

# **Real-time tuning of a Hybrid Nonlinear Energy Sink : simulation and experiment**

Louis Mesny, Roberto Alcorta, Simon Chesné, Sébastien Baguet

## **To cite this version:**

Louis Mesny, Roberto Alcorta, Simon Chesné, Sébastien Baguet. Real-time tuning of a Hybrid Nonlinear Energy Sink : simulation and experiment. European Nonlinear Dynamics Conference, Jun 2024, Delft, Netherlands. hal-04727146

## **HAL Id: hal-04727146 <https://hal.science/hal-04727146v1>**

Submitted on 10 Oct 2024

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

### Real-time tuning of a Hybrid Nonlinear Energy Sink : simulation and experiment

### Louis Mesny† , Roberto Alcorta† , Simon Chesné† and Sébastien Baguet † † *INSA Lyon, CNRS, LaMCoS, UMR5259, 69621 Villeurbanne, France*

*Summary*. This article studies a hybrid nonlinear vibration absorber. It consists of a passive part (a Nonlinear Energy Sink) and an active part (electromagnetic transducer). This work aims to tune the internal properties of the NES in real time, such that the activation threshold and effective energy pumping mechanism are maintained under varying operating conditions. To this end, numerical tools are developed for the design of suitable control laws which are then validated on a structure with varying parameters.

#### Inroduction

In contrast with conventional linear vibration absorbers, whose effectiveness depends on precise tuning at a given frequency, Nonlinear Energy Sinks (NES) are able to perform robustly over a wide frequency range. Indeed, their nonlinear nature enables an irreversible energy transfer for sufficiently high oscillation amplitudes, effectively pumping energy from the main structure regardless of vibration frequency [1, 2]. This *activation threshold* is thus a key parameter for NES design, which depends strongly on the properties of the structure to be controlled. Consequently, the practical implementation of such an absorber can be quite tricky and its performance is likely to decrease upon variations of, e.g. the natural frequency of the primary structure.

In this work, we consider hybridization as a means to overcome this limitation, by including an active controller coupled to the passive NES and developing an algorithm for performance stabilization. The resulting Hybrid Nonlinear Energy Sink (HNES) allows for real-time modifications to the absorber settings. This ensures that irreversible energy transfer is activated, providing consistent protection to the latter.

#### **Theory**

Let us consider a reduced-order model representing the forced response of a linear structure near its first natural frequency  $\omega_0$ , on which the HNES is positioned. Following modal projection, a single structural mode is retained and the absorber is idealized as a mass-spring-damper system with cubic stiffness, giving the coupled equations of motion:

$$
\begin{cases}\nm_1\ddot{x}_1 + c_1\dot{x}_1 + k_1\dot{x}_1 + k_2(x_1 - x_2) + c_2(\dot{x}_1 - \dot{x}_2) + k_{nl2}(x_1 - x_2)^3 = \Gamma\sin(\omega t) + F_a(t) \\
m_2\ddot{x}_2 + k_2(x_2 - x_1) + c_2(\dot{x}_2 - \dot{x}_1) + k_{nl2}(x_2 - x_1)^3 = -F_a(t)\n\end{cases} \tag{1}
$$

where  $x_j$  denote displacements, and the subscripts  $j = 1$ ,  $j = 2$  correspond to the primary structure and the absorber, respectively. In practice, the modal parameters are identified experimentally. Moreover, Γ is the forcing amplitude and the term  $F_a(t)$  represents the active force from the controller, which introduces a *synthetic* nonlinear absorber of the form:

$$
F_a(t) = k_c(t)(x_1 - x_2) + c_c(t)(\dot{x}_1 - \dot{x}_2) + k_{nlc}(t)(x_1 - x_2)^3
$$
\n(2)

where  $k_c(t)$ ,  $k_c(t)$  and  $k_{nlc}(t)$  are the time-dependent stiffness, damping and nonlinear stiffness control gains, respectively. A successful HNES implementation depends on deducing the optimal form of these parameters.

To this end, we propose two different approaches. The first one is analytical, based on the computation of the Slow Invariant Manifold (SIM) of the controlled system, Eq. (1), under the assumption of 1:1 resonance ( $\omega \approx \omega_0$ ) [3, 4]. This allows us to assess the impact of active control on the activation threshold and pumping mechanism by studying the SIM's critical points. When a generic control term is incorporated into the system, it can be shown that the corresponding SIM in real coordinates is defined by:

$$
\sqrt{\left(N_2 - \frac{3}{4}K_{nl2}N_2^3 - K_2N_2 + \text{Im}(\hat{F}_a(N_2))\right)^2 + \left(\lambda_2N_2 + \text{Re}(\hat{F}_a(N_2))\right)^2} = N_1
$$
\n(3)

where  $N_1$  and  $N_2$  are related to the displacement of the primary structure and to the absorber stroke, respectively. The parameters  $K_{n/2}$ ,  $K_2$  and  $\lambda_2$  are the dimensionless counterparts of  $k_{n/2}$ ,  $k_2$  and  $c_2$ , respectively, following a re-scaling of time:  $\tau_0 = \omega_0 t$ . The term  $\hat{F}_a(N_2)$  corresponds to the active force in the complex domain.

The second approach is based on numerical bifurcation tracking. Indeed, the activation threshold directly correlates to Neimark-Sacker bifurcations on a frequency-response curve, which enclose an interval of quasi-periodic responses characteristic of strong modulation induced by energy pumping. Therefore, an implementation of pseudo arc-length continuation based on the harmonic balance method [5] is used to detect and track these bifurcations with respect to the primary structure's parameters. The stability boundaries generated in this manner are exploited to construct the functions  $k_c(t)$ ,  $c_c(t)$  and  $k_{nlc}(t)$ .

#### Experimental application

As a concrete example, we consider a primary structure representing a one-story building 50 cm high and 25 cm wide (see Fig. 1 a)), whose natural frequency is approximately 13 Hz. An HNES is installed on top of this structure (Fig. 1 b)), and the mass ratio between both sub-systems is  $\approx 7\%$ . Following [6], the absorber nonlinearity is realized by embedding two metal strings on each side of the moving mass. The active part is managed by electromagnetic transducers, used both as sensors and actuators.



Figure 1: Illustration of experimental setup of HNES on primary structure: a) global view, b) absorber close-up view, c) numerical SIM for varying primary system frequency.

Fig. 1 c) shows the evolution of the SIM upon variation of the natural frequency of the primary structure. It can be seen that the quantity  $||N_1^2(D_1) - N_1^2(D_2)||$ , i.e. the structure stroke, increases considerably with natural frequency, meaning that the activation threshold is higher. In this application, a robustness study is carried out by modifying the mass of the primary structure and therefore its natural frequency and therefore the activation threshold, in order to validate the proposed algorithm.

#### **Conclusions**

In this study, a performance-stabilization algorithm for a new type of absorber called Hybrid Nonlinear Energy Sink (HNES) was developed. The active control enables real-time modification of the absorber's parameters. The effectiveness of this algorithm was tested numerically, and experimental studies allowed validation of its efficiency and robustness on an example structure. In the present case, the control was used to maintain the activation threshold at a given level, but this same approach could be used to control any other parameter of the absorber. This work gives rise to a large number of perspectives and potential applications. Indeed, this study focused on adapting and optimizing the robustness of the passive NES; on the other hand, it may be possible to further improve the absorber's performance by dissipating electrical energy via the active control, similarly to what was previously realized on hybrid mass dampers [7].

#### References

- [1] A. F. Vakakis, O. V. Gendelman, L. A. Bergman, D. M. McFarland, G. Kerschen Y. S. Lee (2009). Nonlinear Targeted Energy Transfer in Mechanical and Structural Systems. Springer Dordrecht.
- [2] Kerschen, G., Lee, Y. S., Vakakis, A. F., McFarland, M. D., Bergman, L. A. (2005). Irreversible Passive Energy Transfer in Coupled Oscillators with Essential Nonlinearity. SIAM Journal on Applied Mathematics, 66(2), 648-679.
- [3] Manevitch, L. (2001). The description of localized normal modes in a chain of nonlinear coupled oscillators using complex variables. Nonlinear Dynamics, 25, 95-109.
- [4] Y. Starosvetsky O.V. Gendelman (2008). Dynamics of a strongly nonlinear vibration absorber coupled to a harmonically excited two-degree-offreedom system. Journal of Sound and Vibration, 312, 234-256.
- [5] Xie, L., Baguet, S., Prabel, B., Dufour, R. (2017). Bifurcation tracking by Harmonic Balance Method for performance tuning of nonlinear dynamical systems. Mechanical Systems and Signal Processing, 88, 445-461.
- [6] Pennisi, G., Mann, B. P., Naclerio, N., Stephan, C., Michon, G. (2018). Design and experimental study of a nonlinear energy sink coupled to an electromagnetic energy harvester. Journal of Sound and Vibration, 437, 340-357.
- [7] Collette, C., Chesne, S. (2016). Robust hybrid mass damper. Journal of sound and vibration, 375, 19-27.