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## ► To cite this version:

Theo Boutin, Issam Bendaoud, Josselin Delmas, Damien Borel, Cyril Bordreuil. Development of a machine learning model for deviation from trajectory detection in multi-pass TIG welding in a narrow gap. Welding in the World, 2023, 68, pp.743 - 753. 10.1007/s40194-023-01626-z . hal-04725347

## HAL Id: hal-04725347 https://hal.science/hal-04725347v1

Submitted on 21 Oct 2024

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#### **RESEARCH PAPER**



## Development of a machine learning model for deviation from trajectory detection in multi-pass TIG welding in a narrow gap

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Received: 20 September 2023 / Accepted: 3 November 2023 © International Institute of Welding 2023

#### Abstract

This paper presents a novel approach to detect deviations in welding trajectory, using a Machine Learning model that predicts the occurrence of defects during narrow gap operations. A regression model is developed and trained using experimental data acquired from a number of synchronised sensors used in the TIG process with filler metal. Two primary sources of data are employed. The voltage, current and other electrical parameters are monitored as well as the changing dynamics of the weld pool, which are monitored by two cameras stationed behind and in front of it. An image processing algorithm is then used to extract the contour and geometrical characteristics of the weld pool, including its length, width, and area. The data is organized, stored, and referenced in a database for swift and easy processing. The database is created to train, test and validate the Machine Learning (ML) model. During welding, artificial deviations are introduced to examine the ML model's ability to predict the welding trajectory deviation. Various training models are evaluated to analyse their predictability within a defined set of parameters, in order to anticipate the occurrence of defects.

Keywords Welding · Welding physics · Image processing · Machine learning · Monitoring

## **1** Introduction

The safety regulations within the nuclear industry require thorough monitoring of manufacturing processes and weld repairs. Currently, non-destructive testing is conducted postoperation and corrective action must adhere to specific protocols in the event of non-conformity. The use of an online welding monitoring system capable of detecting deviations

Recommended for publication by Commission XII - Arc Welding Processes and Production Systems.

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would enable identification of changes in the bead geometry during manufacturing. This would safeguard the assembly's integrity from the impact of insufficient fusion or porosity. It is essential to monitor process parameters, such as heat input and filler metal rate, to comply with welding procedure specifications. In reality, alterations to process parameters will impact how energy is distributed to the component, thus affecting the shape of the welding pool (length, width, and penetration).

During on-site welding operations, components may be large and may have geometric defects, such as inaccurate cylindricity at the bottom of a narrow gap. Distortion can cause geometric changes and alter the conditions for deposited material as the weld moves. Applying too much energy to the edge may cause the bead to move up the groove (refer to Fig. 1). This defect can lead to problems such as porosity, cracking, or lack of penetration when subsequent layers are added.

Various sensors can be used to monitor fusion welding processes, obtaining information on weld bead profile or geometry [1, 2], arc variation pressure [3, 4], or arc emission [5–7]. Currently, industrial welding operations are monitored by observing the molten pool through a camera, with the operator providing feedback on the bead's regularity.



Fig. 1 Diagram of arc energy transfer in a narrow gap deviation to the left

In this paper, a sensor is used for monitoring the weld pool and for storing the resulting data in a database. Machine learning (ML) models are trained on, using this database.

Privious research [8–10] has proven the efficacy of using ML regression algorithms with sensor data to enhance welding activities. Regression models have proven to be especially proficient in predicting intermediate values, specifically for transient phenomena [8].

Sensor-based monitoring should be supplemented by experienced welders performing on-line checks. There are several types of deviations that can occur when welding, including sub-optimal positioning of the torch electrode in the narrow gap, resulting in undesirable arc behaviours. It is particularly important to position the torch correctly when working with large pipes because this can cause the weld pool to lose symmetry and modify arc behaviour.

The objective of this study is to accurately detect first pass deviations in a narrow gap. This is operationalized as identifying tungsten position deviations in the narrow gap, a configuration of torch deviation that has not been previously discussed in the literature. A method utilizing experimental data and machine learning is outlined in this paper, aimed at detecting tungsten position deviations using a simplified geometry in a narrow gap. The process is equipped with multiple measurements, including process parameters and weld pool observation. An image processing algorithm detects the contour of the weld pool in real-time to measure its shape. The database is fed by tests involving programmed torch exit from the centre of the narrow gap and a deviation to the left and right. A regression model determines the electrode's position in case of any deviations. In order to improve understanding of the database and regression model's effectiveness, the experiment aims to evaluate their efficiency. The regression model used geometrical characteristics of weld pools (ML features) to predict torch placement.

### 2 Experimental device

#### 2.1 Sensors

The experimental set-up for this investigation is shown in Fig. 2. Synchronized data acquisition is carried out using the acquisition card to be on the same time base. Process parameters are monitored by sensors attached to the welding generator. Two cameras capture the dynamics of the weld pool. One camera is situated so as to observe the weld pool's rear due to limited accessibility, while the other captures the front. All data is recorded and stored in a database to support input for the machine learning mode.

## 2.2 Materials, geometry, and dimensions of the weld test

The metallurgical composition of both the base and filler metals must conform to construction standards and codes. 316L austenitic stainless steel was used as the base and filler metals for the welding process with a wire diameter of 1 mm.

To facilitate the study, a simplified narrow gap geometry was employed, as illustrated in Fig. 3, which depicts the welding specimen's dimensions. The specimen was created using 150 mm in lenght, 70 mm in width, and 40 mm in thickness. The dimensions were selected to illustrate welding on a large part and to avoid distortion of the part. The depth of the small gap was determined to ensure easy access for the welding torch and effective shielding for the weld pool. This design could potentially enable the deposition of at least five beads. In this configuration, it is the torch that moves.

Forming the bead in a narrow gap using a torch as a heat source is complex. To position test specimens in a robotic reference frame, a device with the male shape of the gap is placed on the nozzle and electrode. This device, manufactured via additive techniques, ensures accurate positioning of the component at position 0, the midpoint of the narrow gap. The torch is positioned at the starting point, followed by the endpoint to create the trajectory. Next, the part is clamped, and the positioning is verified before welding to ensure alignment with the narrow gap, guaranteeing repeatable results across tests. Panasonic's anthropomorphic robot moves the torch, utilizing its 6-axis capability to maintain a linear path with an accuracy of less than 0.1 mm between the two points.

#### 2.3 Choice of variation domain

In order to demonstrate the cumulative impact and establish the extent of torch deviation, a test was conducted during the initial phase using a programmed torch deviation of 1 mm. The cameras are attached to the welding torches. Figure 4 depicts the observations made during the deviation at t = 50

## **Fig. 2** Overall wiring diagram of the experimental set-up



seconds. The weld pool had asymmetric, and the melt zone was misaligned due to improper arc alignment. The initial images show a smooth deposition process. However, at t = 78 seconds, the images show changes in the transfer resulting in the formation of a significant drop. A discontinuity of the weld pool is formed. The images show that the edge of the narrow gap has melted much more in this first pass.



Fig. 3 Dimensions of the narrow gap and definition of torch any offset

After the initial bead is created, a second bead is created at its centre. Figure 5 depicts the macrograph of the sample and an image of the front face captured at t = 78 seconds. A lack of fusion is visible between pass 1 and pass 2 at the base of the bead.

A relationship is identified between the position of the electrode, energy transfer, and the behaviour of the weld pool. The objective is to develop a method for determining the electrode's value in relation to the narrow gap centre, by examining weld pool contours.

A parametric domain will be established with torch offsets ranging from 0 to 0.6 millimetres in both directions (refer to Fig. 6) which will serve as the learning base (cf. Section 4). This domain selection enables the creation of a regression model that can calculate deviations within a range of variation that ensures bead continuity.

The range of parameters is defined by torch deviations within the range of -0.6 to 0.6 millimetres so as to guarantee the absence of defects. A database is created which includes the minimum, maximum and 0 boundaries of the range (see Fig. 7). A test was conducted to verify the positioning symmetry of the robot by introducing right and left deviations. The wide applicability of the database is an advantage that takes account of any asymmetry caused by the operational conditions.

**Fig. 4** Operation with a torch deflection of 1 mm. (a) Weld pool rear view at 50 seconds. (b) Weld pool front view at 50 seconds. (c) Weld pool rear view at 78 seconds. (d) Weld pool front view at 78 seconds



Rear view (a)



Behind view



Rear view (c)



Behind view (d)

## 2.4 Welding parameters

A TIG welding process was performed and observed as described previously, with the aim of analysing variations in the geometry of the weld pool during operation with constant welding parameters (voltage, current, wire feed rate, gas flow). The welding parameters used are set out in Table 1.

The acquisition card inputs have an acquisition frequency of 1 kHz for the process parameters, whilst the cameras have an acquisition frequency of 50 Hz.

## 3 Method

## 3.1 Image processing

Figure 8 (a) shows raw image that can be used to extract various data, including the size of the weld pool. The investigation of weld pool geometry is essential, as it changes according to process parameters. Periodic evaluations of the weld pool's geometry are necessary to precisely determine changes and dynamics of its geometrical characteristics. Also, an algorithm is used to detect the edges of the weld pool

**Fig. 5** Weld pool front view and macrographic section during welding of the second pass





Front view : Pass 2

Lack of fusion between Pass 1-2



Fig. 6 Definition of torch offset

allowing this particular part of the image to be extracted. For this purpose, an image processing algorithm utilizing contour detection [11] is employed to evaluate the geometry, as illustrated in Fig. 8. The processing steps for the weld pool image are also presented in the Fig. 8. The convex hull contour (highlighted in red in Fig. 8 (d)) is subsequently utilized to extract geometric characteristics.

A polygonal contour is then generated. The geometric characteristics of the welding process can be captured as a function of the location of the torch heat input and the filler metal. Figure 9 (a) illustrates all outline features in a clear schematic representation. The welding torch has a camera mounted on it, and the camera frame is aware of the torch position. Automated systems are used for image acquisition, processing and extraction of geometric features to ensure reliable online control [12]. The processing steps and their respective durations are displayed in Fig. 9 (b).

The algorithm must be adaptable and strong enough to account for the grey level fluctuations resulting from the arc. This fluctuation is mainly determined by the programmed current in the welding generator [12].

#### 3.2 Weld pool features

Figure 9 shows one image whilst Fig. 10 shows three sets of images (three different operations) of the geometric features extracted by the image-processing algorithms. These quantities can be used to derive information about the stability and repeatability of the weld pool. The study of geometric variation in relation to heat input is facilitated by contour extraction. The scatter plots shown in Fig. 10 show clear separation for certain characteristics, including "weld pool min - tungsten position", "weld pool max - tungsten position" and area asymmetry, while others are mixed. These deviations show that the weld pool has different wetting characteristics in the narrow gap.

Finally, ML algorithms are tested on experimental data to predict any possible deviations. A new database will be created based on the geometric features of the weld pool. This approach helps to streamline the database, reduces storage size, and reduces training time for testing various ML algorithms.

## 4 Machine learning

#### 4.1 Regression

A regression model [13, 14] consists of a commonly used and easily explicable supervised learning approach. The primary goal of regression is to predict a real valued output (also known as the target variable: Y) using a real-valued data vector as input (variable X). When the input is one-dimensional, the model is approximated by the linear equation 1 that correlates the input to the output.

$$f(x) = Ax + B \tag{1}$$

The most commonly used method is the least squares approach, as applied in this paper. Figure 11 presents a linear regression, with vertical bars depicting variations between observed output (blue circle) and expected value (orange cross) for each input. The optimal line minimises the sum of the discrepancies.

The fundamentals of machine learning involve the learning model, also known as the f function, that relies upon a database. The process of machine learning consists of multiple stages, including a learning phase designed to train or modify the model and a test phase to assess the chosen model's performance.

#### 4.2 Training, testing

In this study, the geometric properties of the weld pool shape are utilised as input models to forecast the welding robot's trajectory position. The data is labelled with the torch offset setpoint in the narrow gap. Table 2 provides a summarisation of the data which is inserted into the ML model.

In order to enhance the visualisation of test data, a single geometric characteristic from the contour is extracted. This pertains to the maximum distance between the electrode's



Fig. 7 Parametric domain without defects for machine learning model

Table 1TIG weldingparameters used

Weld Nr.	Current	Voltage	Welding	Wire feed	Gas flow	Torch offset
	I [A]	U [V]	Ws [m/min]	Wf [m/min]	[L/min]	[mm]
1	180	9.5	0.08	1.3	12.0 (Ar)	0
2	180	9.5	0.08	1.3	12.0 (Ar)	0.6
3	180	9.5	0.08	1.3	12.0 (Ar)	- 0.6

Fig. 8 (a) Image rectification via a calibration test pattern. (b) Use of an adaptive threshold filter. (c) Use of a Canny filter. (d) Edge detection using the shape method coupled with a graph algorithm (blue) and convex contour (red)



(a)

(c)





(d)





(a)





Fig. 10 Geometric features from weld pool contour feeding the ML models

location and a point on the contour's end in a direction perpendicular to the weld (refer to Fig. 12). This distance allows to track the torch's position in the narrow gap, taking into consideration the inherent fluctuations of the weld pool.



Fig. 11 Simple regression, which consists of minimizing the sum of the squared residuals [15]

The distances for each programmed offset setpoint (0, -0.6, 0.6) are illustrated in Fig. 13. The colour of the points indicates the programmed offset for the linear trajectory. Each operation consists of 2500 observations to ensure equal representation of the various classes in the database. The xaxis indicates the torch position in relation to the contour's end, and the y-axis indicates the robot position setpoint. Notable fluctuations in the weld pool are evident for any deviation. The variations in the weld pool mainly occur due to changes in energy transfer, which can cause the estimated torch position in relation to the contour to vary by up to 0.2 mm. Despite the range of torch positions relative to the contour, there is a distinct linear relationship between the programmed position and the associated feature. As a reminder, the robot's position between two points is 0.1 mm. The corresponding data can be labelled using the torch position setpoint.

Regression analysis is performed on all data points. The coefficient of determination is close to one, confirming that the geometric feature can represent the position of the elec-

Characteristics number	Input	Output
0	Distance weld pool min - Position tungsten (Dist Ymin - Posi_W)	0
1	Distance weld pool max - Position tungsten (Dist Ymax - Posi_W)	
2	Length	0.6
3	Width	
4	Area asymmetric left (Area DIssy left)	
5	Area asymmetric right (Area DIssy right)	
6	Centroid X (gravity centre X)	
7	Centroid Y (gravity centre Y)	- 0.6
8	Perimeter	
9	Area	

trode. However, the regression line does not intersect the origin. This indicates that there is an asymmetry between right and left. This may be due to the weld pool behaving differently on each side or the robot not positioning itself accurately at the intended point.

A simple approach is evaluated given the linear trend observed in Fig. 13. Multiple linear regression is used, using the attributes of the input contour (as outlined in Table 2) to determine the programmed position of the torch.



**Fig. 12** Distance between the tungsten position and weld pool extremity and distance between the tungsten position and top of the narrow gap for verification purposes

The model is trained using the geometric characteristics of the weld pool for three categories (Ma1, Ma2 and Ma3) as input, as well as the robot's trajectory position (0 mm, -0.6 mm, 0.6 mm) in output. The regression model achieves a high accuracy of 98%, suggesting its robustness for predicting specific configurations. The learning time is 0.64 seconds.

## 4.3 Prediction

A welding procedure tests the regression model's prediction. The tungsten's position is altered in both directions in order to artificially program trajectory deviation during the welding process. Initially, the torch shows no deviation, but after about 25 seconds, it moves to the left and then to the right at approximately 50 seconds. At the end of the procedure, the robot trajectory returns to its initial position. There could



Fig. 13 Training database (only one feature presented) and model selection



Fig. 14 Deviation of the welding torch during the operation

potentially be a discrepancy between the programmed and actual position of the electrode. In order to confirm that the torch was indeed misaligned, the position of the tungsten electrode relative to the centre of the narrow gap is measured (see Fig. 14). A slight offset was observed, which could be attributed to interpolation or a gap in the robot's swivel.

Figure 15 illustrates the outcome of the linear regression and the measured distance between the edge of the restricted gap and the electrode (identified as "top narrow gap - posi W" on the illustration). The robot moved 0.6 mm to the left and 0.6 mm to the right. When the robot moves to the left of the narrow gap, the model accurately predicts the transition zone until it anticipates a deviation of approximately -0.5 mm However, the points captured in the image indicate a deviation of 0.6 mm. hen the robot moves 1.2 mm (between



Fig. 16 Significance of geometrical characteristics of the regression model shown as a histogram. Feature numbers are referenced in Table 2

-0.6 and 0.6 mm), it is predicted to deviate by 0.4 mm, but it actually deviates by 0.5 mm. When the robot then returns to its original position, the model can predict this condition. Based on direct observation, this study demonstrates that the geometrical features of the weld pool can serve as an effective indicator of welding deviations. The inherent characteristics of the weld pool and gaps or interpolation in the robot's joints are possible sources of underestimation in torch position prediction.



Any deviation

Left deviation

**Right deviation** 

**Fig. 15** Prediction of the linear regression used to detect a variation in the position of the tungsten on the left and right. The images shown are acquired by the camera with the field of view of the front panel

#### 4.4 Significance of geometrical characteristics

The regression model's training coefficients are shown. Figure 16 shows the significance features in the tested machine learning-model using a histogram. Each number on the abscissa from 0 to 9 corresponds to a weld pool feature and the corresponding numbers are cross-referenced in Table 2.

It can be observed that for all the models, the distances between the tungsten and the end of the melt/weld pool have the highest coefficients. We can also observe that the asymmetric areas also have high coefficients. The width, length, centroid (X and Y), areas, and perimeter have very low coefficients, but this is not very surprising. For instance, although moving the tungsten alters the streamlines of the narrow gap, the width is limited by the gap's size. This modification, however, affects how far the base metal is penetrated along the triple lines.

## **5** Conclusion

This approach employs physical data pertaining to weld pool dynamics (specifically, the weld pool contour) to forecast any deviations from a nominal trajectory. Through this study, a linear regression model designed to identify such deviations is validated, which further enables us to predict all the intermediate positions. This results in a continuous aspect of the prediction based on weld pool images. This model can detect even the slightest disparity of 0.6 mm and every intermediate position, that is a local data point. It should be acknowledged that attempting to extrapolate results beyond the ranges of the training set may result in significant errors. Implementing this methodology based on geometrical characteristics has led to a reduction in storage size, as it is less costly to store a vector of features over time than complete images. Furthermore, this approach has shown that geometrical characteristics support the identification of variations in the welding torch's position during operation. Additional regression techniques, such as GPR or SVR, may yield comparable outcomes, yet linear models are usually simpler to execute and interpret. The advantage of this regression model, which relies on the geometrical features of the weld pool, is that it can be adjusted to detect alterations in the motion of a carriage or any other welding method.

**Acknowledgements** The authors would like to acknowledge the financial support of the Agence Nationale de la Recherche and Electricite de France.

#### **Author Contributions**

**Theo Boutin:** Conceptualization (lead), Data curation (lead), Formal analysis (lead), Investigation (lead), Methodology (lead), Software (lead), Supervision (lead), Project administration (lead), Validation(lead), Visualization (lead), Writing - original draft (lead), Writing - review (lead). **Issam Bendaoud:** Conceptualization (supporting), Data curation (supporting), Formal analysis (supporting), Investigation (supporting), Methodology (supporting), Software (supporting), Validation (supporting), Visualization (supporting), Writing - review (supporting).

**Josselin Delmas:** Project administration (lead), Funding acquisition (lead), Resources (lead), Supervision (supporting), Conceptualization (supporting), Data curation (supporting), Formal analysis (supporting), Investigation (supporting), Methodology (supporting), Software (supporting), Validation (supporting), Visualization (supporting), Writing - review (equal).

**Damien Borel:** Conceptualization (supporting), Data curation (supporting), Formal analysis (supporting), Investigation (supporting), Methodology (supporting), Software (supporting), Validation (supporting), Visualization (supporting), Writing - review (supporting).

**Cyril Bordreuil:** Supervision (lead), Project administra- tion (equal), Resources (supporting), Conceptualization (supporting), Data curation (supporting), Formal analysis (supporting), Investiga- tion (supporting), Methodology (supporting), Software (supporting), Validation (supporting), Visualization (supporting), Writing - review (equal), Writing original draft (supporting).

#### Declarations

**Competing interest** The authors declare that they have no known competing financial interests or personal relationships that may have appeared to influence the work reported in this paper. This work is funded by Agence Nationale de la recherche and Electricite de France.

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