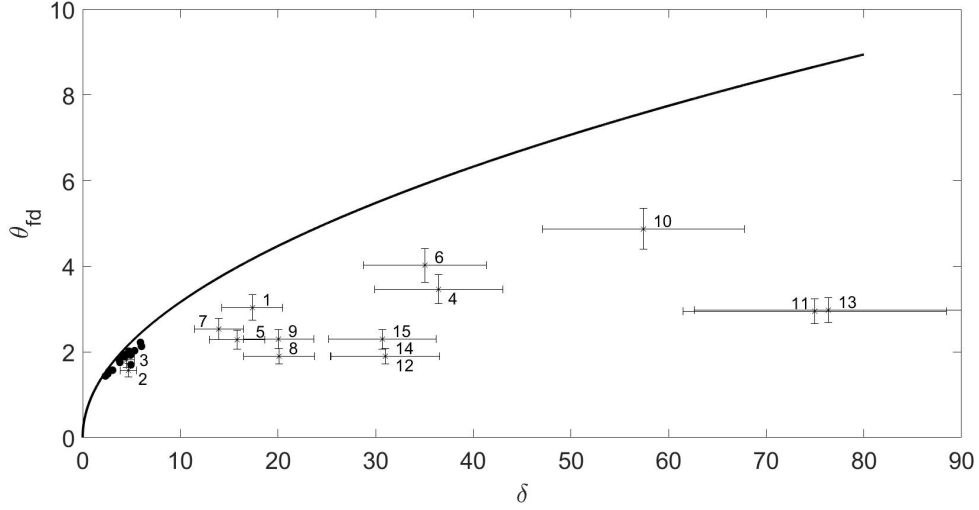


Figure 11: Location in the  $(\theta_{fd}, \delta)$  plane of the 15 runs. Each run number is printed at its location with the corresponding error bars. The black line is the [Montalvo et al., 2013 a] limit (location of the critical  $\theta_{fd}^c$  values) where there is no amplification by the wind. Black dots are the points from the Australian Shallow Water Experiment [Donelan et al., 2006].

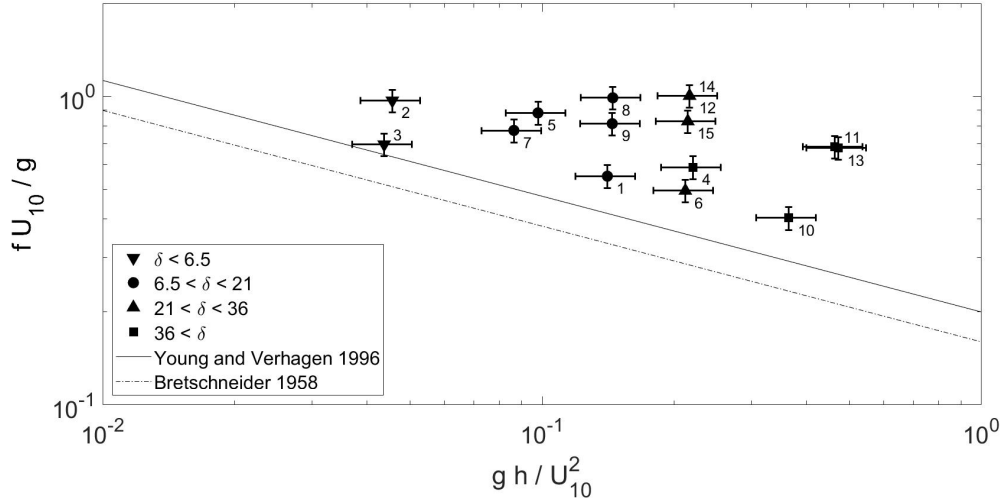


Figure 12: Non-dimensional frequency  $f_0 U_{10}/g$  evolution with the non-dimensional depth  $gh/U_{10}^2$ . Run numbers are written on the plot. Full symbols with error bars : measured values for  $\delta \in [0 \ 6.5]$  ( $\blacktriangledown$ ),  $\delta \in [6.5 \ 21]$  ( $\bullet$ ),  $\delta \in [21 \ 36]$  ( $\blacktriangle$ ),  $\delta \in [36 \ \infty]$  ( $\blacksquare$ ). Straight black line: [Young and Verhagen, 1996 a] depth asymptotic limit :  $\frac{f_0 U_{10}}{g} = 0.20 \left(\frac{gd}{U_{10}^2}\right)^{-0.375}$  ; Straight dotted line: [Bretschneider, 1958] limit :  $\frac{f_0 U_{10}}{g} = 0.16 \left(\frac{gd}{U_{10}^2}\right)^{-0.375}$ .

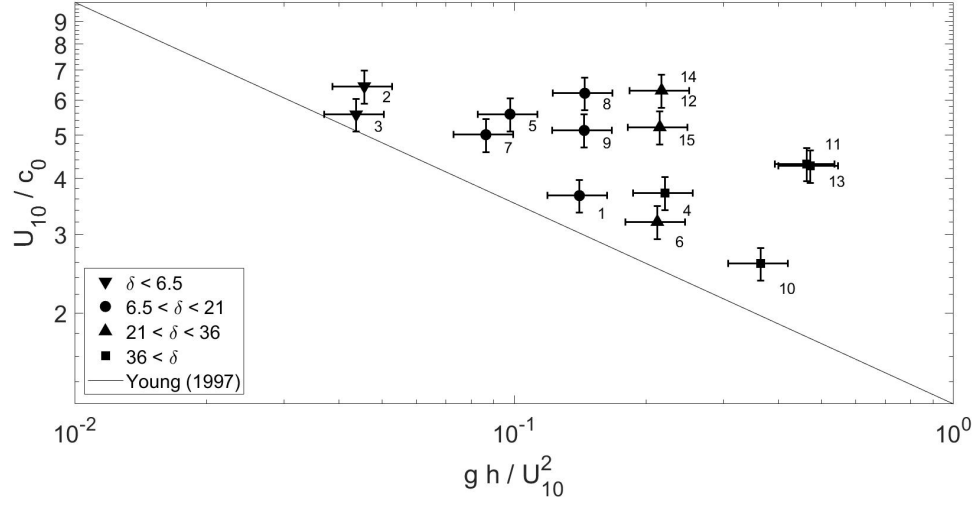


Figure 13: Inverse wave age  $U_{10}/c$  evolution with the non-dimensional depth  $gh/(U_{10}^2)$ . Run numbers are written on the plot. Full symbols with error bars : measured values for  $\delta \in [0 \ 6.5]$  ( $\blacktriangledown$ ),  $\delta \in [6.5 \ 21]$  ( $\bullet$ ),  $\delta \in [21 \ 36]$  ( $\blacktriangle$ ),  $\delta \in [36 \ \infty]$  ( $\blacksquare$ ). Straight black line: [Young, 1997] limit :  $\frac{U_{10}}{c} = 1.25 \left(\frac{gd}{U_{10}^2}\right)^{-0.45}$ .

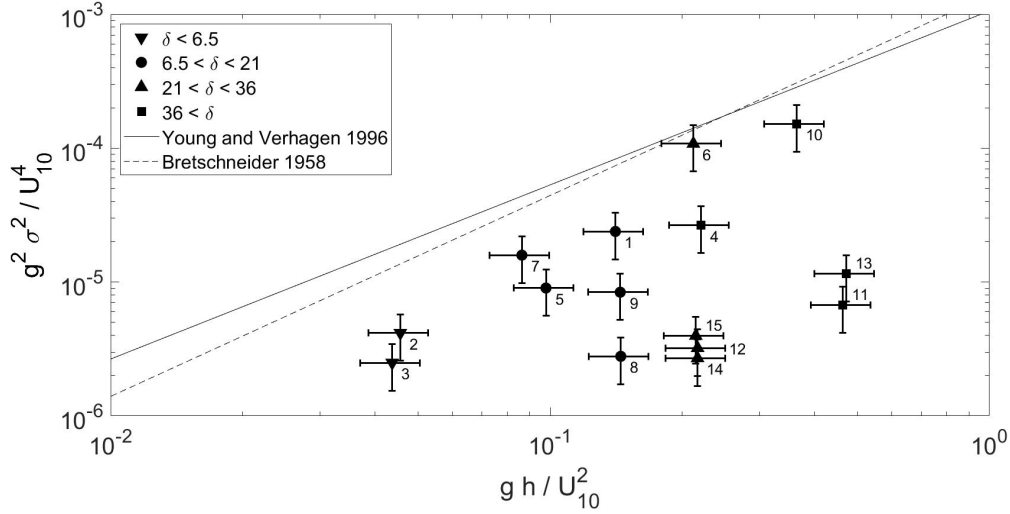


Figure 14: Non-dimensional energy  $g^2 \sigma^2 / U_{10}^4$  evolution with the non-dimensional depth  $gh/(U_{10}^2)$ . Run numbers are written on the plot. Full symbols with error bars : measured values for  $\delta \in [0 \ 6.5]$  ( $\blacktriangledown$ ),  $\delta \in [6.5 \ 21]$  ( $\bullet$ ),  $\delta \in [21 \ 36]$  ( $\blacktriangle$ ),  $\delta \in [36 \ \infty]$  ( $\blacksquare$ ). Straight black line: [Young and Verhagen, 1996 a] limit :  $\frac{g^2 \sigma^2}{U_{10}^4} = 1.06 \ 10^{-3} \left(\frac{gd}{U_{10}^2}\right)^{1.3}$  ; Straight dotted line: [Bretschneider, 1958; CERC, 1977] limit :  $\frac{g^2 \sigma^2}{U_{10}^4} = 1.40 \ 10^{-3} \left(\frac{gd}{U_{10}^2}\right)^{1.5}$ .

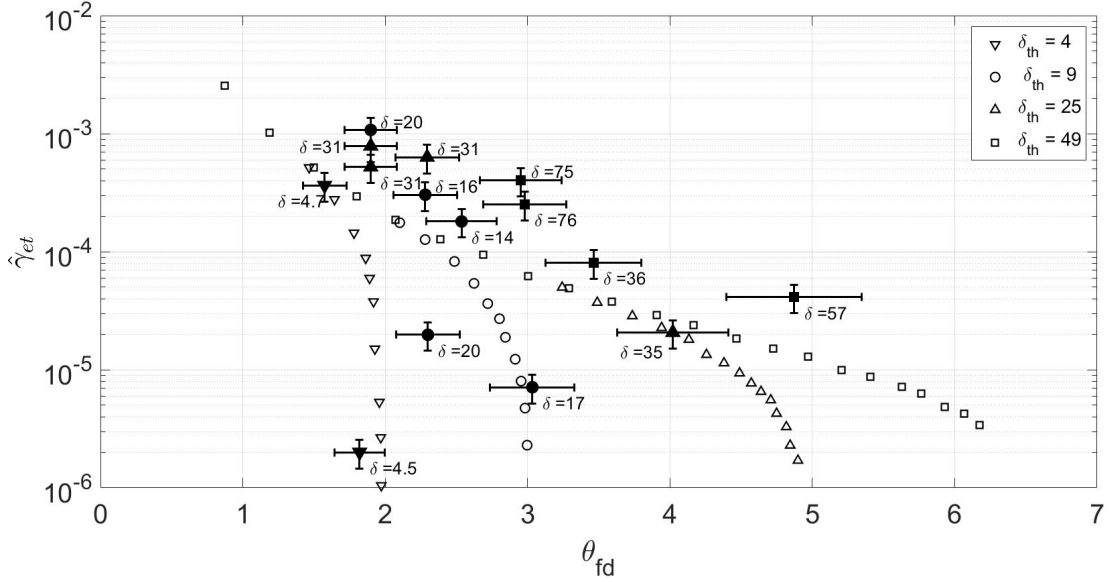


Figure 15: Comparison of the non-dimensional growth rate  $\hat{\gamma}_{et}$  values measured in the facility with the theoretical curves of [Montalvo et al., 2013 a]. Empty symbols: theoretical values for  $\delta_{th} = 4$  ( $\nabla$ ),  $\delta_{th} = 9$  ( $\circ$ ),  $\delta_{th} = 25$  ( $\triangle$ ),  $\delta_{th} = 49$  ( $\square$ ). Full symbols with error bars : measured values for  $\delta \in [0 \ 6.5]$  ( $\blacktriangledown$ ),  $\delta \in [6.5 \ 21]$  ( $\bullet$ ),  $\delta \in [21 \ 36]$  ( $\blacktriangle$ ),  $\delta \in [36 \ \infty]$  ( $\blacksquare$ ). The value of  $\delta$  for each experimental run is written inside the figure.

growth rate in deep water in a *family of curves* indexed by  $\delta = gh/U_1^2$ , i.e., a curve for each value of  $\delta$ . Figure (15) shows a theoretical family of  $\hat{\gamma}_{et}$  for four values of  $\delta$ :  $\delta_{th} = 4, 9, 25, 49$  against  $\theta_{fd}$  parameter. In the same figure are plotted the experimental values of  $\hat{\gamma}_{et}$ . These data are partitioned into four ranges of  $\delta$  i.e.,  $\delta \in [0 \ 6.5]$ ,  $[6.5 \ 21]$ ,  $[21 \ 36]$ ,  $[36 \ \infty]$ . Small finite values of  $\theta_{fd}$  correspond to short surface waves characterizing a young wind waves. For time increasing the wave age  $\theta_{fd}$  increases which corresponds to mild or moderate wavelengths. The waves develop and the growth rate  $\hat{\gamma}_{et}$  gradually decreases due to the effects of finite depth. As  $\hat{\gamma}_{et}$  goes to zero, each  $\delta$ -curve approaches the theoretical  $\theta_{fd}$ -limited growth,  $\theta_{fd}^c$ , given by  $\theta_{fd}^c = \delta^{0.5}$ . Hence for  $\delta_{th} = 4, 9, 25, 49$  the values of  $\theta_{fd}^c$  for  $\hat{\gamma}_{et}$  are:  $\theta_{fd}^c = 2, 3, 5, 7$ . The wave reaches a final state of linear progressive wave with zero growth. In others words, for a given  $\delta$  the surface wave does not grow old any more beyond a determined  $\theta_{fd}^c$ .

The experimental measurements follow more or less the general tendencies of the theoretical curves: a) the grow rate is lower for low values of  $\delta$  and large values of  $\theta_{fd}$ ; b) the growth rate with low  $\delta$  can be sometimes two order of magnitudes lower than run with higher  $\delta$ . c) usually, but not always, experimental values of  $\hat{\gamma}_{et}$  are higher than the theoretical values. If we draw linear regression lines for several bins of  $\delta$  values, we find however a good general trend of the experimental measurements with a decrease towards no amplification at high  $\theta_{fd}$  and low  $\delta$ . These regression lines are drawn in figure (16). The intersection of the regression lines with quite zero non-dimensional growth-rate (here  $\hat{\gamma}_{et} = 10^{-6}$ ) gives the critical  $\theta_{fd}^c$  experimental value for the bin of  $\delta$  data considered, and thus the corresponding  $\delta^c = (\theta_{fd}^c)^2$  value. We note that for each bin of  $\delta$ , the critical  $\delta^c$  belongs to that bin :  $3.42 \in [0 \ 6.5]$ ,  $12.7 \in [6.5 \ 21]$ ,  $34.3 \in [21 \ 36]$ ,  $68.6 \in [36 \ \infty]$ . This decrease towards zero amplification at  $\theta_{fd}^c$  was the key point developed by the [Montalvo et al., 2013 a,b; Latifi et al., 2017] theory. It is the first time that such a theory is illustrated by waver tank experiments.

In order to compare the measured wave growth with previous well known experiments, we have plotted our computed values of  $\gamma_{et}/f_0$  versus inverse wave age  $u^*/c$  in figure (17), together with data compiled by [Plant, 1982], as they are shown in figure 4.2, page 52 of [Young, 1997]. Data came from

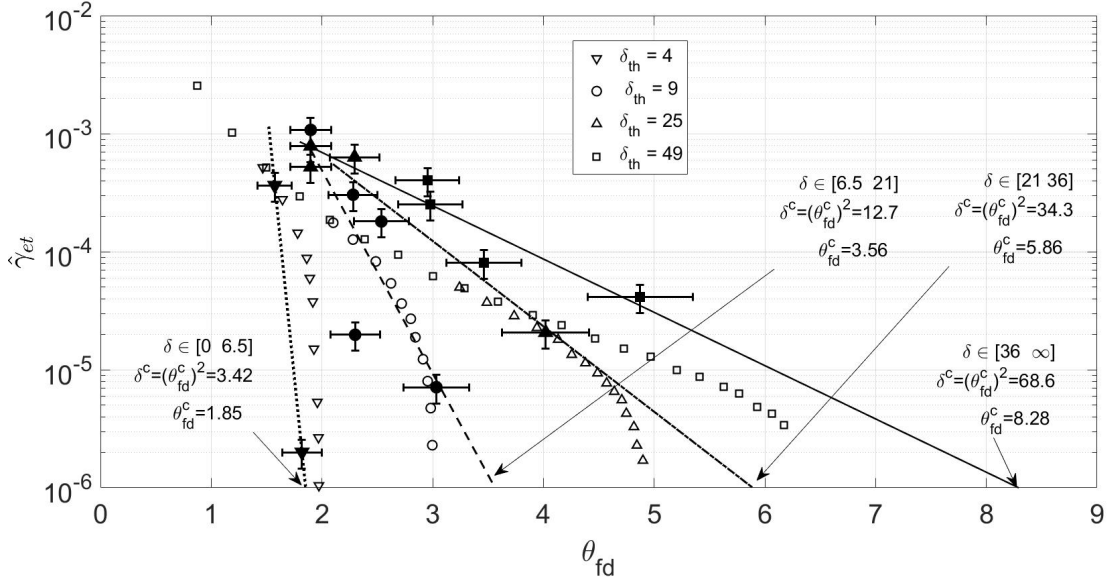


Figure 16: Same as figure (15), but we have added linear regression lines between  $\log_{10} \hat{\gamma}_{et}$  and  $\theta_{fd}$ . Empty symbols: theoretical values for  $\delta_{th} = 4$  ( $\nabla$ ),  $\delta_{th} = 9$  ( $\circ$ ),  $\delta_{th} = 25$  ( $\triangle$ ),  $\delta_{th} = 49$  ( $\square$ ). Full symbols and regression lines: experimental values: dotted line and ( $\blacktriangledown$ ) :  $\delta \in [0, 6.5]$ , dashed line and ( $\bullet$ ) :  $\delta \in [6.5, 21]$ , dashed-dotted line and ( $\blacktriangle$ ) :  $\delta \in [21, 36]$ , straight line and ( $\blacksquare$ ) :  $\delta \in [36, \infty]$ . The critical experimental values of  $\theta_{fd}^c$  and  $\delta^c$  for which there is no amplification ( $\hat{\gamma}_{et} = 10^{-6}$ ) by bins of  $\delta$  are written in the figure.

[Larson and Wright, 1975; Wu et al., 1977, 1979; Snyder et al., 1981; Shemdin and Hsu, 1967]. We see clearly that some of measured growth rates are right in the [Plant, 1982] tendency, lying along Miles and [Snyder et al., 1981] curves, but others are completely below the proposed range, close to zero amplification. As we saw before, according to [Montalvo et al., 2013 a] theory, this happens because  $\delta$  is not high and  $\theta_{fd}$  is not small. In shallow water environment, amplification factor by the wind may be sometimes not efficient.

The comparison between the non-dimensional experimental  $\beta$  Miles parameter and the theoretical ones of [Montalvo et al., 2013 a] is shown in figure (18). Some measured  $\beta$  values seems much larger than theoretical  $\beta$  values. We have done those runs several times and we always found these large  $\beta$  values. In fact, those values seem high in comparison with the [Montalvo et al., 2013 a] theory, but they are not so high if we compare them to previous experimental studies done in the past on the wave growth of wind waves, as it is shown on figure(19). Our  $\beta$  values are more or less on the lower part of the figure, and we see clearly that some values are very close to zero, if compared with previous published data. In another paper [Troitskaya et al., 2018], figure (2-c), all the  $\beta$  values computed from a precise DNS model, and also by a quasi-linear model of waves amplified by wind, are all above 20, i.e. above our  $\beta$  measured data.

Regression lines between  $\beta$  and  $\theta_{fd}$ , for several bins of  $\delta$  values, are shown in figure(20). The intersection of these lines with the  $\beta = 0$  axis gives the critical  $\theta_{fd}^c$  experimental value for the bin of  $\delta$  data considered, and thus the corresponding  $\delta^c = (\theta_{fd}^c)^2$  value. Like in figure(16), we note again that for each bin of  $\delta$ , the critical  $\delta^c$  belongs to that bin :  $3.3 \in [0, 6.5]$ ,  $8.6 \in [6.5, 21]$ ,  $26.0 \in [21, 36]$ ,  $50.4 \in [36, \infty]$ . This experiment shows that, following [Montalvo et al., 2013 a,b; Latifi et al., 2017] development, wind-wave growth rate by Miles's mechanism in finite water depth environment could be very low.

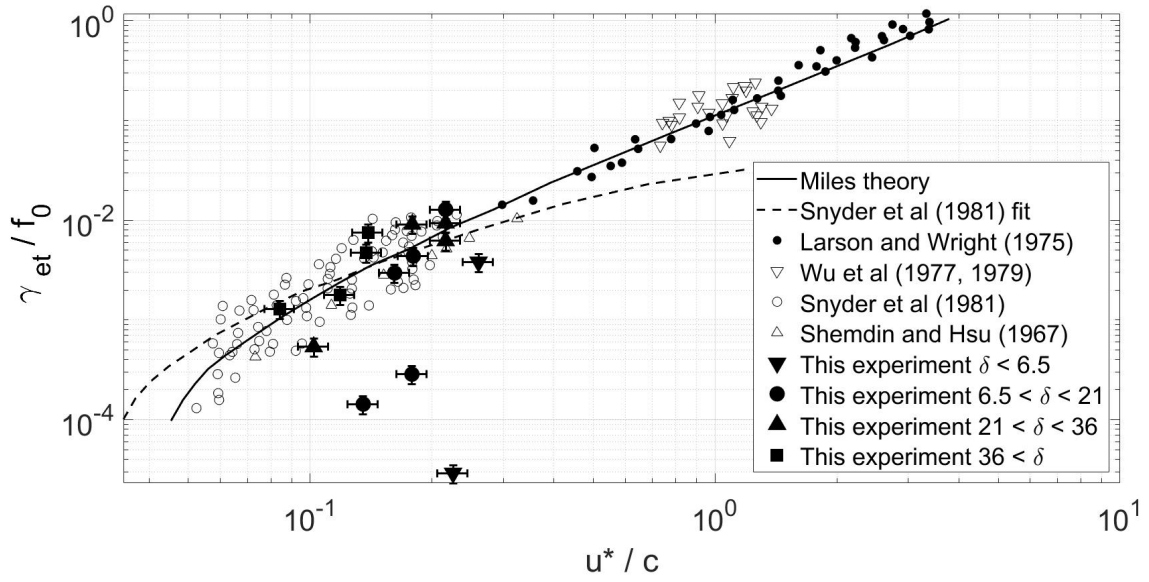


Figure 17: Non-dimensional growth rate  $\gamma_{et}/f_0$  versus inverse wave age  $u^*/c$ . Solid symbols : our experiment. Open symbols : other experiments, data gathered by [Plant, 1982] and displayed in [Young, 1997] page 52 (see also [Snyder et al., 1981; Larson and Wright, 1975; Wu et al., 1977, 1979; Shemdin and Hsu, 1967]). Solid line: Miles Theory, Dashed line : [Snyder et al., 1981] interpolation.

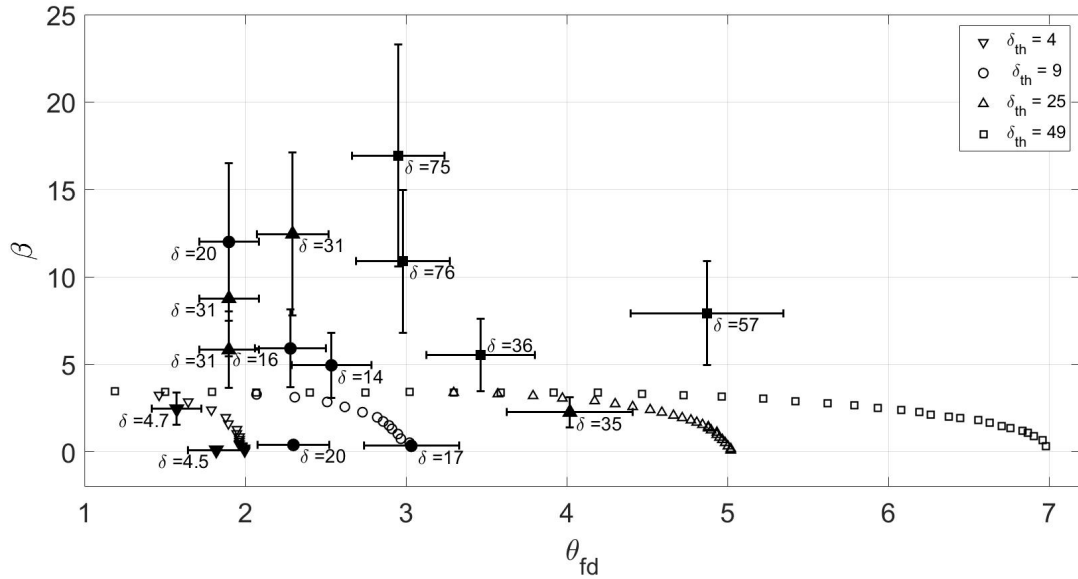


Figure 18: Comparison of the non-dimensional growth rate  $\beta$  measured in the facility with the theoretical curves of [Montalvo et al., 2013 a]. Empty symbols:  $\beta$  theoretical values for  $\delta_{th} = 4$  ( $\nabla$ ),  $\delta_{th} = 9$  ( $\circ$ ),  $\delta_{th} = 25$  ( $\triangle$ ),  $\delta_{th} = 49$  ( $\square$ ). Full symbols with error bars :  $\beta$  measured values for  $\delta \in [0 \ 6.5]$  ( $\blacktriangledown$ ),  $\delta \in [6.5 \ 21]$  ( $\bullet$ ),  $\delta \in [21 \ 36]$  ( $\blacktriangle$ ),  $\delta \in [36 \ \infty]$  ( $\blacksquare$ ). The value of  $\delta$  for each experimental run is written in the figure.

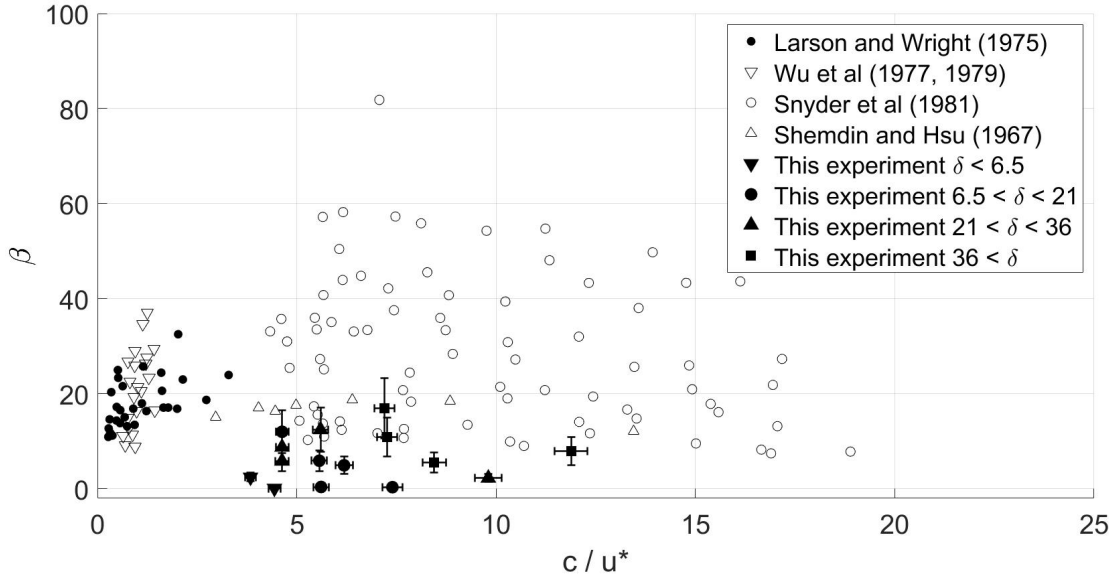


Figure 19: Non-dimensional growth rate  $\beta$  versus wave age  $c/u^*$ . Solid symbols : our experiment. Open symbols : other experiments (see legend), data gathered by [Plant, 1982] and displayed in [Young, 1997] page 53 (see also [Snyder et al., 1981; Larson and Wright, 1975; Wu et al., 1977, 1979; Shemdin and Hsu, 1967]).

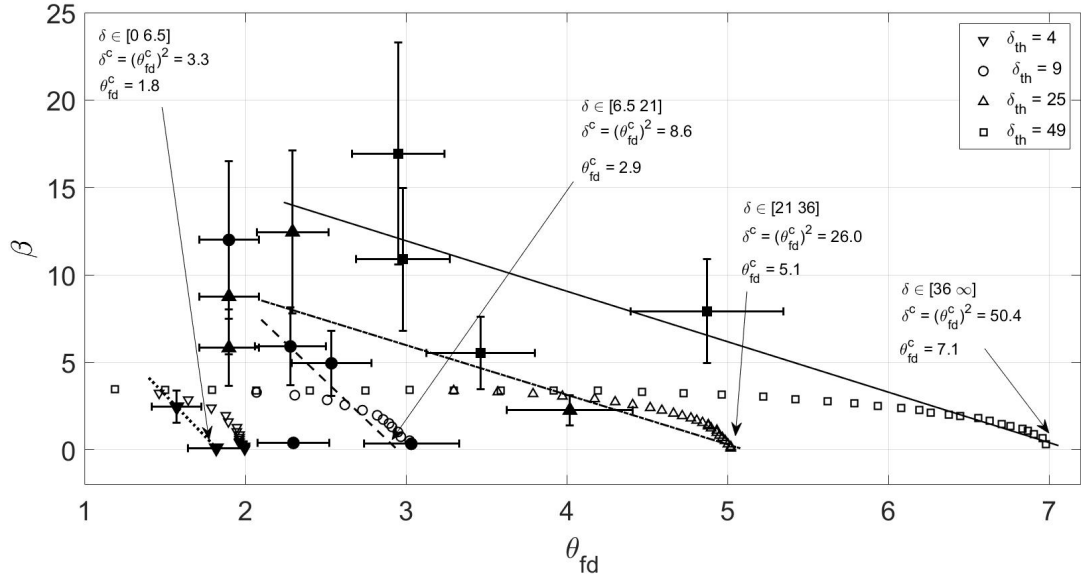


Figure 20: Same as figure (18), but we have added linear regression lines between  $\beta$  and  $\theta_{fd}^c$ : Empty symbols: theoretical values for  $\delta_{th} = 4$  ( $\nabla$ ),  $\delta_{th} = 9$  ( $\circ$ ),  $\delta_{th} = 25$  ( $\Delta$ ),  $\delta_{th} = 49$  ( $\square$ ). Full symbols and regression lines: experimental values: dotted line and ( $\blacktriangledown$ ) :  $\delta \in [0, 6.5]$ , dashed line and ( $\bullet$ ) :  $\delta \in [6.5, 21]$ , dashed-dotted line and ( $\blacktriangle$ ) :  $\delta \in [21, 36]$ , straight line and ( $\blacksquare$ ) :  $\delta \in [36, \infty]$ . The critical experimental values of  $\theta_{fd}^c$  and  $\delta^c$  for which there is no amplification ( $\beta = 0$ ) by bins of  $\delta$  are written in the figure.



## 5. Conclusions

Experiment were conducted in a wind-wave facility to study momentum transfer from air to water and the growth rate of the waves by the wind for different configurations of water depth. Fifteen different runs were done. Some runs were done many times to check the uncertainties on the measurements. All the error bars on the different graphs were plotted. Wind speeds and wave steepnesses were relatively low to avoid wave breaking, aerodynamics effect of high steepnesses on the air-flow. Several studies have been conducted in the past on the effect of shallow water on drag coefficients and momentum transfers, but no specific work, until this one, has been done before on water tank experimental evidence of wave growth by the wind when the water depth is low without breaking events. Although still limited, the results derived from this unique wind wave tank experiment shed new light on the relationship between drag,  $U_{10}$ , friction velocity, roughness length, wave growth ( $\gamma$ -types or  $\beta$ -types), and other non-dimensional parameter (depth, energy, frequency)

We confirm that the dependence of  $C_d$  on wind speed is modified by water depth. Low depth increases Drag coefficient  $C_d$ , roughness length  $z_0$ , Charnock parameter  $\alpha_{Ch}$ . This implies that for a given wind speed, momentum transfer from air to water is higher when the water depth is low.

All data are inside the [Young, 1997; Young and Verhagen, 1996 a; Bretschneider, 1958] depth asymptotic limits. So, even if our measurements are not open field measurements, they follow well the natural limits observed in real open seas.

When the water depth is very low, the growth rate decreases zero amplification for some combinations of non-dimensional parameters  $\delta = \frac{gh}{U_1^2} = \frac{gh\kappa^2}{u_*^2}$  and  $\theta_{fd} = \frac{\kappa}{u_*} \sqrt{\frac{g}{k} \tanh kh}$ . The non-dimensional water depth parameter  $\delta$  depending on depth and on a characteristic wind speed, induces theoretically a family of curves representing the wave growth as a function of the wave phase velocity and the wind speed Montalvo et al. [2013 a,b]; Latifi et al. [2017]. Our experimental data follow those theoretical curves, with a decrease towards null amplification for some low depth value in conjugation with particular wind speeds. To date, no fully consistent and conclusive theory of wave amplification by wind exists. We are focused here on the Miles' mechanism, because we used quite low wind speed and low wave steepnesses conditions to avoid breaking and/or air flow separation that could be generated by steep waves, but probably the Miles' mechanism is not the only mechanism involved in the amplification of waves in finite depth conditions.

At the water surface the momentum transfer  $\tau = \rho_a u_*^2$  is split into two distinct parts: the form drag  $\tau_{form}$ , and the viscous drag  $\tau_{viscous}$  [Makin and Kudryavtsev, 2002; Donelan et al., 2012]:  $\tau = \tau_{form} + \tau_{viscous}$ . The form drag  $\tau_{form}$  is the part of the momentum transfer that goes to the amplification of the waves, i.e. it induces directly the wave growth by the wind [Grare et al., 2013]. The form drag arises from the form of the waves and becomes larger as the steepness of the waves increases. The steeper the waves the larger the contribution of the wave drag to the total drag. The viscous drag  $\tau_{viscous}$ , also called surface tangential viscous stress or skin friction drag [Veron et al., 2007; Buckley et al., 2020], comes about through the direct molecular interaction at the interface. This viscous drag is the part of the momentum transfer that goes to the generation and amplification of the surface current, i.e. the surface drift. in this study, we have shown that, in low depth conditions, a) momentum flux  $\tau$  increases and b) wave growth decreases, i.e.  $\tau_{form}$  decreases. Then, by simple deduction from  $\tau_{viscous} = \tau - \tau_{form}$ , because  $\tau$  increases and  $\tau_{form}$  decreases, the viscous drag  $\tau_{viscous}$  should increase significantly when the water depth is low.

In the future, it should be nice to conduct experiments in this wave tank to measure the influence of water depth on the surface wind drift current. This kind of study, to our knowledge, was never done. More data and analysis are needed to confirm these findings. Another point is that all the theories and developments on wave growth over finite depth involve water of constant depth, which is very restrictive. Even in the most ideal of field basins such a situation never exists. Variable depth effect

on air-sea interactions is still an open issue.

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*Data statements:* Experimental data collected and analysed in this article can be obtained upon request from author Hubert Branger at [branger@irphe.univ-mrs.fr](mailto:branger@irphe.univ-mrs.fr)

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