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Learning smooth graphs with sparse temporal variations to explore long-term financial trends

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Abstract—The return of inflation raises the issue of assessing cross-dependencies in the interest rates of long-term government bonds. Learning cross-dependencies directly from data is framed here as a graph learning problem that requires to address the issues of bond rates heterogeneous and nonstationary evolutions, with sharp changes along time and across countries, and of managing missing samples. As a first contribution, the present work devises a data driven time-dependent graph for bonds markets, specifically based on risk premia. As a second contribution, it shows the relevance of such constructions when applied to a broad database of 29 countries over 6 decades (1960-2020), that includes the high inflation episode of the 1970s.

I. INTRODUCTION

Recent works have studied multivariate time series through the lens of dynamic graphs [1]–[6]. In these works, one seeks to capture the inter-dependencies of the series by fitting a dynamic graph to model the connections between them across times. Learning such a graph is a question of dynamic network inference. Static network inference is now a well studied problem, either from graphical model inference [7], or from the learning of Laplacian or Adjacency matrices from a constraint of smoothness of the data on the associated graph – see reviews in [8], [9]. Other works have tackled the challenge of generalizing existing approaches to time-varying settings. This is the objective of Time-Varying Graphical lasso [2], [10] for graphical models, of the works in [3], and of works based on a smoothness criteria for the signals on the graph to be learned, leveraging the static case of [11], [12], to propose dynamic methods in [1], [4]–[6].

The objective of the present work is the study of global, long-term financial series of international bond markets. We will show how we can leverage the work of [4] to expose a descriptive structure as a dynamic network. We have adapted existing network inference methods to deal with the usual features of series: missing samples, nonstationary evolution of the number of available countries, and global heterogeneity

across both countries and times of the data. The present study will extract, from the resulting dynamic network of inter-dependencies between bond rates, specific networks features such as the number of edges, of clusters, their sizes and the centrality of countries in the network, to describe the long-term evolutionary patterns of these international bond markets.

Economic context and rationale for the study. After 4 decades of steady decline, global long-term interest rates rise sharply from 2022, in the context of resurgent inflation and growing public deficits associated with the current polycrisis. Public policies designed to mitigate the effects of this rise in long-term interest rates require to understand interdependencies between bond markets. The literature in financial cliometrics and macroeconomics focuses on the historical dynamics of international bond markets and their effects on portfolio composition and investment financing [13]–[21]. A narrower literature in financial networks studies the interdependencies of international bond markets using graph models [22]–[26].

Related work in financial cliometrics and financial macroeconomics. Interest rates on long-term loans are based on bond rates (i.e. interest rates on debt securities). A major feature is the difference between bond rates for different maturities, i.e. the “price of time”, referred to as the yield curve. This means that differences in bond rates is what matters most. Long-term bond rates also depend on, especially, the nature of the issuer [13]. Government bonds offer the lowest risks, i.e. “sovereign risk”. Any other borrower has to pay a risk premium. Moreover, not all sovereign borrowers offer the same level of risk [20]. Throughout the Gold standard period (1870-1930’s), British consols were the lowest sovereign risk [14]. At present, the lowest risk is German debt, while the benchmark global borrower is the United States, whose debt is denominated in the dominant currency [17]. Generally speaking, advanced economies have lowest risk levels and lowest bond rates [21].

In optimal portfolios derived from the Capital Asset Pricing Model (CAPM) [15], sovereign bonds are “risk-free”. The separation into two funds, “risk-free” and risky, has been

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corroborated by cliometric studies highlighting portfolio diversification since the British and French imperialisms [16], [18], [19]. This portfolio model is still widely used today by deposit banks and institutional investors. This raises the question of accurately assessing, with a data-driven technique, which sovereign bonds can be considered as the core and risk-free assets at a given time. For that, we propose to leverage time-varying graph learning methods [1]–[6].

Related work in financial networks. Interdependencies between asset prices are the subject of a specific literature on financial networks, dominated by studies on equity markets. A few works cover sovereign bond markets. They replicate the standard approach for equity markets, i.e. correlation-based networks; e.g., [24] combines cDCC-FIAPARCH and stable networks. Correlation measures may take into account nonlinear dynamics and tail-dependency: [26] employs partial correlations associated with a graphical lasso; and [25] conditional correlations associated with various network filtering methods. An alternative approach to network models involves clustering from sovereign bond mapping, e.g. [22]. Studies often involve around 15 debt securities, mostly European [24]–[26], dating back only to the early 2000s. No paper studies bond market dynamics specific to periods of high inflation.

Goals, contributions and outline. Our contribution is twofold: i) we study the international inter-dependencies of long-term interest rates by leveraging a specific time-varying graph learning framework which is not based on correlations but on yield differentials, allowing us to characterize “risk premia”; ii) we employ a broad database of 29 countries, including the high inflation episode of the 1970s–1980s, allowing us to discuss current issues on inflation from a long-term economic (i.e. cliometric) perspective. For signal processing, our contribution is to showcase that the dynamic graph learning framework is fruitful for the study of long series with many missing data, a structure of several clusters, and a notion of core and periphery for the nodes. Section II presents the studied data and associated challenges. Section III recalls the dynamic graph learning setting elaborated in [4], [5] and the learning algorithm under hypotheses of signal smoothness on the graphs and of sparseness of the temporal variations. Section IV presents some features about the obtained dynamic graphs. We conclude by discussing the results.

II. LONG-TERM FINANCIAL SERIES OF BOND MARKETS

A. Main dataset

The present study deals with sovereign world bond markets over a long period, and the evolution of their inter-dependencies. We use a database of monthly sovereign bond yields from 29 advanced, emerging and developing economies, from 1960 to 2020. The data source is the International Monetary Fund, and the data is retrieved from FRED (Federal Reserve Bank of St. Louis: <https://fred.stlouisfed.org/>). Fig. 1 shows the series by country. For easier reading, they are divided into 5 groups: Eurozone, Non Eurozone European Union (EU), Non-EU Europe, Other advanced economies, Developing and emerging economies.

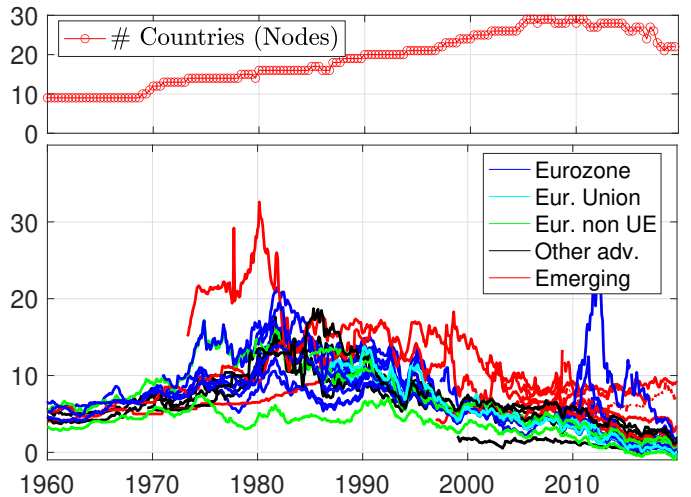


Fig. 1. **Long-term interest rates.** Top: Number N_t of countries whose data is known at t . Bottom: Government bond yields, in %, for the countries in the database (10 to 29 countries) grouped into 5 economic groups, 1960–2020.

B. Challenges: nonstationarity and missing samples

Fig. 1 illustrates that, until the end of the 1970s, interest rates slowly increase (by around 5 to 10%), with little dispersion for the countries in the database (10 initially). With the culmination of inflation in the late 1970s, they rise sharply, peaking at over 30% for developing economies, and 20% for advanced economies. They then steadily decline until the recent resurgence of inflation. We also note a marked convergence of European rates, notably the Eurozone (with the exception of the 2011 debt crisis, especially affecting Greece); and a persistent gap of around 10% points between advanced and developing or emerging economies. This emphasizes the first challenges: the global nonstationary behavior of the series, the fact that they neither change globally at some times, nor smoothly along the time, and the fact that globally bond rates are decreasing. Hence, a given difference is not perceived the same at a later time.

The second challenge is that there is a lot of missing samples, both over extended periods of time for some countries (e.g., when not in the international bond markets) and locally for others (e.g., some countries report bond rates with gaps). We write N_t the number of countries with available data at t .

III. METHOD: DYNAMIC GRAPH LEARNING

A. Smoothness-based methods for static graph learning

Since correlations are sensitive to sharp fluctuations, we exclude the use of graphical model inference [2], [7] to represent inter-dependencies between countries over time. Instead, assuming that countries are inter-dependent when their interest rates are close, we consider graph inference by maximising the smoothness of these signals (the rates) on the graph [11], [12].

A graph is denoted by $\mathcal{G} = (V, E)$ where V is a set of N nodes (here, countries) and E is a set of edges. $W \in \mathcal{W}$ is the adjacency matrix, meaning that W_{ij} is the weight of the edge between nodes v_i and v_j . The admissible set is $\mathcal{W} = \{W \in$

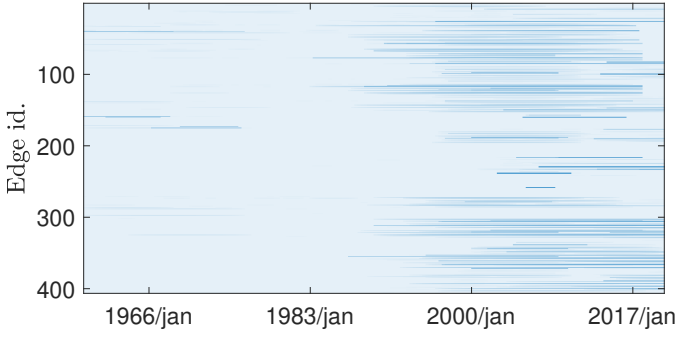


Fig. 2. Vector form of temporal adjacency matrices W_k , showing the edge persistence along time. Edges are arbitrarily numbered.

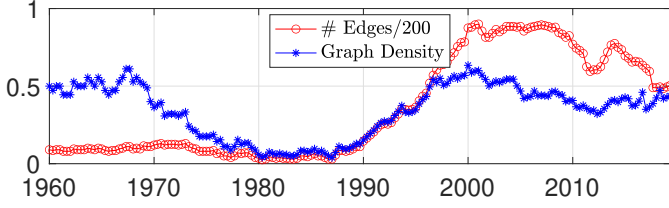


Fig. 3. Number of edges and graph density along time.

$\mathbb{R}_+^{N \times N}$ s.t. $W = W^\top$, $\text{diag}(W) = 0$ for undirected weighted graphs without self-loops. Given $Z_{ij} = \|x_i - x_j\|^2$, the basic term capturing the global variations of signals $X \in \mathbb{R}^{N \times m}$ (having m components per node, e.g. 12 for a monthly signal spanning a year) on a graph is written as:

$$\frac{1}{2} \sum_{(i,j) \in V \times V} W_{ij} \|x_i - x_j\|^2 = \|W \circ Z\|_1. \quad (1)$$

In the work [11], the optimization problem to find a graph \mathcal{G} on which the signals X are smooth, is formulated as:

$$\min_{W \in \mathcal{W}} f(W) = \min_{W \in \mathcal{W}} \|W \circ Z\|_1 - \alpha \mathbf{1}^\top \log(W\mathbf{1}) + \beta \|W\|_F^2. \quad (2)$$

The first term controls the smoothness of the signals on the inferred graph \mathcal{G} (see eq. (1)). To exclude the trivial solution $W = 0$, the second term constrains each node to have at least one neighbour, and α controls the average degree node. The third term permits to control the density of the graph independently from signals; for a fixed α , denser graphs are obtained with larger β .

This formulation is well-suited to the technical challenges raised by economic data:

- Some countries have missing data. Using a window Ω_k spanning m months starting at time k enables us to propose a proxy to compute distances despite some missing samples:

$$Z_{ij,k} = \sum_{t \in \Omega_k} (x_{i,t} - x_{j,t})^2 \frac{m}{m - \text{Card}(\text{isNaN}(x_{i,t} - x_{j,t}))}. \quad (3)$$

isNaN is the operator returning that the value cannot be computed (i.e., $x_{i,t}$ or $x_{j,t}$ is missing) and Card is the cardinal. When all months in Ω_k are missing, this term is obviously infinite and this prohibits this edge to exist in window Ω_k .

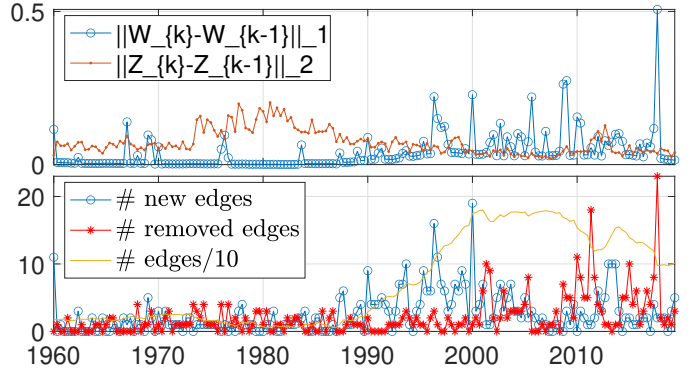


Fig. 4. Temporal variations and changing edges. Top: terms of temporal variations of the inferred graph $\|W_k - W_{k-1}\|_1$, and of the original distances $\|Z_k - Z_{k-1}\|_2$. Bottom: number of added or removed edges at each time k in \mathcal{G}_k (counting in non-zeros unweighted edges).

- We impose that nodes with no data at all on the considered time window to be disconnected (they are not put in the optimization problem). For the other nodes, having data, the log barrier on the degree (second term of (2)) forces them to be connected to at least one other node.

- However, the graph is not necessarily connected (like what would happen with Minimum Spanning Trees [25]). As will be shown in Fig. 6, there exists time periods with disconnected clusters. This is a valuable outcome of the tools used here.

- The differences in bond interest rates are what interests us, contrary to studying correlations as it is usually done for equity securities. Still, one could generalize it to any distance, e.g., the ℓ_p distances: $Z_{ij,k}^{(p)} = (\sum_{t \in \Omega_k} |x_{i,t} - x_{j,t}|^p)^{\frac{1}{p}}$.

B. Dynamic graph learning with sparse edge edition

In [1], the method has been extended to a time-varying setting by imposing smooth variations of the inferred graphs between successive time windows. This amounts to add, in the optimisation problem, a Tikhonov regularization term: $\sum_{k=2}^T \|W_k - W_{k-1}\|_F^2$. However, as argued in [2] for a graphical lasso setting, and in [4], [5] for a smoothness-based one, a temporal sparsity prior on the variations of W_k is better suited when sharp changes occur rather than smooth ones. [5] introduced a group lasso term when graphs are expected to globally change at sparse time points, and a fused lasso term when only some edges change at sparse time points. For long-term financial time series, the latter case is more adapted as global restructuring are expected to be rare.

The dynamical problem that we consider here is then:

$$\min_{W_k \in \mathcal{W}} \sum_{k=1}^T f_k(W_k) + \eta \sum_{k=2}^T \|W_k - W_{k-1}\|_1, \quad (4)$$

where $f_k(W_k)$ is the expression of eq. (2) for the graph at time k , with distance Z_k defined in (3) and parameters α_k and β_k . The algorithm of [27] solves the optimization problem (4). It relies on the primal dual optimization algorithm as in [4], [5] using the primal-dual splitting framework of [28]. This provides a dynamic weighted adjacency matrix W_k which defines temporal graphs \mathcal{G}_k .

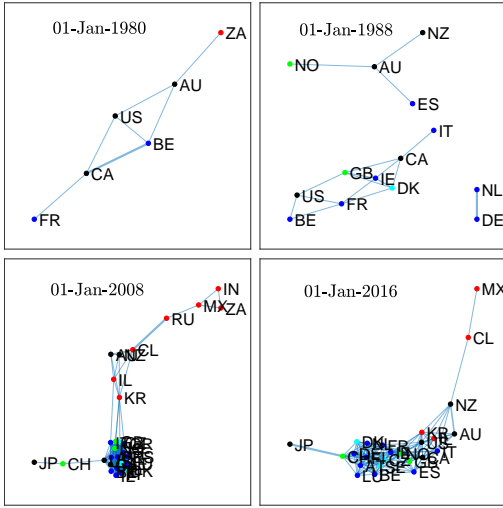


Fig. 5. **Graphs learned dynamically, selected at certain dates.** The layout is obtained by the ForceAtlas method. Node colors: blue = Eurozone, cyan = non-eurozone European Union (EU), green = non-EU Europe, black = other advanced, red = emerging.

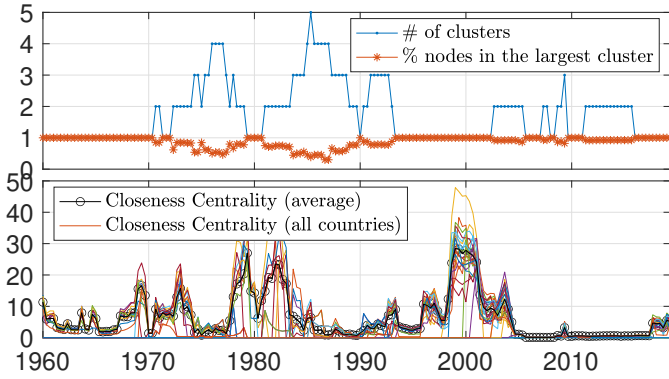


Fig. 6. **Number of clusters in W_k and Closeness Centrality along time.** This shows the various levels in integration, with periods of fragmentation of bond markets (more than 1 cluster). Yet there almost always exists a large cluster of more than 50% of countries.

We refer to [4], [5] for the details on the derivation of the algorithm, and the pseudo-code to solve (4). Note though that the ℓ_1 proximal operator uses η to correctly penalize the problem. Iterations are stopped when the relative changes become smaller than a given threshold. We force the solution to be zero on the diagonal, and to be sparse by thresholding to the desired numerical precision (here, set to 10^{-5}).

The parameters are set by following [11] and its implementation in the gsp toolbox [29]. Adaptations have been necessary, as the dynamic version has been coded from scratch. The window length is $m = 4$ months. Parameter α_k scales as α_0/N_k^2 which imposes that $\|\alpha_k Z_k\|_1$ would be constant across times for stationary distances with a changing number of nodes. With the scaling theorem (Prop. 12 of [11]), we can set $\beta_k = \beta_0 N_k^2$. Then, imposing $\alpha_0 = 1$ and an averaged density of W_k of $1/3$, we tested several values of β_0 to achieve this density, while changing η to promote sparse temporal variations.

IV. STUDY OF THE DYNAMICS OF GRAPHS OF LONG-TERM FINANCIAL EVOLUTIONARY PATTERNS

A. Features of dynamical graphs for world bond markets

The inference framework of Section III is used on the data presented in Section II. Remember that the present work is focused on the application and interpretability of the time-varying graph learning method. Fig. 2 shows the persistence of edges along time. It highlights the obtention of stable structures over long periods.

The learning problem (4) controls well the density of the dynamic graphs (defined as $|E_t|/(N_t(N_t - 1)/2)$ as displayed on Fig. 3. Indeed, the number of edges (also on Fig. 4 bottom) changes dramatically, yet the time-averaged density is controlled while the actual instantaneous density can vary. It is roughly stationary around 0.5 in the two main periods of high bond market convergence (of the data at hand at each t), between 1960 and 1970 (only developed countries in the data) and 1999-2008, despite rates which are more spread out than in the 1960s.

The second point is to check the usefulness of solving the time-varying optimization problem of eq. (4) instead of solving only eq. (2) on sliding windows. Fig. 4 gives a first answer: the temporal term $\|W_k - W_{k-1}\|_1$ is indeed sparser than $\|Z_k - Z_{k-1}\|_2$. We clearly see specific instants of changes (upper plot), associated to both addition of new edges in the graphs and removal of edges. Sticking to the raw data (Fig. 1) or the distances Z_k , it is harder to extract periods of stability. On Fig. 4, the temporal evolution of $\|Z_k - Z_{k-1}\|_2$ only points to a larger integration (i.e. reduction of differences) of bond markets from 1990 on, while Fig. 3 pinpoints extended periods of integration, and others of its regression (e.g., after the repeated crises in 2008, 2011, and the 2010s).

B. Clusters and centralities

A second lesson extracted from the learned \mathcal{G}_k comes from the study of the structure of the graphs at each instant. First, we show instances of these dynamic graphs on Fig. 5. Globally, they are made up, from core to periphery, of: (i) Europe; (ii) other advanced economies (close or connected to the core: USA, Switzerland, Japan, etc.); (iii) emerging economies (including Greece after 2011). At the start of the 2000s, (i) and (ii) almost overlap. Post 2008 they are more spread out and more separated. Note also, as from the top of Fig. 6, that \mathcal{G}_t is not always a connected graph (e.g., see Jan-1988 in Fig. 5). While there is often a large cluster, which contains a dominant fraction of the existing nodes at k (orange curve on top of Fig. 6), the number of clusters itself becomes as high as 4 or 5 at some times. This adds weights to the comment about Fig. 3 that, despite a controlled density, the graph reveals changes in the integration of bond markets. The periods (70-95 and again post 2007) where there is more than one cluster have less integration than in between (even if integration before 1970 is also linked to the absence of emerging countries).

We compute the centrality of each country in \mathcal{G}_k , so as to determine if there are countries having a more central role in the

series of \mathcal{G}_t . To keep things simple, Fig. 6 reports the Closeness Centralities: $C_k(i) = (R_{i,k}/(N_k-1))^2 / \sum_{j \in V_k} d_k(i,j)$ where $d_k(i,j)$ is the shortest path distance in $\mathcal{G}_k = (V_k, E_k)$, and $R_{i,k}$ is the number of reachable nodes through \mathcal{G}_k from node i . The evolution of the average of the centrality on all nodes V_k (in black) further shows periods of high integration. If one would look also at the ranking of the various countries (not displayed), often hubs like The Netherlands or Belgium, benchmark countries like the USA or France (because in EU) are ranked among the first (and are in the center of graphs as seen in Fig. 5). Also, EU countries in the 2000s and 2010s have a high Closeness Centrality as they are in majority close one to another (exception: Greece in the 2010s).

V. DISCUSSION AND CONCLUSION

We have used the time-varying graph learning optimization framework of [4], [5], based on signal smoothness on the graphs and sparseness of the temporal variations of the graphs, to extract relevant patterns from long-term financial series of sovereign bond markets. The method has been adapted to missing data, and to be able to extract edges when there is a varying number of nodes with a controlled density.

The obtained dynamic graph, as in Fig. 5 and having the specific features reported in the other Figs., illustrates well the very strong nonstationarity in the increasing integration of bond markets of advanced economies in the 1980s and 1990s, fuelled by financial deregulation and European integration. Then, under the impact of repeated crises (2008, 2011, pandemic, etc.), integration regresses.

The connections and density of the graph (Fig. 3) show a strong progression of integration in the 1990s. Specifically, the number of edges (Fig. 3 top) increases rapidly. A plateau of around 10 years is then observed, followed by a first decline. From the beginning of the 2010s, as a result of repeated crises, connections decline again (see Fig. 4). Density (Fig. 3 bottom) sheds further light: until the early 1970s, it is essentially made up of advanced economies with high long term interest rates, and comparable to that of the global graph of the 2000s with much lower rates, highlighting period-specific forms of integration. These results illustrate that, in the current period of high inflation, long term interest rates are doubly affected: by the rise in US rates, which are the benchmark, as the direct consequence of inflation; and by the rise in risk premia.

A second conclusion is that the graph leaning method can be efficiently leveraged for complex datasets, avoiding the many pitfalls of missing data and nonstationarities, while providing interpretable dynamic graphs.

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