



**HAL**  
open science

# Stabilizing control design for liquid propelled rocket engines

Jules Gibart, Piet-Lahanier H el ene, Francois Farago

► **To cite this version:**

Jules Gibart, Piet-Lahanier H el ene, Francois Farago. Stabilizing control design for liquid propelled rocket engines. Space Propulsion conference 2024, May 2024, Glasgow, United Kingdom. hal-04724741

**HAL Id: hal-04724741**

**<https://hal.science/hal-04724741v1>**

Submitted on 7 Oct 2024

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destin ee au d ep ot et  a la diffusion de documents scientifiques de niveau recherche, publi es ou non,  emanant des  tablissements d'enseignement et de recherche franais ou  trangers, des laboratoires publics ou priv es.

## STABILIZING CONTROL DESIGN FOR LIQUID PROPELLED ROCKET ENGINES

### SPACE PROPULSION 2024

GLASGOW, SCOTLAND | 20 – 23 MAY 2024

Jules Gibart <sup>(1)</sup>, Piet-Lahanier Hélène <sup>(1)</sup>, Farago François <sup>(2)</sup>

<sup>(1)</sup> ONERA DTIS, Palaiseau, France

<sup>(2)</sup> CNES, Paris, France

**KEYWORDS:** propulsion control, non-linear systems, port-hamiltonian systems, contraction theory

#### ABSTRACT:

The recent development of reusable rocket launchers has led to new requirements for liquid propelled rocket engines as the reusable trajectory planning and landing of the engine stage leads to performing complex maneuvers. In this paper, a Port-Hamiltonian model is derived from the state space equations of a liquid propelled rocket engine. From this modeling, conditions are searched out to show the passivity of the system around a functioning point of the engine. This paper additionally proposes the use of contraction theory to build a suiting stabilizing controller for the system. The controller is illustrated with simulations.

#### 1. INTRODUCTION

In the wake of reusable rocket engines, the requirements for maneuverability and knowledge on Liquid Propelled Rocket Engines (LPRE) have increased. Following the need of completing more complex maneuvers, the development of electrical actuators has left room for the design of new control laws. LPREs constitute complex non-linear systems that contain several unknown parameters, which are often approximated. As the modeling of LPREs becomes more precise [1] and evolves, the control law methods used require more flexibility. In a previous work [2], the stability of a transient model of a rocket engine has been tackled, however, the determination of a Lyapunov function for this type of system proves to be a difficult task. Moreover, such method for the determination of a Lyapunov function copes badly with the addition of new dynamics to the system. This can lead to the stability demonstration not to be viable anymore. A solution would be to develop an approach in which the dynamics added present guarantees for the system to remain stable, as well as an intrinsic Lyapunov function that can be added to the previous one. Following this observation, the choice has been made to introduce a Port-Hamiltonian [3] formulation of the non-linear systems constituting

an LPRE. Such a formulation allows for a closer analysis of the different elements and the simplified introduction of new dynamics. Port-Hamiltonian systems present an intrinsic energy function, that under certain conditions prevails as a Lyapunov function, guaranteeing the stability of the system around a chosen functioning point.

To perform complex maneuvers however, fixed-point stability is not sufficient. In this paper, the determination of the appropriate thrust for the maneuver is not computed, and we only consider pre-set scenarios for the thrust. For such trajectory tracking, it is possible to define enough intermediary points to ensure the stability along the trajectory; however, such a method requires additional computation during the process. An alternative solution is to analyse the convergence of the trajectories themselves. Contraction theory [4] considers the flow of possible trajectories of a system and relies on proving the convergence of the flow. Recent progress in contracting control design [5] has led to the development of control methods using contraction theory. In [6], a contraction based controller is used for friction compensation in trajectory tracking, while in [7] the control of mechanical systems using contraction theory is depicted. In this paper, we use contraction theory on an affine input nonlinear system, for trajectory tracking objectives. A previous work [8] tackled the problem on a simplified model of an LPRE, while providing a good understanding of the contracting controller.

The paper is organized as follows. The model used is presented in section 2. The Port-Hamiltonian approach is then depicted in section 3 and the stabilizing control using contraction theory is described in section 4. Concluding remarks close the paper in section 5.

#### 2. MODEL PRESENTATION

##### 2.1. Principle of an LPRE

An LPRE is a complex system that provides a thrust to a body through a combustion. For a reusable engine, liquid propellers are used, in this case oxygen O<sub>2</sub> and hydrogen H<sub>2</sub> are chosen for the fuel and oxidizer species. Storage of the liquid species

requires cryogenic temperatures, leading to low temperatures in the supply lines. The fuels are supplied using turbo-pumps situated between the liquid tanks and the combustion chambers (see figure 1). To power the turbo-pumps, a gas generator (GG) consumes a portion of the fuels into a secondary combustion chamber and the resulting hot gases are redirected into the turbine end of the turbo-pumps. Each pump is independent and the fraction of gas supplied to each pump is controlled using a valve ( $Z_{HG}$  on figure 1). Four additional valves ( $S_{VGH}, S_{VGO}, S_{VCH}, S_{VCO}$ ) are placed to control the

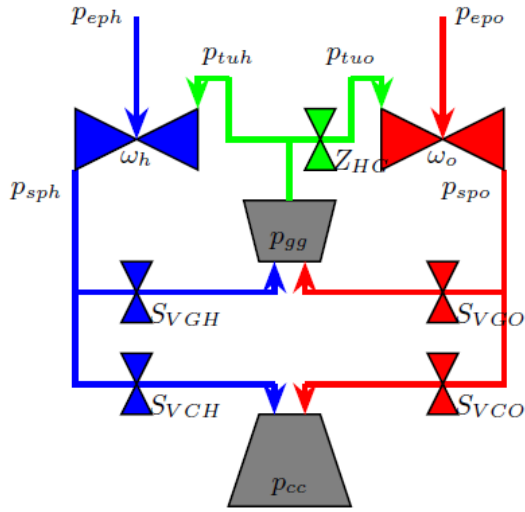


Figure 1: LPRE model considered

mass flows in the different pipes (into the combustion chamber and into the gas generator). The modeling of the LPRE stands in the modeling of several physical phenomena, divided here in different domains:

- The **mechanical** domain is composed of the equation of the conservation of the angular moment of the turbo-pump axis of rotation.
- The **hydrodynamical** domain contains the evolution of the mass flows in the pipes, as well as the pressure increase in the cavities due to the hydraulic effect of the flows.
- The **chemical** domain represents the evolution of the pressures in the combustion chambers following the chemical reaction of the combustion.
- The **thermic** domain, containing the evolution of the heat exchanges between the different elements.

For simplification, several hypothesis are considered for the modeling of the LPRE:

- The thermic phenomena are neglected in this approach, since the impact on the chamber temperature of the heat exchange with the cooling circuit is negligible. However, a static temperature increase is modeled by raising the temperature of the fluid flowing into the chamber.

- The different densities of the fluids are considered constant over time in the delivery circuit to the combustion chambers. The density is approximated in the hot gases lines using the perfect gas equation.
- The actuators dynamics are neglected, and the controller is set to provide a control surface to the system with no additional dynamics.
- The symmetry of the system is exploited, and by supposing that the Oxygen half of the system is well-regulated, it is possible to fix the mixture ratios in order to only consider the Hydrogen part of the system.
- The turbo-pumps' dynamics are computed using manufacturer data, under the form of polynomial functions. The functions' validity domain is supposed to be the study domain.

## 2.2. State equations of the model

In this paragraph, the state equations of the system are given following the modeling in [2]. The equations are expressed for the Hydrogen half of the model, with the complete equations given in 2.3.

### 2.2.1. Turbo-pumps

The dynamics of the turbo-pump rotational speed  $\omega_H$  is obtained using the conservation of the kinetic moment:

$$\dot{\omega}_H = \frac{1}{J_H} (T_{mH} - T_{rH}) \quad \text{Eq.1}$$

where  $J_H$  is the angular inertia of the turbine,  $T_{mH}$  and  $T_{rH}$  are the motor and resistive torques respectively. The modeling of the turbo-pump dynamics requires to determine the resisting torque  $T_{rH}$  applied by the fluid in the line and the motor torque  $T_{mH}$  generated by the turbine. The resistive torque is expressed following

$$T_{rH} = - \left| \frac{a_{cH}}{\rho_H} q_H^2 + b_{cH} \omega_H q_H + c_{cH} \rho_H \omega_H^2 \right|, \quad \text{Eq.2}$$

where  $a_{cH}, b_{cH}, c_{cH}$  are manufacturer parameters for the turbo-pump,  $\omega_H$  is the rotational speed of the pump,  $\rho_H$  is the density of the fluid and  $q_H$  is the mass flow through the pump with  $q_H = q_{cCH} + q_{ggH}$  ( $q_{cCH}, q_{ggH}$  are the mass-flows in the CC and GG respectively).  $T_{mH}$  is then expressed considering the evolution of the hot gases produced by the GG,

$$N_R = \frac{\omega_H \text{Rad} T}{\sqrt{\gamma R \Theta}} \quad \text{Eq.3}$$

$$W = q_{outH} \frac{\sqrt{\gamma R T}}{\gamma - 1} \left( 1 - \pi_T^{\frac{1-\gamma}{\gamma}} \right) \text{Rad} T \quad \text{Eq.4}$$

$$ST = (a_1 + N_R (A_2 + N_R a_3) + \pi_T (a_4 + \pi_T a_5) + a_6 \pi_T N_R + a_7 \ln \pi_T + a_8 \ln N_R). \text{Corr} \quad \text{Eq.5}$$

$$T_{mH} = ST \cdot W \quad \text{Eq.6}$$

$$\dot{p}_{tuH} = k_{1tuH}\Theta_{gg}q_{tuH} - k_{2tuH}\sqrt{\Theta_{gg}}p_{tuH} \quad \text{Eq.13}$$

$$q_{outH} = \frac{p_{tuH}A_{tuH}}{\gamma} \sqrt{\frac{R\Theta}{\gamma} \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}} \quad \text{Eq.7}$$

Where  $q_{outH}$  is the mass flow through the turbine,  $a_1, \dots, a_8$  are specific coefficients for the turbo-pump and  $N_r$  is a reduced rotational speed.

### 2.2.2. Combustion chamber and gas generator

The chamber pressure's evolution follows the energy conservation equation:

$$\frac{d}{dx} \left( \frac{pV}{\gamma-1} \right) = (h_{in}q_{in} - h_{out}q_{out}), \quad \text{Eq.8}$$

where  $h$  is the enthalpy of the species consumed and created. The thermodynamic parameter  $\gamma$  is assumed constant during the evolution of the system. The enthalpy of the system is composed of the formation enthalpy  $h_f$ , the kinetic energy  $e_c$  and the loss due to the heating of the fluid  $C_v L_v \theta$ ,

$$h = h_f + e_c + C_v L_v \theta \quad \text{Eq.9}$$

0

where  $C_v L_v$  is the vaporisation heat of the fluid. The equation Eq.8 can be written as follows,

$$\dot{p}_{cc} = (k_{1cc} + k_{3cc} - k_{4cc}\Theta_{cc})(q_{ccH} + q_{ccO}) - k_{2cc}p_{cc} \quad \text{Eq.10}$$

Where  $k_{1cc}$  to  $k_{4cc}$  are obtained by first order Taylor expansion of the equation Eq.8,  $\Theta_{cc}$  is the temperature in the CC and  $q_{ccH}, q_{ccO}$  are the mass flows of the species entering the CC. With the use of the mixture ratio, the equation is

$$\dot{p}_{cc} = (k_{1cc} + k_{3cc} - k_{4cc}\Theta_{cc})(MR + 1)q_{ccH} - k_{2cc}p_{cc} \quad \text{Eq.11}$$

The evolution of the gas generator pressure  $p_{gg}$  is modeled with Eq.8, using the output mass flows rather than the internal pressure.

$$\dot{p}_{gg} = (k_{1gg} + k_{3gg} - k_{4gg}\Theta_{gg})(q_{ggH} + q_{ggO}) - k_{2gg}(q_{tuH} + q_{tuO}) \quad \text{Eq.12}$$

Where  $k_{1gg}$  to  $k_{4gg}$  are also obtained by first order Taylor expansion of Eq.8,  $\Theta_{gg}$  is the temperature,  $q_{ggH}, q_{ggO}$  are the entering mass flows in and  $q_{tuH}, q_{tuO}$  are the output mass flows of the GG. The temperature  $\theta$  of the combustion chambers is approximated by a function  $f_\theta$  of the mixture ration, i. e.  $\theta = f_\theta(MR)$ .

#### Turbine pressures

The evolution of the turbine pressures follows equation Eq.8, where no chemical reaction occurs.

Where  $k_{1tuH}, k_{2tuH}$  are obtained as the other coefficients.

#### Mass-flows

The dynamics of the mass flows are derived from the equation of momentum-conservation applied to the lines,

$$I \dot{q} = p_{in} - p_{out} - Res_{eq}q|q|. \quad \text{Eq.14}$$

Where  $I$  is the inertia of the line, of the form  $I = \frac{A}{L}$ ,  $A$  the area and  $L$  the length of the line,  $p_{in}, p_{out}$  are the upstream and downstream pressures,  $Res_{eq}$  is the equivalent resistance coefficient associated with pressure drops and  $q$  is the mass flow considered. Pressure drops are due to the valves  $S_{VGH}, S_{VGO}, S_{VCH}, S_{VCO}$ , injectors and friction in the lines.

Pressure drop due to the valves is explicated by

$$\Delta p = \frac{1}{2\rho A_v^2} q^2 \quad \text{Eq.15}$$

$$Res_v = \frac{1}{2\rho A_v^2}$$

Where  $A_v$  is the surface associated with the opening angle of the valve,  $\rho$  is the density of the fluid in the line. Pressure drops in the exhaust of the GG express as

$$\Delta p_{ptuH-gg} = \frac{Khy_{tuH}R_{gg}\Theta_{gg}}{p_{gg}} q_{tuH}^2,$$

$$\Delta p_{ptuO-gg} = \frac{Z_{res}R_{gg}\Theta_{gg}}{p_{gg}} q_{tuO}^2, \quad \text{Eq.16}$$

Where  $Z_{res}, Khy_{tuH}$  are equivalent resistance coefficients for the valve and the line. Due to the burned gases,  $\rho$  is not supposed constant in the turbine lines ( $q_{tuH}$ ). Using the perfect gas formula, its variation is expressed as a function of  $p_{gg}$ . Pressure drops associated with friction in the lines also follows equation Eq.15.

The pressure increase due to the pumps follows the equation

$$p_{spH} = p_{epH} + \frac{a_{pH}}{\rho_H} q_H^2 + b_{pH}\omega_H q_H + c_{pH}\rho_H\omega_H^2. \quad \text{Eq.17}$$

Where  $p_{spH}$  is the output pressure of the pump,  $p_{epH}$  is the output pressure of the tank. The other parameters are similar to Eq.2.

### 2.3. State equations of the global system:

$$\dot{p}_{cc} = (k_{1cc} + k_{3cc} - k_{4cc}\Theta_{cc})(q_{ccH} + q_{ccO}) - k_{2cc}p_{cc}$$

$$\dot{p}_{gg} = (k_{1gg} + k_{3gg} - k_{4gg}\Theta_{gg})(q_{ggH} + q_{ggO}) - k_{2gg}(q_{tuH} + q_{tuO})$$

$$\dot{p}_{tuH} = k_{1tuH}\Theta_{gg}q_{tuH} - k_{2tuH}\sqrt{\Theta_{gg}}p_{tuH}$$

$$\begin{aligned}
\dot{p}_{tu0} &= k_{1tu0}\Theta_{gg}q_{tu0} - k_{2tu0}\sqrt{\Theta_{gg}p_{tu0}} \\
\dot{q}_{ccH} &= \frac{A_{VCH}}{L_{CH}}\left(p_{spH} - p_{cc} - \left(\frac{1}{2\rho A_{VCH}^2} + Res_{cch}\right)q_{ccH}^2\right) \\
\dot{q}_{cco} &= \frac{A_{VCO}}{L_{CO}}\left(p_{spO} - p_{cc} - \left(\frac{1}{2\rho A_{VCO}^2} + Res_{cco}\right)q_{cco}^2\right) \\
\dot{q}_{ggH} &= \frac{A_{VGH}}{L_{GH}}\left(p_{spH} - p_{gg} - \left(\frac{1}{2\rho A_{VGH}^2} + Res_{ggH}\right)q_{ggH}^2\right) \\
\dot{q}_{ggo} &= \frac{A_{VGO}}{L_{GO}}\left(p_{spO} - p_{gg} - \left(\frac{1}{2\rho A_{VGO}^2} + Res_{ggo}\right)q_{ggo}^2\right) \\
\dot{q}_{tuH} &= \frac{1}{I_{tuH}}\left(p_{gg} - p_{tuH} - \frac{Khy_{tuH}R_{gg}\Theta_{gg}}{p_{gg}}q_{tuH}^2\right) \\
\dot{q}_{tuO} &= \frac{1}{I_{tuO}}\left(p_{gg} - p_{tuO} - \frac{ZresR_{gg}\Theta_{gg}}{p_{gg}}q_{tuO}^2\right) \\
\dot{\omega}_H &= \frac{1}{J_H}(T_{mH} - T_{rH}) \\
\dot{\omega}_O &= \frac{1}{J_O}(T_{mO} - T_{rO})
\end{aligned} \tag{Eq.18}$$

### 3. PORT-HAMILTONIAN FORMULATION OF AN LPRE

The LPRE's state's evolution requires complex and highly non-linear equations, that are not adequate for an easy design of a control law. A modeling solution proposed here is to make use of the Port-Hamiltonian systems framework, and reformulate the LPRE model established under this formulation.

#### 3.1. Port-Hamiltonian Modeling

Port-Hamiltonian systems, described in [9], [10] and [3], used for the modeling of mechanical systems, have been extended to various physical domains as hydrodynamics in [11] and chemical processes in [12], [13]. Port-Hamiltonian modeling makes use of physical invariants of a system, such as the total energy, to highlight the energy transfers between the different elements composing a system. The complete model is broken down to the level a simple elements that each represent a simple equation. They are represented as independent subsystems, with two links: the effort  $e$  and the flow  $f$ . Both the effort and flow represent coupled physical quantities (e.g. pressure and flow). Elements can be classified into three distinct functions:

- Energy-storing, denoted by  $e_s, f_s$ , are elements that store energy (kinetic, potential, ...).
- Energy-dissipating, denoted by  $e_r, f_r$  dissipate the energy, using friction for example.
- Energy-routing, denoted by  $e_t, f_t$  transfers energy, with no storage nor loss. These elements are by construction perfect

transformers, therefore are seldom used in real systems.

Additionally, interactions with the exterior environment is denoted by  $e_p, f_p$ . The product  $e^T f$  is the power resulting of the flow and effort. Reconstructing an equation consists in regrouping the efforts to build the equation for the flow.

An important property resides in the energy balance of such a system:

$$e_s^T f_s + e_r^T f_r + e_p^T f_p = 0 \tag{Eq.19}$$

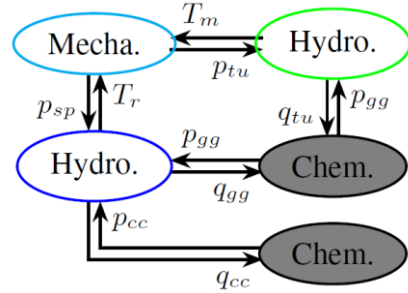


Figure 2: Physical domains of an LPRE

By definition, the dissipative elements yield  $e_r^T f_r \leq 0$ , therefore

$$e_s^T f_s \leq e_p^T f_p \tag{Eq.20}$$

Equation Eq.20 is a classical equation for the passivity of a system where the energetic input is  $e_p$  and the output  $y$  is  $f_p$ . Let the function  $H$  be defined by:

$$\frac{dH}{dt} = e_s^T f_s = \frac{\partial H}{\partial x} \dot{x} . \tag{Eq.21}$$

Such function  $H$  is called the Hamiltonian of the system that represents the total energy of the system. For  $H(x) > 0$  for all  $x$ ,  $H$  is a candidate storage function for the passivity of the system. From these definitions, a system under PH form is expressed

$$\dot{x} = (J(x) - R(x)) \frac{\partial H}{\partial x} + G(x)u \tag{Eq.22}$$

Where the matrix  $J(x)$  expresses the energy storage terms,  $J = -J^T$  and  $R(x)$  expresses the energy dissipation  $R(x) \geq 0$ .

#### 3.2. LPRE formulation

In this section, a reformulation under Port-Hamiltonian form is proposed for the system in Eq.18. The different physical domains as expressed in 2.2. can be connected using the different energy transfers between the domains. On figure 2 the energy transfers are represented between the different domains. While each domain can be represented on its own, the interconnection between the domains has to be studied to provide a complete PH system. In previous works [14], the PH framework has been extended to cover many

physical domains. Moreover, the interconnection of several physical domains has been proposed in [15].

### 3.2.1. Expression of subsystems under PH formulation

In this subsection, the individual subsystems are expressed under Port-Hamiltonian form, referring to previous works in the literature. The hydraulic part of the system is composed of the evolution of the mass flows and pressures. The coupled flow and efforts  $(e, f)$  are  $(q, I\dot{q})$  and  $(p, k_p\dot{p})$ , which both have the dimensions of a pressure/mass flow couple. From Eq.14, the different elements that drive the mass flow behavior are the resistive inertia of the pipe and the upstream and downstream pressures, respectively a resistive effort, the energetic input and the pressure effort (stoarge effort). This reasoning leads to the structure:

$$\begin{bmatrix} I\dot{q} \\ k_p\dot{p} \end{bmatrix} = \begin{bmatrix} -R_q & -1 \\ 1 & -R_p \end{bmatrix} \begin{bmatrix} q \\ p \end{bmatrix} + \begin{bmatrix} p_{in} \\ 0 \end{bmatrix} \quad \text{Eq.23}$$

where  $R_q$  and  $R_p$  are the resistive terms associated to the evolution of the mass flow and  $p_{in}$  is the input pressure of the pipe. This equation is applicable to the different mass flows of the LPRE, using the expressions of the input and output pressures of the flow. The equations of the pressures in the combustion chambers  $p_{cc}$  and  $p_{gg}$  allow for the chemical domain to be expressed under the same form provided the expression of  $R_p$  and  $k_p$ . In the case of the turbine pressure  $p_{tu}$ , no chemical domain is involved as there is no combustion. The mechanical subsystem is expressed using the port variables for a torque and a rotational speed

$$[J\dot{\omega}] = [-R_\omega][\omega] + [T_{mot}] \quad \text{Eq.24}$$

### 3.2.2. Interconnection of subsystems

Interconnection of several Port-Hamiltonian systems has been described in [3] [16]. Two Dirac structures  $D_A$  and  $D_B$  are considered on product spaces respectively  $F_1 \times F_2$  and  $F_2 \times F_3$ , where  $F_2$  is the space of shared flow variables. The interconnection is realized by equating the flows and efforts on the shared space of variables. The interconnection of the different hydraulic subsystems can be performed directly as the shared space of variables covers both the mass flows and pressures. However, interconnection between mechanical and hydraulic spaces requires to find a shared space of variables. Similarly to the work performed in [15], the method used consists in finding a function to express the torque and the pressure under a common equation leading to the creation of the shared space. In this paper, the pressure and torque have non-reversible equations. A proposition to solve the interconnection is to use an upper value of the torque, and considering the other terms as resistive terms.

Let  $\alpha_q = \frac{b_p \rho}{a_c}$ . Let also  $\bar{p} = b_p q \omega$  be an upper value for the pressure increase of the pump (see Eq.17), and  $\underline{T} = a_c (q_{cc} + q_{gg})^2$  be a lower value of the resistive torque generated by the fluids on the pump (see Eq.2). By multiplying the equation of the resistive torque by  $\alpha_q$ , the evolution equations reads:

$$\begin{aligned} \alpha_q J \dot{\omega} &= -b_p q^2 - \alpha_q (b_c q + c_c \omega) \omega + \alpha_q T_m \\ I \dot{q} &= b_p q \omega - \left( Z_{eq} q - \frac{a_p}{\rho} q - \frac{c_p \omega^2}{q} \right) q + p_{ep}. \end{aligned} \quad \text{Eq.25}$$

This yields the PHS interconnection:

$$\begin{bmatrix} \alpha_q J \dot{\omega} \\ I \dot{q} \end{bmatrix} = \begin{bmatrix} -R_\omega & -b_p q \\ b_p q & -R_q \end{bmatrix} \begin{bmatrix} \omega \\ q \end{bmatrix} + \begin{bmatrix} \alpha_q T_m \\ p_{ep} \end{bmatrix} \quad \text{Eq.26}$$

where  $R_q = \left( Z_{eq} q - \frac{a_p}{\rho} q - \frac{c_p \omega^2}{q} \right)$  and  $R_\omega = \alpha_q (b_c q + c_c \omega)$  have to be positive to satisfy the upper and lower value hypothesis. Here two conditions are derived for the validity of the PH formulation of the system, which can be used as validation test for the stability of the system. Similarly, by posing  $\alpha_t = \alpha_q S T(\omega) w_{tu}$ , where  $w_{tu} = \frac{W}{p_{tu}}$  it is possible to obtain the interconnection for the turbine:

$$\begin{bmatrix} \alpha_q J \dot{\omega} \\ k_{ptu} \dot{p}_{tu} \end{bmatrix} = \begin{bmatrix} -R_\omega & \alpha_t \\ -\alpha_t & -R_{ptu} \end{bmatrix} \begin{bmatrix} \omega \\ p_{tu} \end{bmatrix}, \quad \text{Eq.27}$$

Where  $R_{ptu}$  is computed to match the original equation Eq.13.

The complete Hamiltonian function expresses as :

$$\begin{aligned} H(x) &= \frac{1}{2} \alpha_q J \omega^2 + \frac{1}{2} I_{cc} q_{cc}^2 + \frac{1}{2} I_{gg} q_{gg}^2 + \\ &\frac{1}{2} I_{tu} q_{tu}^2 + \frac{1}{2} k_{pcc} p_{cc}^2 + \frac{1}{2} k_{pgg} p_{gg}^2 + \frac{1}{2} k_{ptu} p_{tu}^2 \end{aligned} \quad \text{Eq.28}$$

The hydrogen subsystem under Port-Hamiltonian form is given in annex.

### 3.3. Derivation of passivity conditions

Denoting that the Hamiltonian function given in Eq.28 is positive,  $H$  is a candidate Lyapunov function, if the system verifies the conditions for the PHS structure. The construction of the interconnection focuses on the structure of the energy transfers, yielding  $J = -J^T$ . However, to justify the dissipativity of the system it is necessary to verify that the terms  $R_\omega, R_q, R_{ptu}$  are positive and do not lead to the creation of energy. With the three conditions fulfilled, the system is passive with respect to the energetic input  $p_{ep}$ .

The condition for the passivity relies on the construction of the model. During the model construction, it is possible to optimize the passivity analysis when given a choice between different representations. The most adapted form for passivity analysis is the overestimation of a parameter, and using correction terms that consume energy.

#### 4. STABILIZATION AND CONTRACTION THEORY

In the section 3., a method to prove stability around a functioning point of the system has been given. However, in the case of a maneuver to be performed, it is necessary to ensure stability along the trajectory and between the known functioning points. To address this problem, two solutions are available:

- Study the stability around each functioning point on the trajectory even by adding rallying points along the trajectory.
- Study the stability around the trajectory

The use of contraction theory allows the latter option to be chosen. The idea is to generate a trajectory that corresponds to the maneuver to be performed, and stabilize the system around this reference trajectory. In this section, a method for the application of contraction theory is presented.

##### 4.1. Contraction theory

Let a system of the form

$$\dot{x} = f(x, t) \quad \text{Eq.29}$$

We define a virtual displacement  $\delta x$  [17],

$$\delta \dot{x} = \frac{\partial f}{\partial x}(x, t) \delta x \quad \text{Eq.30}$$

A trajectory in the flow field  $\dot{x} = f(x, t)$  is a possible solution of the system Eq.29. The squared distance between two neighbouring trajectories is defined in [17] as  $\delta x^T \delta x$  and its rate of change as

$$\frac{d}{dt} (\delta x^T \delta x) = 2 \delta x^T \delta \dot{x} = 2 \delta x^T \frac{\partial f}{\partial x} \delta x \quad \text{Eq.31}$$

**Definition 1.** A contraction region [17], is a region of the state if the Jacobian  $\frac{\partial f}{\partial x}$  is uniformly negative definite in that region,

$$\exists \beta > 0, \forall x, \forall t \geq 0, \frac{1}{2} \left( \frac{\partial f}{\partial x} + \frac{\partial f^T}{\partial x} \right) \leq -\beta I < 0 \quad \text{Eq.32}$$

**Theorem 1.** [17] Given the system equations  $\dot{x} = f(x, t)$ , any trajectory which starts in a ball of constant radius centered about a given trajectory and contained at all times in a contraction region, remains in that ball and converges exponentially to this trajectory.

This sufficient condition for contraction is seldom fulfilled with a system, therefore, we introduce a sufficient condition introduced in [18]

If there exists a constant matrix  $P \in \mathbb{R}^{n \times n}$ ,  $P^T = P$ , such that:

$$P \frac{\partial f}{\partial x} + \frac{\partial f^T}{\partial x} P < -\beta I. \quad \text{Eq.33}$$

Then the system defines a contraction on the

region. Moreover, for nonlinear systems, it is possible to decompose the system equation:

$$\dot{x} = f(x, t) = Ax + M\phi(y, t), y = Lx \quad \text{Eq.34}$$

where the linear and nonlinear parts of the system are separated,  $y$  is a vector composed of the states used in the nonlinear part of the equation,  $\phi$  is a  $C^1$  mapping and  $A, L, M$  are matrices of appropriate dimensions.

The condition Eq.33 reads:

$$P \left( A + M \frac{\partial \phi}{\partial y}(y, t) L \right) + \left( A + M \frac{\partial \phi}{\partial y}(y, t) L \right)^T P < -\beta I \quad \text{Eq.35}$$

**Remark 1.** Such expression of the system pairs efficiently with a PH system, as the matrix  $J$  used for the energy storing is often taken as close as the identity as possible, resulting in a good candidate for the matrix  $A$  in Eq.34.

In the case of the system studied not being contracting, it is possible to define a control to force the contraction of the closed-loop system. In [5], such a design is proposed, using an LMI solving approach to answer the problem. The benefit of using an LMI approach is the reduction of computation time, which is appropriate when dealing with embarked systems.

Let a system under the form Eq.34. Assume that for all  $(y, t)$ .

$$0 \leq \frac{\partial \phi}{\partial y}(y, t) + \frac{\partial \phi^T}{\partial y}(y, t) \quad \text{Eq.36}$$

Then, for a controlled system such that,

$$\dot{x} = f(x, t) = Ax + M\phi(y, t) + Bu, y = Lx \quad \text{Eq.37}$$

$u = Kx + N\phi(y, t)$  is a contracting controller if we can find three matrices  $Z, W, N$  such that:

$$\begin{aligned} K &= ZW^{-1} \\ (AW + BZ) + (AW + BZ)^T &\leq 0, W > 0 \\ WL^T &= -(M + BN). \end{aligned} \quad \text{Eq.38}$$

##### 4.2. Design of a contracting controller for an LPRE

In this section, the method for designing a contracting controller is depicted for the application of the LPRE. The controller must regulate the mass flows that enter the different combustion chambers. However, rather than a energy input to the system, such interaction with the system is a friction-based controller, where the controller action performs as a friction coefficient. The term associated to the friction exercised by the controller is under the form:

$$\frac{1}{2\rho A_b^2} q^2. \quad \text{Eq.39}$$

For simplification, the equivalent resistance of the

valve is used to represent the controller:

$$Z_{eq} = \frac{1}{2\rho A_v^2} \quad \text{Eq.40}$$

Denoting  $u = Z_{eq}$  in Eq.37, the matrix  $B$  is time-varying;

$$B(x) = [0 \ 0 \ -q^2 \ 0 \ \dots \ 0]^T, \quad \text{Eq.41}$$

which leads to the LMI Eq.38 being state dependent. This is a contradiction with the hypothesis Eq.33, where we suppose that the matrix  $P = W^{-1}$  is time-invariant. Note that a time-variant  $\mathcal{P}(t)$  matrix consists in an other formulation of the problem [5].

The method proposed here consists in fixing the LMI parameter  $P = P_0$ , which locks the value  $W = W_0 = P_0^{-1}$ . The value  $P_0$  is called the constant metric for the system. Additionally, let the introduction of the notion of virtual systems [19].

Let a system

$$\Sigma : \dot{x} = f(x) + g(x)u \quad \text{Eq.42}$$

And introduce the associated virtual system  $\Sigma_v$  with associated virtual state  $x_v$  and virtual control  $u_v$ ,

$$\Sigma_v : \dot{x}_v = f(x_v) + g(x)v_u \quad \text{Eq.43}$$

Here the state  $x$  is considered a constant value regarding the variations of virtual state  $x_v$ . Now suppose a contracting control  $u_v(x, x_v, t)$  for the virtual system Eq.43, for all  $x$  with respect to  $x_v$ . Then the controller  $u_v(x, x, t)$  is contracting for the real system Eq.42, (see [19], Theorem 2).

### 4.3. Affine controlled system

In the event of an affine controlled system of the form Eq.42, the decomposition looking forward to the LMI resolution Eq.37 yields:

$$\begin{aligned} \dot{x} &= f(x, t) + g(x)u = Ax + M\phi(y, t) + B(x)u \\ y &= Lx \end{aligned} \quad \text{Eq.44}$$

Where  $x$  is considered constant regarding the variations of  $x_v$ . In particular, for  $x = x_0$ , we are able to solve the LMI:

$$\begin{aligned} K_0 &= Z_0 W_0^{-1} \\ (AW_0 + B(x_0)\hat{B}(x)B(x_0)Z_0) &+ (AW_0 + B(x_0)Z_0)^T \leq 0, W_0 > 0 \\ W_0 L^T &= -(M + B(x_0)\hat{B}(x)B(x_0)N_0). \end{aligned} \quad \text{Eq.45}$$

$P_0 = W_0^{-1}$  is a constant metric of contraction for the virtual system Eq.44 for  $x = x_0$ . It is possible to conserve such constant metric of contraction for different values of  $x$ . Let  $\hat{B}(x)$  be the generalized left inverse of  $B(x)$ , then posing  $Z(x) = \hat{B}(x)B(x_0)Z_0$  and  $N(x) = \hat{B}(x)B(x_0)N_0$  yields

$$\begin{aligned} K_0 &= Z_0 W_0^{-1} \\ (AW_0 + B(x)\hat{B}(x)B(x_0)N_0) &+ (AW_0 + B(x)\hat{B}(x)B(x_0)N_0)^T \leq 0, W_0 > 0 \\ W_0 L^T &= -(M + B(x)\hat{B}(x)B(x_0)N_0). \end{aligned} \quad \text{Eq.46}$$

which by replacing the terms gives back Eq.45. Therefore, then  $Z(x) = \hat{B}(x)B(x_0)Z_0$  and  $N(x) = \hat{B}(x)B(x_0)N_0$  are solutions of the LMI problem, with a constant  $P_0 = W_0^{-1}$ . To design this controller, no state-dependent LMI is computed. The LMI is computed once for the chosen state  $x_0$ , for example a particular functioning point, and then the control action can be deduced from  $K_0, N_0$ .

For the LPRE system, it can be shown that the control action  $u = K_0 \frac{q_{gg}^2}{q_{gg}^2} x + N_0 \frac{q_{gg}^2}{q_{gg}^2} \phi(y, t)$  is a contractive controller in the sense of [5].

### 4.4. Results

In this subsection, application of contraction theory to the model described in section 2.3. and reformulated in 3.2. is presented. The model used is valid for the range 90% – 150% of the states' nominal value  $x_0$ . The different results and graphs are given in normalized form. The normalized form is  $x_{norm} = \frac{x}{x_0}$ . This paper focuses on the stability of the engine and the provided controller. The scenario proposed are computed beforehand, and are designed to cover different situations, and highlight the limits of controllers.

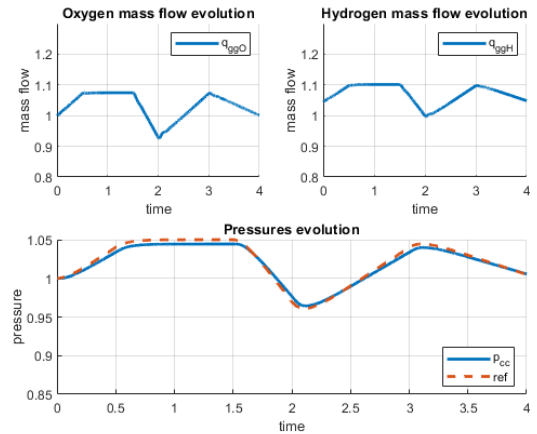


Figure 3: Tracking performances of the states of an LPRE for a simple reference using a contracting controller



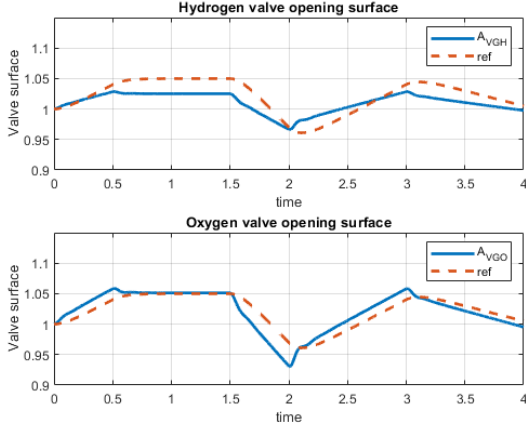


Figure 4: Evolution of the actuators of an LPRE for a simple reference using a contracting controller

To illustrate the tracking performance of the controller, a first non-realistic simple trajectory is applied to the system. The objective of this trajectory is to evaluate the tracking performance of the controller. On figure 3, the evolution of both mass flows incoming in the GG is represented, as well as the main combustion chamber pressure evolution. Figure 4 completes the illustration by depicting the evolution of the surface actuators during the same scenario.

The next illustration of the controller proposed makes use of a realistic reference for a maneuver. For a typical landing scenario, the returning module of the rocket handles three phases [20]. A first re-entry burn is performed, for the safety of the returning module. Due to air friction, it is necessary to reduce speed in the first phase of the landing. The second phase of the scenario consists in atmospheric gliding using the fins of the craft, while the rocket engine is kept at a lower thrust value. Finally, the final burn drastically reduces speed for a safe landing. The challenge of such a scenario relies on the amplitude of the changes in the

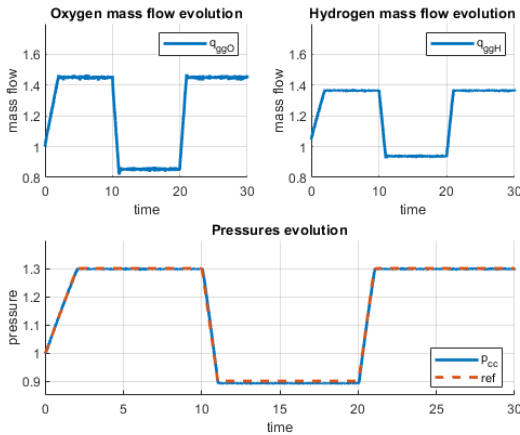


Figure 5: Evolution of the states during a re-entry scenario using a contracting controller

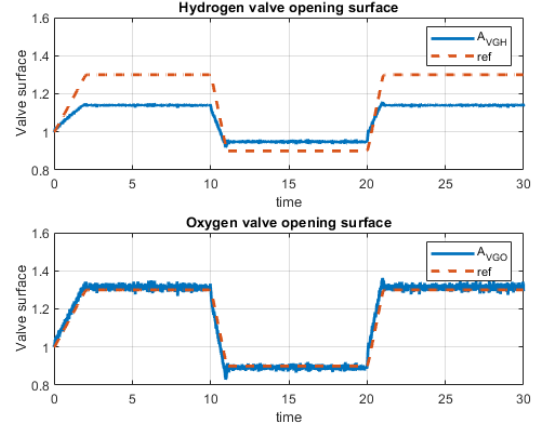


Figure 6: Evolution of the actuators during a re-entry scenario using a contracting controller

trajectory of the states. Figure 5 and 6 represent the contracting controller action following such a reference. Noise has been added on the chamber temperature to represent combustion fluctuations. As illustrated on figure 5, the controller provides a good precision and speed for the scenario, and the actuators' evolution on figure 6 remains feasible.

The controller's contracting property is illustrated on figure 7. In this scenario, the reference is reaching the validity limit of the model. Additionally, a noised signal has been added on the chamber temperature to illustrate uncertainty on the parameter  $\theta_{cc}$ . Two controllers are proposed to illustrate the behavior. First, the contracting controller design provides a stable trajectory. The second controller is a PID that is designed around the functioning point  $x_0$ . The second controller fails to provide stability to the system and the simulation is unable to complete, due to divergence of states (see  $q_{cc}$  on figure 6). In this scenario, the contracting controller fulfills its role as it manages to maintain the system in the contraction region, and thus ensuring the stability of the LPRE.

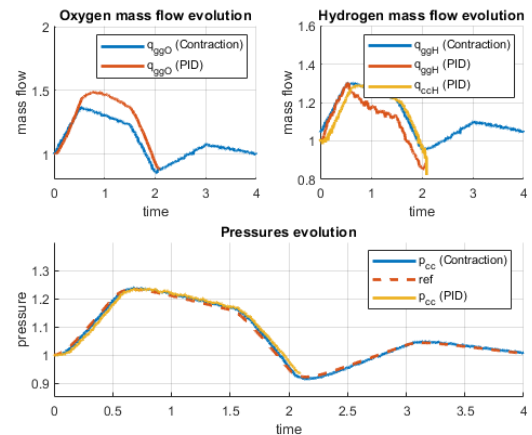


Figure 7: Evolution of the states in an unstable scenario for a contracting controller and a PID controller

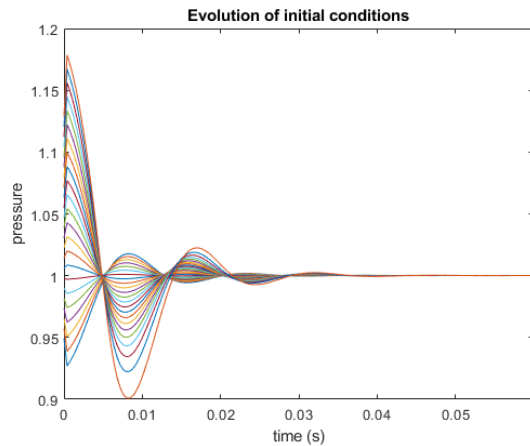


Figure 8: Evolution of the pressure for different initial conditions using a contracting controller

A property of contraction theory is to forget initial conditions of a system. On figure 8 are represented different initial conditions for a system regulated around a nominal trajectory  $p_{cc} = 1$ ). All trajectories converge to the same reference trajectory, validating the notion of forgetting initial conditions.

## 5. CONCLUSION

This paper summarizes the work performed on the stability of an LPRE. Two approaches were considered, for fixed-point stability and state-transition stability.

Using Port-Hamiltonian systems framework, it is possible to reformulate a complex non linear model under a form optimized for passivity analysis. Using the reformulation, a Hamiltonian function of the LPRE has been deduced, additionally proving the passivity of the system. The framework also enables flexibility in the design, as the addition of elements is simplified. As an example, this paper provides the addition of actuator dynamics. Port-Hamiltonian framework opens for new control methods and the derived conditions for the passivity enables the user to verify the controller proposed. Contraction theory is a powerful alternative to feedback controller, ensuring the stability of the system over a range of trajectories defined by the user. The analytical formulation of the controller is suitable for embedded systems, and provides good performances in terms of speed and precision. Moreover, the reference provided by the user is subject to optimization, leading to different solutions for the problem.

Future work includes the design of a passivity base controller.

## ACKNOWLEDGEMENTS

This study was partly conducted under co-funding by CNES and ONERA, to whom the authors are thankful.

## 6. REFERENCES

- [1] S. Zhang, J. Wei, and O. J. Haidn J. Liu, "Rans based numerical simulation of a gch4/go2 rocket engine combustion chamber with film cooling and improvement of wall heat flux prediction," *Applied Thermal Engineering*, 2023.
- [2] S. Perez Roca, "Model-based robust transient control of reusable liquidpropellant," *Ph.D. dissertation*, 2020.
- [3] A. Macchelli, S. Stramigioli, and H. Bruyninckx V. Duindam, *Modeling and control of complex physical systems: the port-Hamiltonian approach.*: Springer Science & Business Media, 2009.
- [4] W. Lohmiller and J.-J. E. Slotine, "On contraction analysis for nonlinear systems," *Automatica*, p. 1998.
- [5] V. Andrieu and S. Tarbouriech, "Lmi conditions for contraction and synchronization," *FAC-PapersOnLine*, pp. 616-621, 2019.
- [6] A. C. Sanchez, and J. Rodriguez-Resendiz S. Thenozhi, "A contraction theory-based tracking control design with friction identification and compensation," *IEEE Transactions on Industrial Electronics*, pp. 6111-6120, 2021.
- [7] W. Lohmiller and J.-J. Slotine, "Control system design for mechanical systems using contraction theory," *IEEE Trans.on Autom. Contr.*, pp. 984-989, 2000.
- [8] H. Piet-Lahanier, F. Farago, and M. Galeotta J. Gibart, "Regulation of a liquid propelled rocket engine using contraction theory," *IFAC PapersOnLine*, pp. 307-312, 2023.
- [9] B. M. Maschke and A. J. van der Schaft, "Port-controlled hamiltonian systems: modelling origins and systemtheoretic properties," *Nonlinear Control Systems Design*, pp. 359-365, 1992.
- [10] D. Jeltsema et al. A. Van Der Schaft, "Port-hamiltonian systems theory: An introductory overview," *Foundations and Trends® in Systems and Control*, pp. 173-378, 2014.
- [11] R. Pasumarthy and A. van der Schaft, "A port-hamiltonian approach to modeling and interconnections of canal systems," *16th International Symposium on Mathematical*, pp. 1436-1443, 2006.
- [12] B. Maschke, and A. Van Der Schaft D. Eberard, "An extension of hamiltonian systems to the thermodynamic phase space: Towards a geometry of nonreversible processes," *Reports on mathematical physics*, pp. 175-198, 2007.
- [13] B. Maschke, and D. Sbarbaro H. Ramirez, "Irreversible port-hamiltonian systems: A general formulation of irreversible processes with application to the cstr," *Chemical Engineering Science*, pp. 223-234, 2013.
- [14] T. Janus and B. Ulanicki, "Hydraulic modelling for pressure reducing valve controller design

addressing disturbance rejection and stability properties," *Procedia Engineering*, pp. 635-642, 2017.

- [15] M. Muñoz-Arias, H. Phillips-Brenes, and R. Pereira-Arroyo L. M. Esquivel-Sancho, "A reversible hydropump-turbine system," *Applied Sciences*, 2022.
- [16] A. J. van der Schaft, and A. Baños J. Cervera, "Interconnection of port-hamiltonian systems and composition of dirac structures," *Automatica*, pp. 212-225, 2007.
- [17] W. Lohmiller, "Contraction analysis of nonlinear systems," *PHD Dissertation*, 1999.
- [18] B. Jayawardhana, and L. Praly V. Andrieu, "Transverse exponential stability and applications," *IEEE Transactions on Automatic Control*, pp. 3396-3411, 2016.
- [19] A. van der Schaft, and B. Jayawardhana R. Reyes-Baez, "Virtual contractivity-based control of fully-actuated mechanical systems in the port-hamiltonian framework," *Automatica*, 2022.
- [20] B. Hérisse, and F. Jean C. Leparoux, "Optimal planetary landing with pointing and glide-slope constraints," *IEEE 61st Conference on Decision and Control*, pp. 4357-4362, 2022.

Expression of the constant scaling terms:

$$\begin{aligned}\kappa_{tu} &= k_{2gg}k_{gg} \\ k_{ptu} &= \frac{\kappa_{tu}}{k_{1tu}} \\ k_{pcc} &= \frac{1}{k_{1cc} + k_{3cc} - \Theta_{cc}k_{4cc}} \\ k_{pgg} &= \frac{1}{k_{1gg} + k_{3gg} - \Theta_{gg}k_{4gg}}\end{aligned}$$

Expression of the interconnection terms:

$$\begin{aligned}\alpha_q &= \frac{b_p\rho}{a_c} \\ \alpha_t &= \alpha_q ST(\omega)w_{tu}\end{aligned}$$

## ANNEX: PORT-HAMILTONIAN FORMULATION OF THE LPRE

$$\begin{bmatrix} \alpha_q J \dot{\omega} \\ I_{cc} \dot{q}_{cc} \\ I_{gg} \dot{q}_{gg} \\ I_{tu} \kappa \dot{q}_{tu} \\ k_{pcc} \dot{p}_{cc} \\ k_{pgg} \dot{p}_{gg} \\ k_{ptu} \dot{p}_{tu} \end{bmatrix} = [J(X) - R(X)] \begin{bmatrix} \omega \\ q_{cc} \\ q_{gg} \\ q_{tu} \\ p_{cc} \\ p_{gg} \\ p_{tu} \end{bmatrix} + \begin{bmatrix} 0 \\ p_{ep} \\ p_{ep} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$J(X) = \begin{bmatrix} 0 & -\lambda & -\lambda & 0 & 0 & 0 & \alpha_t \\ \lambda & 0 & 0 & 0 & -1 & 0 & 0 \\ \lambda & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \kappa_{tu} & -\kappa_{tu} \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -\kappa_{tu} & 0 & 0 & 0 \\ \alpha_t & 0 & 0 & \kappa_{tu} & 0 & 0 & 0 \end{bmatrix}$$

$$R(X) = \text{diag}[R_\omega R_{qcc} R_{qgg} R_{qtu} R_{pcc} R_{pgg} \beta_t]$$

Where  $\lambda = b_p(q_{cc} + q_{gg})$ .

Expression of the resistive terms:

$$R_\omega = \alpha_q (b_c(q_{cc} + q_{gg}) - c_c \rho \omega)$$

$$R_{qcc} = Z_{eqcc} q_{cc} - \frac{a_p}{\rho} (q_{cc} + q_{gg}) - \frac{c_p \rho \omega^2}{q_{cc}}$$

$$R_{qgg} = Z_{eqgg} q_{gg} - \frac{a_p}{\rho} (q_{cc} + q_{gg}) - \frac{c_p \rho \omega^2}{q_{gg}}$$

$$R_{qtu} = \kappa_{tu} Z_{hg} q_{tu}$$

$$R_{pcc} = k_{2cc} k_{pcc}$$

$$R_{pgg} = 0$$

$$R_{ptu} = \beta_t$$