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WP 2414 – October 2024

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Benoît Le Maux, Sonia Paty

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Representative Democracy, Decentralization, Local public sector, Cube Root Law.

### JEL codes:

D72, D73, H7, H11.

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Benoît Le Maux<sup>‡</sup>  
Sonia Paty<sup>§</sup>

October 4, 2024

## Abstract

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## 1. Introduction

The growing distrust in politics prompts a crucial inquiry into the optimal organization of modern democracies. Scholars in political science, political economy, and social choice often address this issue by exploring ways to enhance citizen representation, either through changes in voting rules or by decentralizing power to local councils. A common challenge in both approaches is determining the optimal size of assemblies. Despite its importance, however, the question of the level of representation remains highly debated, especially at the subnational level, where multiple entities such as counties or municipalities possess their own set of governmental responsibilities.

The question of optimal assembly sizes has been investigated primarily at the national level. Taagepera's (1972) *cube root law*, which prescribes the optimal number of seats in a legislature as the cube root of the population, has been a subject of normative debate (e.g., Auriol and Gary-Bobo, 2012; Godefroy and Klein, 2018; Margaritondo, 2021; Gamberi et al., 2021). Empirical studies also indicate that the number of parliament members tends to increase with population size, although actual estimates often deviate from theoretical predictions (see De Santo and Le Maux, 2023, for a review). Furthermore, questions persist regarding the applicability of these findings to decentralized systems. When a country is divided into multiple local authorities (e.g., provinces, states, regions, or counties), does the overall number of representatives increase?

Drawing on Taagepera's theory, we investigate how territorial fragmentation enhances political representation. Our model demonstrates that the total number of local representatives exhibits elasticity to both population size (with an elasticity of  $e = 1/3$ ) and the number of jurisdictions (with an elasticity of  $e = 2/3$ ), a relationship we refer to as the *law of 2/3*. We validate this theory using multiple datasets. First, we establish that the *cube root law* applies at the municipal level across 13 countries. Second, our *law of 2/3* is tested and

validated using census data from the lowest government tier in France, covering over 34,000 municipalities. Third, our analysis reveals that smaller French municipalities (with less than 10,000 inhabitants) have 11 times more councillors than larger ones (with more than 10,000 inhabitants), despite having similar total populations. This observation aligns well with the predictions of the *law of 2/3*. Last, our findings provide insights into the implications of merging reforms, based on data from French regions.

The remainder of the paper is structured as follows: Section 2 develops the model, Section 3 presents our empirical investigations, and Section 4 provides the conclusion.

## 2. The model

As a starting point, let us consider a single local jurisdiction  $j$  of population size  $p_j$ . This local population is divided into two groups:  $a_j$  elected representatives and  $p_j - a_j$  non-representatives. Following Taagepera (1972), two types of democratic costs are considered: intra- and extra-assembly communication channels. First, each assembly member is involved in the jurisdiction’s assembly “either as a participant or, more often, as an attentive bystander to the discussion,” so that the total number of intra-assembly communication channels amounts to  $d_j = \frac{1}{2}a_j(a_j - 1)$ . Second, assuming the assemblymen share the workload (e.g., by discussing and sharing their views with the  $p_j - a_j$  non-representatives), the number of extra-assembly communication channels per representative is computed as  $c_j = \frac{p_j - a_j}{a_j}$ .

Taagepera (1972) posits that each elected representative must allocate equal time and effort to the assembly and to non-representatives, ensuring a balance such that  $c_j = d_j$  (see Appendix A.1 for an example). We obtain:

$$p_j = \frac{1}{2}a_j^3 - \frac{1}{2}a_j^2 + a_j. \tag{1}$$

Let  $a_j^* = f(p_j)$  denote the solution. From the inverse function theorem, we get:<sup>1</sup>

$$\frac{df}{dp_j} > 0 \text{ and } \frac{d^2f}{dp_j^2} < 0. \quad (2)$$

Function  $f$  can reasonably be specified as:

$$f(p_j) = \mu(p_j)^\lambda \text{ with } \lambda < 1. \quad (3)$$

There are two reasons for this specification. First, equation 1 can be approximated by  $p_j \approx \frac{1}{2}a_j^3$  or, equivalently,  $a_j \approx (2p_j)^{\frac{1}{3}}$ . Second, as evidenced in several studies, the relationship is shown to be linear in logarithm (Taagepera, 1972; Tufte, 1974; Taagepera and Shugart, 1989; Auriol and Gary-Bobo, 2012; Jacobs and Otjes, 2015; Godefroy and Klein, 2018; De Santo and Le Maux, 2023) and US States (Stigler, 1976), with an estimated elasticity close to 1/3. This yields the following testable proposition:

**Proposition 1** (Cube root law). *The number of local representatives  $a_j^*$  in jurisdiction  $j$  scales with its population size  $p_j$  according to  $a_j^* = \mu(p_j)^\lambda$ , where  $\lambda = 1/3$ .*

Now, let  $J$  denote the total number of local jurisdictions in a government tier,  $P = \sum_{i=1}^J p_j$  stand for the whole population size, and  $A^* = \sum_{i=1}^J a_j^*$  be the total number of local representatives. From equation 3, we have:

$$A^* = \sum_{j=1}^J f(p_j) = \mu \times \sum_{j=1}^J (p_j)^\lambda. \quad (4)$$

Dividing by  $P^\lambda$  on both sides yields  $A^*/P^\lambda = \mu \times \sum_{j=1}^J (\alpha_j)^\lambda$ , where  $\alpha_j = p_j/P$  is the share

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<sup>1</sup>From equation 1, we have  $p_j' = \frac{3}{2}a_j^2 - a_j + 1$ , which is positive. Therefore  $a' = 1/p_j'$  is positive as well. Moreover, we have  $p_j'' = 3a_j - 1$ , which is positive. From the second derivative rule for inverse function, we have  $a_j'' = -p_j''/(p_j')^3$ , which is negative.

of the country population living in jurisdiction  $j$ . In other words:

$$A^* = \mu P^\lambda \times X \text{ with } X = \sum_{j=1}^J (\alpha_j)^\lambda. \quad (5)$$

Following Padovano and Rocaboy (2018),  $X$  can be written as:

$$X = \alpha_1^\lambda + \dots + \alpha_l^\lambda + \dots + \alpha_m^\lambda + \dots + \alpha_J^\lambda, \quad (6)$$

where  $\alpha_m = 1 - \sum_{j \neq m} \alpha_j$ . Assuming a population shift from jurisdiction  $l$  to  $m$ , we get:

$$\frac{dX}{d\alpha_l} = \lambda \left( \alpha_l^{\lambda-1} - \alpha_m^{\lambda-1} \right). \quad (7)$$

For  $\lambda \in (0, 1)$ , this derivative is strictly positive if  $\alpha_l < \alpha_m$ , i.e.,  $X$  increases when a smaller jurisdiction grows at the expense of a larger jurisdiction. In other words,  $X$  stands for a fragmentation index ranging from 1 (when a single jurisdiction encompasses all inhabitants) to  $J^{1-\lambda}$  (all jurisdictions have the same size).<sup>2</sup>

Equation 5 shows that the total count of local representatives  $A^*$  grows not only with the population size  $P$  but also with territorial fragmentation  $X$ . If we consider the simpler case where the population is uniformly distributed across jurisdictions (such that  $X = J^{1-\lambda}$ ), we obtain  $A^* = \mu P^\lambda \times J^{1-\lambda}$ . In that case, a rise in the number of jurisdictions automatically escalates the number of representatives across the entire country. If  $\lambda < 0.5$ , as the empirical evidence suggests, then the total number of local representatives is more elastic to the number  $J$  of jurisdictions than to the population size  $P$ . Specifically, if we set  $\mu = 0$  and  $\lambda = 1/3$  as traditionally assumed, we get:

$$A^* = P^{1/3} \times J^{2/3}. \quad (8)$$

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<sup>2</sup>A discussion on the well-known fragmentation indexes can be found in Le Maux and Rocaboy (2012).

This equation provides a straightforward method for predicting the number of representatives in a given government tier, and clarifies the link between the level of representation and the number of jurisdictions:<sup>3</sup>

**Proposition 2** (Law of 2/3). *The elasticity of the number of local representatives  $A^*$  with respect to the number of jurisdictions  $J$  is 2/3.*

The model becomes particularly insightful when comparing the representation of residents living in smaller jurisdictions versus those in larger ones. Consider the cumulative distribution function (CDF) of the population shares across jurisdictions:

$$F(x) = \sum_{\alpha_j \leq x} \alpha_j. \quad (9)$$

This function indicates the share of agents living in jurisdictions with less than  $x$  percent of the total population. Let  $x^*$  be such that  $F(x^*) = 0.5$ . This split point divides the population into two groups of equal size: Group 1 (fragmented group) consists of jurisdictions of size smaller than  $x^*$  while Group 2 (consolidated group) consists of jurisdictions of size larger than  $x^*$ . As a result, Group 1 will have more councillors compared to Group 2, despite having similar total populations. For instance, consider the simpler case where the population is uniformly distributed within each group, so that Group 1 consists of  $J_1$  small jurisdictions, while Group 2 consists of  $J_2$  large jurisdictions, with  $J_1 > J_2$ . From equation 8, the total number of representatives in each group amounts to  $A_1^* = \mu P_1^\lambda \times J_1^{1-\lambda}$  and  $A_2^* = \mu P_2^\lambda \times J_2^{1-\lambda}$  where  $P_1 = P_2 = P/2$  stands for the population size in each group. Assuming that  $\lambda = 1/3$ , the relative difference between the representation levels is given by:

$$\frac{A_1^*}{A_2^*} = \frac{\mu P_1^{1/3} \times J_1^{2/3}}{\mu P_2^{1/3} \times J_2^{2/3}} = \left( \frac{J_1}{J_2} \right)^{2/3}. \quad (10)$$

---

<sup>3</sup>With multiple government tiers, say  $k = 1 \dots K$ , the total count of local representatives becomes  $\sum_{k=1}^K A_k^* = P^{1/3} \times \sum_{k=1}^K J_k^{2/3}$ . The expression  $\sum_{k=1}^K J_k^{2/3}$  cannot be simplified into a single algebraic term in a straightforward manner.



This equation specifies the ratio of representatives as a function of the ratio of jurisdiction counts. To sum up:

**Proposition 3** (Relative representation of smaller cities). *When splitting the population into two groups of equal size, those living in smaller cities and those in larger cities, the fragmented group (smaller cities) will have a proportionally larger number of representatives relative to the consolidated group (larger cities).*

### 3. Empirical tests

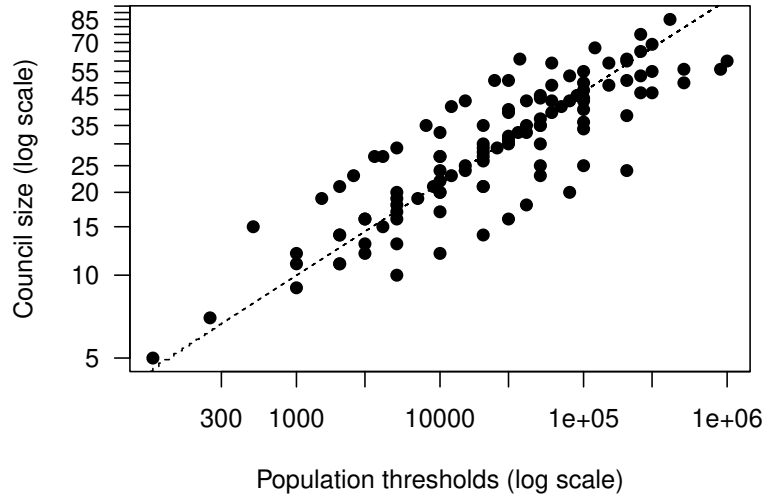
#### 3.1. *The cube root law applied to local assemblies*

The *cube root law* (proposition 1) is a fundamental component of our theory and must be considered as a necessary condition for the validity of the *law of 2/3*. Specifically, the exponent  $\lambda = 1/3$  has been observed to apply empirically on national assemblies. Figure 1 extends this result on local assemblies using seat-allocation formulas from 13 countries (see Appendix A.2 for data description). The estimated elasticity stands at  $e = 0.26$  (95% confidence interval: 0.24; 0.30), which closely aligns with the traditionally predicted value depicted by the dashed line. Note that differences are observed across countries as shown in Figure 2. Countries such as Finland, Sweden, and France have a higher number of seats than anticipated by theoretical predictions. Similarly, some nations provide fewer seats than predicted (e.g., Poland, Italy, Norway, Buenos Aires Province).

#### 3.2. *Testing the law of 2/3 on French data*

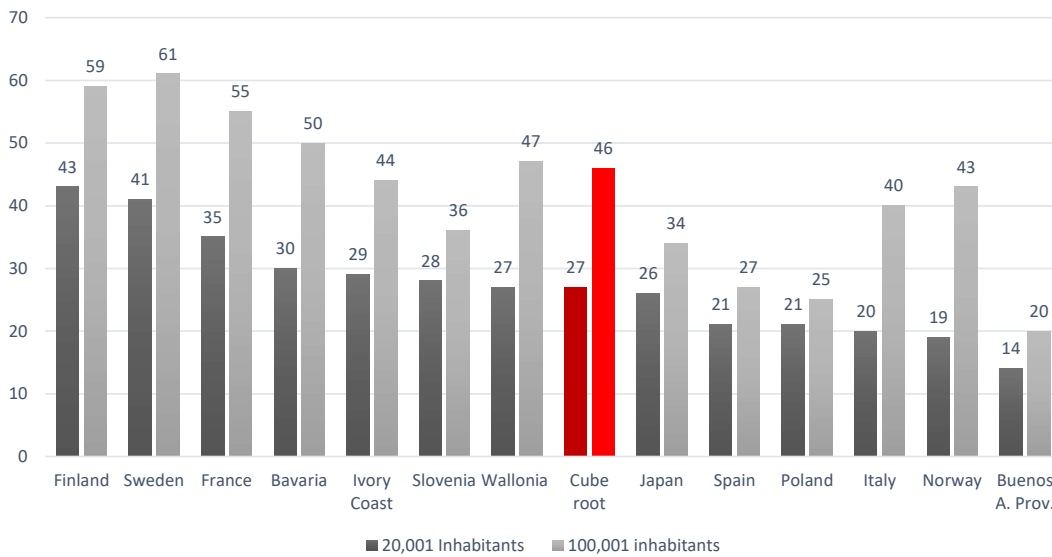
We test our *law of 2/3* (proposition 2) using census data at the French municipal level. Metropolitan France comprises 95 Departments, constituting the secondary level of local governance, with each containing a distinct subset of municipalities and, therefore, a specific number of municipal councillors. This allows us to estimate elasticities, as described in equation 8.

**Figure 1:** Cube root law at the municipal level in 13 countries



Notes: based on seat-allocation formulas in 13 countries: Bavaria, Buenos Aires Province, Finland, France, Ivory Coast, Italy, Japan, Norway, Poland, Slovenia, Spain, Sweden, Wallonia. Cube root law in dashed line. For comparability, minimum population thresholds are excluded. See Appendix A.2 for data description.

**Figure 2:** Municipal council size at specific thresholds



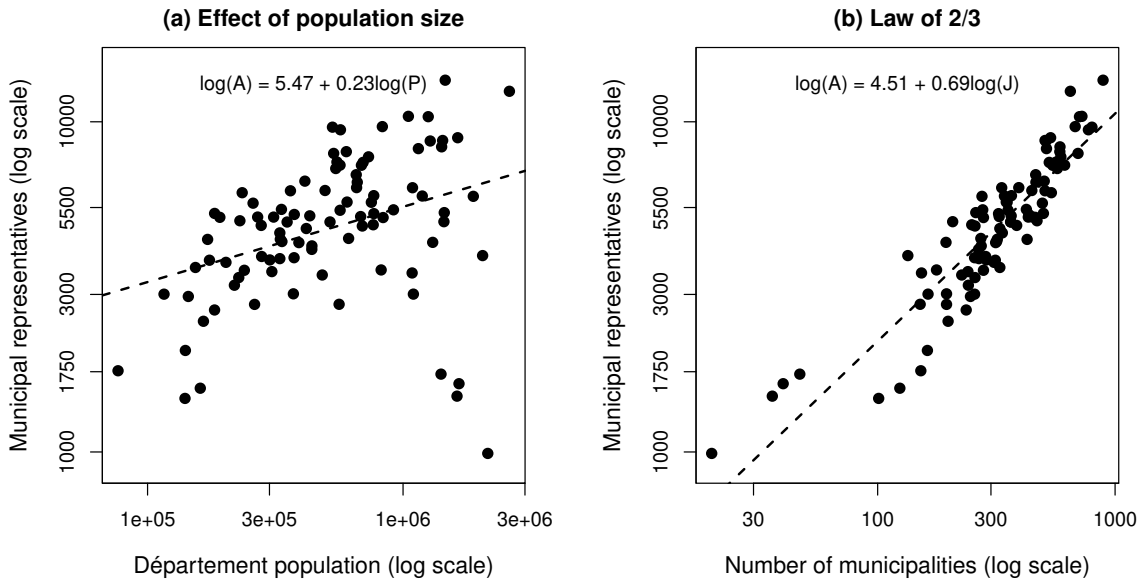
Note: see Appendix A.2 for data description.

Figure 3 offers an illustration. The population elasticity stands at  $e = 0.22$  (95% confidence interval: 0.10; 0.35), while the elasticity concerning the number of jurisdictions reaches  $e = 0.69$  (95% confidence interval: 0.62; 0.75). Hence, the total number of local representatives exhibits a stronger relationship with the total count of jurisdictions compared to the population. Overall, the OLS estimations yield:

$$\ln A_i = 0.97 + 0.26 \ln(P_i) + 0.71 \ln(J_i), \text{ (Adj. R}^2=0.99) \quad (11)$$

where  $i$  stands for Department  $i$ . These findings align closely with our theoretical predictions, i.e.,  $\lambda = 1/3$  and  $1 - \lambda = 2/3$ .

**Figure 3:** Number of municipal representatives: elasticity estimates at the Département level



Note: Based on 95 Departments in metropolitan France. Data are from Insee, population census at 2022. Lines of best fit (OLS) are in dashed lines.

### 3.3. Representation levels in small versus large municipalities

France is an interesting case as it is characterized by a tremendous number of local jurisdictions:  $J = 34,980$  in 2022 for a population size equal to  $P = 67,162,154$ . As a result,

the number of local representatives is quite large as well:  $A = 509,548$  councillors. Table 1 offers an illustration. If we apply the *cube root law* to each municipality separately, taking into account their own population, we reach a similar range, i.e.,  $A^* = 316,163$  officials (Table 1, column 6). The *law of 2/3* established in this paper (equation 8) provides a more straightforward method for estimating this figure, as it requires census data solely at the national level:  $A^* = 406 \times 34,980^{2/3} = 434,765$  (column 7). As can be seen, both methods underestimate the true count to some extent. This discrepancy arises from the seat-allocation formula employed at the municipal level, which mandates a larger number of seats than what is predicted by the *cube root law* (see Appendix A.2).

**Table 1:** Number of local representatives at the municipal level in France

(1)	(2)	(3)	(4)	(5)	(6)	(7)
Muni. size	Muni. number	Population	% Rural	Real local rep.	Cube root law	Law of 2/3
All size	34,980	67,162,154	88%	509,548	316,163	434,765
< 10,000 inhab.	33,925	33,283,594	90.6%	470,295	285,178	337,098
> 10,000 inhab.	1,055	33,878,560	1.97%	39,253	30,984	33,533

Data source: Insee, population census at 2022.

In France, the population is inequitably distributed among municipalities of different size, which in turn strongly affects the distribution of elected representatives (as established in proposition 3). To show this, we split the total population into two equivalent subsets composed of 33,925 municipalities with less than 10,000 inhabitants and 1,055 municipalities with more than 10,000 inhabitants. Both subsets have about the same number of inhabitants, and yet we observe a huge difference in their true number of officials: 470,295 councillors for the smallest cities versus 39,253 for the largest ones, resulting in a representation ratio of 11.98. Those ranges are fairly (yet not perfectly) approximated using either the *cube root law* and the *law of 2/3* (see Table 1, columns 6 and 7). Replacing  $J_1$  and  $J_2$  in equation 10 yields:

$$\frac{A_1^*}{A_2^*} = \left( \frac{33,925}{1,055} \right)^{2/3} = 10.11, \quad (12)$$

which accurately approximates the relative representation of smaller cities. Note that only 1.97% of the largest municipalities (with more than 10,000 inhabitants) are rural, whereas 90.6% of the smaller municipalities (with less than 10,000 inhabitants) belong to rural areas (column 4). Therefore, it is reasonable to conclude that, due to their greater fragmentation, rural areas are overrepresented compared to urban areas.

### 3.4. Insights from Merger Reforms: Case Study of French Regions

**Table 2:** Number of local representatives at the regional level in France

(1) Year	(2) Regions number	(3) Pop. in thousands	(4) Real local rep.	(5) Cube root law	(6) Law of 2/3
2014 (before merger)	26	65,499	1,758	3,260	3,538
2021 (after merger)	17	67,419	1,758	2,449	2,691

Note: Data come from Insee, population census. Regions number includes 13 (after merger) or 22 (before merger) metropolitan regions and 4 overseas territories.

Our findings also provide insights for merger reforms. Take, for instance, the case of French regions, the upper tier of government in France, which saw a reduction in number from 22 to 13 in metropolitan France (from 26 to 17 including overseas territories) in 2016. While six regions retained their boundaries, five resulted from the merger of two former regions, and two were formed from three former regions. This reform aimed to create European-sized regions (Seys, 2017) and capitalize on economies of scale (Orange, 2019). Intriguingly, although the population size naturally increased due to the mergers, the reform did not alter the total number of regional councilors. Instead, each council's size was determined as the sum of the merged regional seats, thus running counter to our theoretical expectations. Last, in Table 2, we can see that both the *cube root law* and the *law of 2/3* significantly overestimate the actual number of regional representatives, prompting inquiries into the factors influencing assembly size within this specific government tier.

## 4. Conclusion

Taagepera's (1972) *cube root law* appears to hold true at the local level, as observed in the seat-allocation formulas identified across a set of 13 countries, which exhibit a logarithmic relationship with population size. As demonstrated on French municipalities, when such concave relationships are present, the number of elected officials in a specific area will naturally increase with territorial fragmentation and decrease with mergers. Specifically, we anticipate an elasticity of  $e = 2/3$  in relation to the number of jurisdictions, our so-called *law of 2/3*. It is worth noting that exceptions may exist, such as in the case of French regions. Despite a merging reform in 2016, the total number of regional councillors remained unchanged. This highlights the influence of historical factors in determining assembly sizes at the local level.

Our results prompt important questions about the optimal organization of democracies. For instance, as highlighted in Oates' (1972) decentralization theorem, territorial fragmentation can enable a more thorough consideration of geographically distributed preferences. It can also lead to increased pork barrel spending and distributive policies, as described by the 'law of 1/n' (Weingast et al., 1981). Our findings reveal that these effects may be further amplified by seat-allocation formulas: territorial fragmentation not only enhances the consideration of local preferences but also significantly increases their political representation through a greater number of seats, thereby inducing a stronger form of both the decentralization theorem and law of 1/n.

### Data availability statement

The datasets generated and analysed during the current study are available from the corresponding author on reasonable request.

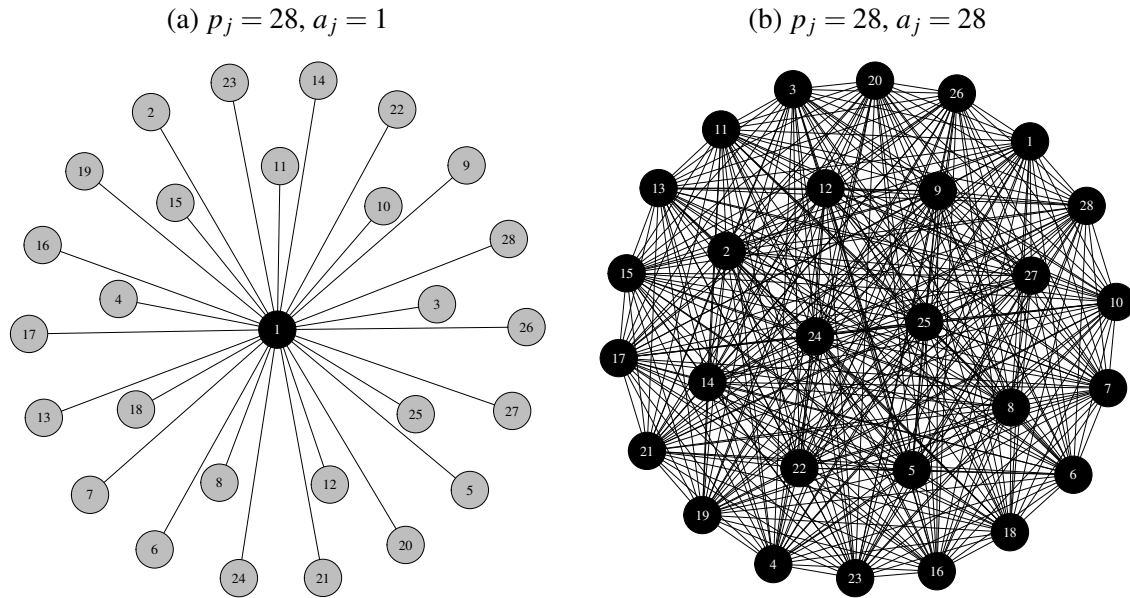
## A. Appendix

### A.1. Optimal assembly size: an example

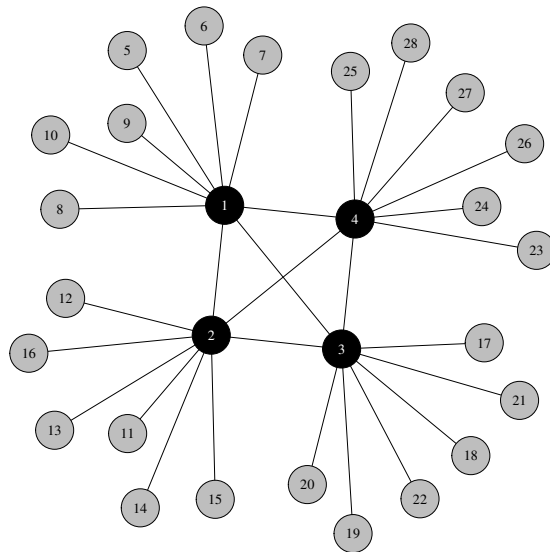
Figure 4 depicts two extreme scenarios for a local population of size  $p_j = 28$ . Each agent is represented by a circle, and a communication channel is represented by a line (or edge) linking one circle to another. Elected representatives are displayed in black. In panel (a), the number of communication channels is minimized, resulting in an assembly with a single representative for the entire population (hence  $d_j$  is zero), but with the representative shouldering the entire workload ( $c_j = 28$ ). Panel (b) illustrates the consequences of an all-population assembly, resulting in  $\frac{1}{2} \times 28 \times (28 - 1) = 378$  intra-assembly communication channels and, therefore, high decision-making costs.

Given these two extreme cases, Taagepera (1972) assumes that an optimal democracy should balance the intra-assembly and extra-assembly channels, so that  $c_j = d_j$ . Figure 5 illustrates this normative result with an optimal assembly size of  $a_j^* = 4$ . As shown, agent 1 is connected to agents 2, 3, and 4; agent 2 is also connected to agents 3 and 4; and agents 3 and 4 are connected. This configuration yields a total of  $d_j = \frac{1}{2} \times (4 \times 3) = 6$  intra-assembly communication channels. Each representative has access to  $c_j = 6$  citizens. Hence, in this scenario, there is a perfect balance between the number of communication channels, representing the optimal solution.

**Figure 4:** Two extreme cases.



**Figure 5:** Optimal assembly size:  $p_j = 28, a_j^* = 4$ .





## A.2. *Seat-allocation formulas in 13 countries*

This subsection presents the seat-allocation formulas we have identified, along with their respective sources of reference. Note that acquiring the data was challenging. First, there is no centralized dataset. While certain countries may publish guidelines or legislation outlining the principles underlying seat allocation, the specifics of these formulas often remain dispersed across diverse sources, including legal documents, government reports, and academic literature. Even within a single country, different regions may employ distinct methodologies for seat allocation. Sometimes, no formula exists: in the UK, the number of local elected representatives varies considerably and is, in large part, the result of historical accidents (Purdam et al., 2007). A similar statement applies to the French regions.

Sources of data are the following:

- Bavaria: Egger, P., & Koethenbueger, M. (2010). Government Spending and Legislative Organization: Quasi-Experimental Evidence from Germany. *American Economic Journal: Applied Economics*, 2(4), 200-212.
- Buenos Aires Province: <https://normas.gba.gob.ar/documentos/OVG48SW0.html>
- Finland and Sweden: Pettersson-Lidbom, P., 2012. Does the size of the legislature affect the size of government? Evidence from two natural experiments. *J. Public Econ.* 96 (3), 269-278.
- France: <https://www.interieur.gouv.fr/Archives/Archives-elections/Dossier-elections-municipales-2014/Annexe-4-Nombre-de-conseillers-municipaux-selon-la-population-de-la-commune>
- Ivory Coast: <http://dgddl.gouv.ci/documentation/201208131523012012081315230120100930120439201009301204391.pdf>

- Italy: Benedetto, M.A.. (2018). The effect of council size on municipal expenditures: evidence from Italian municipalities.
- Japan: Hirota, H., Yunoue, H., 2012. Local Government Expenditure and Council Size: Quasi-Experimental Evidence from Japan. MPRA Paper 42799, University Library of Munich, Germany.
- Norway: [https://lovdata.no/dokument/NL/lov/2018-06-22-83/KAPITTEL\\_1-2#%C2%A72-1](https://lovdata.no/dokument/NL/lov/2018-06-22-83/KAPITTEL_1-2#%C2%A72-1)
- Poland: [https://pl.wikipedia.org/wiki/Rada\\_gminy](https://pl.wikipedia.org/wiki/Rada_gminy)
- Slovenia: <http://pisrs.si/Pis.web/pregledPredpisa?id=ZAK04301>
- Spain: <https://elecciones.locales2023.es/en/informacion-general/resuelva-sus-dudas/elecciones-locales/>
- Wallonia: <https://www.uvcw.be/fonctionnement/focus/art-2450>

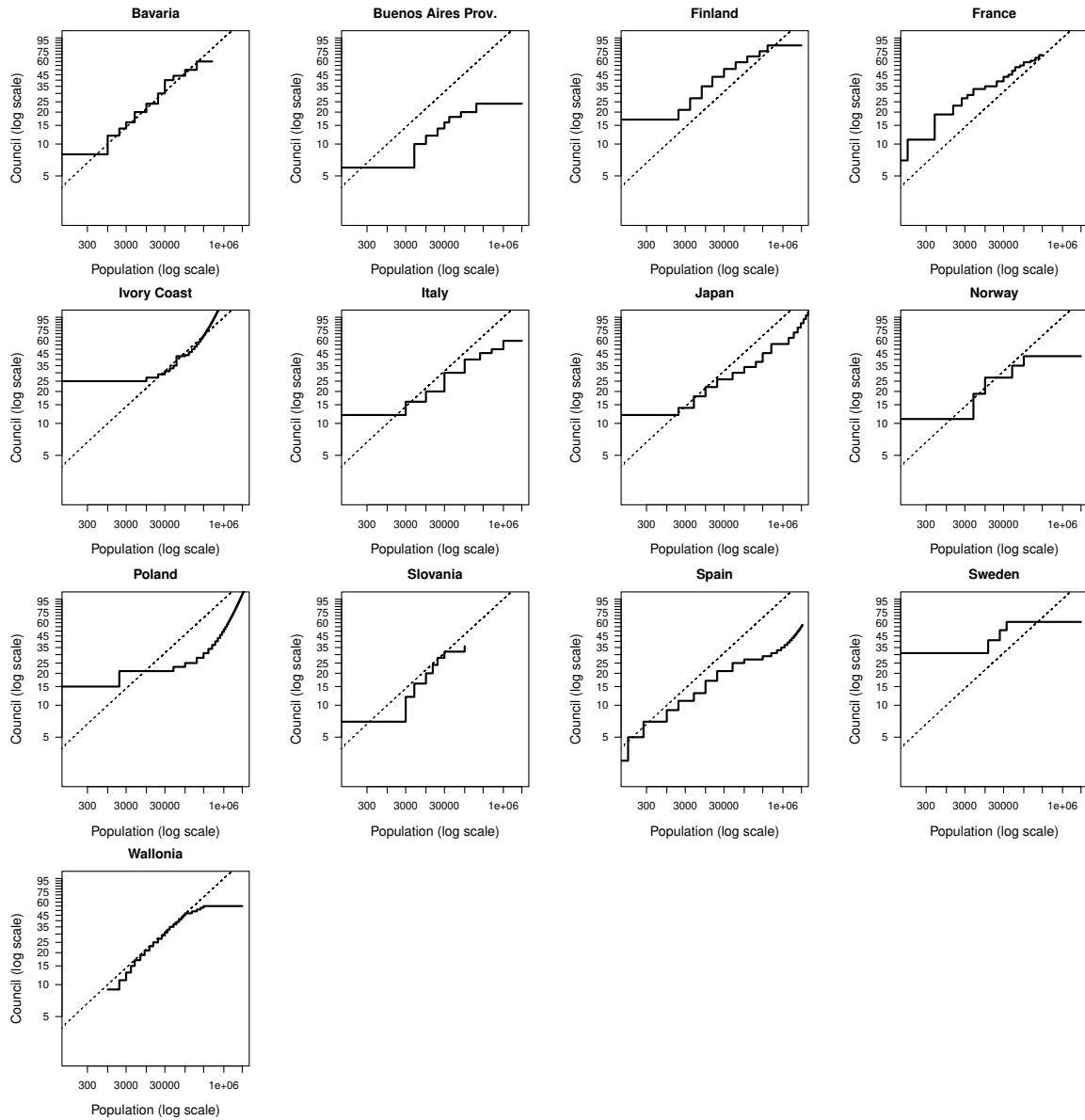
Table 6 displays the raw data. A few stylized facts can be identified. First, some formulas are more precise as they consider smaller threshold intervals, e.g., Wallonia or France. Second, the minimums and maximums of the formulas differ, e.g., the smaller threshold starts at 100 inhabitants in France, and at 12,000 inhabitants in Sweden. Third, some countries have an even number of seats per council : Bavaria, Japan, Italy, Buenos Aires Province, Slovenia. Last, some countries employ linear extrapolation above a certain population size: Spain, Poland, Japan, Ivory Coast. As seen in Figure 6, most of the formulas exhibit a linear relationship when plotted on a logarithmic scale, closely following the cube root law indicated by the dashed line.

**Table 3:** Seat-allocation formulas at the municipal level in 13 countries

	<b>Wallonia</b>		<b>France</b>		<b>Bavaria</b>		<b>Japan**</b>		<b>Finland</b>	
<b>1</b>	Below	9	Below	7	Below	8	Below	12	Below	17
<b>2</b>	2000	11	100	11	1000	12	2000	14	2000	21
<b>3</b>	3000	13	500	15	2000	14	5000	18	4000	27
<b>4</b>	4000	15	1500	19	3000	16	10000	22	8000	35
<b>5</b>	5000	17	2500	23	5000	20	20000	26	15000	43
<b>6</b>	7000	19	3500	27	10000	24	50000	30	30000	51
<b>7</b>	9000	21	5000	29	20000	30	100000	34	60000	59
<b>8</b>	12000	23	10000	33	30000	40	200000	38	120000	67
<b>9</b>	15000	25	20000	35	50000	44	300000	46	250000	75
<b>10</b>	20000	27	30000	39	100000	50	500000	56	400000	85
<b>11</b>	25000	29	40000	43	200000	60	900000	56		
<b>12</b>	30000	31	50000	45	Nueremberg	70	Each 500,000	" +8"		
<b>13</b>	35000	33	60000	49	Munich	80				
<b>14</b>	40000	35	80000	53						
<b>15</b>	50000	37	100000	55						
<b>16</b>	60000	39	150000	59						
<b>17</b>	70000	41	200000	61						
<b>18</b>	80000	43	250000	65						
<b>19</b>	90000	45	300000	69						
<b>20</b>	100000	47	Lyon	73						
<b>21</b>	150000	49	Marseille	101						
<b>22</b>	200000	51	Paris	163						
<b>23</b>	250000	53								
<b>24</b>	300000	55								
<b>1</b>	<b>Spain</b>		<b>Italy</b>		<b>Buenos Aires Prov.</b>		<b>Slovenia*</b>		<b>Ivory Coast</b>	
<b>2</b>	Below	3	Below	12	Below	6	Below	7	Below	25
<b>3</b>	100	5	3000	16	5000	10	3000	12	10000	27
<b>4</b>	250	7	10000	20	10000	12	5000	16	20000	29
<b>5</b>	1000	9	30000	30	20000	14	10000	20	30000	31
<b>6</b>	2000	11	100000	40	30000	16	15000	24	40000	33
<b>7</b>	5000	13	250000	46	40000	18	20000	28	50000	35
<b>8</b>	10000	17	500000	50	80000	20	30000	32	60000	43
<b>9</b>	20000	21	1000000	60	200000	24	100000	36	100000	44
<b>10</b>	50000	25							Each 25,000	" +1"
<b>11</b>	Each 100,000	" +1"								
<b>12</b>	Even number	" +1"								
<b>1</b>	<b>Sweden*</b>		<b>Norway*</b>		<b>Poland</b>					
<b>2</b>	Below	31	Below	11	Below	15				
<b>3</b>	12000	41	5000	19	20000	21				
<b>4</b>	24000	51	10000	27	50000	23				
<b>5</b>	36000	61	50000	35	100000	25				
<b>6</b>	Stockholm	101	100000	43	Each 100000	" +3"				
<b>7</b>					Warsaw	60				

\* Minimum requirement. \*\* Maximum requirement.

**Figure 6:** Formulas at the municipal level



Note: Cube root law in dashed line.

## References

- Auriol, E. and Gary-Bobo, R. J. (2012). On the optimal number of representatives. *Public Choice*, 153(3/4):419–445.
- De Santo, A. and Le Maux, B. (2023). On the optimal size of legislatures: An illustrated literature review. *European Journal of Political Economy*, 77:102317.
- Gamberi, L., Förster, Y.-P., Tzanis, E., Annibale, A., and Vivo, P. (2021). Maximal modularity and the optimal size of parliaments. *Scientific Reports*, 11(1).
- Godefroy, R. and Klein, N. (2018). Parliament shapes and sizes. *Economic Inquiry*, 56(4):2212–2233.
- Jacobs, K. and Otjes, S. (2015). Explaining the size of assemblies. A longitudinal analysis of the design and reform of assembly sizes in democracies around the world. *Electoral Studies*, 40:280–292.
- Le Maux, B. and Rocaboy, Y. (2012). A simple microfoundation for the utilization of fragmentation indexes to measure the performance of a team. *Economics Letters*, 116(3):491–493.
- Margaritondo, G. (2021). Size of national assemblies: The classic derivation of the cube-root law is conceptually flawed. *Frontiers in Physics*, 8.
- Oates, W. E. (1972). *Fiscal federalism*. Harcourt Brace Jovanovich, New York.
- Orange, G. (2019). Les régions issues de la fusion vont-elles connaître des économies d'échelle ? *Gestion & Finances Publiques*, pages 46–58.
- Padovano, F. and Rocaboy, Y. (2018). How defense shapes the institutional organization of states. *Public Choice*, 175(1):111–134.

Seys, F.-O. (2017). Les nouvelles régions françaises dans le contexte de l'union européenne: Définition et analyse cartographique et statistique. *Pôle Sud*, n° 46:97.

Stigler, G. J. (1976). The sizes of legislatures. *The Journal of Legal Studies*, 5(1):17–34.

Taagepera, R. (1972). The size of national assemblies. *Social Science Research*, 1(4):385–401.

Taagepera, R. and Shugart, M. S. (1989). *Seats and votes : the effects and determinants of electoral systems*. Yale University Press New Haven.

Tufte, E. R. (1974). *Data Analysis for Politics and Policy*. Pearson College Div; 1st edition.

Weingast, B. R., Shepsle, K. A., and Johnsen, C. (1981). The political economy of benefits and costs: A neoclassical approach to distributive politics. *Journal of Political Economy*, 89(4):642–664.