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Alejandro Arceo, Rodrigo Gutierrez-Garza, Jorge de Jesus Lozoya-Santos, Mauricio A. Ramirez-Moreno, Luis C. Felix-Herran, et al.. Compensators Design using Laguerre Polynomials Applied to a Real Steer-by-Wire System of Autonomous Vehicle. CDC 2024 - 63rd IEEE Conference on Decision and Control, IEEE, Dec 2024, Milan, Italy. hal-04735234

HAL Id: hal-04735234

<https://hal.science/hal-04735234v1>

Submitted on 14 Oct 2024

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Compensators Design using Laguerre Polynomials Applied to a Real Steer-by-Wire System of Autonomous Vehicle

Alejandro Arceo, Rodrigo Gutierrez-Garza, Jorge de J. Lozoya-Santos, Mauricio A. Ramirez-Moreno, Luis C. Felix-Herran, Olivier Sename, and Juan C. Tudon-Martinez*

Abstract—This paper provides two main contributions to the field of control systems design. Firstly, an algorithm is proposed for computing digital compensators aimed at stabilizing a specific class of linear time-invariant discrete-time systems. The method leverages pole placement to devise a control system that uses a digital compensator within a closed-loop configuration. The poles crucial to this design are determined as roots of sequences of Schur polynomials, constructed via a linear combination of Laguerre polynomials orthogonal to a modified version of the classical Laguerre measure. This design yields an infinite family of compensators capable of stabilizing strictly proper plants by manipulating parameters within the coefficients of orthogonal polynomials. Secondly, the effectiveness of this approach is demonstrated through configuring a compensator for a real Steer-by-Wire platform using an industrial and embedded vehicle control unit with the CAN communication protocol. Additionally, experimental tests show improved performance compared to traditional tuning for PID controllers. The Error-to-Signal Ratio index is reduced by the novel algorithm by up to 35.5% and 65.6% in contrast with the PID controller against DLC and Fishhook procedures.

I. INTRODUCTION

Every year, road traffic accidents are a prominent cause of death, claiming millions of lives due to factors such as reckless driving and speeding. The 2023 Global Status Report on Road Safety indicates a marginal decrease in the yearly count of road traffic fatalities, now standing at 1.19 million [1]. However, many countries persist in designing mobility systems primarily for vehicles rather than prioritizing the safety and accessibility of vulnerable road users.

An alternative is to transform vehicle control by substituting traditional mechanical links with electronic or electromechanical systems, i.e., to implement Drive-By-Wire (DBW) technology [2]. These systems encompass elements such as Electronic Throttle Control, Steer-By-Wire (SBW), Brake-by-Wire, and Shift-by-Wire. DBW offers several benefits, including reduced complexity, customizable inputs, greater flexibility in interior design, reduced noise, and improved safety features. However, the adoption of this technology results in an increase in energy consumption due to the incorporation of additional actuators.

This work was supported in part by the CampusCity Future Clean Mobility Initiative at Tecnológico de Monterrey and CONHACYT.

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As SBW systems evolve, advanced control strategies are becoming increasingly necessary. Linear and nonlinear control methods are tailored to handle the complex dynamics inherent in SBW devices, whereas adaptive control techniques enable real-time adjustments to varying driving conditions and uncertainties. For example, nonlinear model predictive control can assess model uncertainty for robust stability analysis [3]. Moreover, nonlinear adaptive sliding mode approaches offer adaptive sliding gains, reducing the need for precise uncertainty limits and minimizing computational complexity [4]. Furthermore, neural networks streamline computational processes in fuzzy logic, and adaptive controllers effectively address the nonlinear dynamics of SBW systems [5]. One of the most important issues to be accomplished in the controller design task is the feasibility of being implemented in a real-time embedded control system.

In this sense, Vehicle Control Units (VCUs) are crucial components of modern electric and electronic systems, functioning as the central nervous system for automotive electronic control. The VCU is an embedded system, i.e. a compact computing device designed for specific tasks and power-dedicated functions within larger mechanical or electronic systems in the vehicle. VCU coordinates various functions, including engine management, transmission control, braking systems, steering assistance, and climate control. Therefore, these devices are used for vehicle safety and reliability where their primary objective is to mitigate possible accidents by avoiding collisions with obstacles, whereas optimizing the vehicle's overall performance [6].

In the framework of stochastic processes, a method for identifying continuous-time Wiener models is introduced, employing a Laguerre expansion to describe linear dynamics and approximating static nonlinearity with an inverse polynomial function [7]. Moving into the PID domain, a computer-oriented algebraic approach is proposed for designing digital PID controllers, utilizing discrete Orthogonal Polynomials (OPs) [8]. In the transition to signal processing, the use of discrete functions based on Laguerre polynomials is advocated for the design of the controller on constrained servo turntables [9]. In electric systems, Laguerre polynomial-based control techniques are devised to regulate single-phase grids connected to voltage source converters [10] and to design a centralised linear model predictive controller [11]. Laguerre polynomials, along with other orthogonal polynomials, have been utilized in neural operators across various machine learning applications, as mentioned in [12], [13]. In the field of medicine, an adaptive backstepping controller is

engineered for multi-input, multi-output cancer immunotherapy systems, focusing on the approximation property of Laguerre polynomials [14]. Shifting the focus to automotive applications, an adaptive functional-link-based neural fuzzy control system is developed, employing orthogonal Hermite polynomial-based nonlinear combinations to approximate an ideal complementary Sliding-Mode Controller (SMC) [15]. On the basis of a state-of-the-art review and to the best of the authors' knowledge, the latter is the only work concerning the utilization of OPs in automotive applications.

However, certain kinds of OPs, particularly Laguerre polynomials, can be used to explicitly compute families of Hurwitz polynomials [16]. Taking into account that the weakness of the pole placement technique lies precisely in the location of the poles of the control system, the initial focus of this work is to construct a new family of Schur polynomials that is used to solve the problem of assigning poles and stabilizing a specific class of Linear Time-Invariant (LTI) discrete-time plants. The presented methodology incorporates parameters into the coefficients of the compensators that can be manipulated to improve the performance of linear control systems, which is applied to a real-world platform. In this context, the contributions to this paper are:

- Development of a new algorithm for computing digital compensators to stabilize a class of LTI discrete-time systems. This method employs pole placement within a closed-loop configuration, utilizing Schur polynomials derived from Hurwitz polynomials. These Hurwitz polynomials are constructed as a linear combination of Laguerre polynomials, which are orthogonal with respect to a modified Laguerre measure.
- Formulation of a design approach to generate an infinite family of compensators capable of stabilizing strictly proper plants. This is achieved by manipulating the parameters within the coefficients of the OPs considered.
- Implementation of the proposed method through the practical scenario of configuring a compensator for a real SBW platform using an industrial and embedded VCU via the CAN communication protocol. The efficacy of the approach is demonstrated by comparing its performance with a traditional tuning method of a PID controller, showing superior results.

The manuscript is structured as follows. Section II provides an overview of the role of a VCU in the automotive industry and the framework of the experimental setup. The implementation of the technique is described in Section III. Theoretical and experimental results are presented in Sections IV and V. Finally, the main discussion and conclusions can be found in Sections VI and VII, respectively.

II. EXPERIMENTAL PLATFORM

This section provides an overview of the VCU's characteristics and communication setup, including the compilation and flashing process using designated software. In addition, it outlines the experimental platform comprising a SBW system and its subsystems tailored for steering.

A. Ecotron® Software and Hardware

Ecotron® specializes in the development of controllers for electric and autonomous vehicles, offering three key software components: EcoCoder, an auto-code generation library integrated with MATLAB®/Simulink; EcoFlash, for flashing VCU executable files generated in EcoCoder; and EcoCal, a calibration tool for managing batteries or motors.

The VCU EV2106B01 features a microprocessor with 121 pins and 3 Controller Area Network (CAN) bus channels, operating within a voltage range of 9 to 16 V. Communication between the VCU and computer involves using a Peak CAN cable. The control code is compiled within Simulink using the EcoCoder library. This process yields two files: a “.mot” file for flashing using EcoFlash via CAN and a “.a2l” file for accessing program variables in real-time using EcoCal, with the possibility of reading, deploying, and exporting data.

B. SBW Platform

The experimental platform is composed of a SBW system as shown in Fig. 1. This type of systems is used in vehicles for allowing the steering system to be mechanically separated from the steering wheel. Its main application is to use it in autonomous vehicles, since the steering system is also capable of autonomously driving the vehicle using only electrical signals instead of needing a mechanical input.

The platform has three main subsystems. The first subsystem consists of the Ecotron® EV2106B01 VCU, which will perform all the control operations. The VCU, along with a PWM to analogous and relay modules, is able to control the second subsystem, i.e., the velocity driver module for the motor. Lastly, the third subsystem is composed of the steering column with a JTEKT JJ001 12VDC motor and a precision potentiometer for detecting the steering angle.

III. MATHEMATICAL PRELIMINARIES

A LTI discrete-time system is a mathematical representation of various phenomena, characterized by a linear and time-invariant difference equation, which refers to events occurring at specific time intervals, unlike the continuous-time domain. In discrete-time control, a transfer function

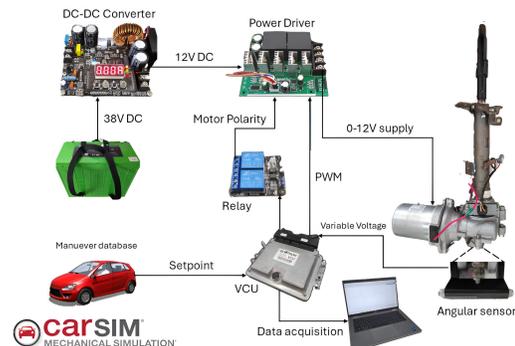


Fig. 1. Experimental closed-loop steer-by-wire system.

representing the physical phenomena (or plant) is given by

$$\mathcal{P}(z) = \frac{N(z)}{D(z)}, \quad (1)$$

where $N(z) = \sum_{k=0}^n \mathbf{n}_k z^k$ and $D(z) = \sum_{k=0}^d \mathbf{d}_k z^k$ are polynomials with real coefficients. $\mathcal{P}(z)$ is said to be a proper and a strictly proper plant of invariant degree d if $0 \leq n \leq d$ and $0 \leq n < d$, respectively. In both cases, it is required $\mathbf{d}_d \neq 0$. Furthermore, the output signal of closed-loop control system of $\mathcal{P}(z)$ associated with the controller $\mathcal{C}(z)$, and denoted by $\mathcal{Y}(z)$, is defined by

$$\mathcal{Y}(z) = \frac{\mathcal{C}(z)\mathcal{P}(z)}{1 + \mathcal{C}(z)\mathcal{P}(z)} \mathcal{X}(z), \quad (2)$$

where $\mathcal{C}(z)$ and $\mathcal{X}(z)$, the input signal, are rational functions of two polynomials with real coefficients.

A. Pole Placement Methodology

Pole Placement is a control strategy used in the design of control systems. It involves placing the poles of the closed-loop transfer function in specific locations in the complex plane to achieve desired performance characteristics like stability, dynamic response, and damping [17]. In particular, $\mathcal{Y}(z)/\mathcal{X}(z)$ is asymptotically stable if its denominator polynomial has roots within the unit circle, which is known as a Schur polynomial [18].

Taking into account (2), suppose $\mathcal{P}(z)$ is a strictly proper plant as in (1) of invariant degree $d \geq 1$ such that $N(z)$ and $D(z)$ are polynomials with real coefficients of degrees n and d , respectively, having no common roots. For $m \geq 0$, a proper compensator is represented by

$$\mathcal{C}(z) = \frac{B(z)}{A(z)}, \quad (3)$$

where $A(z) = \sum_{k=0}^m \mathbf{a}_k z^k$ and $B(z) = \sum_{k=0}^m \mathbf{b}_k z^k$ are polynomials with real coefficients. So (2) becomes

$$\mathcal{Y}(z) = \frac{B(z)N(z)}{A(z)D(z) + B(z)N(z)} \mathcal{X}(z). \quad (4)$$

If $S(z) = \sum_{k=0}^{d+m} \mathbf{s}_k z^k$ denotes a Schur polynomial of degree $d+m$ with $m \geq d-1$ and real coefficients, then the pole placement technique aims to find the coefficients of $A(z)$ and $B(z)$ such that

$$S(z) = A(z)D(z) + B(z)N(z) \quad (5)$$

by solving the following linear equation (see [17, Th. 9.2])

$$\begin{pmatrix} \mathbf{a}_0 & \mathbf{b}_0 & \cdots & \mathbf{a}_m & \mathbf{b}_m \end{pmatrix} = \begin{pmatrix} \mathbf{s}_0 & \mathbf{s}_1 & \cdots & \mathbf{s}_{m+d} \end{pmatrix} M_d^\dagger \quad (6)$$

with

$$M_d = \begin{pmatrix} \mathbf{d}_0 & \mathbf{d}_1 & \cdots & \mathbf{d}_d & 0 & \cdots & 0 \\ \mathbf{n}_0 & \mathbf{n}_1 & \cdots & \mathbf{n}_d & 0 & \cdots & 0 \\ 0 & \mathbf{d}_0 & \cdots & \mathbf{d}_{d-1} & \mathbf{d}_d & \cdots & 0 \\ 0 & \mathbf{n}_0 & \cdots & \mathbf{n}_{d-1} & \mathbf{n}_d & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & \mathbf{d}_0 & \cdots & \mathbf{d}_d \\ 0 & 0 & \cdots & 0 & \mathbf{n}_0 & \cdots & \mathbf{n}_d \end{pmatrix},$$

where, for any matrix M of any dimension, $M^\dagger = M^T(MM^T)^{-1}$ is the right Moore–Penrose pseudo-inverse of M , which always exists if and only if the rows of M are linearly independent. Here, M^T is the transpose of M .

Notice that the existence of M_d^\dagger is guaranteed by $d > n$ and $\mathcal{P}(z)$ is a reduced polynomial fraction with invariant degree. Moreover, if the matrix M is square, the linear independence of its rows implies that M is an invertible matrix, and $M^{-1} = M^\dagger$.

B. Relation between Stable and Orthogonal Polynomials

Once the conditions of existence for the solution of (5) are given, the remainder of this Section will be devoted to providing some class of Schur polynomials. It is noteworthy that this approach aims to incorporate parameters (associated with the desired closed-loop performance) into the coefficients of Schur polynomials that are constructed using OPs associated with modified classical measures.

As well as Schur polynomials play a pivotal role in characterizing the stability of discrete-time linear systems, Hurwitz polynomials characterise the stability of continuous-time linear systems. Generally, both families are called stable polynomials. As a consequence, determining the roots of polynomials holds crucial importance in various applications in control theory [19].

On the other hand, OPs are a special class of polynomials that satisfy a particular orthogonality condition on the real line \mathbb{R} . Suppose $\omega(s)$ is a positive weight function defined on an interval I of \mathbb{R} . A sequence $\{P_n(s)\}_{n \geq 0}$ of polynomials such that $P_n(s)$ has real coefficients and degree n is the sequence of OPs on \mathbb{R} associated with $\omega(s)$ if for $m \geq 0$, there exists $K_m > 0$ such that (see [20])

$$\int_I P_m(s) P_n(s) \omega(s) ds = K_m \delta_{m,n}, \quad n \geq 0.$$

It is well-known that any Schur polynomial is an OP on the unit circle, and vice versa [21]. However, it is not evident that any Hurwitz polynomial can be decomposed into two OPs on \mathbb{R} of consecutive degree [22].

A fundamental challenge in control system design is devising a compensator to improve the performance of a closed-loop system. This process entails identifying the roots of the characteristic polynomial within a specific part of the stability region [23]. Since

$$S(z) = (z-1)^q H\left(\frac{z+1}{z-1}\right) \quad (7)$$

is a Schur polynomial provided that $H(\cdot)$ is a Hurwitz polynomial with degree $q \geq 1$ [24], it is possible construct stable polynomials using OPs. Specifically, we will be dealing with linear combinations of modified Laguerre polynomials due to the dynamic of their roots with respect to parameters can be explicitly described by [23, Eq. (10)]. Furthermore, this family of OPs has been utilized to propose a stabilization method for interval plants with uncertain time-delay, showing improvements in the performance of the controlled system [25, Algo. 1]. Following the notation of [16], the modified

Laguerre polynomials, $\{L_n^{\alpha,t}(s)\}_{n \geq 0}$, are orthogonal with respect to $\omega(s,t) = s^\alpha e^{-s/t}$ when $\alpha > -1$ and $t > 0$. Notice that $\omega(s,1)$ is the classical Laguerre weight.

The algebraic and analytic properties of OP have been extensively examined in the literature due to their relevance in various fields, including approximation theory, signal processing, quantum mechanics, and stochastic processes, among others. For a comprehensive exploration of the general properties of OP on \mathbb{R} , refer to [26].

IV. COMPENSATORS DESIGN BASED ON LAGUERRE POLYNOMIALS

This section presents the proposed method for stabilizing LTI discrete-time systems by utilizing modified Laguerre polynomials. The goal is to create compensators that ensure stability in closed-loop configurations. The proposed approach capitalizes on the robust stability of stable polynomials derived from these techniques across various parameter values. This, alongside pole placement technique, forms the basis for the results presented in this work.

Given $d \geq 1$ and a strictly proper plant $\mathcal{P}(z)$ as in (1), where $N(z)$ and $D(z)$ are polynomials with no common roots, (4) implies that the characteristic polynomial of the closed-loop control system is

$$S(z) = A(z)D(z) + B(z)N(z).$$

In the remainder of this section, (7) is used to construct $S(z)$ using linear combinations of modified Laguerre polynomials in order to present the obtained results.

For $n \geq 1$, $\xi \geq 0$, $\alpha > -1$, $t > 0$, $\lambda_1 > 0$, and $\lambda_2, \lambda_3, \lambda_4 \in \mathbb{R}$, set

$$\ell_n^{\alpha,t}(s; \lambda_1, \lambda_2) = \lambda_1 L_n^{\alpha,t}(s) + \lambda_2 L_{n-1}^{\alpha,t}(s).$$

If $\lambda_1 \lambda_4 - \lambda_3 \lambda_2 \neq 0$, then the roots of $\ell_n^{\alpha,t}(s; \lambda_1, \lambda_2)$ and $\ell_n^{\alpha,t}(s; \lambda_3, \lambda_4)$ are real, simple, and interlaced. By the Hermite-Biehler theorem,

$$H_{2n}(s; \boldsymbol{\lambda}) = (-1)^n \ell_n^{\alpha,t}(-s + \xi)^2; \lambda_1, \lambda_2 + (-1)^{n-1} \times \text{sgn}[\lambda_3] (s + \xi) \ell_n^{\alpha,t}(-s + \xi)^2; 0, \lambda_4 \quad (8)$$

and

$$H_{2n+1}(s; \boldsymbol{\lambda}) = (-1)^n \ell_n^{\alpha,t}(-s + \xi)^2; \lambda_1, \lambda_2 + (-1)^n \times \text{sgn}[\lambda_3] (s + \xi) \ell_n^{\alpha,t}(-s + \xi)^2; \lambda_3, \lambda_4 \quad (9)$$

are Hurwitz polynomials of degree $2n$ and $2n + 1$, respectively, for all values of $\lambda_1, \lambda_2, \lambda_3$, and λ_4 that satisfy certain inequalities (see [23]). Here, $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \lambda_3, \lambda_4, \xi, \alpha, t)$, and $\text{sgn}[x]$ denotes the sign function of the real number x . Notice that ξ is simply a leftward shift of the respective roots.

As a straightforward consequence,

Proposition 1: For $n \geq 1$, $\xi \geq 0$, $\alpha > -1$, $t > 0$, and $\lambda_1 > 0$,

$$S_{2n}(z) = (z - 1)^{2n} H_{2n} \left(\frac{z + 1}{z - 1}; \boldsymbol{\lambda} \right) \quad (10)$$

is a Schur polynomial of degree $2n$ if and only if $\lambda_3 = 0$, $\lambda_4 \neq 0$, and

$$\lambda_2 < -\lambda_1 \frac{L_n^{\alpha,t}(\xi)}{L_{n-1}^{\alpha,t}(\xi)}.$$

Also,

$$S_{2n+1}(z) = (z - 1)^{2n+1} H_{2n+1} \left(\frac{z + 1}{z - 1}; \boldsymbol{\lambda} \right) \quad (11)$$

is a Schur polynomial of degree $2n + 1$ if and only if $\lambda_2 < -\lambda_1 L_n^{\alpha,t}(\xi)/L_{n-1}^{\alpha,t}(\xi)$,

- $\lambda_3 > 0$, and

$$\lambda_4 < \min \left\{ \frac{\lambda_3 \lambda_2}{\lambda_1}, -\lambda_3 \frac{L_n^{\alpha,t}(\xi)}{L_{n-1}^{\alpha,t}(\xi)} \right\},$$

or

- $\lambda_3 < 0$, and

$$\lambda_4 < \max \left\{ \frac{\lambda_3 \lambda_2}{\lambda_1}, -\lambda_3 \frac{L_n^{\alpha,t}(\xi)}{L_{n-1}^{\alpha,t}(\xi)} \right\}.$$

Proof: Since the Möbius function $s = (z + 1)/(z - 1)$ is a conformal mapping between the open unit disk and the open left half-plane, that (10) and (11) are Schur polynomials follows from [27, Propositions 6 and 7]. ■

Then, the next formulation is established:

Theorem 1: Consider (2) and assume that $\mathcal{P}(z)$ is a strictly proper plant with invariant degree d as in (1) such that $N(z)$ and $D(z)$ are polynomials with real coefficients having no common roots. For $m \geq d - 1$, $\mathcal{C}(z)$ as in (3) stabilizes to $\mathcal{P}(z)$ in closed-loop if $A(z)D(z) + B(z)N(z) = S_{d+m}(z)$, where $S_{d+m}(z)$ is computed using Proposition 1. The coefficients of $A(z)$ and $B(z)$ are computed using (6).

Proof: From Proposition 1, $S_{d+m}(z)$ is a Schur polynomial. Thus, there exist two real polynomials, say $A(z)$ and $B(z)$, with degree m and almost degree m , respectively, such that $S_{d+m}(z) = A(z)D(z) + B(z)N(z)$, or equivalently, (6) is satisfied. Taking $\mathcal{C}(z) = B(z)/A(z)$, the proof is complete. ■

The principal significance of Theorem 1 is to explicitly compute $\mathcal{C}(z)$ that stabilizes any given strictly proper plant. However, the coefficients of $A(z)$ and $B(z)$ also inherit the parameters of the coefficients of the OPs involved in the construction of Schur polynomials. Thus, the following methodology computes a family of proper compensators that stabilizes in closed-loop to $\mathcal{P}(z)$. Suppose $\mathcal{P}(z)$ is a strictly proper plant as in (1) of invariant degree $d \geq 1$ such that $N(z)$ and $D(z)$ are real polynomials of degrees n and d , respectively, having no common roots. Then,

Algorithm 1: Proper compensator design for strictly proper plants via Prop. 1.

Input: Strictly proper plant $\mathcal{P}(z)$ as in (1) of invariant degree $d \geq 1$ with $N(z)$ and $D(z)$ with no common roots; sampling period $T > 0$; number of iterations I_0 .

Output: $\mathcal{C}(z)$.

Initialisation:

- 1: Choose $m \in \mathbb{N}$ such that $m \geq d - 1$.

- 2: Compute $S(z) = \frac{1}{h}(z-1)^{d+m} H_{d+m}((z+1)/(z-1); \boldsymbol{\lambda})$ using Proposition 1, h being the leading coefficient of $(z-1)^{d+m} H_{d+m}((z+1)/(z-1); \boldsymbol{\lambda})$.
- 3: $\lambda_5 \leftarrow \xi$, $\lambda_6 \leftarrow \alpha$, and $\lambda_7 \leftarrow t$.
- 4: Assign values to all uncertain parameters involved in the coefficients of $S(z)$ according to Proposition 1, except for $\lambda_{k_1}, \lambda_{k_2}, \dots, \lambda_{k_p}$, where $k_1, k_2, \dots, k_p \in \{1, 2, \dots, 7\}$ and $1 \leq p \leq 7$.
- 5: Solve (6) to obtain the coefficients of $A(z)$ and $B(z)$ as functions of $\lambda_{k_1}, \dots, \lambda_{k_p}$, and set $\mathcal{C}(z) = B(z)/A(z)$.
- 6: **for** $0 \leq n \leq I_0$ and some values for $\lambda_{k_1}, \dots, \lambda_{k_p}$, **do**
- 7: simulate $y[n] = \mathcal{Z}^{-1}(\mathcal{Y}(z))$ for $\lambda_{k_1}, \dots, \lambda_{k_p}$, where $\mathcal{Y}(z)$ as in (4), $\mathcal{X}(z)$ is the unit step function, and $\mathcal{Z}^{-1}(\cdot)$ denotes the inverse Z -transform.
- 8: **end for**
- 9: **return** $\mathcal{C}(z)$ for the selected values of $\lambda_{k_1}, \dots, \lambda_{k_p}$.

V. IDENTIFICATION AND CONTROL

In this section, Algorithm 1, developed in this study, is used to control the discrete-time steering angular position, denoted as $p[n] = \mathcal{Z}^{-1}(\mathcal{P}(z))$ and measured in degrees, of the JTEKT JJ001 12VDC motor shown in Fig. 1. Consequently, the objectives are to compute and apply the computed compensator to $\mathcal{P}(z)$, with initial condition $p[0] = 0$. By conducting simulations and experiments on the SBW platform depicted in Fig. 1, the validity of the proposed methodology through the design and implementation of a digital compensator is confirmed.

A. Simulation Results

In order to obtain the discrete transfer function of the 12VDC motor, $\mathcal{P}(z)$, raw data is collected from an open-loop configuration by direct excitation of the system and obtaining the transient response from one steady state of angular velocity to another by the variation of the PWM signal delivered to the motor, avoiding sensor saturation. The data collection was done using EcoCAL with a sample time of 0.02 s. The process involves interpreting the open-loop system code and flashing procedure in Simulink and EcoFlash and this sampling interval guarantees the accurate representation of the system's dynamics by using a 1:10 ratio to the rise time [28]. After collecting the data, it is exported to a ".csv" file and filtered to remove noise. Using the System Identification Toolbox from MATLAB® with the filtered data, the following model was obtained

$$\mathcal{P}(z) \approx \frac{0.0369z - 0.03615}{z^2 - 1.99292z + 0.99293},$$

which is a strictly proper plant and exhibits a fit to estimated data of 95.26%. Hence, Fig. 2 depicts the comparison between the model output based on input data and the actual output behavior of $p[n]$. The experimental system was identified as non-linear. Hence, for simplicity purposes, the model has lesser adjustment to the real dataset at certain operative conditions, such as higher speed.

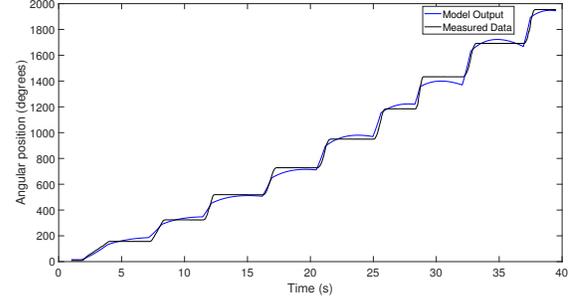


Fig. 2. Comparison between the linear model with the real acquired data.

In Step 1 of Algorithm 1, $m = 1$ is selected for simplicity since $d = 2$. Step 2 now gives

$$S(z) = \frac{1}{h}(z-1)^3 H_3((z+1)/(z-1); \boldsymbol{\lambda})$$

with

$$h = \lambda_1((\alpha+1)t + (\xi+1)^2) - \lambda_2 + (\xi+1)sgn[\lambda_3](\lambda_3((\alpha+1)t + (\xi+1)^2) - \lambda_4)$$

and

$$H_3(s; \boldsymbol{\lambda}) = |\lambda_3|s^3 + (3|\lambda_3|\xi + |\lambda_1|)s^2 + ((\alpha+1)|\lambda_3|t + 3|\lambda_3|\xi^2 + 2|\lambda_1|\xi - \lambda_4sgn[\lambda_3])s + |\lambda_3|\xi^3 + |\lambda_1|\xi^2 + (\alpha+1)|\lambda_3|\xi t - \xi\lambda_4sgn[\lambda_3] - \lambda_2 + (\alpha+1)|\lambda_1|t.$$

Following Steps 3, 4, and 5, we obtain, for $\lambda_5 = \xi \geq 0$,

$$A(z) = z - \frac{1.01\lambda_5^3 - 5.43841\lambda_5^2 + 864.201\lambda_5 - 78631.7}{\frac{1}{2}h}$$

and

$$B(z) = \frac{0.08027\lambda_5^3 - 80.3048\lambda_5^2 + 23824.6\lambda_5 - 766313}{\frac{1}{2}h}z - \frac{0.08017\lambda_5^3 - 80.2166\lambda_5^2 + 23816\lambda_5 - 776734}{\frac{1}{2}h},$$

with $\lambda_1 = 1$, $\lambda_2 = 100000$, $\lambda_3 = 2$, $\lambda_4 = 10$, $\lambda_6 = 1$, $\lambda_7 = 1/20$, and $h = 2\xi^3 + 7\xi^2 - \frac{9\xi}{5} - \frac{1000067}{10}$. Next, the simulated unit-step responses are shown in Fig. 3 when $50 \leq \lambda_5 \leq 120$ with a step size of 14 and $T = 0.02$ s. Finally, if $\lambda_5 = 50$,

$$\mathcal{C}(z) = \frac{2.79789z - 2.67078}{z - 0.922767}. \quad (12)$$

B. Experimental Results

1) *Scenarios*: During the experimentation, two main test configurations were performed, the Double Line Change (DLC) and a Fishhook procedure. The first one, defined by ISO 3888-2 as a maneuver, allows to test a vehicle's performance in avoiding obstacles. The Fishhook procedure is used for vehicle stability testing given rapid angle changes on the steering wheel of a vehicle. Using Carsim®, the desired angular position is obtained by importing the steering wheel angle dataset reported from a simulated test. Both

maneuvers datasets are obtained using a B-class Hatchback vehicle. Next, the controller receives as set-point signal the calculated steering wheel angle and executes it on the real SBW system, emulating on the real-platform the same track as on Carsim® (see Fig. 4). The velocity settings examined in the tests include speeds of 45 km/h, 55 km/h, 65 km/h, 75 km/h, and 85 km/h for DLC, and 80 km/h for Fishhook. All experiments are based on a sample time of 0.02 s.

2) *Data Use:* The CAN settings within the EcoFlash program are as follows. A Peak CAN device is utilized for communication between the PC and the VCU, operating at a baud rate of 500 kbps and employing the CCP protocol.

3) *Results Analysis:* The experimental performance control, the error, and the control signal measured in volts (V) for the angular steering position in a DLC test are shown in Fig. 5. When a sudden change occurs, the error deviates from zero, indicating a shift in the controlled angular position from the desired position. Consequently, the compensator responds by amplifying the voltage supplied to the motor in an attempt to compensate for this change.

Analogously, Fig. 6 illustrates the experimental performance control, error, and control signal for the $y[n]$ during the Fishhook procedure. Also, any abrupt changes lead to the error straying from zero, indicating a deviation in the controlled angular position from the desired one. Subsequently, the compensator reacts by increasing the voltage supplied to the motor to counteract this alteration.

In order to obtain the efficacy of the compensator, the Error-to-Signal Ratio (ESR) is computed as described by [29]. This metric ranges between 0 and 1, where 0 indicates a “trivial” model capable only of predicting average output values, while 1 signifies a “perfect” model that accurately predicts exact output values. Since a displacement between the desired and controlled position of the steering wheel due to the delay in actuation is noticeable even when the path appears to be correct, two different ESR calculations are performed. The first uses the raw data obtained directly from the VCU, and the Adjusted ESR (AESR) uses an adjusted data set that eliminates the delay and juxtaposes both signals. The calculation of these indexes is shown on Table I, where

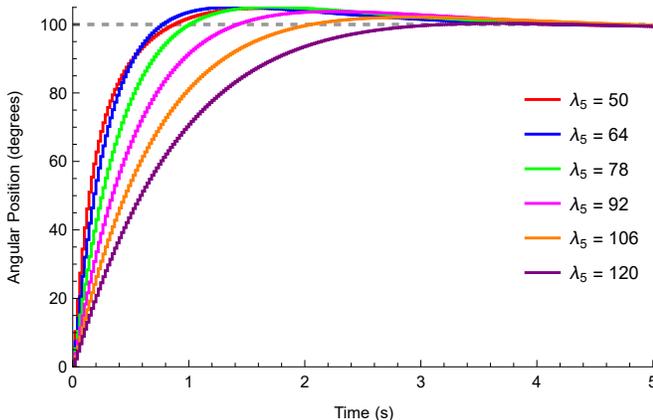


Fig. 3. The simulated unit-step response of $y[n]$ for some values of λ_5 .

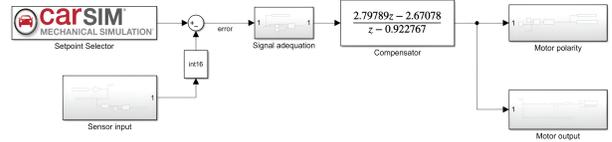


Fig. 4. Simulink block diagram of the closed-loop control system.

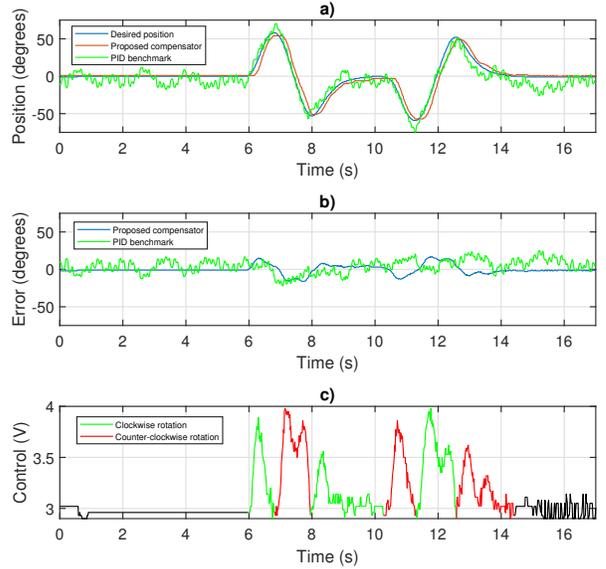


Fig. 5. Control systems performance for a DLC test at 55 km/h: a) behavior of the proposed control system compared to the PID, b) error signal in both control systems, c) control signal considering the proposed compensator.

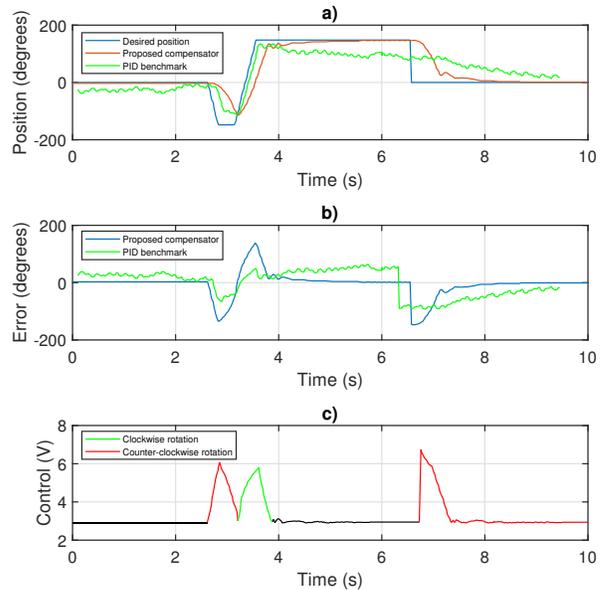


Fig. 6. Control systems performance for a Fishhook test at 80 km/h: a) behavior of the proposed control system compared to the PID, b) error signal in both control systems, c) control signal considering the proposed compensator.

the AESR index provides a better perspective of the increased value of the ESR metric for higher speed configurations.

TABLE I
EXPERIMENTAL ERROR-TO-SIGNAL RATING

Test	Configuration	Proposed controller		PID controller	
		ESR	AESR	ESR	AESR
DLC	45 km/h	0.05191	0.005775	0.17450	0.17363
	55 km/h	0.08255	0.007756	0.23268	0.22528
	65 km/h	0.11402	0.011714	0.51347	0.50554
	75 km/h	0.26760	0.025041	0.78608	0.73924
	85 km/h	0.47875	0.098925	0.41330	0.36490
Fishhook	80 km/h	0.31746	0.081935	0.48390	0.46544

Finally, Figs. 7 and 8 show a comparison between the control performances when the delay is reduced in the DLC and Fishhook tests, respectively.

VI. DISCUSSION

The proposed method offers a notable advantage in the substantial flexibility it provides for selecting the number of parameters within the components of λ , which directly impacts the selection of $H_{d+m}(s; \lambda)$. Increasing the number of parameters within the coefficients of Schur polynomials offers enhanced flexibility in positioning their roots, a favorable trait for designing control systems. Additionally, this work focused the attention on cases where m is small, indicating a lower degree for the compensator's denominator polynomial, which is good for implementation. Nevertheless, a disadvantage of this methodology is the challenge of determining the optimal value for m to meet specific performance criteria effectively.

The experimental results expose an efficient approach to the desired output, resulting in ESR values below 0.1 for the adjusted data set. Comparing the performance of (12) against an ideal PID discrete controller, the proposed methodology decreases the metric ESR by 29.75%, 35.48%, 22.2%, and 34.04% when a DLC test at 45 km/h, 55 km/h, 65 km/h, and 75 km/h is considered, respectively. The PID benchmark controller was tuned using the Ziegler-Nichols method for which the proportional gain and integral and derivative time constants are 5.4340, 0.8833, and 7.95, respectively. However, (12) increases this index by 15.84% when a DLC test is implemented at 85 km/h. On the other hand, (12) decreases the ESR index by 65.6% when the

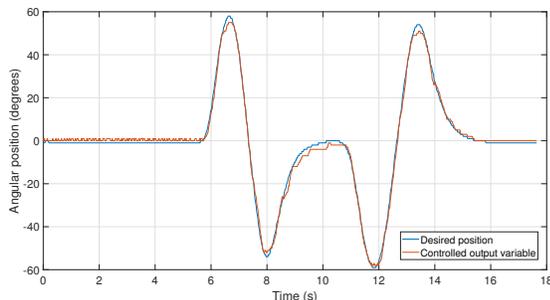


Fig. 7. DLC test at 45 km/h with displacement reduction.

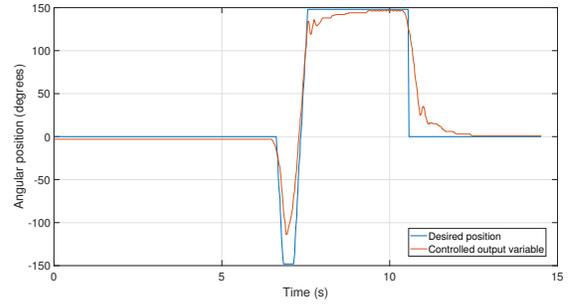


Fig. 8. Fishhook test at 80 km/h with displacement reduction.

Fishhook procedure is considered. For the PID control, the obtained ESR and AESR indexes are similar, in contrast with the proposed compensator. This comparison may be further appreciated in Fig. 9, clearly demonstrating that the proposed compensator outperforms the traditional PID controller.

Regarding the complexity of the methodology applied to the steer-by-wire technology reported in the literature, this research and [30], both use linear control methods. The approach based on [31] is Linear Parameter-Varying (LPV), indicating a method adaptable to changing system parameters. Notice that the proposed technique is also flexible in selecting the compensator to adjust the control system's performance despite changes in the plant parameters. The methodology presented in [32] stands out by proposing an innovative nonlinear SMC approach, reflecting a higher level of complexity in terms of implementation since a dual-motor SBW system is required. More details related to this conceptual comparison are shown in Table II.

VII. CONCLUSIONS

A novel approach to control system design has been introduced, particularly focusing on digital compensators and their application to stabilize LTI discrete-time systems. Using OP manipulations, a diverse family of compensators has been generated, showcasing the efficacy of stabilizing

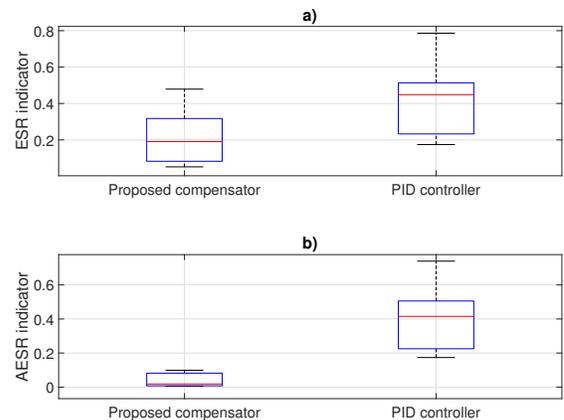


Fig. 9. Controllers performance comparison: a) ESR indicator, b) AESR indicator.

