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# The power balance equation for converter control - Application to a three-level neutral point clamped rectifier

Manon Doré, Yassine Ariba, Germain Garcia

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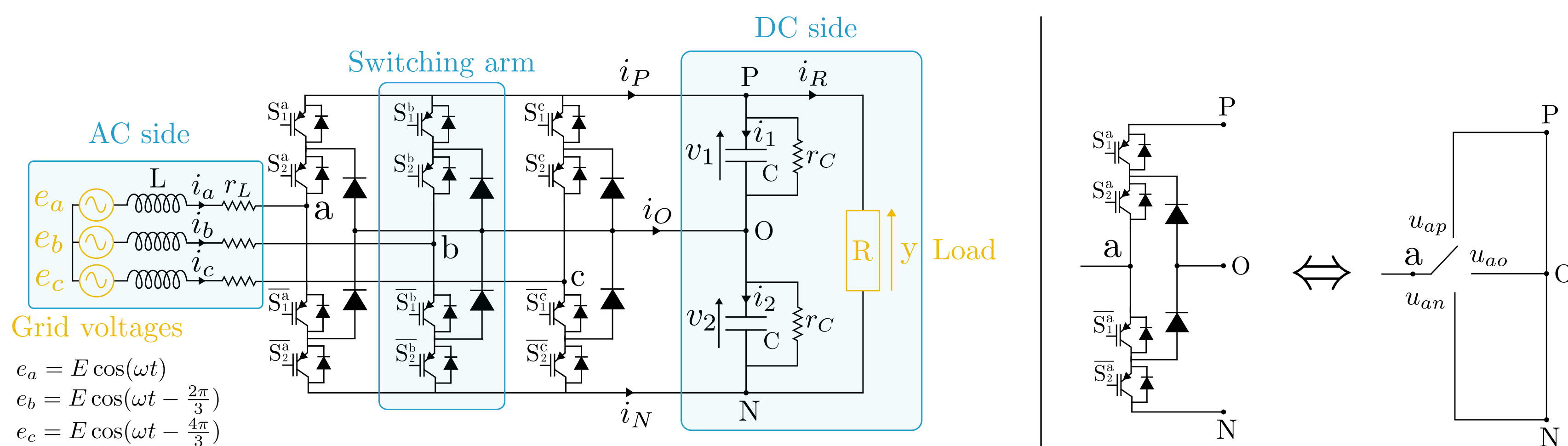
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## Neutral point clamped (NPC) rectifier model



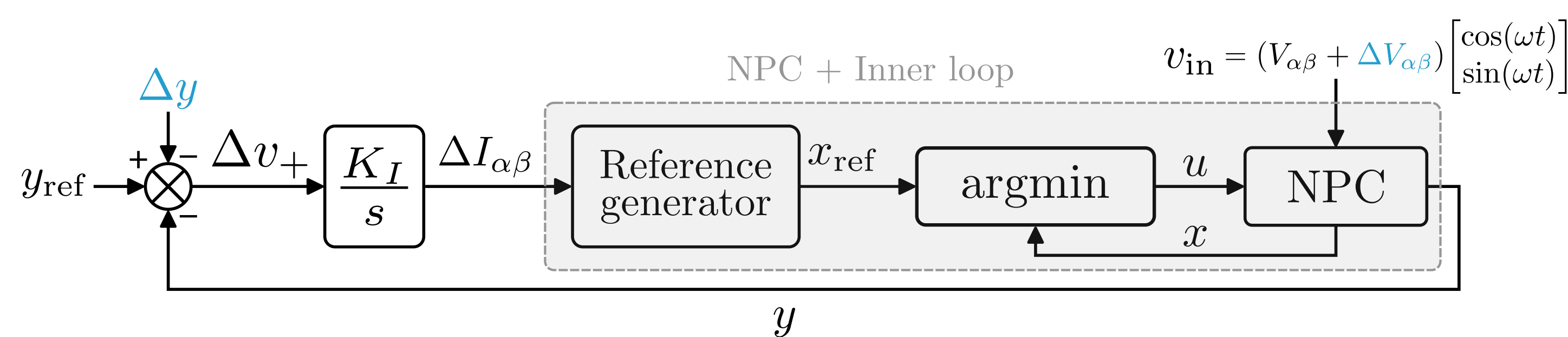
- Assume **balanced signals** :  $e_a + e_b + e_c = 0$  and  $i_a + i_b + i_c = 0$
- Modeling of **switches** : **boolean variables**,  $u_{ij} = 1 \Leftrightarrow$  phase  $i$  is connected to point  $j \Rightarrow 9$  control variables,  $u = [u_{ap} \ u_{ao} \ u_{an} \ u_{bp} \ u_{bo} \ u_{bn} \ u_{cp} \ u_{co} \ u_{cn}]$
- For  $i = a, b, c$ ,  $u_{ip} + u_{io} + u_{in} = 1 \Rightarrow 27$  possible values of  $u$ , defined in  $\mathcal{U}_{com}$
- Using Kirchoff's laws, we obtain a **bilinear model**

$$\begin{cases} L \begin{bmatrix} \dot{i}_a \\ \dot{i}_b \\ \dot{i}_c \end{bmatrix} = \begin{bmatrix} e_a \\ e_b \\ e_c \end{bmatrix} - r_L \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} - \frac{1}{3} \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix} \begin{bmatrix} u_{ap} \\ u_{bp} \\ u_{cp} \end{bmatrix} v_1 - \begin{bmatrix} u_{an} \\ u_{bn} \\ u_{cn} \end{bmatrix} v_2 \\ C\dot{v}_1 = u_{ap}i_a + u_{bp}i_b + u_{cp}i_c - \frac{v_1 + v_2}{R} - \frac{v_1}{r_C} \\ C\dot{v}_2 = -(u_{an}i_a + u_{bn}i_b + u_{cn}i_c) - \frac{v_1 + v_2}{R} - \frac{v_2}{r_C} \end{cases}$$

- Using Clarke-Concordia transform  $\mathcal{C}$ , we obtain a bilinear state-space model of the form

$$\begin{bmatrix} g_\alpha \\ g_\beta \end{bmatrix} = \mathcal{C} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} \quad \begin{cases} \dot{x} = A(u)x + B_0 v_{in} \\ y = C_0 x \end{cases}, \quad x = \begin{bmatrix} i_\alpha \\ i_\beta \\ v_+ = v_1 + v_2 \\ v_- = v_1 - v_2 \end{bmatrix}, \quad v_{in} = \begin{bmatrix} e_\alpha = V_{\alpha\beta} \cos(\omega t) \\ e_\beta = V_{\alpha\beta} \sin(\omega t) \end{bmatrix}$$

## Control



### Inner loop - argmin control [2]

- Quadratic stability condition**:  $\exists P = P^T > 0$  for  $Q_C > 0$  s.t.  $\forall u \in \mathcal{U}_{com}$
- Argmin control law**: For  $V = e^T P e$ ,  $u$  **minimizes**  $\dot{V}(e, u)$ , s.t.  $e = 0$  where  $e = x - x_{ref}$  is asymptotically stable for

$$A(u)^T P + P A(u) + 2Q_C < 0$$

$$u \in \operatorname{argmin}_{d \in \mathcal{U}_{com}} e^T P (A(d)x + B_0 v_{in})$$

### Outer loop - Integral action

- The system is affected by constant perturbations on the load  $\Delta y$ /input voltages  $\Delta V_{\alpha\beta}$
- Assume inner loop is much faster than the outer loop.
- The outer loop with an **integral action** adapt  $I_{\alpha\beta}$  with an extra term

$$\Delta I_{\alpha\beta}(t) = K_I \int_0^t (y_{ref} - y(u)) du \quad \text{s.t.} \quad \begin{bmatrix} i_{\alpha ref}(t) \\ i_{\beta ref}(t) \end{bmatrix} = \begin{bmatrix} (I_{\alpha\beta} + \Delta I_{\alpha\beta}) \cos(\omega t) \\ (I_{\alpha\beta} + \Delta I_{\alpha\beta}) \sin(\omega t) \end{bmatrix}$$

- Evaluating the power balance equation (1) with  $i_\alpha = (I_{\alpha\beta} + \Delta I_{\alpha\beta}) \cos(\omega t)$ ,  $i_\beta = (I_{\alpha\beta} + \Delta I_{\alpha\beta}) \sin(\omega t)$ ,  $v_+ = y_{ref} - \Delta v_+$  et  $v_- = 0 - \Delta v_-$ , taking into account (2) and retaining first order terms, we get the **transfer function**

$$G_{v_+}(s) = \frac{\Delta v_+}{\Delta I_{\alpha\beta}} = \frac{(V_{\alpha\beta} - 2r_L I_{\alpha\beta}) R_{eq}}{y_{ref} \left(1 + \frac{R_{eq} C}{2} s\right)} := \frac{K}{1 + T s}$$

- Then, the open-loop transfer function is  $\frac{K K_I}{s(1 + T s)}$ . For example, a phase margin of  $\frac{\pi}{3}$  is obtained in closed-loop for  $K_I = \frac{2}{3KT} = \frac{4y_{ref}}{3R_{eq}^2 C (V_{\alpha\beta} - 2r_L I_{\alpha\beta})}$ .

## References

- [1] M. Dore, Y. Ariba, and G. Garcia, "Observer-based switched control of the three level neutral point clamped rectifier," in *2023 62nd IEEE Conference on Decision and Control (CDC)*, pp. 3580–3585, 2023.
- [2] G. Deaecto, J. Geromel, F. Saldanha Garcia, and J. Pomilio, "Switched affine systems control design with application to DC-DC converters," *Control Theory & Applications, IET*, vol. 4, pp. 1201–1210, Aug. 2010.

## Power balance equation

- Converters are power electronics systems using switches and storage elements to modulate electrical energy
- Energy exchanges and system trajectories are constrained by the **power balance equation**

$$P_{sources} - P_{stored} - P_{losses} - P_{load} = 0$$

- For inductors and capacitors, the stored power is

$$P_L = L \dot{i}_L i_L \quad P_C = C \dot{v}_C v_C$$

- As the state variables are usually inductor currents and capacitor voltages, the power balance equation **can be derived from the state-space model**.
- The power balance equation is a **powerful tool for reference design and control**

## Reference design

- Using the state-space model of the converter, we retrieve its power balance equation:

$$L i_\alpha \dot{i}_\alpha + L i_\beta \dot{i}_\beta + \frac{C}{2} (v_+ \dot{v}_+ + v_- \dot{v}_-) = e_\alpha i_\alpha + e_\beta i_\beta - r_L (i_\alpha^2 + i_\beta^2) - \frac{v_-^2}{2r_C} - \frac{v_+^2}{2R_{eq}} \quad (1)$$

- Control objectives :

- Ensuring an unit power factor (voltages/currents in phase)
- Stabilize  $y$  to a reference  $y_{ref}$
- Balanced capacitors voltages

$$x_{ref}(t) = \begin{bmatrix} I_{\alpha\beta} \cos(\omega t) \\ I_{\alpha\beta} \sin(\omega t) \\ y_{ref} \\ 0 \end{bmatrix}$$

- Evaluating (1) with  $x = x_{ref}$  for  $y_{ref} < V_{\alpha\beta} \sqrt{\frac{R_{eq}}{2r_L}}$ , we get

$$0 = -r_L I_{\alpha\beta}^2 + V_{\alpha\beta} I_{\alpha\beta} - \frac{y_{ref}^2}{2R_{eq}} \quad (2)$$

$$\Leftrightarrow I_{\alpha\beta} = \frac{V_{\alpha\beta}}{2r_L} \left( 1 \pm \sqrt{1 - \frac{2r_L y_{ref}^2}{V_{\alpha\beta}^2 R_{eq}}} \right)$$

## Simulation

Parameter	Value	Parameter	Value
$L$	15 mH	$r_L$	0.4 $\Omega$
$C$	1500 $\mu$ F	$r_C$	20 k $\Omega$
$E$	72 V	$R$	30 $\Omega$
$y_{ref}$	150 V	$\omega$	100 $\pi$ rad/s

