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► **To cite this version:**

Minh Tu Pham, Maxime Gautier, Philippe Poignet. Identification of joint stiffness with bandpass filtering. ICRA 2001 - IEEE International Conference on Robotics and Automation, May 2001, Seoul, South Korea. pp.2867-2872, 10.1109/ROBOT.2001.933056 . hal-04741134

**HAL Id: hal-04741134**

**<https://hal.science/hal-04741134v1>**

Submitted on 17 Oct 2024

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# IDENTIFICATION OF JOINT STIFFNESS WITH BANDPASS FILTERING

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## Abstract

*This paper proposes a method to identify the joint stiffness of a robot using a bandpass filter. It is based on moving one axis at a time. The dynamic model reduces to a model which is linear in relation to a minimum set of dynamical parameters which have to be identified. These parameters are estimated using the least squares solution of an over determined linear system obtained from the sampling of the dynamic model along a closed loop tracking trajectory. Conditions for a good data processing before identification are exhibited through practical aspects concerning data sampling and data filtering. An experimental study shows the efficiency of the method with two sets of data depending on motor and joint position measurements.*

## 1 Introduction

Accurate dynamic models of robots are required to control or simulate their motions. These models are function of the geometric parameters of robots (length of links, angle between joint axis, ...) and the dynamic parameters of links (inertia, first moments, masses) and drive chain (stiffness, friction). In the last years, subspace methods for estimating flexible modes of mechanical systems with the direct dynamic model have received a lot of attention [1][2]. An other alternative consists in performing identification in the frequency domain with a spectral analysis [3]. However, most of the investigations do not underline the difficulty to estimate the physical parameters of the system using the identified state space or transfer function model. This paper is focused on estimating the joint stiffness which is the major source of flexibility in many applications. The method is based on special test motions moving one joint at a time while the others are locked. The dynamic model reduces to a model which is lin-

ear in relation to gravity parameters (first moments), joint moment of inertia, friction and stiffness parameters. These parameters are estimated from sampled data of motor current reference and available position measurements using a least squares identification technique. In the case of flexible systems, flexible degrees of freedom are not all measured. This is a main difference and difficulty compared with the rigid multi body systems where all the joint positions are measured. The method which is presented takes into account this problem. This article is divided into four sections. Section 2 describes the dynamic modeling of a flexible joint robot. Section 3 is devoted to the identification using the inverse dynamic model. Some practical aspects about the data filtering necessary to perform the least squares identification algorithm are exhibited. Experimental results of the dynamic identification of a joint prototype robot will illustrate theoretical results in section 4.

## 2 Modeling

### 2.1 The dynamic model

The description of the geometry of the robot is carried out using the modified Denavit and Hartenberg notation [4]. The inverse dynamic model of a flexible joint robot composed of  $n$  moving links calculates the motor torque vector (the control input) as a function of the generalized coordinates (the state vector and its derivative). It can be written as [5][6]:

$$0 = M(q)\ddot{q} + N(q, \dot{q}) - K(q_m - q) \quad (1)$$

$$\Gamma_m = J_m\ddot{q}_m + \Gamma_{fm} + K(q_m - q) \quad (2)$$

where :  $q$ ,  $\dot{q}$ ,  $\ddot{q}$  are the  $(n \times 1)$  vectors of joint positions, velocities and accelerations respectively,  $M(q)$

is the (nxn) robot inertia matrix,  $N(q, \dot{q})$  is the (nx1) vector of centrifugal, Coriolis, gravitational and joint friction torques  $\Gamma_f$ .  $\Gamma_f$  is usually modeled at non zero velocity as following:

$$\Gamma_{fj} = FS_j \text{sign}(\dot{q}_j) + FV_j \dot{q}_j \quad (3)$$

$\dot{q}_j$  is the joint j velocity.  $\text{sign}(x)$  denotes the sign function.  $FV_j$  and  $FS_j$  are the viscous and Coulomb friction coefficients of joint j.  $q_m, \dot{q}_m, \ddot{q}_m$  are the (nx1) vectors of motor positions, velocities and accelerations respectively.  $\Gamma_{fm}$  is the drive chain slip friction torque given as following:

$$\Gamma_{fmj} = FS_{mj} \text{sign}(\dot{q}_m) + FV_{mj} \dot{q}_m \quad (4)$$

$FV_{mj}$  and  $FS_{mj}$  are the viscous and Coulomb friction coefficients of drive chain j.  $J_m$  is a (nxn) diagonal matrix with the drive chain inertia moment on its diagonal (polar moments of motor and gears).  $K$  is a (nxn) diagonal matrix with the joint stiffness on its diagonal, assuming that the elastic joints can be approximated by pure torsional springs that is to say taking into account only the first mode of the flexible joint. Moving one axis j at a time while the others are locked reduces the dynamic model to the following:

$$0 = J_{Lj}(q_L) \ddot{q}_j + Q_j(q_L) + \Gamma_{fj} - K_j(q_{mj} - q_j) \quad (5)$$

$$\Gamma_{mj} = J_{mj} \ddot{q}_{mj} + \Gamma_{fmj} + K(q_{mj} - q_j) \quad (6)$$

Where:  $q_L$  are the locked joints from j+1 to n.

$J_{Lj} = M_{jj}(q_L)$  is the  $j^{\text{th}}$  diagonal coefficient in M,  $Q_j$  is the gravity torque:

$$Q_j = MX_{Lj}(q_L)g \cos(q_j) + MY_{Lj}(q_L)g \sin(q_j) \quad (7)$$

$MX_{Lj}$  and  $MY_{Lj}$  are the first moments, g is the gravity acceleration.

$J_{Lj}$ ,  $MX_{Lj}$  and  $MY_{Lj}$  depend on the locked joint  $q_L$ .

## 2.2 The standard identification model

The dynamic model (5) and (6) can be rewritten in a relation linear to the dynamic parameters as following, where the subscript j is omitted for simplicity:

$$y = D_s X_s \quad (8)$$

with:

$$y = \begin{pmatrix} \Gamma_m \\ 0 \end{pmatrix} \quad (9)$$

$$D_s = \begin{pmatrix} \ddot{q}_m \dot{q}_m \text{sign}(\dot{q}_m) & (q_m - q) & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -(q_m - q) & \ddot{q} & g \cos(q) & g \sin(q) & \dot{q} \text{sign}(\dot{q}) \end{pmatrix} \quad (10)$$

$$X_s = (J_m \ FV_m \ FS_m \ K \ J_L \ MX_L \ MY_L \ FV \ FS)^T \quad (11)$$

There are 9 parameters to identify which are called the standard parameters of the moving link j.

## 2.3 The minimal identification models

The minimal model is the model which calculates  $y$  with the minimum set of parameters  $X$  in (12) (see Section 3.1). It is computed using the expression of the standard model (8), (9), (10), (11) and depends on the available measurements. It is noted as following:

$$y = DX \quad (12)$$

Two minimal models are considered.

### 2.3.1 Identification model using joint and motor positions

The minimal model corresponds to the standard model (8), (9), (10), (11) and it is given by:

$$D = D_s \quad X = X_s \quad y = \begin{pmatrix} \Gamma_m \\ 0 \end{pmatrix} \quad (13)$$

### 2.3.2 Identification models with only the motor position

Assuming that flexible degree of freedom  $q$  and its derivatives are not measured which is the case for industrial robots,  $q$  and its derivatives must be calculated from (6). The flexible position is given by:

$$q = \frac{J_m \ddot{q}_m + FS_m \text{sign}(\dot{q}_m) + FV_m \dot{q}_m + K q_m - \Gamma_m}{K} \quad (14)$$

The first and second derivatives of (14) give  $\dot{q}$  and  $\ddot{q}$  where  $\text{sign}(\dot{q}_m)$  is removed because its derivative is not defined:

$$\dot{q} = \frac{J_m \dddot{q}_m + FV_m \ddot{q}_m + K \dot{q}_m - \dot{\Gamma}_m}{K} \quad (15)$$

$$\ddot{q} = \frac{J_m \ddddot{q}_m + FV_m \dddot{q}_m + K \ddot{q}_m - \ddot{\Gamma}_m}{K} \quad (16)$$

By substituting (16), (15) and (14) in (5), (7) and (3) do not allow to rewrite (5) as a linear relation to the dynamic parameters because of  $\cos(q)$  and  $\sin(q)$  in (7) and  $\text{sign}(\dot{q})$  in (3). We propose to overcome this

difficulty by using a chirp signal as a trajectory which is exciting for all the parameters of  $X_S$  but less exciting for  $MX_L$ ,  $MY_L$ . Working near the rest position  $q_o$  of the robot with respect to the gravity force is a solution to decrease the effect of the gravity in the model. If a priori knowledge is available  $q_o$  can be calculated as:

$$q_o = -\arctan\left(\frac{MX_L}{MY_L}\right) \quad (17)$$

Choosing this value implies that the contribution of the gravity torque decreases in (5) because the term (7) remains small. Moreover, we assume that the chirp trajectory does not excite  $FS$ . This assumption will be verified in the experimental validation (see Section 4). In the case where the effects of the gravity terms and  $FS$  can be neglected on the used trajectory, the substitution of (16), (15) and (14) in (5) lead to a minimal linear model as (12), with:

$$D = (-\ddot{\Gamma}_m - \dot{\Gamma}_m \ddot{q}_m \ddot{q}_m \ddot{q}_m \ddot{q}_m \dot{q}_m \text{sign}(\dot{q}_m)) \quad (18)$$

$$X = (X_1 \ X_2 \ X_3 \ X_4 \ X_5 \ X_6 \ X_7)^T \quad (19)$$

$$y = \Gamma_m \quad (20)$$

$$\begin{aligned} X_1 &= \frac{J_L}{K} & X_2 &= \frac{FV}{K} \\ X_3 &= \frac{J_m J_L}{K} & X_4 &= \frac{J_m FV + J_L FV_m}{K} \\ X_5 &= \frac{J_m K + J_L K + FV FV_m}{K} & X_6 &= FV + FV_m \\ X_7 &= FS_m \end{aligned}$$

where:

### 3 Identification method and filtering aspects

#### 3.1 Identification method

The identification method developed for robot manipulators is applied for flexible systems. From a sampling of the dynamic model along different trajectories [6][7][8], at different times  $t$ ,  $i=1, \dots, n$ ,  $X$  can be estimated as the least squares (L.S.) solution  $\hat{X}$  of the linear system:

$$y = WX + \rho \quad (21)$$

where:  $W$  is a (rxNp) observation matrix, which is a sampling of the regressor (10) or (18),  $Y$  is a (rx1)

vector which is a sampling of (9) of (20),  $\rho$  is a (rx1) vector of errors due to model error and noise measurements,  $r > Np$  is the number of equations.

The L.S. solution minimizes the 2 norm  $\|\rho\|$  of the vector of errors  $\rho$ . The unicity of the L.S. solution depends on the rank of the observation matrix  $W$ . The rank deficiency of  $W$  can come from two origins:

- structural rank deficiency which stands for any samples of  $(q_m, \dot{q}_m, \ddot{q}_m)$  and  $(q, \dot{q}, \ddot{q})$  samples constrained by (5). This is the structural parameters identifiability problem which is solved using base (or minimal) parameters [9].

- data rank deficiency due to a bad choice of the trajectory  $(q, \dot{q}, \ddot{q}, q_m, \dot{q}_m, \ddot{q}_m)$  which is sampled in  $W$ . This is the problem of optimal measurement strategies which is solved using closed loop identification to track exciting trajectories [8][10].

Calculating the L.S. solution of (21) from noisy discrete measurements or estimations of derivatives, may lead to bias because  $W$  and  $Y$  may be non independent random matrices [6]. Then it is essential to filter data in  $Y$  and  $W$ , before computing the L.S. solution.

#### 3.2 Filtering aspects

The derivatives of  $q$  in (21) are obtained without phase shift using a central difference algorithms of the low-pass filtered position. The low pass filter without phase shift and without magnitude distortion into the bandwidth is easily obtained with a non causal zero-phase digital filtering by processing the input data through an IIR lowpass Butterworth filter in both the forward and reverse direction using a 'filtfilt' procedure from Matlab. The recursive equation of the central difference algorithms is given by:

$$q_{cd}[k] = \frac{q[k+1] - q[k-1]}{2T_s} \quad (22)$$

where  $q$  is the joint position and  $q_{cd}$  the central difference of  $q$ . The discrete transfer function corresponding to (22) is given by:

$$\frac{q_{cd}}{q}(z^{-1}) = \frac{1}{2T_s} \frac{1 - z^{-2}}{z^{-1}} \quad (23)$$

$z^{-1} = e^{-T_s s}$  is the unit delay in the time domain. Taking  $z^{-1} = e^{-T_s j\omega}$  in (23) gives the transfer function  $D(j\omega)$  in the frequency domain:

$$D(j\omega) = \frac{q_{cd}}{q}(j\omega) = j\omega \frac{\sin(\omega T_s)}{\omega T_s} \quad (24)$$

(24) is an approximation of the continuous derivative transfer function,  $s = j\omega$ , without phase distortion, but with the amplitude distortion  $\frac{\sin(\omega T_s)}{\omega T_s}$ . In the case where the Nyquist frequency  $\omega_s/2$  is larger than the cutoff frequency  $\omega_n$  of the lowpass Butterworth filter, the magnitude of the forward and reverse discrete Butterworth filter can be well approximated by the continuous filter  $H(j\omega)$  in the range frequency close to  $\omega_n$  and it is given by:

$$|H(j\omega)| = \frac{1}{1 + \left(\frac{\omega}{\omega_n}\right)^{2m}} \quad (25)$$

where  $m$  is the order of the forward filter. The product of the lowpass filter  $H(j\omega)$  and the central difference filter  $D(j\omega)$  transfer functions gives a bandpass filter  $F(j\omega)$  which is a band-limited derivative filter. Figure 1 shows the Bode's diagram of the different filters with  $\omega_s/2 = 10^3 \text{ rad/s}$ .

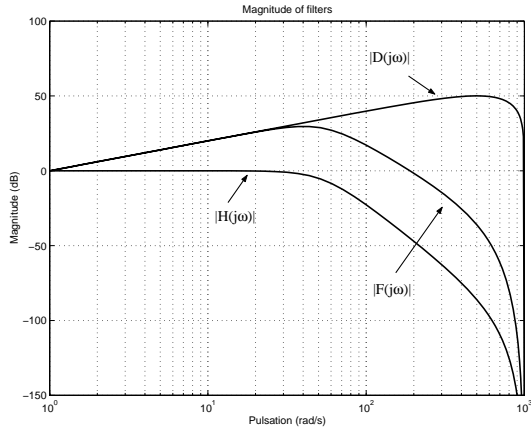


Figure 1: Magnitude of filters ( $m=2$ )

Therefore, two parameters have to be tuned to define the bandpass filter: The frequency  $\omega_n$  of the Butterworth filter  $H(j\omega)$  and the sampling time  $T_s$ . These parameters must be tuned in order to avoid magnitude distortion inside the bandwidth of the derivatives because some distortion of data in the dynamic range of the system yields errors in the observation matrix  $W$  and lead to bias estimation. Let us define the magnitude distortion  $A(\omega)$  of the bandpass filter  $F_{nd}(j\omega)$  which approximates the derivative of order  $nd$ :

$$A(\omega) = \frac{1}{1 + \left(\frac{\omega}{\omega_n}\right)^{2m}} \left(\frac{\sin(\omega T_s)}{\omega T_s}\right)^{nd} \quad (26)$$

The cutoff frequency  $\omega_c$  bounds the magnitude distortion of  $F_{nd}(j\omega)$  to  $A_c$  such  $A(\omega) \leq A_c$  for any  $\omega$

inside the bandwidth  $[0, \omega_c]$ , and  $A_c = A(\omega_c)$ . The frequency  $\omega_c$  is chosen into the dynamic range of the flexible joint between  $2\omega_1$  and  $10\omega_1$ , where  $\omega_1$  is the first mode of the joint. Let us define  $r_c$  as the ratio between the amplitude distortion introduced by the lowpass filter and the distortion due to the derivative filter:

$$r_c = \frac{\frac{1}{1 + \left(\frac{\omega_c}{\omega_n}\right)^{2m}}}{\left(\frac{\sin(\omega_c T_s)}{\omega_c T_s}\right)^{nd}} \quad (27)$$

Straightforward computations show that:

$$\omega_n = \omega_c \left( \frac{1}{\sqrt{r_c A_c}} - 1 \right)^{-\frac{1}{2m}} \quad (28)$$

and  $\omega_s = \frac{2\pi}{T_s} = \frac{2}{x_c} \omega_c$ , where  $x_c$  solves the equation:

$$\text{sinc}(x_c) = \frac{\sin(\pi x_c)}{\pi x_c} = \frac{\sin(\omega_c T_s)}{\omega_c T_s} = \left(\frac{A_c}{r_c}\right)^{\frac{1}{2nd}} \quad (29)$$

A typical value for  $r_c$  is 1 which equals the distortion between the lowpass filter and the central difference.  $A_c$  equal  $\frac{\sqrt{2}}{2}$  defines the usual  $-3\text{dB}$  bandwidth of the filter  $F_{nd}(j\omega)$ . Starting from these initial values,  $\omega_n$  and  $T_s$  can be improved by optimization in order to bound the estimation error relative to a priori values [11].

The torque  $\Gamma_m$  is perturbed by high frequency torque ripple of the joint drive chain and has also to be filtered. Then  $y$  and each column of  $W$  in (21) are filtered by a lowpass filter and then are resampled at a lower rate (decimate Matlab procedure), keeping one sample over  $Nd$  to get a new filtered linear system. It is to be noted that the distortion of this filtering process doesn't introduce any error in the linear relation (21).

## 4 Experimental validation

Experimental identification has been carried out on one axis of a prototype robot, starting from two sets of data measurements. A closed loop identification, using classical position and velocity feedback PI control, has been performed, tracking a chirp function sweeping between  $0.05\text{Hz}$  and  $30\text{Hz}$ , in order to excite the system around its first natural frequency,  $f_1 = 3.9\text{Hz}$  with a payload. It is recommended to load the robot in order to increase the excitation of the stiffness parameter, which is better estimated than without payload. For the prototype, the experimental sampling frequency is imposed by the data acquisition system at  $F_s = 1/T_s = 350.87\text{Hz}$ .

## 4.1 Identification results with both joint and motor position measurements

The maximum order of derivatives is  $nd = 2$ . Choosing  $\omega_c = 6\omega_1 = 12\pi f_1$ ,  $A_c = \frac{\sqrt{2}}{2}$ ,  $r_c=1$ ,  $m = 4$  gives the Butterworth frequency  $\omega_{n2} = 181rad/s$ . The sampling frequency obtained with (29) is equal to  $F_{s2} = 205Hz$ . Experimental results show that a good estimation is obtained with a Butterworth frequency  $\omega_{n2} = 176rad/s$ . This value is close to the theoretical value. The decimate procedure is tuned to keep one sample over  $Nd = 26$  which gives a cutoff frequency equal to  $5.4Hz$  and close to  $f_1$ .

The minimal model is given by (13) and the estimated values are given in Table 1 with their confidence interval  $2\sigma_{\hat{x}_i}$  and their relative standard deviation  $\% \sigma_{\hat{x}_{ri}} = \frac{\sigma_{\hat{x}_i}}{\hat{x}_{ri}} 100$ .

Parameters	$X$	$2\sigma_{\hat{x}}$	$\% \sigma_{\hat{x}_r}$
$J_m$	174.7996	2.7741	0.7952
$K$	20341.0458	437.1569	1.0746
$J_L$	47.4458	1.6281	1.7157
$FS_m$	41.4254	5.5393	6.6858
$FV_m$	235.9100	127.9280	27.1137
$FV$	83.1508	41.4113	24.9013
$MX$	-0.7241	1.1948	82.5056
$MY$	-53.4201	10.3659	9.7023
$FS$	8.4612	2.5726	15.2025

Table 1: Identification results (ISO units)

The relative standard deviations show that the joint stiffness  $K$  and the parameters  $J_m$ ,  $J_L$ ,  $FS_m$  are accurately identified.  $MY$  is identifiable but at less accuracy while the others are not significant on the used trajectory.

## 4.2 Identification results without joint position measurement

The maximum order of derivatives is  $nd = 4$ . Choosing  $\omega_c = 6\omega_1 = 12\pi f_1$ ,  $A_c = \frac{\sqrt{2}}{2}$ ,  $r_c=1$ ,  $m = 6$  gives the Butterworth frequency  $\omega_{n4} = 169rad/s$  which is close to  $\omega_{n2}$ . The sampling frequency obtained with (29) is equal to  $F_{s4} = 289Hz$ . For the comparison of both methods we use the same bandpass filter as previously that is to say  $\omega_{n4} = \omega_{n2} = 176rad/s$  and  $Nd = 26$ .

The minimal model is given by (18), (19) and (20). It should be noted that the identified parameters  $X_i$ ,

$i=1,\dots,7$ , are non linear in relation to the physical ones, which makes difficult to compute the standard deviations of the inertial and friction estimated values in the identification procedure. Table 2 exhibits estimated values with their confidence interval and their relative standard deviation.

Parameters	$X$	$2\sigma_{\hat{x}}$	$\% \sigma_{\hat{x}_r}$
$X_1$	0.0021	0.0001	2.5977
$X_2$	0.0046	0.0022	23.6303
$X_3$	0.3554	0.0176	2.4774
$X_4$	3.1739	0.3569	5.6218
$X_5$	205.4558	7.9221	1.9279
$X_6$	1246.5690	165.1359	6.6236
$X_7$	5.1624	7.4232	71.8975

Table 2: Identification results (ISO units)

The relative standard deviations show that the identification model given by (18), (19) and (20) leads to estimated parameters which are accurately identified except for  $X_2$  and  $X_7$ .

The physical dynamic parameters are calculated inverting the relations  $X_1, X_2, X_3, X_4, X_5, X_6, X_7$  and are given in Table 3. It is to be noted that the values of  $J_m$ ,  $K$  and  $J_L$  are close to the values obtained with the model given by (13), despite of the effect of  $M_Y$  which has been neglected, while friction parameters are not identified. The direct validation on Figure 2 shows that the predicted motor torque is close to the measured one.

Parameters	Estimation	Values
$J_m$	$\hat{J}_m = X_3/X_1$	172.1890
$K$	$\hat{K} = \hat{F}\hat{V}/X_2$	19552.6214
$J_L$	$\hat{J}_L = X_1\hat{K}$	40.3601
$FS_m$	$F\hat{S}_m = X_7$	5.1624
$FV_m$	$F\hat{V}_m = (X_4 - \hat{J}_m X_2)/X_1$	1157.4716
$FV$	$\hat{F}\hat{V} = X_6 - F\hat{V}_m$	89.0974

Table 3: Physical parameters (ISO units)

We get a good estimation of the joint stiffness for both methods but each of them have their advantages and drawbacks. On the one hand the method based on motor and joint position measurements has the advantage to use the complete inverse model which gives the opportunity to identify the physical parameters. Unfortunately the major drawback of this method is to use

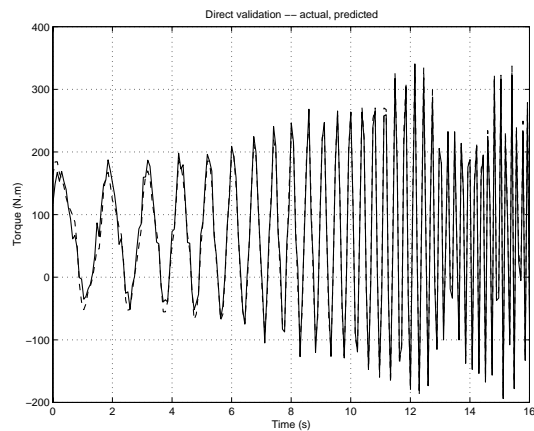


Figure 2: Direct validation

two encoders which is expensive and is not the case in most industrial robots. On the other hand, the identification method using only the motor position is based on a simplified model which identifies parameters as non linear relations to the physical ones. Then only the joint stiffness and the inertia can be accurately identified. The main drawback is that some parameters as friction parameters and gravity parameters are not identified.

## 5 Conclusion

This paper has proposed a method to identify the stiffness of robot joint. It is based on moving one axis at a time. The dynamic model reduces to a model which is linear in relation to a set of parameters which are estimated as the least squares solution of the sampled and filtered dynamic model. Conditions for a good data processing before identification are exhibited through practical aspects concerning data sampling and data filtering. Finally, an experimental study shows the efficiency of the identification procedure with two sets of data depending on motor and joint position measurements and highlights advantages and drawbacks of each method.

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