



HAL
open science

Artificial Fog Modelling for Synthetic Lidar Data

Ammar Ridzuan, Youri Mikhail Noutatien Guiafaing, Otmane Attou,
Sylvestre Prabakaran

► **To cite this version:**

Ammar Ridzuan, Youri Mikhail Noutatien Guiafaing, Otmane Attou, Sylvestre Prabakaran. Artificial Fog Modelling for Synthetic Lidar Data. Société des Ingénieurs de l'Automobile (SIA), Société des Ingénieurs de l'Automobile (SIA), Oct 2024, PARIS, France. hal-04729093

HAL Id: hal-04729093

<https://hal.science/hal-04729093v1>

Submitted on 9 Oct 2024

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Artificial Fog Modelling for Synthetic Lidar Data

Ammar RIDZUAN^{1,2}, Youri NOUTATIEM², Otmane ATTOU^{2,3}, Sylvestre PRABAKARAN^{1,2}

1: AVSimulation, 1 Cr de l'Île Seguin, 92100 Boulogne-Billancourt, France

2: IRT SystemX, 2 Bd Thomas Gobert, 91120 Palaiseau, France

3: VALEO, 6 rue Daniel Costantini, 94000 Créteil, France

Abstract: LiDAR sensors are now a main component in perception systems of vehicles. With the advancements in technology of ADAS and autonomous vehicle systems that rely on LiDAR sensors, it is necessary to thoroughly test and validate these systems before deployment into the real world. Among the method is via the use simulation that allows the system to traverse millions, if not billions, of kilometres with thorough testing and validation including scenarios that cover a larger scope of situations and cases. However, despite the accuracy of LiDAR for mapping the environment and estimating precise distances, it is known to be affected by adverse weather conditions, thus reducing the operational design domain of vehicles equipped with this type of sensor. This paper aims to introduce a methodology of acquiring real world data, and through data analysing and processing be able to create and improve the current LiDAR models in simulation by implementing the effects of perturbation, whether it be hardware or weather caused.

Keywords: LiDAR, Fog, Weather Perturbations, AD/ADAS, Simulation

1. Introduction

LiDAR (Laser Imaging Detection And Ranging) is more and more commonly on autonomous vehicles used in the perception systems. However, even though lidars are capable of providing detailed information about the environment, diverse weather perturbations such as rain, snow and fog introduce a significant functional limitation to its usage. In the cases where the target object has a high reflectivity, this issue is mitigated (Trautmann, Blechschmidt et Friedrich). Due to the variety in results of the feedback information depending on weather and environmental factors, it is necessary that the nuances and complexity of the technology are taken into account during the modelling and validation of the recognition algorithm as well as the AD/ADAS systems using LiDAR.

In order to be able to ensure reliable validation of the autonomous system, it is necessary to drive billions of kilometres, time consuming and expensive. (Kalra et Paddock). Simulation based testing is considered an efficient method to validate the ADAS functionalities,

where the number of tested scenarios can increase exponentially as the combination of parameters increase. This allows the validation of the system in a wide range of situations that are easily reproduce and repeated (Duy Son, Awatsu et Jubrechts).

However, current LiDAR sensors are limited in terms of technology and representativeness. Ray Tracing models exist however the perturbation models are not well implemented enough to produce realistic results in disturb weather conditions such as rain, snow and fog. Under Project CVH (Tool-Based Chain for the Validation and Homologation of Automated and Connected Vehicles) it aims to resolve issues on sensor models and their behaviours when confronted with weather perturbations. Most sensor models currently in existence are modelled as perfect sensor and thus do not account error and disturbances due to environmental factors.

This paper proposed a model that has been developed, used and implemented in AVSimulations' SCANer Studio software, with the use of a specific semi-rotative LiDAR.

2. Overview of LiDAR sensor modelling

A LiDAR sensor measures the distance of the different points in the environment. In automotive systems, LiDAR is used to plot a 3D Cartography by sending Lasers in several directions and calculating the distances of each point. The output is a 3D point cloud representing the environment around the vehicle.

2.1 Types of lidars

Different technologies are used depending on the system and usage, however all LiDARs depend on having a laser ray emitter directing the device towards different directions and an emitter to receive the reflected rays from the environment. The delay between the emitting ray and the received ray is used to calculate the distance based on the time of flight of the ray. In the automotive industry, three main types of LiDARs are commonly used:

Rotative LiDAR: A motor is used to mechanically rotate the laser emission beam to scan with a specific pattern in every direction

Semi-rotative LiDAR: Electro-mechanical parts (such as mirrors or MEMS) are used to scan the environment in different directions. Usually, the field of view of this LiDAR is restricted to specific angles.

Solid-state LiDAR: A laser beam is sent with specific phases and patterns to broadcast rays in each direction.

2.2 Perturbations

Numerous noise factors can impact the performance of a LiDAR sensor. (Chan, Dhadyalla et Donzella) propose a list of 16 factors, organized into five categories. These factors may involve interactions with other systems (such as coexistence with other LiDARs or malicious attacks) and the gradual degradation of electronic or mechanical components over time, among others. Adverse weather conditions, particularly rain, snow, and fog, have a significant impact on the resulting point clouds. Airborne particles, such as snowflakes or raindrops, can affect the propagation of LiDAR signals in the atmosphere due to phenomena like absorption, reflection, and scattering.

Similarly, dry and wet surfaces interact differently with incoming LiDAR rays. Consequently, under adverse weather conditions such as rain, snow, or fog, LiDAR point clouds are affected in terms of the number of detected points, their intensity, and the appearance of noisy points (Zhang, Ang et Rus)

Several research groups have examined the effects of adverse weather on LiDAR sensors. (Filgueira, Gonzales-Jorge et Lagüela) conducted outdoor experiments with a 360°, 16-layer LiDAR, successfully collecting data under varying rain intensities. By analysing segmented portions of point clouds, they concluded that rainy conditions result in a reduction in the number of detected points and attenuation of the returning signal. (Kutilla, Pyykönen et Holz Hüter) gathered data in a controlled environment that simulated artificial rain and fog, specifically comparing two different wavelengths (905nm and 1550nm). They observed a slight decrease in LiDAR signal intensity in the presence of rain and a significant reduction in the presence of dense fog, with visibility less than 40 meters.

As a result, the operation of perception systems using LiDAR sensors in intelligent vehicles is impaired by adverse weather conditions. Therefore, it is crucial to replicate these effects in simulation models to make virtual tests more realistic and reliable.

Many authors have proposed models addressing noise factors related to airborne particles in the atmosphere. (Goodin, Carruth et Doude) introduced a mathematical model that incorporates noise into range measurements and reduces point intensity

based on the scattering coefficient and rainfall rate. (Byeon et Yoon) took a microscopic approach, incorporating raindrop characteristics specific to different regions, such as size and shape, into their model. . (Espineira, Robinson et Groenewald) developed a LiDAR model using a ray-casting method in Unreal Engine, simulating backscattering effects from rain by adding false positive points to the point cloud.

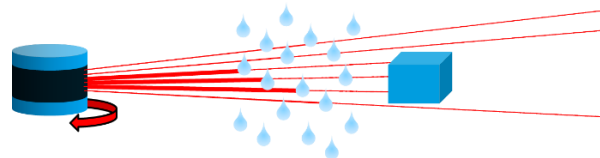


Figure 1: A simplified view of why perturbation affect LiDAR sensor

2.3 Sensor outputs

LiDAR sensor outputs can be divided into two layers:

The first layer consists of raw data, represented as a three-dimensional point cloud. Regardless of the LiDAR technology used, the sensor produces a 3D point cloud of the environment. For each specific direction of a laser ray, one or more points (in the case of multi-echo) are added to the point cloud. The sensor's receivers measure the attenuation of the ray between its emission and reception, resulting in an intensity value for each point in the cloud. This intensity is influenced by factors such as the target material's reflectivity, the incident angle of the ray, the distance, and more. This intensity data provides additional information that helps the software recognize and classify objects in the environment.

The second layer, which is optional, involves a detection software layer available in some LiDARs for object detection and classification. Modelling perturbations in simulation can also be applied to this second layer, focusing on the detection and classification outputs.

2.4 Simulation Tools

In order to be able to acquire realistic data, we must ensure that all aspects of the Simulation is reflective of real life data and properties. This is dependent on three main factors: The environment, the actors and the sensor model.

The environment refers to the surrounding conditions in where we can find the vehicle driving. The environment must contain objects and materials that reflect the scenario being tested. In the case of simulation with a LiDAR sensor, the material of the environments must be defined since LiDAR data is heavily based on the reflected ray due to the material of the object. Another factor that is important while

modelling the environment is how these objects are modelled.

The actors in the scenario refer to the vehicle and/or surrounding cars and pedestrians. These must also be well described, including their geometry, materials, and behaviour (how the actors move around in the environment).

3. LiDAR Sensor Fog Modelling in Simulation

The proposed methodology presented is based on existing simulation frameworks and adding upon current sensor model to consider the missing aspects that ensure greater realism, perturbations caused by weather disturbances. The data used to construct the new sensor model is obtained by capturing real data measurements with a physical LiDAR, obtained at the CEREMA Laboratory.

3.1 Real Measurements

The initial goal of collecting real data that would fit the needs of the project: placing the physical LiDAR sensor in a perturbed environment that was physically descriptive for measurements and having its equivalence in simulation. It is also necessary to have the physical environment in a non-perturbed state/scenario.

Platform of simulation of degraded climatic conditions

The first series of data acquisition was executed at the CEREMA Laboratory. The simulation equipment used is referenced within the CEREMA laboratory. This simulation equipment used is referenced within the LABEX ImobS3 "innovative mobility: intelligent and sustainable solutions" among the regional platform "PAVIN B-P", Auvergne platform for intelligent vehicles Fog - Rain.

The site consisted of a tarmac track thirty meters in length, fitted and equipped with various weather systems (artificial vision systems, transmissometers, rain and fog generators, photometers, granulometers, radiometers, video photocalorimeters, spectropluviometers) under a roofed dome to be able to carry out experiments day and night.

The different objects within the CEREMA Laboratory allows for the testing of different shapes, forms and materials, enabling the measurements of the photometric characteristics of the studied objects, namely luminance and colorimetry.

In the case of the project, the CEREMA Laboratory allows the generation of controllable meteorological disturbances which can be physically describes. Physical description allows us to understand the

nature of the current weather perturbation and its replication in other scenarios.

The LiDAR is positioned in such a way to replicate its position on a car and data was acquired during the generation of various weather perturbations.

Fog Characterization

In the meteorology domain, fog density is quantified using a value of meteorological visibility defined by the International Commission on Illumination (CIE) (Commission.) as the distance beyond which a black object of an appropriate dimension is visible with a specific contrast limit.

The change between the different fog visibility is considered to be slow enough to be pseudo-static perturbation. For each visibility, the data generated by the LiDAR is in the form of a PCD file (point cloud file), where we can associate each PCD file to a visibility.

Generation of Scenarios

The nature of the properties of the CEREMA Platform allows us to generate multiple scenarios, anticipated in advanced to be able to extract data and create our model. The scenarios consisted of placing objects at different positions and orientations from the LiDAR as well as the generation of different levels of rain and fog as well as no perturbations.

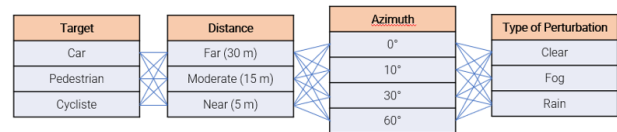


Figure 2: Combination of statistical scenarios

Within the CEREMA platform are sensors that enable to measure in real time the exact fog density giving the meteorological visibility distance. During the scenario which lasted between 10 to 30 minutes, the fog density gradually changed from 10m of visibility and ended at 1000m of visibility.

3.2 Data Extraction

For each point cloud generated, there is a precise number of points (or a variation of this number of points in the case of multi-echo systems). By understand the nature of these points, we are able to distinguish the points generated due to the fog and the points that belong to the physical environment (target). Points generated to real targets will be referred to as hard targets in the following chapters, and the fog points caused by the backpropagation effect will be referred to as soft points.

Data is extracted from the point cloud by first extracting the rays and further slicing the rays to understand how the fog behaves within the system.

Ray Extraction

A sub point cloud of the main point cloud is extracted in clusters. Each ray (noted R) is examined and the number of points within each ray is extracted. The reasoning behind having a ray selection is to ensure that the two distinct point being extracted do not stem from the same ray (except in the case of multi echoes). Any rays belonging to hard targets are not accounted for and ignored.

Ray Slicing

After extracting the points, the rays are then divided in multiple segments. The density of each segment is then calculated, referring to a distance from the LiDAR. This allows us to create a model of density of points, characterizing the backpropagation effects in function of distance from the LiDAR.

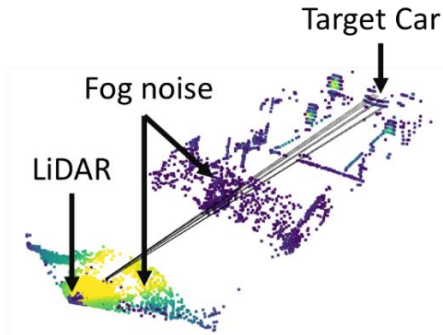


Figure 3: Example of measured point cloud with the selected rays oriented towards the target car

For each ray (noted i in a direction defined by an elevation angle θ_i and an azimuthal angle φ_i), zero to three echoes can be detected at different ranges or distance from the LiDAR, also containing the intensity of each echo:

$$echoes(\theta_i, \varphi_i) = \begin{cases} echo_1(\theta_i, \varphi_i) \\ echo_2(\theta_i, \varphi_i) \\ echo_3(\theta_i, \varphi_i) \end{cases} \quad [1]$$

With each $echo_1(\theta_i, \varphi_i)$ being the first echo or the nearest distance corresponding to the laser (θ_i, φ_i) and each $echo_3(\theta_i, \varphi_i)$ being the third echo or the farthest distance corresponding to the laser (θ_i, φ_i).

Density Repartition Representation

For each point cloud, and each selected rays R , a cumulative distribution function can be defined as the following:

$$\forall j \in \{1,2,3\}, F_j(x) = \frac{1}{|R|} \sum_{i \in R} [echo_j(\theta_i, \varphi_i) < x] \quad [2]$$

For each given visibility, 3 cumulative distributions can be computed each referring to either the first, second or third echo

Plane Slicing

A similar approach to the ray slicing was done in order to observe the behaviour of the soft points along the azimuthal axis of the LiDAR. A series of rays through the azimuthal range (Figure 4 in red) on a fixed polar range were broadcasted and the density of each ray for each echo was calculated.

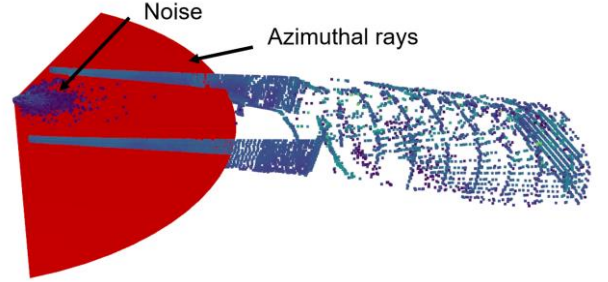


Figure 4: Example of the measured azimuthal rays

For each ray (noted k in a direction defined by a fixed elevation angle, and an azimuthal angle φ_k), zero to three echoes can be detected and their density along each ray is calculated.

$$Plane\ echoes(\theta_k, \varphi_k) = \begin{cases} Plane\ echo_1(\varphi_k) \\ Plane\ echo_2(\varphi_k) \\ Plane\ echo_3(\varphi_k) \end{cases} \quad [3]$$

With the distribution function defined as the following

$$\forall j \in \{1,2,3\}, F_l(x) = \frac{1}{|R| * F_{lmax}} \sum_{i \in R} [Plane\ echo_j(\varphi_k) < x] \quad [4]$$

4. Perturbated Lidar Model Construction

Following the analysis of real-world data, the aim of the study is to develop a model that simulates the fog backpropagation phenomenon (soft target).

4.1 The LASERMETER model

The simulation tool used is AVSimulation's SCANer Studio software, which operates on a modular architecture, with each module handling a specific task within the simulation (e.g., vehicle dynamics, traffic management, pedestrians, sensors, etc.). One of these modules, LASERMETER, is responsible for the LiDAR modelling, utilizing ray tracing. The ray tracing pattern is configured based on the LiDAR's laser emission patterns, which can be adjusted within the model.

The current version of the LASERMETER model represents a perfect sensor of the LiDAR output, unaffected by environmental anomalies. Any target within the field of view of the simulated laser beam is

detected, regardless of weather conditions. The objective of this study is to enhance the existing LiDAR model by incorporating these environmental disturbances, as demonstrated in the following proof of concept, which implements fog effects in the simulation.

4.2 Perturbation generation

The improvement of the LASERMETER model is achieved through a plugin that acts as an additional layer on the existing model. This involves adjusting the generated point cloud to simulate the effects of fog disturbances.

Step 1: Obtaining LASERMETER output: The output consists of a point cloud, with each laser beam generating one, two, or three points. The coordinates of these points are produced relative to the sensor frame.

Step 2: Introducing precision noise: The distance for each point can be modified by applying an error based on a probability distribution. A simple approach is to use Gaussian error to introduce a percentage of uncertainty in the distance estimation.

Step 3: Introducing soft targets (backpropagation points): The backpropagation effect occurs when points are generated between the LiDAR and the target. This effect can be simulated by generating points for each laser beam. The position of the backpropagation points is determined based on a probability distribution. If the randomly generated position is beyond the target object, no modification is made. However, if the position falls between the LiDAR and the object, the new point is assigned to the laser beam, and the original target point is removed.

The different steps are illustrated in figure 9

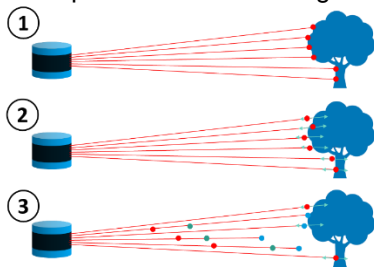


Figure 5: Different steps from the LASERMETER output (1) to add precision noise (2) and then backpropagation due to fog (3)

Distribution of backpropagation

The third step (**Step 3**) outlined in the previous section must be parameterized using a probability distribution. This is because the occurrence of

backpropagation is determined by a selected position, extracted from the real fog data.

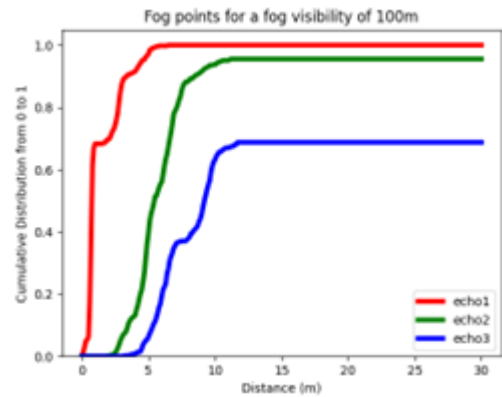


Figure 6: Mean cumulative distribution function for each echo and a fog visibility of 100m

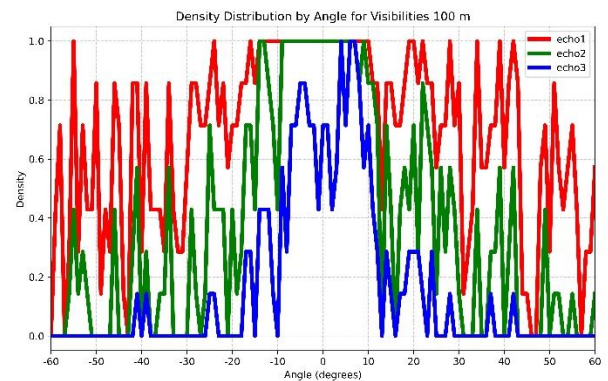


Figure 7: Distribution function for each echo along each azimuthal angle for a fog visibility of 100m

Using the cumulative distribution functions

The results from CEREMA measurements have demonstrated that the soft target, representing fog backpropagation, can vary in complexity based on positions and multi-echoes, influenced by fog density. This complexity allows us to define a probability distribution based on the density data obtained from these metrics. By using this density distribution as a probability law, a new method for generating soft targets can be established.

$$\text{For each echo } j, P(X > x) = F_j(x) * F_l(\varphi) \quad [5]$$

The intensities of the soft target points follow the same probability law and are derived from the mean intensity function. In addition to positioning the soft target points based on multi-echoes, the intensities of the generated points are scaled to align with the real-world data used in the simulation, ensuring accuracy.

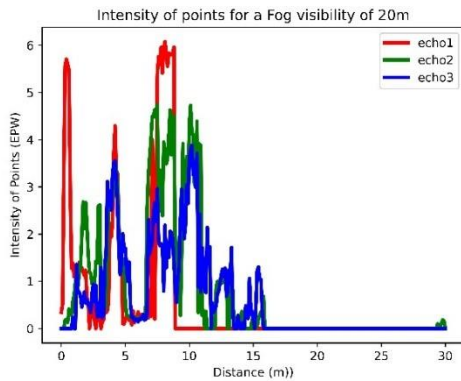


Figure 8: Mean intensity functions for each echo and a fog visibility of 20m

5. Simulation with the perturbation

Simulated scenario

To compare simulated data with real data, the CEREMA Pavin platform was recreated to run an equivalent scenario in SCANer Studio. Additionally, the appropriate parameters were selected to configure the simulated sensor in alignment with the specifications outlined in the LiDAR's datasheet.



Figure 9: The simulated environment in SCANer Studio

Perturbation model construction

A module in SCANer is utilized to integrate the previously generated perturbation model into the point cloud produced by the LASERMETER model. This is done via SCANer Studio's communication bus, referred to as the "Network." The modified point cloud is then published as PCD files, maintaining the same format as the LiDAR's original point cloud outputs.

6. Results

In order to compare the results of our model two types of analysis was conducted. First a qualitative analysis was carried out to visually observe the results between the real data acquired at CEREMA compared to the simulated data. The second

6.1 Qualitative analysis

The presented methodology consists of three components to generate the noisy point cloud: the collected data and the method used to analyse density, the simulation environment featuring the virtual LiDAR and the virtual CEREMA Pavin Platform, and the noise model based on a defined probability distribution.

Merging these three components results in a scaled noise model based on the collected data. The resulting output is displayed in the figure below.

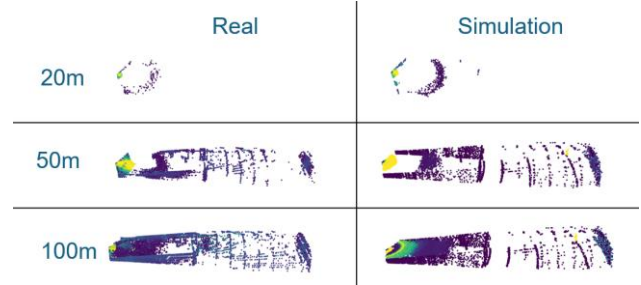


Figure 10: Comparison via visualisation of the LiDAR points, left being the real data acquisition and right being the simulation data

Both presented point clouds show a noticeable similarity, with the inclusion of the fog model adding significant realism. However, further improvements can be made by addressing the isotropy of the model.

6.2 Quantitative analysis

A more quantitative approach can be utilized to compare the generated point clouds. The purpose of such metrics is threefold: to ensure a visually coherent point cloud, to achieve precision in object representation, and to maintain similarity between point clouds for software detection. An initial analysis has been introduced, focusing on evaluation the point cloud density.

This metric is designed to analyse the entire point cloud by dividing it into voxels along the x,y,z axis, with each voxel containing the voxel of the LiDAR point found in the contained surface area. (as shown in Figure 11).

By comparing the densities between the real and simulated point clouds, the difference can be calculated. The metric is established using two formulas that count the number of points $N_{i,real}$ or $N_{i,simulated}$, in each voxel i . The **Global Difference Metric [6]** allows a global view on the difference between the two compared files, while the **Voxel Difference Metric [7]** allows a more refined view on how each individual Voxel compares to one another.

Global difference Metric:

$$\frac{\sum_i(|N_{i,real} - N_{i,simulated}|)}{\sum_i(N_{i,real})} \quad [6]$$

Voxel difference Metric :

$$\frac{1}{Nb_voxels} \sum_i \frac{|N_{i,real} - N_{i,simulated}|}{N_{i,real}} \quad [7]$$

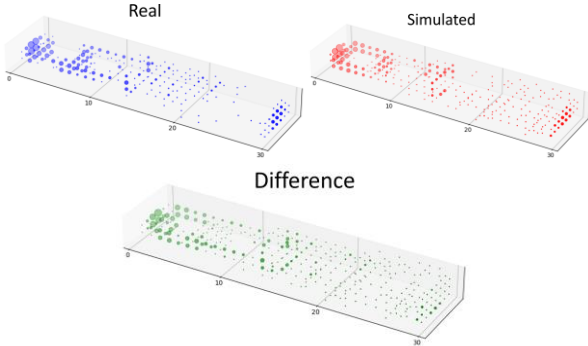


Figure 11: Representation of the comparison metric

The metrics were calculated for several of the generated point clouds, resulting in the following table.

	Without model		With fog model	
Fog	Metric1	Metric2	Metric1	Metric2
20m	194%	147%	4%	6%
50m	118%	84%	76%	43%
100m	101%	46%	68%	40%

Table 1: Obtained metrics for different level of fog

The calculated metrics vary depending on the formula used and can be enhanced by considering factors such as intensities and the grid cell size. The results indicate that the noise model reduces the discrepancy between the virtual and real data.

The **Localised Difference Metric [8]** focuses on a specific, unique box corresponding to an object in the environment. This box is used to analyse the point cloud data associated with that particular object, isolating it for more detailed examination. In the figure, the selected box surrounds the object, allowing for precise comparison between the real and simulated point clouds. By examining the density and distribution of points within this box, we can determine how accurately the simulation captures the object's characteristics in the virtual environment. This method enables targeted analysis, ensuring that the object in the environment is well-represented in the simulation.

The formula is:

$$metric_3 = \frac{|N_{real} - N_{simulated}|}{N_{real}} \quad [8]$$

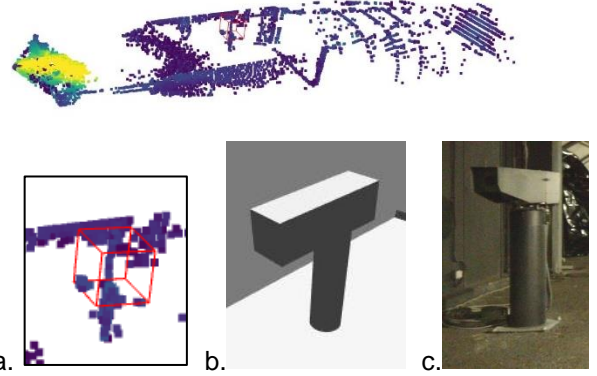


Figure 12: Visual representation of the Localised difference metric and object being studied. (a.) being the object in PCD format, (b) being the simulated instance of the object and (c) being the real physical object.

Fog Visibility	Without fog noise model	With fog noise model
20m	3033%	55.5%
50m	95.83%	2%
100m	83.3%	28%

Table 2: Localised Difference Metric (comparison with CEREMA data)

The calculated metrics show significant variation depending on whether the fog noise model is applied. As the table demonstrates, without the fog noise model, the discrepancies between the virtual and real data are considerably higher, particularly at 20m visibility. The inclusion of the fog noise model drastically reduces these discrepancies across all visibility ranges, indicating that the model improves the realism of the simulation. Further refinements, such as adjusting for intensities and grid cell sizes, could enhance the accuracy of the metrics even more, leading to a closer match between virtual and real data.

7. Discussion

Future improvements

The model presented in this paper provides simulation users with an initial representation of degraded fog measurements, while leaving space for further improvements.

Dynamic study

The data collected so far pertains to static scenarios with fixed targets and a stationary vehicle. However, weather perturbations (especially fog) can vary significantly when the vehicle is in motion, and dynamic studies would provide a more realistic representation of such scenarios.

Metrics of comparison

The comparison of point clouds needs to be enhanced using additional metrics and applied to other point clouds. Furthermore, comparing the results with scenarios acquired outside the CEREMA platform would allow for a more comprehensive validation of the modelling methodology.

8. Conclusion

This paper presents a methodology for integrating adverse weather conditions into a simulated LiDAR sensor model, designed for the validation and homologation of autonomous vehicles. The approach involves collecting and recording real-world LiDAR data under various adverse weather conditions, which is then used to generate a perturbation model. This model is applied to the ideal sensor model, resulting in a more realistic sensor for testing purposes. It is crucial that the collected data accurately reflects the weather conditions to correctly simulate the appropriate perturbations. This method can be extended to simulate different types of weather disturbances and is applicable not only to LiDAR sensors but also to cameras, radar, and even ultrasonic sensors. Additionally, it is less resource-intensive compared to other physics-based sensor models and can be implemented for real-time simulations.

9. Acknowledgement

This work was supported by the French government under the "France 2030" program, as part of the SystemX Technological Research Institute.

10. Reference

- [1] Byeon, Mijung and Sang Won Yoon. "Analysis of automotive lidar sensor model considering scattering effects in regional rain environments." *15th European Conference Antennas and Propagation* (2021).
- [2] Chan, Pak Hung, Gunwant Dhadyalla and Valentina Donzella. "A Framework to Analyze Noise Factors of Automotive Perception Sensors." *IEEE Sensors Letters* (2020).
- [3] Commission. *International Lighting Vocabulary*. Vol. 17.4. Vienna, 1987.
- [4] Duy Son, Tong, et al. "A simulation-based testing and validation framework for ADAS development." *Proceedings of 7th Transport Research Arena TRA* (2018).
- [5] Espineira, Juan P., et al. "Realistic LiDAR With Noise Model for Real-Time Testing of Automated Vehicles in a Virtual Environment." *Fourth IEEE International Conference on Software Testing, Verification and Validation* (2011).
- [6] Filgueira, A, et al. "Quantifying the influence of rain in LiDAR performance." *Measurement, Volume 95* (2017): 143-148.
- [7] Goodin, Christopher, et al. "Predicting the Influence of Rain on LIDAR in ADAS." *Electronics* (2019).
- [8] Kalra, Nidhi and Susan Paddock. "Driving to safety: How many miles of driving would it take to demonstrate autonomous vehicle reliability?" *Transportation Research Part A: Policy and Practice* (2016): 182-193.
- [9] Kutilla, M., et al. "Automotive LiDAR performance verification in fog and rain." *21st International Conference on Intelligent Transportation Systems* (2018).
- [10] Trautmann, Toralf , et al. "Possibilities and Limitations of Object Detection Using Lidar." *23. Internationales Stuttgarter Symposium*. Springer Fachmedien Wiesbaden, 2023. 36-43.
- [11] Zhang, Chen, Marcelo H Ang and Daniela Rus. "Robust lidar localization for autonomous driving in rain." *IEEE/IROS* (2018).