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# Constrained Tensor Decomposition Reveals Lever Arms Synergies in the Musculoskeletal System

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**Abstract**—This study investigates the muscular synergies of the lower limb during human walking, estimated using a constrained canonical polyadic decomposition of muscle lever arms. The results confirm the existence of synergies complementary to synergies previously derived from non-negative matrix factorization of computed muscle activations or measured electromyographic signals. Furthermore, we present a method for computing musculotendon forces based on the use of ‘synergistic forces’, computed using lever arm synergies. This method is evaluated through the estimation of knee contact forces for one subject equipped with an instrumented knee prosthesis.

## I. INTRODUCTION

Rigid multi-body modeling in biomechanics is a valuable tool for understanding the functioning and loading of the musculoskeletal system. However, a significant issue is muscular redundancy, namely for state-of-the-art musculoskeletal models there are infinitely many valid musculotendon forces that produce the same motion. Estimation of individual musculotendon forces during human locomotion can be of great help in diagnosing, planning treatment and monitoring patients suffering from joint pathologies such as osteoarthritis. To address this problem, a common approach is to solve a regularized linear system. This involves selecting the variables to be estimated, typically musculotendon forces or muscle activations [1], and choosing an objective function that has a significant impact on the outcome, typically ridge or sparse regression [2], [3].

While regularized least squares allows to take into account the system’s dynamics and physiological limits and to reduce the solution space, their output is difficult to interpret as the priors are ill-fitted. A possible approach is to first reduce the problem’s dimension by considering muscle synergies. Synergies are groupings of muscles often identified from experimentally measured electromyography (EMG) signals [4], [5] or previously computed muscle activations [6], [7], with typically four to six synergies identified in the lower limb for activities like walking. Muscle synergies are usually computed using non-negative matrix factorisation or principal component analysis [8].

Muscle synergies can represent neural coordination but can also reflect biomechanical affordances (i.e. musculoskeletal

structure) as well as biomechanical constraints (i.e. task requirements) [9]. Therefore, there are likely other synergies at the level of musculoskeletal mechanics, in addition to those related to neuromuscular control. Moreover, these synergies also have the potential to reduce problem complexity when computing the musculotendon forces.

The objective of this work is twofold. Firstly, it aims to infer synergies by factorizing the muscle lever arms tensor with a low-rank constrained Canonical Polyadic (CP) decomposition. Secondly, it aims at computing musculotendon forces and joint contact forces by considering these synergies for dimensionality reduction. This concept will be validated using the experimental walking data of a subject from the Grand Challenge Competition to predict in vivo knee loads [10], thereby demonstrating the relevance and applicability of this approach in the field of biomechanics.

## II. METHODS

### A. Musculoskeletal model

The 3D lower limb musculoskeletal used in this study is described in [11], [12]. The musculoskeletal geometry used in this model is based on the work of Delp [13] and is composed of five segments (pelvis, femur, patella, fibula, foot), six joint degrees-of-freedom (DoFs: hip modeled as spherical, knee modelled as hinge with coupled 2-D translations, and ankle modeled as universal joint) and forty-three muscular lines of action representing thirty-three muscles. The joint kinematics is predicted through a multibody kinematics optimization [14] that minimizes, under rigid body and kinematic constraints, the distances between measured and model-determined skin marker positions. This model is scaled to the length of the participant segments.

Using this model, musculotendon and joint contact forces can, in principle, be estimated simultaneously through a time-dependent linear system inversion. The dynamic equilibrium equation can be written as a linear system at a given time step:

$$\mathbf{G}\ddot{\mathbf{Q}} + \mathbf{K}^T\boldsymbol{\lambda} = \mathbf{R} + \mathbf{P} + \mathbf{L}\mathbf{f} \quad (1)$$

where  $\mathbf{G}$  is the generalized mass matrix,  $\ddot{\mathbf{Q}}$  is the vector of generalized accelerations,  $\mathbf{K}$  is the Jacobian matrix of

constraints,  $\lambda$  is the vector of Lagrange multipliers,  $\mathbf{R}$  is the generalized vector of ground reactions,  $\mathbf{P}$  is the generalized vector of weights,  $\mathbf{L}$  is the matrix of generalized lever arms and  $\mathbf{f}$  is the vector of musculotendon forces.

To reframe this system in a more conventional form, we project the solution on the kernel of the Jacobian constraint matrix  $\mathbf{K}^T$  as described in [15]. The problem can be rewritten as

$$\mathbf{A}\mathbf{f} = \mathbf{b}, \quad (2)$$

with  $\mathbf{A} = \mathbf{Z}_{\mathbf{K}^T}\mathbf{L}$  and  $\mathbf{b} = \mathbf{Z}_{\mathbf{K}^T}(\mathbf{G}\ddot{\mathbf{Q}} - \mathbf{R} - \mathbf{P})$  and  $\mathbf{Z}_{\mathbf{K}^T}$  the projector on the Jacobian kernel. At each time step of the gait cycle,  $\mathbf{A}$  is the  $n \times m$  matrix of muscle lever arms for all  $m$  muscle lines of action across the  $n$  degrees of freedom,  $\mathbf{b}$  is the vector of joint torques for each degree of freedom, and  $\mathbf{f}$  is the unknown vector of musculotendon forces.

Typical dimensions for  $n$  and  $m$  would be respectively six and forty-three, and generally,  $n$  is much smaller than  $m$ . Note that  $\mathbf{A}$ ,  $\mathbf{f}$ , and  $\mathbf{b}$  depend on time, which means that we in fact aim at solving a tensorial linear system

$$\mathcal{A} \times_2 \mathbf{F} = \mathbf{B} \quad (3)$$

with  $\mathcal{A}$  the tensor of size  $n \times m \times t$  of muscle lever arms stacking matrices  $A$  at  $t$  time steps (conveniently  $t = 100$ ) along a third mode,  $\mathbf{B}$  is the matrix of joint torques along time, and  $\mathbf{F}$  is the unknown matrix of musculotendon forces along time. The product  $\times_2$  is a contraction of the second mode of a tensor with a matrix [16]. Note that the duration of the gait cycle was normalized to a percentage in order to more easily compare gait trials. Finally, to take muscle contraction dynamics into account and make  $\mathcal{A}$  and  $\mathbf{F}$  more readily interpretable, we scale the muscle lever arms tensor by the corresponding maximal active force of each muscle. This force corresponds to the force produced by the muscle when fully activated and depends on the current length and contraction velocity of the muscle [1], [13].

### B. Dimensionality reduction through tensor decomposition

The dimensions of the linear system (3) make it clear that the problem of estimating  $\mathbf{F}$  is underdetermined. Our proposed approach is to perform dimensionality reduction to group entries of columns of  $\mathbf{F}$  together based on biologically baked-in synergies in the musculotendon forces. We will look for these synergies in the muscle lever arms tensor  $\mathcal{A}$ . Canonical Polyadic/PARAFAC decomposition [16] is therefore performed on tensors  $\mathcal{A}$  (there is one  $\mathcal{A}$  for each recorded gait trial of the patient in the dataset).

The lever arms tensor  $\mathcal{A}$  is decomposed through a rank  $r$  third-order CP decomposition that separates the muscle lever arms tensor into three factors,

$$\mathcal{A} \approx \sum_{q=1}^r \mathbf{M}_1[:, q] \circ \mathbf{M}_2[:, q] \circ \mathbf{M}_3[:, q],$$

where  $\mathbf{M}_1 \in \mathbb{R}^{n \times r}$ ,  $\mathbf{M}_2 \in \mathbb{R}^{m \times r}$ , and  $\mathbf{M}_3 \in \mathbb{R}^{t \times r}$ .

Each rank-one component in the CP decomposition models a group of muscles working together to produce a part of the

movement, and there should be few such groups, typically no more than seven for lower limb movements [4], [5], [9]. Factor  $\mathbf{M}_1$  is the degrees of freedom weightings, factor  $\mathbf{M}_2$  is the muscle weightings (which are the synergies we are looking for) and factor  $\mathbf{M}_3$  is the temporal modules. To ensure physiological coherence, we enforced an empirical sparsity constraint on each column of  $\mathbf{M}_2$ , stipulating that only a limited number of muscles should be grouped within each synergy; specifically, the constraint mandates that no more than 55% of the muscles are utilized in each synergy. This is the typical sparsity percentage of the lever arms tensor  $\mathcal{A}$  as numerous muscles only span one joint.

Further, factors  $\mathbf{M}_1$  and  $\mathbf{M}_2$  are normalized so that their maximum in each synergy is equal to 1. We set the  $\mathbf{M}_3$  factor to be positive for each synergy to enforce interpretability. We report the normalization factors in  $\mathbf{M}_3$  and the sign corrections in  $\mathbf{M}_2$ .

The constrained decomposition problem may be formalized as follows:

$$\underset{\substack{\mathbf{M}_1 \in \mathbb{R}^{n \times r} \\ \mathbf{M}_2 \in \mathbb{R}^{m \times r} \\ \|\mathbf{M}_2\|_0 \leq \frac{55m}{100} \\ \mathbf{M}_3 \in \mathbb{R}_+^{t \times r}}}{\text{argmin}} \quad \|\mathcal{A} - \sum_{q=1}^r \mathbf{M}_1[:, q] \circ \mathbf{M}_2[:, q] \circ \mathbf{M}_3[:, q]\|_{\mathbf{F}}^2 \quad (4)$$

The chosen initialization method for decomposition is the High-Order Single Value Decomposition (HOSVD) [17]. The algorithm used to solve problem (4) is a combination of Alternating Optimization (AO) and the Alternating Direction Method of Multipliers (ADMM), referred to as AO-ADMM, as developed in [18] and available in the Tensorly toolbox [19]. Tensor  $\mathcal{A}$  is a difficult tensor to decompose because it features highly correlated rows, columns and fibers while being sparse (about half of the entries are zeros because numerous muscles only span one joint). Therefore the constraints we use are critical in obtaining a reasonable decomposition in terms of residual loss and interpretability. The HOSVD initialization is also important, random initialization would often lead to unsatisfactory results.

### C. Computation of muscle forces

The numerical optimization used to compute muscle forces studied in this paper is based on the change of variable  $\mathbf{F} = \mathbf{M}_2^+ \mathbf{S}$ . The  $\mathbf{M}_2^+$  matrix is of size  $m \times 2r$  and is computed from the  $\mathbf{M}_2$  factor extracted from the decomposition of  $\mathcal{A}$ . In matrix  $\mathbf{M}_2^+$ , the positive and negative entries are split into separate columns to form  $2r$  synergies with only positive coefficients. We thus obtain  $2r$  synergies associated with muscular coefficients that are all positive. The  $\mathbf{S}$  matrix therefore represents the unknown, collecting the  $2r$  'synergistic forces' associated with each synergy across time. The least squares problem may still be ill-posed but the number of unknowns has been reduced as long as  $2r \leq m$ . Moreover, muscle contributions in the synergies are scaled, allowing to use a simple ridge penalization for controlling the musculotendon

forces. We obtain the following quadratic program that can be solved efficiently with interior point methods:

$$\min_{\mathbf{S} \in \mathbb{R}^{2r \times t}} \|\mathbf{M}_2^+ \mathbf{S}\|_F^2 \text{ so that } \begin{cases} \mathbf{M}_2^+ \mathbf{S} \geq 0 \\ \mathcal{A} \times_2 \mathbf{M}_2^+ \mathbf{S} = \mathbf{B} \end{cases} \quad (5)$$

Constraints represent respectively the positivity of musculo-tendon forces and conformity to dynamic equilibrium. Once the musculotendon forces are computed, the knee medial and lateral contact forces can be obtained as components of the the vector of Lagrange multipliers  $\lambda$  by inverting the transpose of the Jacobian matrix in equation (1), typically using the Moore–Penrose pseudo-inverse.

### III. EXPERIMENTAL RESULTS

#### A. Dataset

The experimental walking data used in this study is taken from the First Grand Challenge Competition to Predict in vivo Knee Loads [10]. The subject (PS, male, left total knee arthroplasty with neutral leg alignment, age 86, mass 75 kg, height 1.80 m) is implanted with a force-measuring tibial prosthesis. We used data from four gait cycles carried out under similar conditions. In practice we perform four tensor decompositions with  $[n, m, t, r] = [6, 43, 100, 6]$ . The experimental walking data set include the trajectories of markers placed on the skin, ground reaction forces and contact forces in both compartments of the knee.

#### B. Synergy analysis

The results of the four rank-six constrained tensor decompositions of the muscle lever arms tensor are very similar for all gait cycles of the subject, as highlighted in Fig.1. The average reconstruction error is 10.7 %. We observe four synergies (synergies 1, 4 and 6) associated with the three hip degrees of freedom (DoFs 4-5-6), one synergy (synergy 3) associated almost exclusively with the knee degree of freedom (DoF 3) and two synergies (synergies 2 and 5) corresponding to the ankle degrees of freedom (DoFs 1-2). Some temporal modules also reflect the hip, knee, and ankle joint kinematics. Synergies 1, 2, and 3 match typical flexion-extension curves during gait.

#### C. Musculotendon and joint contact forces

Consideration of musculoskeletal mechanics, namely the synergic evolution of muscle lever arms, has an impact on the distribution of musculotendon forces while emphasizing muscles with larger lever arms and strength. For example, in the triceps surae, taking into account the 'synergic forces' modifies the distribution between soleus ( $m = 34$ , ankle mono-articular muscle) and gastrocnemii ( $m = 32, 33$ , knee and ankle bi-articular muscles), thus reducing the estimated contact force on the knee (Fig. 2a).

Similar conclusions can be drawn from the muscular forces of the thigh muscles (Fig. 2b): the forces developed by the 3 vastii muscles (only the vastus medialis,  $m = 29$ , is shown in the figure, but the results are identical with the vastus intermedialis and the vastus lateralis,  $m = 30, 31$ ) and the tensor fasciae latae ( $m = 21$ , 3 mono-articular knee muscles

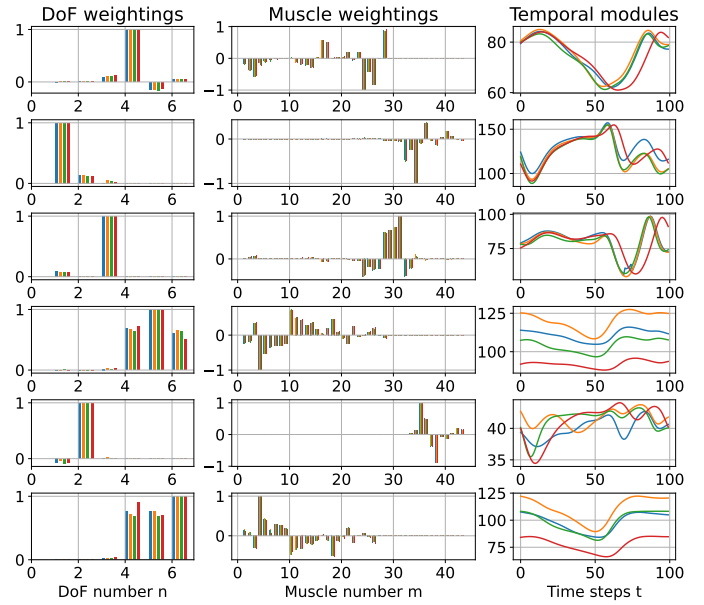


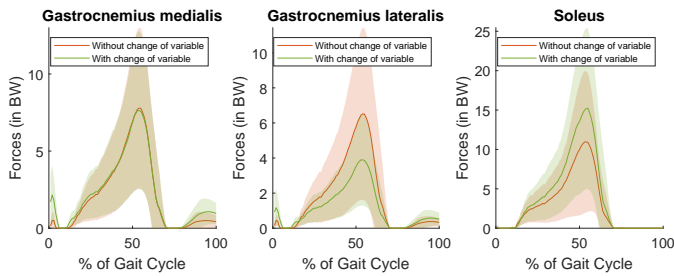
Fig. 1: Tensor decomposition result of order 3 and rank 6. Each color represents the results of one of the subject's four gait cycles. The first column represents the  $M_1$  factor, associated with the degrees of freedom, the second column represents the  $M_2$  factor, associated with the muscular contribution and the third column is the  $M_3$  factor of the temporal modules. Each line therefore corresponds to one of the six synergies.

and one bi-articular hip and knee muscle, respectively) are weaker with the change of variable, and part of their force is taken up by the rectus femoris ( $m = 28$ , bi-articular muscle acting on both the knee and the hip).

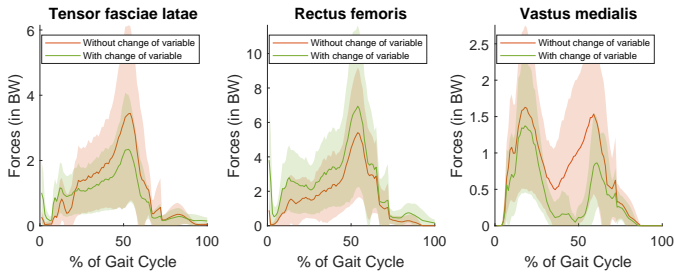
The impact of this altered distribution of musculotendon forces is shown on the estimated knee medial and lateral contact forces (Fig.3). By taking into account muscle lever arms synergies, we therefore can reduce the difference between measured and estimated forces in the second peak (at 55 % of gait cycle) and especially for the lateral compartment of the knee. Medial contact force remains overestimated.

### IV. DISCUSSION

The initial hypothesis has been confirmed: there are indeed synergies at the level of lever arms, specifically six synergies corresponding to combinations of the six degrees of freedom of the lower limb during human walking. However, these synergies can be positive or negative and do not show any possible connection between the hip and the ankle. Indeed bi-articular muscles exist but they do not span simultaneously these two joints. These synergies are thus different in nature from those already analyzed in the literature, typically from experimentally measured EMG [4], [5], [7], [20]. Lever arms provide additional information that had never been studied before. They directly reflect some of the biomechanical affordances (muscle lines of action, forces at maximal muscle activations) and constraints (joint kinematics during gait). Tensor decomposition is a suitable tool for extracting syn-



(a) Musculotendon forces for 3 muscles of the calf.



(b) Musculotendon forces for 3 muscles of the thigh.

Fig. 2: Muscles forces of the subject for different muscle groups, estimated with the change of variable (green) and without the change of variable (red). The solid lines represent the mean values over the 4 gait cycles of the subject, with the standard deviation shown in light color around the mean.

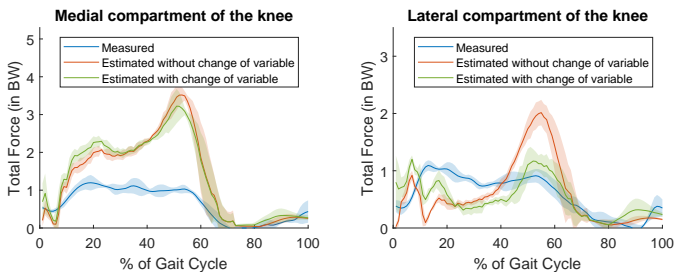


Fig. 3: Contact forces in the knee of the subject, measured by the prosthesis (blue), estimated with the change of variable (green) and without the change of variable (red). The solid lines represent the mean values over the 4 gait cycles of the subject, with the standard deviation shown in light color around the mean.

ergies, although it is rarely used in biomechanics and used for the other variables than lever arms [21], [22]. This method is promising but requires specific algorithms with proper initialization and well-chosen constraints. For experimentally measured EMG signals, which are rectified and normalized, non-negativity is a classical constraint used to directly compute muscle contributions between 0 and 1 [8]. Here, choices were made on the values and signs of the three the factors to obtain similar results. The tensor was also scaled by the maximal active forces. Moreover, the sparsity and sign alternation of the muscle lever arms tensor was translated into the muscle contributions. The rationale to decompose the lever arms

tensor through a rank number of 6 was to match the number of joint degrees of freedom ( $r = n$ ). However, synergies 4 and 6 show similarities which suggests a lower rank  $r = 5$  could eventually be considered. However when testing  $r < 6$ , residual errors rise very quickly (18,5% for  $r = 5$  and 28.1% for  $r = 4$ ) and synergies become more complicated to interpret. In addition, associated musculo-tendinous forces and knee contact forces are less relevant.

Nevertheless, the sign of the lever arm of a muscle on a particular degree of freedom is determined by the combination of the sign of the coefficient associated with that degree of freedom and the one associated with the muscle's contribution. However, it's worth noting that most of the glutei muscles ( $m = 1$  to 9) are represented as extensors, abductors in synergy 1 (DoFs 4-5) and simultaneously as extensors, adductors, and external rotators of the hip in synergies 4 and 6 (DoFs 4-5-6).

This contradiction arises because the value of the lever arm of a muscle on a degree of freedom is the result of combining all the synergies with a nonzero coefficient for that muscle and degree of freedom. Synergy 6 then appears as a kind of correction to Synergy 4, allowing the CP model to reduce the reconstruction error of the tensor across all hip muscles. Increasing the value of rank  $r$  allow to add other "correction synergies" but does not reduce the residual error enough to be taken into account (9% for  $r = 7$  and 7.7% for  $r = 8$ ). To overcome this problem, an unusual sign sharing constraint would need to be implemented in the CP decomposition algorithm. Implementing this constraint is not straightforward and will likely be the subject of future work.

The second hypothesis of this project is also confirmed: dimensionality reduction can be beneficial and can lead to reliable results in terms of musculotendon and joint contact forces. The errors between the estimated and the measured knee contact forces are reduced in the second peak. In the literature, these errors have been reported to range between 0.17 and 1.39 body weights for the medial contact force, and between 0.18 and 0.81 body weights for the lateral contact force [23]. The contact forces estimated with the change of variable method show errors which are in the high and the low range for the medial and lateral compartments, respectively.

The change of variable method is not just a transformation of the solution space to solutions that align with the muscle synergies obtained from tensor decomposition; it also significantly restricts this space. Without the change of variable, the optimization algorithm acts on forty-three variables (the forty-three musculotendon forces), whereas it can only act on the twelve 'synergistic forces'  $\mathbf{S}$  when using the change of variable  $\mathbf{F} = \mathbf{M}_2^+ \mathbf{S}$ .

Therefore, the method may not yield coherent results for some subjects if the solution space becomes too restricted for the algorithm to find a physiologically satisfactory solution. This can be the case typically when using a generic and scaled model, as we process in this study, because the lever arms and maximal active force then depend only on joint kinematics obtained from a multibody kinematics optimization tracking

trajectories of markers placed on the skin [14].

It should be noted that the quality of the kinematics data in the dataset, related to the placement and tracking of marker trajectories on the subject's skin, is therefore crucial for achieving a meaningful substitution.

To overcome this limitation, constraints can be converted into penalties. Another approach is to compute synergies within the optimization problem itself, as done in the SynO approach [20] or similar methods, e.g. [24].

## V. CONCLUSION

In summary, a new method for extracting muscle synergies has been developed using a low-rank constrained Canonical Polyadic decomposition on the lever arms tensor. This method highlights the muscles synergies associated within the muscle lines of action and maximal active forces, and the use of these synergies in the computation of musculotendon forces is showing promising results.

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