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Non-convex interferometric inversion for wave scatters recovery in complex media

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Abstract

In this work, we propose a new approach to recover sparse obstacles q from measurements b of the scattered wave field in a slowly varying medium. In the Born approximation the problem is linear ($Aq = b$) but the unknown wave speed variations induces a phase error on the data b . Classical approaches fail at recovering the scatters in this case. Hence we propose an alternative inversion that consists in considering an interferometric quadratic problem

$$(Aq)_k \overline{(Aq)_\ell} = b_k \overline{b_\ell}, \quad \text{for some } (k, \ell).$$

The interest of such system comes from the better robustness property of some cross-correlations $b_k \overline{b_\ell}$ with respect to the medium uncertainties.

Keywords: Inverse scattering, random media, interferometric inversion, non-convex optimization.

1 The inverse scattering problem

In a domain of interest $\Omega \subset \mathbb{R}^d$, $d \in \{2, 3\}$, we aim at recovering scatters $q \in L^\infty(\Omega)$ of small support, from measurements of the harmonic scattered field u , solution of the Helmholtz equation in the Born approximation

$$\Delta u + \frac{\omega^2}{c^2} u = -\omega^2 q u_I \quad \text{in } \mathbb{R}^d,$$

and satisfying the Sommerfeld radiation condition. The incident field u_I is generated by a controlled source at x_0 and satisfies

$$\Delta u_I + \frac{\omega^2}{c^2} u_I = -\delta_{x_0} \quad \text{in } \mathbb{R}^d.$$

Measurements are defined as a discrete evaluation of the scattered field:

$$b_k := \frac{1}{\omega_k^2} u(r_k, \omega_k), \quad k = 1, \dots, n,$$

at some receivers locations $r_k \in \mathbb{R}^d$ and for chosen frequencies $\omega_k \geq 0$.

The Green function $G(x, y, \omega)$ (unknown) can be used to give an integral expression of the propagation operator. The incident field u_I takes the form $u_I(x, \omega) = G(x, x_0, \omega)$ and the measurements read

$$b_k = \int_{\Omega} G(r_k, y, \omega_k) G(y, x_0, \omega_k) q(y) dy.$$

2 Wave propagation in slowly varying random media

In [2], the authors described the behavior of a wave in a slowly varying random media, where the speed c is given by

$$\frac{1}{c^2(x)} = \frac{1}{c_0^2} \left(1 + \sigma \mu \left(\frac{x}{\ell} \right) \right)$$

where μ is a spatial random variable of autocorrelation $\mathbb{E}(\mu(x)\mu(x')) = \exp(-|x - x'|^2/2)$ for all $x, x' \in \mathbb{R}^d$. In such media, the travel-time function is given by

$$\tau(x, y) := \frac{|x - y|}{c_0} + \nu(x, y), \quad x, y \in \mathbb{R}^d,$$

where ν is a spatially smooth random fluctuation built by integrating μ between x and y .

In a regime where ℓ is large compared to the wavelength, the authors show that the Green function of the medium is approximated by

$$G(x, y, \omega) := G_0(x - y, \omega) e^{i\omega \nu(x, y)}, \quad x, y \in \mathbb{R}^d$$

where G_0 is the well-known Helmholtz Green function in homogeneous media.

Assuming that $\text{supp}(q) \subset B(y_0, \varepsilon)$ with ε small compared to the coherence length of the medium ℓ , one can approximate the travel time fluctuation from any $x \in \mathbb{R}^d$ to $y \in \text{supp}(q)$ by

$$\nu(x, y) \approx \nu(x, y_0).$$

Hence, the measurements reads as

$$b_k = b_k^{\text{ex}} e^{i\omega_k \varphi_k}, \quad \varphi_k := \nu(r_k, y_0) + \nu(y_0, x_0)$$