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Prediction of fatigue crack initiation in rubber with the help of configurational mechanics

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ABSTRACT: The present paper is a first step towards the definition of a new multiaxial fracture criterion for rubber-like materials. Here, the end of life is defined by crack initiation. Motivated by recent experimental observations, it is assumed that elastomers contain a uniform distribution of voids. Then, in order to determine the energy release rate which opens voids, the framework of Mechanics in Physical Space is considered. More precisely, properties of the configurational stress tensor are thoroughly examined to derive the criterion. Predictions are compared with published experimental data and the efficiency of the theory is highlighted.

1 INTRODUCTION

Rubber parts are widely used in many applications such as tires, engine mounts, bumpers or shoes. For the development of new industrial products, both short- and long-term durabilities should be estimated. Depending on applications, crack nucleation or crack growth can be considered. As shown by Mars & Fatemi (2002), only few criteria are used for crack nucleation and their efficiency is questionable whereas crack growth approaches are well-established since the pioneering work of Rivlin & Thomas (1953).

The present paper focuses on the prediction of crack initiation in rubber for which most widely used criteria are the maximum principal stretch and the strain energy density. Nevertheless, none of them is applicable to multiaxial loading conditions (see Mars & Fatemi (2002) and the references herein). More recently, some authors consider that rubber contains intrinsic flaws which can be seen as small cracks that grow under cyclic loading conditions. It leads to the definition of the cracking energy density which represents the portion of the total energy that contributes to the growth of small cracks Mars (2002). For a given surface, it is defined as “the work performed by the tractions on the surface in deforming the surface”. The aim of the present paper is to rationalize this approach using the theoretical framework of the Eshelbian mechanics.

In the next Section, principal works on the Eshelbian mechanics are recalled. Section 3 is devoted to the formulation of the new criterion. Then, some comparisons of the theory with published experimental data are presented in Section 4 to demonstrate the efficiency of

the approach. Finally, concluding remarks are given in Section 5.

2 A BRIEF OVERVIEW OF CONFIGURATIONAL MECHANICS

The concept of configurational stress was first introduced by Eshelby (1951). The author studied forces that applied on defects and introduced the following “stress tensor”

$$\Sigma = w(\text{sym}(\nabla \mathbf{u}), \mathbf{x}) \mathbf{I} - (\nabla \mathbf{u})^T \boldsymbol{\sigma}, \quad (1)$$

where w is the strain energy density which depends on both the symmetric part of the displacement gradient $\text{sym}(\nabla \mathbf{u})$ and the position vector \mathbf{x} , and $\boldsymbol{\sigma}$ is the Cauchy stress tensor. In this equation, the superscript T stands for the transposition. The tensor Σ is classically referred as the configurational stress tensor. For a homogeneous body, i.e. without defects, and in absence of body forces, Σ satisfies a strict conservation law:

$$\text{div } \Sigma = \mathbf{0}. \quad (2)$$

Moreover, in the case of a body (or a part of a body) of volume V and boundary S which contains a defect, the force which acts on this defect is given by the integration of the configurational traction vector over S

$$\mathbf{G} = \oint_S \Sigma \cdot \mathbf{N} dS, \quad (3)$$

where \mathbf{N} is the unit outward normal vector to S . \mathbf{G} is generally referred as the configurational force.

Twenty five years later, both Eshelby (1975) and Chadwick (1975) extended the previous theory to finite strain. The corresponding expression for the configurational stress is

$$\Sigma = W(\mathbf{C}, \mathbf{X}) \mathbf{I} - \mathbf{F}^T \mathbf{P}, \quad (4)$$

where W is the strain energy density per unit volume of the reference configuration and depends on both the right Cauchy-Green stretch tensor \mathbf{C} (to ensure objectivity) and the placement \mathbf{X} in the reference configuration, \mathbf{F} stands for the deformation gradient and \mathbf{P} denotes the first Piola-Kirchhoff stress tensor. Introducing the second Piola-Kirchhoff stress tensor, Σ can be rewritten as

$$\Sigma = \mathbf{W} \mathbf{I} - \mathbf{C} \mathbf{S}, \quad (5)$$

in which \mathbf{S} is the second Piola-Kirchhoff stress tensor. In the special case of isotropic materials, for which \mathbf{C} and \mathbf{S} are coaxial and commute, the configurational stress tensor is symmetric:

$$\Sigma = \Sigma^T. \quad (6)$$

These pioneering works were revisited by contemporary authors who considered configurational stress and forces as basic objects of a new concept of mechanics (see the introduction of the monograph of Gurtin (2000)). This concept is designated as the Eshelbian Mechanics by Maugin (1993) and the Mechanics in Material Space by Kienzler & Herrmann (2000) by contrast to the Newtonian Mechanics or the Mechanics in Physical Space, respectively. In this point of view, the duality between classical mechanical quantities and configurational quantities is highlighted.

In the majority of studies involving the Eshelbian Mechanics, only configurational forces are investigated through the calculation of path-independent integrals around inhomogeneities. Thus, configurational stress only appears in the definition of surface tractions. Majority of works focus on the application of configurational mechanics to fracture problems (see for example Steinmann (2000) and the references herein). Indeed, it is now well-established that, for hyperelastostatic problems, configurational forces which apply on crack tip generalize the classical J-(Rice 1968), L- and M-(Budiansky & Rice 1973) path-independent integrals.

Opposite to the case of configurational forces, only few studies are concerned with the peculiar properties of the configurational stress tensor. For the linear theory, the physical significance of the Cartesian components of this tensor were recently identified by Kienzler & Herrmann (1997), who

explain that [*the ij-component*] of the Eshelby tensor is the change in the total energy density at a point of an elastic continuum due to a material unit translation in x_j direction of a unit surface with normal in x_i -direction. More recently, the same authors introduce local fracture criteria for small strain problems based on the components of the configurational stress tensor Kienzler & Herrmann (2002).

3 A NEW CRITERION TO PREDICT THE FATIGUE LIFE OF RUBBERLIKE MATERIALS

3.1 Physical motivation

As mentioned in the introduction, an efficient fatigue life criterion should reflect the physical phenomena which take place during crack initiation and growth. For rubber, Le cam et al. (2004) recently demonstrated that fatigue crack propagation can be explained by growth and coalescence of small voids, which are the consequence of the decohesion between compounding ingredients, such as zinc oxides or filler aggregates and rubber matrix.

From a macroscopic point of view, fatigue crack initiation is defined by the occurrence of a small crack of a given size (1 mm for André et al. (1999), 2 mm for Ostoja-Kuczynski et al. (2003)). This criterion is not a material intrinsic parameter, because it should be compared with the sample/structure size. Indeed, in regard to the previous paragraph, it is obvious that such small cracks are related to initiation from the macroscopic point of view and to propagation in the microscopic point of view.

Following this idea, a well-defined fatigue life criterion should consider these two points of view. First, it has to be written in terms of continuum mechanics variables with no reference to fracture mechanics because small cracks do not significantly influence the mechanical response of the structure in the macroscopic point of view. Second, it should take into account the energy release rate of small flaws in order to render microscopic phenomena. Such idea was investigated in the past by Rivlin & Thomas (1953), Greensmith (1963), Lindley (1972) and Young (1990) who relate tearing energy density to strain energy density for classical fracture rubber test specimen such as the single edge specimen and the trouser specimen, and by Mars (2002) through his concept of cracking energy density.

3.2 Prediction of crack initiation

In this section, the formulation of our criterion is derived.

From a continuum mechanics point of view, elastomers are considered homogeneous, isotropic, elastic

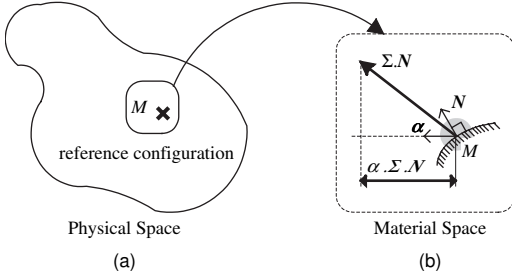


Figure 1. (a) A rubber body in its undeformed configuration. (b) Zoom in near the particle M in the material space.

and incompressible under large strain. The framework of hyperelasticity is adopted. From the microscopic point of view, it is assumed that small pre-existing voids are present everywhere in the material. Thus, each particle of the reference configuration in the Physical Space can be a point defect in the Material Space. The aim of the criterion is to determine the energy release rate of all possible defects (i.e. in each particle of the Physical Space) in order to exhibit the most propitious zone for macroscopic crack occurrence.

Consider a particle M in the undeformed configuration of the Physical Space (see Figure 1(a)). In the Material Space, the point defect in M can be modelled by a small circular void (see Figure 1(b)). In this point, energy properties of the defect are completely defined by the configurational stress Σ given by Eq. (5). More precisely, for a given oriented unit surface of outward normal \mathbf{N} in M , the vector $\Sigma \cdot \mathbf{N}$ represents the configurational traction across the surface. Considering the definition of Σ_{ij} recalled above, the scalar $\alpha \cdot \Sigma \cdot \mathbf{N}$ is the change of the total energy at the point M due to a material unit translation in the unit direction α of the unit surface with normal \mathbf{N} . The configurational stress tensor being symmetric, it admits three real eigenvalues, denoted $(\Sigma_i)_{i=1,3}$ in the following, and three eigenvectors, denoted $(\mathbf{V}_i)_{i=1,3}$:

$$\Sigma = \Sigma_i \mathbf{V}_i \otimes \mathbf{V}_i. \quad (7)$$

These three vectors correspond to the directions in which the energy release rates tend to open or close the void by material “normal” traction without material “shear” traction. Moreover, as the force on a defect is defined by the negative gradient of the total energy with respect to the change in position of the defect (Chadwick 1975; Kienzler & Herrmann 2000), the morphology of the defect evolves with respect to the direction opposite to the material force (Steinmann et al. 2001). So, as the body will ever tend to reduce its total energy, the minimum eigenvalue of Σ can be considered in order to predict the evolution of the defect. If it is negative the defect grows to form

a plane crack which normal is defined by the corresponding eigenvector. If it is positive, the three eigenvalues are positive and the defect tends to shrink. This case can be seen as the theoretical counterpart of the well-known physical “crack-closure effect”. Finally, the new crack nucleation criterion is denoted Σ^* and its formulation can be summarized as follow:

$$\Sigma^* = \left| \min \left[(\Sigma_i)_{i=1,3}, 0 \right] \right|$$

where $(\Sigma_i)_{i=1,3}$ are the principal configurational stresses.

- (i) if $\Sigma^* > 0$, the crack grows in the plane normal to \mathbf{V}^* , the eigenvector of Σ associated with $-\Sigma^*$,
- (ii) if $\Sigma^* = 0$ the defect closes (“crack-closure effect”).

The present theory rationalizes the approach of Mars (2002) who proposed the concept of *cracking energy density* to determine the portion of the energy that is available to be released in crack growth. Nevertheless, even if the physical motivation of both criteria are similar, the work of Mars is more intuitive and can not be included in the general framework of Configurational Mechanics.

3.3 Generalization to cyclic loading conditions

The previous criterion defines an instantaneous value of the damage in the material but it considers no deformation history. In order to take into account the loading history, the increment of the configurational stress tensor should be integrated over the entire history of the material. For fatigue problems that involve several thousands cycles it is impossible to perform this integration. So, in a first attempt the integration of the configurational stress over one cycle is revealed sufficient to predict the fatigue life.

The aim of the following derivation is to propose a suitable method to accumulate the configurational stress during a cycle. In other words, the instantaneous criterion proposed in the previous section is extended to configurational stress increments. Consider an increment of Σ and denote $(d\Sigma_i)_{i=1,3}$ and $(\mathbf{v}_i)_{i=1,3}$ its eigenvalues and eigenvectors, respectively:

$$d\Sigma = d\Sigma_i \mathbf{v}_i \otimes \mathbf{v}_i. \quad (8)$$

Then, recalling that defects grow with respect to the direction opposite to material normal tractions, a *configurational damage stress increment* can be defined as

$$d\Sigma^* = \min(d\Sigma_i, 0) \mathbf{v}_i \otimes \mathbf{v}_i, \quad (9)$$

and the configurational damage stress tensor over a cycle is

$$\Sigma^* = \int_0^T d\Sigma^*. \quad (10)$$

Finally, the crack nucleation prediction method developed above can be applied to this modified tensor to establish the following fatigue life criterion:

$$\Sigma^* = \left| \min \left[(\Sigma_i^*)_{i=1,3}, 0 \right] \right|$$

where $(\Sigma_i^*)_{i=1,3}$ are the principal configurational damage stresses, i.e. eigenvalues of

$$\Sigma^* = \int_0^T \min(d\Sigma_i, 0) \mathbf{v}_i \otimes \mathbf{v}_i$$

with $(d\Sigma_i)_{i=1,3}$ and $(\mathbf{v}_i)_{i=1,3}$ being the eigenvalues and eigenvectors of $d\Sigma$

- (i) if $\Sigma^* > 0$, the crack grows in the plane normal to \mathbf{V}^* , the eigenvector of Σ associated with $-\Sigma^*$,
- (ii) if $\Sigma^* = 0$ the defect closes (“crack-closure effect”).

4 EXAMPLES

4.1 Uniaxial and biaxial data of Roberts and Benzies (1977)

This first is devoted to the first criterion presented in Section 3.2. It concerns the experiments conducted by Roberts & Benzies (1977), who determined the fatigue life of different elastomers for uniaxial and equi-biaxial fatigue loading conditions. Using these results, authors demonstrated that the strain energy density is not a relevant criterion to predict the fatigue life. More recently, Mars (2002) established a similar conclusion for the maximum principal strain.

Here, experimental data are first used to determine the constitutive equation of the materials. The general framework of isotropic, homogeneous and incompressible hyperelasticity is adopted and a second-order Mooney-Rivlin model is considered:

$$W = C_1 (I_1 - 3) + C_2 (I_2 - 3) + C_3 (I_1 - 3)^2, \quad (11)$$

where I_1 and I_2 are the two first stretch invariants. Second, the constitutive equation is used to calculate the criterion. The loading conditions being simple, no cumulation of configurational stress increments is needed and the criterion is equal to the minimum principal configurational stress which corresponds to the maximum deformation.

Wöhler curves in terms of the maximum principal strain and the present criterion are shown in Figures 2 for filled Natural Rubber and 3 for filled Styrene-Butadiene Rubber.

In both cases, uniaxial and equi-biaxial Wöhler curves are superimposed, which shows the superiority

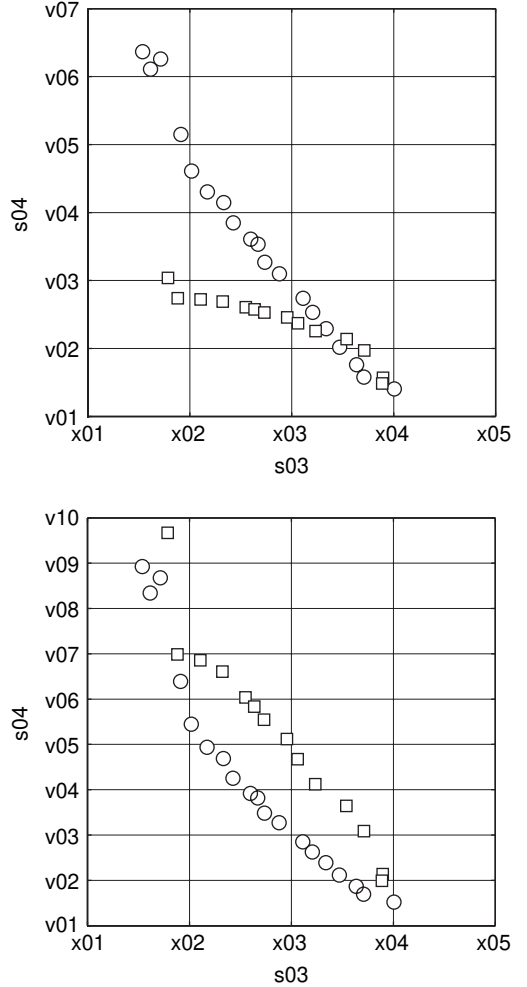


Figure 2. Comparison of uniaxial (○) and equi-biaxial (□) tension fatigue life data of Benzies & Roberts (1977) for NR using the maximum principal strain (first graph) and the present criterion (second graph).

of the configurational stress as compared with classical criteria such as maximum strain, and also maximum stress or strain energy (the corresponding results are not shown here).

4.2 Axial-torsional data of Mars (2001)

The aim of this second example is to validate the cumulation rule proposed in Section 3.3. For this purpose, the remarkable experimental database detailed in the PhD thesis of Mars (2001) is used. The author conducted number of axial tensile/torsion fatigue

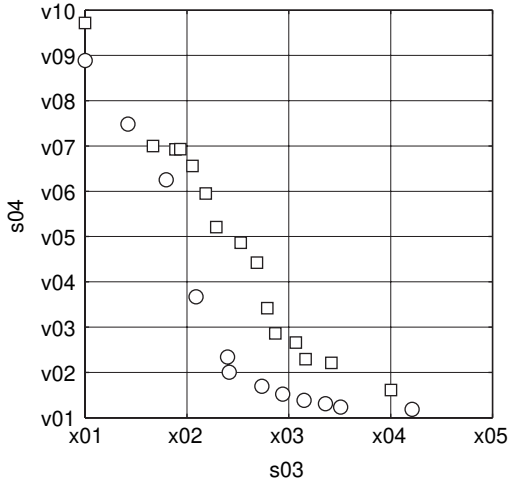
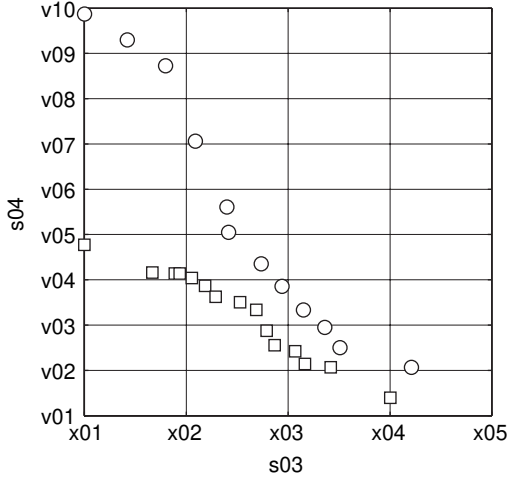


Figure 3. Comparison of uniaxial (○) and equi-biaxial (□) tension fatigue life data of Benzies & Roberts (1977) for SBR using the maximum principal strain (first graph) and the present criterion (second graph).

experiments under proportional and non-proportional loading conditions. Here, the emphasize is laid on out-of-phase tests for which the criterion should be cumulated in a cycle.

To calculate the configurational stress, the analytical solution of the simultaneous axial tensile and torsion of a hyperelastic cylinder is considered (see Green & Adkins (1960) for details). As proposed by Mars, the material is assumed to obey the neo-hookean model:

$$W = C(I_1 - 3) \quad (12)$$

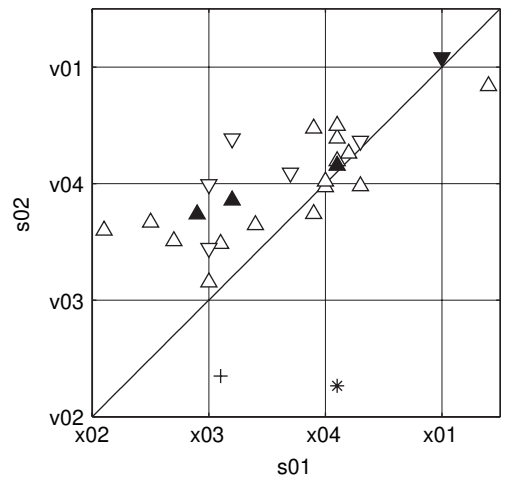
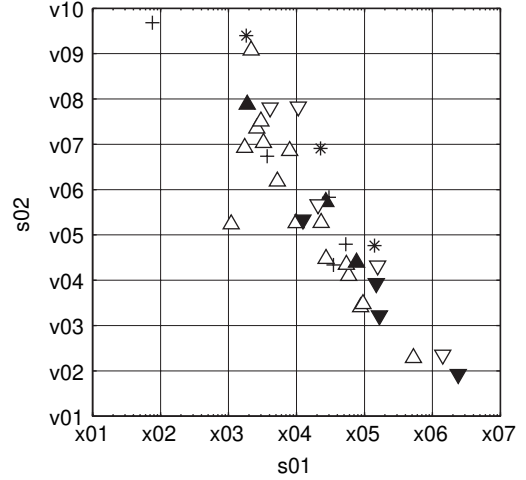


Figure 4. Experiments of Mars (2001): comparison of proportional and non-proportional loading conditions. The first graph presents the Wöhler curve in terms of the present criterion and the second graph compares the experimental and theoretical crack angles. (△) proportional axial tensile/torsion test, (▲) non-proportional 45° axial tensile/torsion test, (▽) non-proportional 90° axial tensile/torsion test, (▼) non-proportional 180° axial tensile/torsion test, (*) proportional axial compression/torsion test, (+) non-proportional 180° axial compression/torsion test.

with $C = 1.5$ Mpa. The time is discretized and configurational stress increments are integrated over a cycle.

Figure 4 presents the Wöhler curve in terms of the present criterion and the crack angle prediction. Results obtained are similar to those of Mars (2002) and lead to good agreement between experimental data and predictions. This validates the cumulation method proposed in this paper.

5 CONCLUDING REMARKS

In this paper, a new criterion for fatigue life prediction of elastomeric parts has been proposed. It is founded on physical considerations and on the theoretical framework of configurational mechanics.

The minimum principal configurational stress is considered: if it is negative the criterion is equal to its absolute value, if it is positive the criterion is null which corresponds to crack closure. Moreover, a simple method to cumulate the fatigue damage over a cycle is also proposed. The first results presented here are very promising.

Nevertheless, this study being only a first step, it leaves unresolved some issues of fundamental importance. As example, we can mention the numerical implementation of the criterion in the finite element context. Moreover, the relationship between the energy release rate of small cracks and the present criteria should be investigated to demonstrate their complete relevance.

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