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The Consequences of Regulating Agency Friction on an Optimal Contract

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Abstract

This paper investigates the impact of changes in the intensity of moral hazard on the optimal provision of incentives during a contractual relationship. We develop a continuous-time principal-agent model in which the agent's benefit from diverting cash flow is subject to a persistent and exogenous shock. We interpret this shock as a new regulation that decreases the fraction of diverted cash flows accessible to the manager. Our results show that managerial compensation becomes compressed following such regulation, as a high-performing manager receives lower compensation, while an underperforming manager receives higher compensation. Furthermore, we demonstrate that this type of regulation leads to the over-retention of underperforming managers. Although reduced agency friction results in increased profitability, the unintended consequences of the regulation include higher compensation for underperformers and their over-retention. Nonetheless, we also show that a competitive labor market can help mitigate these adverse effects arising from a decrease in the severity of agency friction.

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1 Introduction

In the groundbreaking work by Jensen and Meckling (1976) on the separation of ownership and control, the inability of the firm owner to observe the manager’s actions gives rise to moral hazard. Since then, researchers investigate optimal contracts to align the manager’s objectives with those of the firm owner, and the literature provides evidence that moral hazard is a significant and costly financial friction¹. A variety of factors, however, can influence the intensity of this agency friction. Within the firm’s environment, factors such as competition in the product market (Girotti and Salvade (2022)), reputational concerns for managers in the labor market (Chevalier and Ellison (1999)), or regulation can discipline the manager by applying pressure to correct improper behavior.² With the firm’s environment inherently evolving, it is important to understand how changes in moral hazard intensity during the contractual relationship affect the managerial incentives provided by an optimal contract.

In this paper, we consider a principal-agent model in continuous time with cash-flow diversion and assume that a new regulation reduces the fraction of diverted cash flows edible by the manager. Such regulation can be seen as an exogenous change out of the scope of both the firm’s owner and the firm’s manager that persistently changes the ability of the firm manager to misbehave. We focus on regulation not only to help fix ideas but also because it addresses important policy implications regarding the design and consequences of executive compensation regulation.

Specifically, we consider the role of mandatory compensation disclosure in influencing the behavior of a firm’s top executives. Compensation disclosure aims to enhance transparency and accountability in corporate governance. Among the various acts regarding compensation disclosure introduced in the U.S.A., notable examples include the Securities Act of 1934, which was the first to disclose certain elements of executives’ pay, and the 2006 Act, which mandates the disclosure of performance metrics and CFO pay. The literature finds evidence that compensation disclosure reduces executive misbehavior (see Bao et al. (2022)). Hence, this regulation can be viewed as a persistent, exogenous change beyond the control of both the firm’s owner and the firm’s manager, altering the manager’s ability to misbehave.

Our model build upon a dynamic contracting setting in the vein of DeMarzo and Sannikov (2006). We assume that the manager privately observes the firm’s true cash

¹Nikolov et al. (2021) demonstrates that moral hazard is the most severe financial friction in private firms. Ai et al. (2016) reveals that moral-hazard induced incentives account for 52% of managerial compensation

²e.g., see Holmstrom and Tirole (1989) for an overview.

flows and can secretly divert a portion of them before reporting to the firm’s owner. This unobservability of the true cash flows by the firm’s owner creates a moral hazard problem. To incentivize truthful reporting, the owner links the manager’s payoffs to the reported cash flow. The manager receives lump-sum payments (hereafter also referred to as bonuses) following the observation of sufficiently good cash flows, while the manager’s contract is terminated after observing poor cash flows. These deferred compensation and performance-induced contract termination mechanisms are standard incentives in dynamic contracting settings. Our model’s novelty lies in the fact that both the principal and the agent, at the contracting date, anticipate a regulation at an uncertain future date that will persistently change the agent’s benefit from diverting cash flows. By incorporating this regulatory aspect, our model provides a more comprehensive understanding of the dynamics between managerial incentives and changes in agency friction.

We outline our main findings. Due to the agent’s ability to divert cash flow, the principal must steadily link the agent’s expected future compensation to the reported cash flow, which is costly. Anticipating the arrival of a shock affecting the agent’s benefit from diverting cash flows, the principal designs an optimal contract that maintains incentive compatibility throughout the contractual relationship while reducing delegation costs. This optimal contract can be expressed in terms of the agent’s continuation value (the agent’s expected earnings from the contract) and a random variable accounting for the advent of the shock on agency friction. Under the optimal contract, we demonstrate that when such a shock occurs, an agent with a strong performance record (i.e., a high continuation value) will face a decrease in expected future compensation, while an underperformer (i.e., a low continuation value) will see an increase in expected future compensation. This results in a compression of bonuses and a reduced likelihood of contract termination for poor performers. Therefore, a regulation that restricts the agent’s ability to misbehave is beneficial for the firm owner and reduces the compensation of executives performing well. Thus, it achieves the SEC goal to limit too large compensation without imposing salary cap aimed to limit excessive pay.³ However, it also has the unintended consequence of favoring underperformers.

Our paper presents testable implications based on the mechanism of incentive adjustment. First, we anticipate that managerial compensation compression will follow the regulation. This finding aligns with Mas (2019), who demonstrates that the 1934 mandated pay disclosure implemented in the U.S. led to both an increase in average

³SEC chairman Christopher Cox acknowledges that “the SEC lacks statutory authority to impose salary caps on corporate executives and we’d be out of bounds to attempt that through indirection” when he introduced the CD&A act. <https://www.sec.gov/news/speech/spch011706cc.htm>

managerial compensation and a reduction in the earnings of top-performing managers. While the author regards this empirical finding as "intriguing," our dynamic contracting perspective offers a comprehensive explanation for this outcome.

Second, underperformers face a lower risk of dismissal following the shock's advent. In this regard, we introduce the concept of regulation-induced retention, defined as keeping an agent active who would have been dismissed without the shock affecting their cash-flow diversion benefits. This contrasts with the baseline model of DeMarzo and Sannikov (2006), where only good performance moves the agent away from the termination boundary. Although evidence exists that exogenous industry and market shocks influence the decision to retain a manager (e.g., Jenter and Kannan (2015)), our model provides novel insights into how the introduction of regulation on executive pay may also affect this decision. To the best of our knowledge, this effect has not yet been documented in the literature.

Termination payoffs play a crucial role in our results. In fact, while the testable predictions of our main model may suggest that something is missing in the ongoing empirical work, we demonstrate that incorporating into our model a competitive labor market can mitigate the undesirable effects. To achieve this, we extend our primary model by assuming the existence of a competitive labor market with search costs, following the approach of Chemla et al. (2022) (also see section IV.A of DeMarzo and Sannikov (2006)). As a result, termination payoffs become endogenous, allowing both the principal and the agent to match with new counterparts after the agent's contract termination.

The regulatory shock makes delegation more valuable for the principal since it reduces the agent's sensitivity to output. This results in an increase in the principal's termination payoff and a decrease in the agent's termination payoff. Consequently, we show that the presence of a competitive managerial labor market mitigates the undesirable advantage to poor performers – that is, the increase in their compensation and over-retention caused by the shock when termination payoffs are fixed.

Our study is closely related to the reward for luck literature (see Hoffmann and Pfeil (2010), Demarzo et al. (2012), and Li (2017)). Specifically, Hoffmann and Pfeil (2010) develop a similar model but with a shock to the mean of cash flows. They demonstrate that under the optimal contract, the agent is rewarded (or penalized) when a positive (or negative) shock that increases firm's productivity occurs. Their model provides a theoretical explanation for the empirical findings of Garvey and Milbourn (2006), Bertrand and Mullainathan (2001), and Francis et al. (2013), who show that managers are rewarded for luck.

Numerous empirical studies have demonstrated the effectiveness of compensation disclosure in curbing executive misbehavior. Firms with greater executive compensa-

tion disclosure are less likely to have restatements or engage in earnings management (Cheng and Warfield (2005)), and the likelihood of engaging in option backdating is reduced (Larcker et al. (2007)). Recently, the 2006 Compensation Disclosure and Analysis (CD&A) act made CFO compensation disclosure mandatory.⁴ Although CFOs may not directly manage the firm or control its cash flow like CEOs, they still hold crucial decision-making authority within the firm and contribute their financial expertise to financial reporting. As a result, agency friction may arise since shareholders cannot directly observe a CFO’s actions. This aligns with Jiang et al. (2010)’s findings that CFO improper incentives are more strongly associated with accrual-based earnings management and the likelihood of beating analyst forecasts than CEO improper incentives. Bao et al. (2022) shows that the introduction of the 2006 CD&A act resulted in better quality financial reporting, which is a “CFO’s fiduciary duty to shareholders”. These findings suggest that senior executives, such as CFOs, are susceptible to agency problems, and the 2006 CD&A Act serves as a regulatory measure to alleviate moral-hazard friction by restricting their opportunities for misbehavior. Consequently, our paper provides a well-suited theoretical model to address the consequences of such regulation.

Regarding the effect of compensation disclosure on executive compensation, the literature shows that reforms expanding mandatory compensation disclosure for managers have achieved mixed results. First, Morse et al. (2011) argue that additional disclosures mitigate rent extraction and reduce average pay. Empirical investigations of the effect of mandatory compensation disclosure seem to contradict this assertion. Indeed, an increase in the average manager’s pay has been observed following the introduction of the 2006 CD&A act (Gipper, 2021).

Our paper also relates to the literature on the consequences of disclosure in the presence of agency conflicts (see, e.g., Verrecchia (2001), and the reviews by Lambert (2001), Armstrong and Weber (2010), and Roychowdhury et al. (2019) in the accounting literature), as well as the effect of information and disclosure on the firm. For instance, Edmans et al. (2016) investigates the potential detrimental effects of disclosure. Other notable papers in this area include Admati and Pfleiderer (2000), Easley and O’hara (2004), and Iliev (2010).

The paper proceeds as follows. Section 2 presents the model, Section 3 details the optimal contract, and Section 4 provide testable implications. In Section 5, we extend the model and introduce a competitive labor market. All proofs are in the Appendix.

⁴Before 2006, only the five highest-paid executives of the firm had to disclose their compensation. Li and Xu (2016) finds that during the 1999-2006 period, only 40% of firms in the S&P 1500 universe consistently reported CFO compensation, while 15% never reported it.

2 The Model

In this section, we present a model in continuous time where the owner of a firm delegates its management to an agent who can divert cash flow out of the owner's sight and where an exogenous and persistent change can impact the benefit of cash-flow diversion. Both parties fully commit to a long-term contract that characterizes the delegation, and limited liability protects the agent. Let us assume that the cash flow generated by the firm evolves according to the dynamics

$$dX_t = (\mu - a_t)dt + dZt, \quad (1)$$

where $a_t \in \{0; \bar{a}\}$, $\bar{a} > 0$ represents the level of cash flow diversion. We assume that a moral-hazard problem arises, so the principal only observes the reported cash flow. At each instant where the agent diverts cash flow, his consumption increases by $\lambda_t \bar{a}$, with $\lambda_t \in (0, 1]$. μ is the drift parameter of cash flows in the absence of diversion, and Z is a standard Brownian motion on a complete probability space. In addition, λ_t is subject to exogenous and verifiable shock that makes it jump from $\bar{\lambda}$ to $\underline{\lambda} < \bar{\lambda}$ with a probability pdt in any time interval $(t, t + dt]$. Thus, after the advent of the shock, the fraction of diverted cash flows edible by the manager drops permanently. Besides, we assume that both the agent and the principal are risk-neutral, and the agent is protected by limited liability. While the principal discounts at a rate $r > 0$, the agent is more impatient and discounts at $\gamma > r$.

The delegation starts at date 0 and follows the term of a contract $\Pi = \{U; \tau\}$ that consists of nonnegative transfers from the principal to the agent payments $\{U_t : 0 \leq t \leq \tau\}$ (compensation) and a date of termination τ . We assume that the agent cannot save, so the level of agent's consumption is given at any time t by $dU_t + \lambda_t a_t dt$. Once the contract terminate, we assume without loss of generality that the agent has no outside option. For an arbitrary contract Π and a strategy of cash-flow diversion $a = (a_t)_t$, the agent gets in expectation from the contractual relationship a value

$$E^a \left[\int_0^\tau e^{-\gamma t} (dU_t + \lambda_t a_t dt) \right]. \quad (2)$$

The cash-flow diversion strategy induces a unique probability measure Q^a , and E^a is the associated expectation operator. The agent's outside option is normalized to zero without loss of generality. Concerning the principal, he gets in expectation

$$E^a \left[\int_0^\tau e^{-rt} ((\mu - a_t)dt - dU_t) + e^{-r\tau} L \right], \quad (3)$$

where L is the scrap value of the firm at the contract termination. We remark that in the absence of shock on the agent's benefit of cash-flow diversion, the model is similar to the model of DeMarzo and Sannikov (2006).

3 Optimal Contract

At date 0, both parties fully commit on the long-term to contract Π that is contingent on the observed cash flows and the exogenous shock that affects the agent's benefit to divert cash flows. Let us define \mathcal{F}_t as the information set available to the principal at any date t . Formally, we say that $U = (U_t)_t$ is a \mathcal{F} -adapted process, and that τ is a measurable \mathcal{F} -stopping time that can be infinite.

For a fixed contract Π , the agent's expected profit is

$$E^a \left[\int_0^\tau e^{-\gamma t} (dU_t + \lambda_t a_t dt) \right], \quad (4)$$

and the principal's expected profit is

$$E^a \left[\int_0^\tau e^{-rt} ((\mu - a_t) dt - dU_t) + e^{-r\tau} L \right]. \quad (5)$$

The agent chooses a strategy that maximizes his total expected future compensation given in equation (2), and we say that the contract $\Pi = \{U; \tau\}$ is incentive-compatible if it induces the agent to follow a strategy $a^* = (a_t^*(\Pi))_t$ that satisfies

$$a^* = \arg \max_a E^a \left[\int_0^\tau e^{-\gamma t} (dU_t + \lambda_t a_t dt) \right] \quad (6)$$

Hence, the principal's problem is to find an incentive-compatible contract that maximizes the principal's total expected value. It can be formulated as

$$\sup_{\Pi \text{ I.C.}} E^a \left[\int_0^\tau e^{-rt} ((\mu - a_t) dt - dU_t) + e^{-r\tau} L \right] \quad (7)$$

We say that such a contract is optimal.

As it is standard in the dynamic contracting literature, the agent's continuation value is a state variable in our model. It represents how much the agent expects to earn from any date t onwards. To define the agent's continuation value $W = (W_t)_t$

associated with the contract Π , we consider that Π is incentive compatible, so both the principal and the agent have the same set of information. Then, we have

$$W_t = E^a \left[\int_t^\tau e^{-\gamma t} (dU_t + \lambda_t a_t dt) \right]. \quad (8)$$

The agent's continuation value represents the agent's future expected earnings from any date t . As the agent is protected by limited liability, we impose that the contractual relationship stops the first time $W_t = 0$. As a consequence, we introduce

$$\tau_0 = \inf\{t \geq 0 \mid W_t = 0\} \quad (9)$$

and we have that $\tau \leq \tau_0$.

Besides, our model relies on a second state variable that accounts for the advent of the shock on the agent's benefit of cash-flow diversion. This is the purpose of the variable that we denote by $N = \{N_t\}_{t \geq 0}$, a single-jump process that makes λ jumps from $\bar{\lambda}$ to $\underline{\lambda} < \bar{\lambda}$ with intensity p . Specifically, pdt is the probability that the jump occurs during any time interval $(t, t + dt]$. N takes the value 0 when $\lambda = \bar{\lambda}$ and the value 1 when $\lambda = \underline{\lambda}$.

In the following lemma, we apply the martingale representation theorem to find a stochastic representation for the agent's continuation value.

Lemma 1. Representation of the agent's continuation value as a jump-diffusion process

There exists a pair of \mathcal{F} -predictable processes (β, δ) where $\beta = (\beta_t, t \geq 0)$ and $\delta = (\delta_t, t \geq 0)$ such that the agent's continuation value W evolves according to the dynamics

$$dW_t = \gamma W_t dt + \beta_t (dZ_t - a_t dt) + \delta_t (dN_t - pdt 1_{N_t=0}) - dU_t \quad \text{for } t \leq \tau \quad (10)$$

We call β the agent's sensitivity to the cash-flow process and δ the agent's sensitivity to the advent of the shock.

First, we note that the martingale term $\delta_t (dN_t - pdt 1_{N_t=0})$ is zero on expectation, so the agent's continuation value grows at the rate γ if there is no cash-flow diversion. Second, the agent is made sensitive to the realization of the firm's cash flows through the term $\beta_t dZ_t$. Whenever the agent diverts cash-flow and $a_t = \bar{a}$, it impacts the profitability of the firm and thus it reduces his continuation value by $\beta_t \bar{a}$ per unit of time. Also, we note the agent's continuation value instantly jumps at the advent of the shock, and the size of the jump when the shock occurs is given by the value of the process δ .

Now, we derive the optimal contract using the dynamic programming approach. As value functions are forward-looking processes, we use backward induction to solve the principal's problem. Thus, we will derive the solution after the exogenous shock on the agent's benefit of cash-flow diversion and then before the shock. We call the principal's value the highest value that the principal can extract from the delegation to the agent. It depends on both the agent's continuation value and on whether the shock on the agent's benefit of cash flow diversion has occurred. In the rest of this paper, we denote the principal's value by V_0 before the shock and V_1 after the shock. Once the shock has occurred, the agent's benefit of cash-flow diversion remains constant forever, and thus the optimal contract is the one derived in DeMarzo and Sannikov (2006) and characterized in the following proposition.

Proposition 1. ⁵ *The optimal contract after the shock on the agent's benefit of cash-flow diversion*

Suppose that the agent's benefit of cash-flow diversion is equal to $\underline{\lambda}$. Then, under the optimal contract that induces the agent to never divert cash flow, the principal's value function is concave and solves the following Second-Order Differential Equation

$$rV_1(w) = \mu + \gamma w V_1'(w) + \frac{1}{2} \underline{\lambda}^2 V_1''(w) \quad \text{for any } w \in [0; \bar{W}^1]; \quad (11)$$

together with $V_1(0) = L$ (value-matching condition); $V_1'(\bar{W}^1) = -1$ (smooth-pasting condition); and $V_1''(\bar{W}^1) = 0$ (super-contact condition). The value function extends linearly afterward with slope -1. In addition, it is optimal to terminate the contract at τ_0 .

As making the agent sensitive to the project's cash flow is costly, β is set to the minimum value that induces the agent not to divert cash flow, i.e., $\beta_t = \underline{\lambda}$ up to the contract termination. It is maintained constant as the agent's private benefit is fixed after the advent of the exogenous shock.

Moreover, it is valuable for the principal to postpone payment up to a certain threshold denoted \bar{W}^1 . A lump-sum payment ΔU provided at any instant would make the principal proceed with the optimal contract and a remaining continuation value of $w - \Delta U$. Hence, the inequality $V(w) \geq V(w - \Delta U) - \Delta U$ must hold at any instant. It implies that $V'(w) \geq -1, \forall w$, so the marginal benefit of providing incentives through deferred compensation remains greater than the marginal value of making a lump-sum payment to the agent.

⁵The proof is provided in DeMarzo and Sannikov (2006), Section 3, Proposition 7

Finally, the contract is terminated as soon as W reaches 0. Indeed, it implies that the agent no longer expects to earn any benefit from the contractual relationship, so nothing precludes that he diverts cash flow onwards.

We consider now the principal's value before the jump on the agent's benefit of cash-flow diversion. The following proposition characterizes the optimal contract in this situation.⁶

Proposition 2. *The optimal contract before the shock on the agent's benefit of cash-flow diversion*

Assume that both parties foresee the advent of a shock on the agent's benefit of cash-flow diversion that makes the parameter λ drop from $\bar{\lambda}$ to $\underline{\lambda} < \bar{\lambda}$. Then, under the optimal contract that induces the agent to never divert cash flow, the principal's value function is concave and is given by the solution to the following second-order differential equation

$$\forall w \in [0; \bar{W}^0], \quad (p + r)V_0(w) = \mu + (\gamma w - p\delta(w))V_0'(w) + \frac{1}{2}\bar{\lambda}^2 V_0''(w) + pV_1(w + \delta(w)) \quad (12)$$

together with $V_0(0) = L$ (the value-matching condition), $V_0'(\bar{W}^0) = -1$ (the smooth-pasting condition), $V_0''(\bar{W}^0) = 0$ (the super-contact condition), and where the value function V_1 is characterized in Proposition 1. The value function extends linearly afterwards with slope -1. δ is such that $V_1'(W_{t-} + \delta(W_{t-})) = V_0'(W_{t-})$.

3.1 Analysis of the Optimal Contract

Let us discuss the optimal contract in detail. Figure 1 illustrates the principal's value functions associated with the optimal contract and the optimal sensitivity to a shock on the agent's ability to divert cash flow.

At the onset of the shock, the principal's value function jumps from V_0 to $V_1 > V_0$. This jump occurs because the shock reduces the severity of the agency friction by making cash-flow diversion relatively less efficient for the agent. Consequently, providing incentives to the agent becomes cheaper for the principal. As a result, the principal can lower the sensitivity of the agent to the cash-flow process. Additionally, it is no longer necessary to defer compensation to the agent as much as before, so the payment boundary shifts from \bar{W}^0 to $\bar{W}^1 < \bar{W}^0$.

⁶For that purpose, we need to define the left-limit of any continuous-time process Y_t as $Y_{t-} = \lim_{s \uparrow t} Y_s$ together with $Y_{0-} = Y_0$.

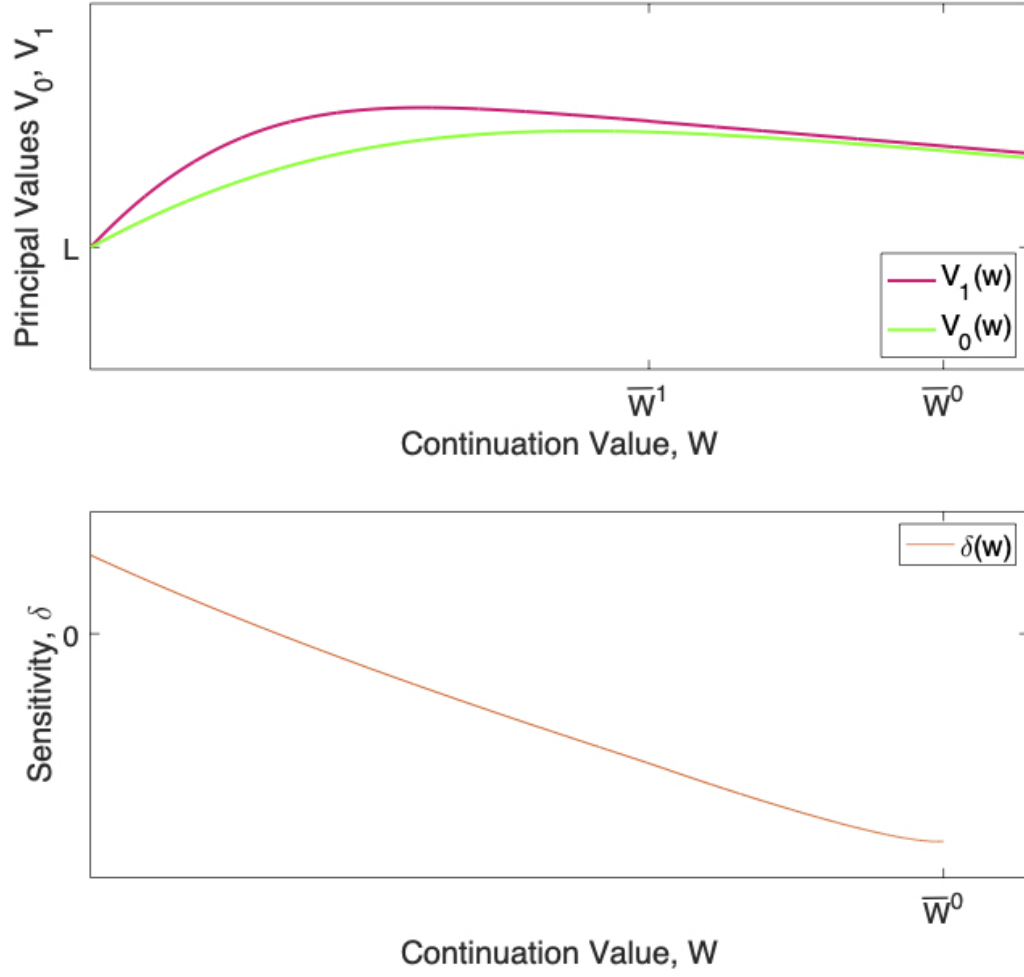


Figure 1: **Principal's value functions $V_0(\cdot)$ and $V_1(\cdot)$ prior to and after the shock on the agent's benefit of cash-flow diversion (upper graph), and a representation of $\delta(\cdot)$ the agent's sensitivity to the advent of the regulation (lower graph).** The parameters value is provided in Table 1.

Next, let us discuss how the agent's sensitivity to the shock is determined to know how the agent's continuation value changes at the onset of the shock. Proposition 2 shows that the sensitivity to the shock maintains the marginal value of delegating to the agent constant ($V_1'(W_{t-} + \delta(W_{t-})) = V_0'(W_{t-})$). We will now explain why this leads to the agent's continuation value compression when the shock occurs.

First, $\bar{W}^1 < \bar{W}^0$, and with the equality $V_0'(\bar{W}^0) = V_1'(\bar{W}^1) = -1$, we obtain $\delta(\bar{W}^0) = \bar{W}^1 - \bar{W}^0 < 0$. As a result, an agent who has performed well enough up to the onset of the shock experiences a drop in their continuation value. Therefore, δ takes negative values when W is large enough. Second, the decrease in the benefit of cash-flow diversion makes contract termination less efficient for the principal ($V_1'(0) > V_0'(0)$)⁷. By continuity of the sensitivity to the shock δ with respect to W , δ takes positive values when W is low, and δ takes negative values when W is high. We summarize this result in the following proposition.

Proposition 3. *Our model predicts that a regulation that reduces the manager's ability to divert cash flow leads to a differential impact on poorly versus well-performing managers. Poorly performing managers benefit from the regulation in terms of higher expected bonuses and lower risk of dismissal. In contrast, well-performing managers suffer from lower expected bonuses due to the compression of the payment structure.*

In the following section, we present several testable implications to investigate how a change in the regulatory environment of the contract impacts the provision of incentives.

4 Testable Implication

In this section, we provide testable implications of our dynamic contracting model in the context of regulatory changes that impact the agent's benefit of cash-flow diversion. Regulators such as the Securities and Exchange Commission (SEC) are implementing reforms to address excessive pay practices (Murphy (2013)). Several U.S. regulatory reforms have been adopted over the past century to ensure that managers' objectives align with those of shareholders. The Securities Act of 1934, implemented during the Great Depression, required disclosure of executives' benefits from trading shares of their own company. In the 1970s, the high use of perquisites due to tax advantages was regulated, and new rules on stock options were passed in the 1990s. More recently, the 2006 Compensation Discussion and Analysis mandates the description of performance metrics and targets for each component of the

⁷We have that $V_0(0) = V_1(0)$ while $V_0 < V_1$ as $\bar{\lambda} > \underline{\lambda}$. Hence, $V_0'(0) = V_1'(0)$

managers' incentive structure. Finally, the Dodd-Frank Act (2010-2011) regulates the pay package for both financial institutions and other publicly traded companies.

We use the expansion of mandatory compensation disclosure (MCD) to contextualize the regulatory shock that affects the agent's benefit to divert cash flow. Arguably, mandatory compensation disclosure affects the managers' benefit to divert cash out of shareholders' sights. Consequently, our theoretical framework sheds light (1) on the empirically observed change in executive compensation and also (2) on the propensity of executives to be dismissed following the new regulations. While the first impact is already documented in the academic literature (Mas 2019; and Gipper 2020, among others), there exists no empirical evidence that such rules may impact the threat of dismissal to the best of our knowledge.

We use the expansion of mandatory compensation disclosure (MCD) to contextualize the regulatory shock that affects the agent's benefit to divert cash flow. MCD regulations require firms to disclose executive compensation, and the literature provides evidence that it reduces the manager's misbehavior (see Cheng and Warfield (2005), Larcker et al. (2007) Jiang et al. (2010), and Bao et al. (2022)). Our theoretical framework sheds light on two empirical observations resulting from these regulations. First, our model can explain the empirically observed changes in executive compensation, as documented in academic literature (see Mas (2019) and Gipper (2021)). Second, our model can explain the potential impact of these regulations on the threat of dismissal for executives, an area where no empirical evidence currently exists to the best of our knowledge.

To provide testable implications, we use the dynamics of the agent's continuation under the optimal contract and numerically solve for the optimal contract using the shooting method. This approach allows us to extract the estimated payment boundaries and the estimated sensitivity to the implementation of MCD. We then perform Monte Carlo simulations to analyze how the introduction of MCD changes the contractual characteristics.

To specify the regulatory shock, we use the expansion of MCD regulations as discussed earlier. Murphy (2013) identified four reforms on MCD implemented by the SEC in the 1970-2011 period. Therefore, we assume that a new reform is expected every ten years and set the intensity of such reforms to $p = 0.1$. The complete list of parameters and variables used in our model is provided in the following table.

Expected bonus following an MCD reform.

We consider a discrete set of values to initialize the agent's continuation value and estimate the expected bonuses received over a fixed period associated with the MCD reform. Formally, the total discounted bonus received over a period of length

Variable	Symbol	Parameter	Symbol	Value
Cumulative cash flows	X	Principal's discount rate	r	0.10
Contract termination date	τ	Agent's discount rate	γ	0.12
Principal's value prior to the MCD reform	V_0	Intensity of the advent of the MCD reform	p	0.1
Principal's value after the MCD reform	V_1	Fraction of the cash-flow diverted consumed by the agent before the MCD reform	$\bar{\lambda}$	0.5
Payment boundary prior to the MCD reform	\bar{W}^0	Fraction of the cash-flow diverted consumed by the agent after the MCD reform	$\underline{\lambda}$	0.25
Payment boundary prior to the MCD reform	\bar{W}^1	Principal's value at contract termination	L	40
Sensitivity to the output	β	Agent's action	\bar{a}	10
Sensitivity to the MCD reform	δ			

Table 1:
Parameter Values and Variables

$t \wedge \tau$ following the MCD reform is given by

$$\phi_1^t(w) = E^a \left[\int_0^{t \wedge \tau} e^{-\gamma s} dU_s \mid W_{0-} = w \ \& \ N_0 = 1 \right], \quad (13)$$

where the agent's continuation value initially jumps from $W_{0-} = w$ to $W_0 = w + \delta(w)$. In the counterfactual scenario where no MCD reform occurs at date 0, the total discounted bonus received over the same period of length $t \wedge \tau$ is given by

$$\phi_0^t(w) = E^a \left[\int_0^{t \wedge \tau} e^{-\gamma s} dU_s \mid W_{0-} = w \ \& \ N_0 = 0 \right]. \quad (14)$$

where $W_0 = w$. Although no reform occurs at date 0 in the counterfactual situation, it does not preclude that it may occur during the interval $]0; t \wedge \tau]$.

We compare the cross-sectional estimates $\phi_1^t(w)$ and $\phi_0^t(w)$ and consider the cumulative bonuses received over 15 years. Our results are presented in Figure 2.

Our simulations suggest a compression of bonuses following the MCD reform. Specifically, while agents with a low continuation value expect to earn more following the MCD reform, those with a large continuation value just before the MCD

reform expect a decrease in their bonuses. We formalize this result in the following implication.

Implication 1. *Enforcing a regulation that limits the agent’s benefit of cash-flow diversion leads to the compression of the bonuses.*

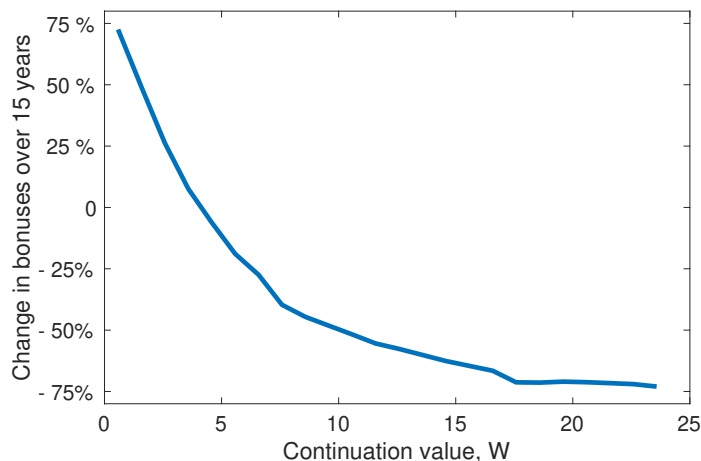


Figure 2: **Compression of bonuses following the drop in agency friction.** Here, the change in expected bonus is computed over 15 years following the shock on the agent’s benefit of cash-flow diversion.

Our findings are consistent with Mas (2019)⁸ who studies the impact of the 1934 SEC reform on manager compensation. Mas finds that the pay of managers above the 97th percentile of the earnings distribution has dropped following the reform, while the rest of the managers have gained from the reform. Here, the initial agent’s continuation value is a good proxy for the manager’s earnings distribution because it measures how much the manager expects to earn in the future from the contractual relationship. While Mas considers the drop in compensation for managers at the right tail of the distribution “intriguing,” our optimal contracting approach provides a potential theoretical explanation for this phenomenon.

Manager dismissal following an MCD reform. We now derive a testable prediction on how the MCD reform impacts the manager’s contract termination.

⁸See Figure 9 10, pp. 43–44 in Mas (2019).

First, we consider the following process:

$$\tau^1(w) = E^a [\inf\{t > 0 \mid W_t = 0 \text{ such that } W_{0-} = w \ \& \ N_0 = 1\}]. \quad (15)$$

It represents the expected contract duration following the implementation of the MCD reform where the agent's continuation value jumps from $W_{0-} = w$ to $W_0 = w + \delta(w)$. In the counterfactual scenario, the expected contract duration is denoted by $\tau^0(w)$ and satisfies

$$\tau^0(w) = E^a [\inf\{t > 0 \text{ such that } W_t = 0, \text{ given } W_0 = w \ \& \ N_0 = 0\}]. \quad (16)$$

Next, we compute the probability that the contract terminates within one year and illustrate our result in Figure 3.

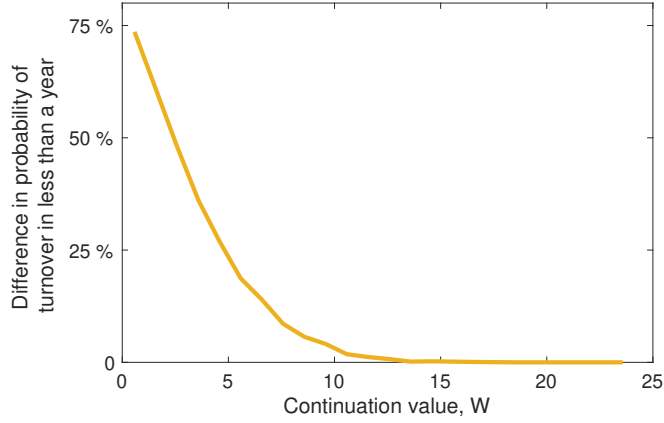


Figure 3: **Regulatory-induced retention of poor performers.** Dismissal is less frequent for poor performers following the drop in the agency friction.

We observe that the MCD reform significantly reduces the risk of termination over the first year for managers with a low continuation value. These agents are at great risk of dismissal prior to the regulatory shock, but the shock moves them away from the termination boundary because termination becomes relatively less efficient compared to continuation. For example, our simulations estimate that for an agent with a very low continuation value just before the MCD reform ($W_0 = 0.5$), the probability of termination over the following year drops from 88

To formalize this phenomenon, we introduce the concept of regulation-induced retention, defined as maintaining an agent active while they would have been dismissed in the absence of the regulatory shock. We present this observation in the following implication.

Implication 2. *Enforcing a regulation that limits the agent’s benefit of cash-flow diversion leads to an over-retention of poor performers.*

We find that our model predicts that regulatory reforms such as the mandatory compensation disclosure, which reduce the agent’s benefit to divert cash flow, will benefit poor performers by increasing their expected bonuses and decreasing their risk of dismissal, while penalizing good performers by decreasing their expected bonuses. Specifically, we expect that following the 1934 SEC reform on managers’ compensation, which led to an increase in compensation for the lowest-paid managers as documented by Mas (2017), a decrease in the dismissal of managers should have followed too. However, recent reforms on pay disclosure seem to have had different effects. For instance, Gipper (2021) finds that the overall effect of the 2006 Compensation Discussion and Analysis (CD&A) reform on manager pay is positive.

In the subsequent section, we expand upon the primary model by endogenizing the values received by the principal and the agent upon contract termination. We show that such an approach helps to alleviate the negative impacts of regulation.”

5 Extension of the Model

In this section, we build upon Section IV.A of DeMarzo and Sannikov (2006) and Chemla et al. (2022), expanding the primary model to include endogenous payoffs for both the principal and the agent upon contract termination. Specifically, we assume that the managerial labor market is competitive, allowing both the principal and the agent to find new counterparts after the termination of the agent’s contract.

With this extension, the principal can now offer an optimal contract to a new agent at a cost of κ_p . Consequently, the principal receives:

$$\begin{aligned} L_0 &= V_0(W_0^*) - \kappa_p, & \text{before the shock,} \\ L_1 &= V_1(W_1^*) - \kappa_p, & \text{after the shock,} \end{aligned}$$

where V_0 (respectively, V_1) represents the principal’s value function before (respectively, after) the shock. In this context, the new contract is initiated at the value W_1^* that maximizes the principal’s payoff $V_i(W_i^*)$ for $i \in 1; 2$.

Next, let us consider the agent’s perspective. We assume that upon termination of the contract, the agent can find a new contract with a different firm by incurring a cost of κ_a . The agent receives a payoff at contract termination which is derived

from their outside option:

$$\begin{aligned} R_0 &= W_0^* - \kappa_a, & \text{before the shock,} \\ R_1 &= W_1^* - \kappa_a, & \text{after the shock.} \end{aligned}$$

All termination values $\{L_0, L_1, R_0, R_1\}$ are assumed to be positive. As in DeMarzo and Sannikov (2006), assuming endogenous termination payoffs only change the boundary conditions of the optimal contract, which is otherwise identical to the one described in the main model. We assume that a new firm and new agent, identical to the previous ones, are always available in the market. We show in the proof of Proposition 4 that there exists a unique equilibrium on the labor market exists.

It is noteworthy that when the shock occurs and thus that the agents' benefit to divert cash-flows is reduced, delegation becomes more valuable and the agent can extract less value from the contract. This translates into $V_1(W_1^*) > V_0(W_0^*)$ and $W_1^* < W_0^*$, so $R_1 < R_0$ and $L_1 > L_0$. Consequently, this translates into diminishing the optimal agent's sensitivity to the shock compared to the baseline model, as illustrated by the lower panel Figure 4.

To see how this compares to the main model, we derive similar testable implications than in Section 4, but for our dynamic contracting model with endogenous termination payoffs. Figures 5 and 6 illustrates our results. In the main model, a manager who performs poorly expects to be more compensated and to be at a lesser risk of dismissal after the regulation is implemented. Here, the presence of a competitive managerial labor market mitigates these two undesirable effects. This is intuitive, because the firm's alternative to continuing the contract in presence of a competitive labor market – which is terminating the current contract and offering a identical contract to a new and otherwise identical agent – is also more valuable after the shock. We formalize this result in the following proposition.

To compare this extended model to the main model, we derive testable implications similar to those presented in Section 4. Our results are illustrated in Figures 5 and 6.

In the main model, a manager who performs poorly expects increased compensation and a lower risk of dismissal after the regulation is implemented. However, in the extended model, the presence of a competitive managerial labor market helps mitigate these two undesirable effects. Intuitively, this is because the firm's alternative option in the presence of a competitive labor market –terminating the current contract and offering an identical contract to a new, otherwise identical agent – also becomes more valuable after the shock. Hence, in the presence of a competitive labor

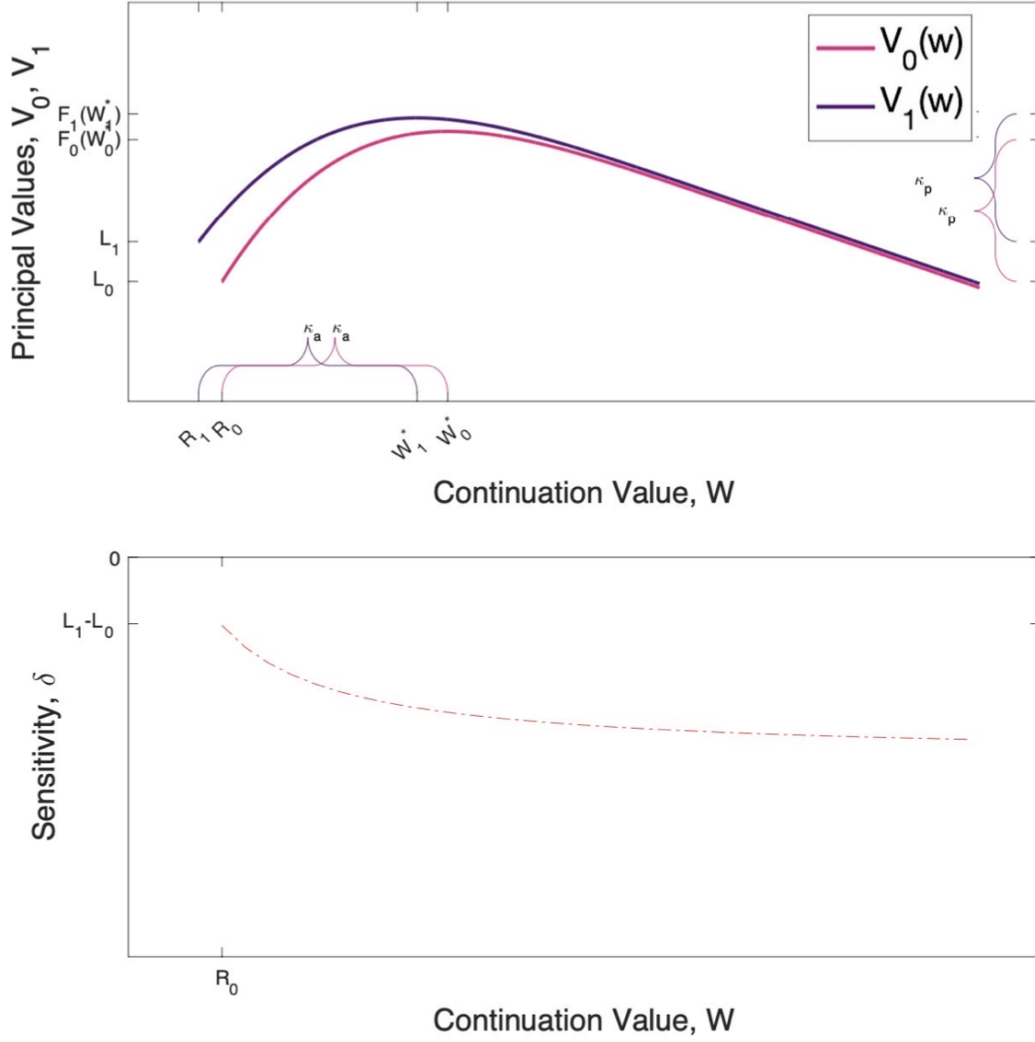


Figure 4: **Determining termination payoffs endogenously.** The upper panel illustrates the principal's value functions V_0 and V_1 – before and after the shock – when the agent can be replaced by the principal at a cost κ_p and when a new firm can be found by the agent at a cost κ_a . The lower panel illustrates the associated agent's sensitivity process to the advent of the shock (lower graph). The parameters value is provided in Table 1, except $\kappa_a = \kappa_p = 1$.

market, contract termination is less inefficient than in the main model. We formalize this finding in the following proposition.

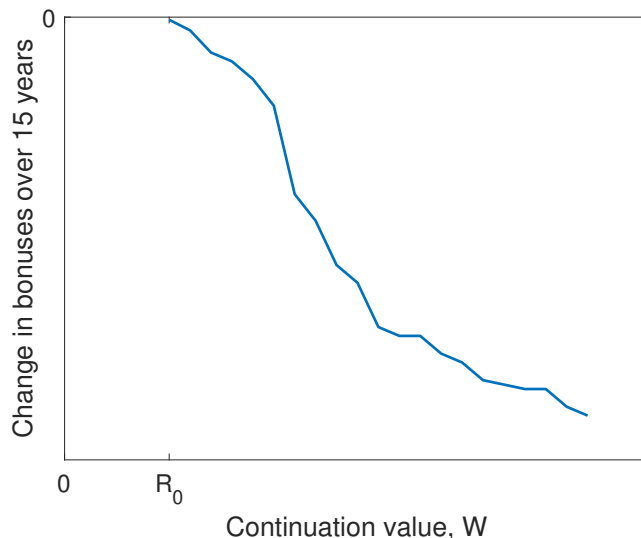


Figure 5: **Bonuses are not compressed in presence of a competitive labor market.** The change in expected bonus is computed over 15 years following the shock on the agent’s benefit of cash-flow diversion. We assume endogenous termination payoffs. The parameters value is provided in Table 1, except $\kappa_a = \kappa_p = 1$.

Proposition 4. *The presence of a competitive labor market mitigates the undesirable over-retention and over-compensation of poor performers that follow the implementation of the regulation.*

6 Conclusion

In conclusion, our paper provides novel insights into the relationship between managerial incentives and changes in agency friction arising during the contractual relationship. While several factors may induce such a change, we focus on regulation to not only help fix ideas but also because it addresses important policy implications regarding the design and consequences of executive compensation regulation.

Incorporating a regulatory shock that affects a firm manager’s ability to misbehave into a dynamic principal-agent model, we identify unintended consequences of the regulation. Specifically, while increased transparency benefits firm owners or

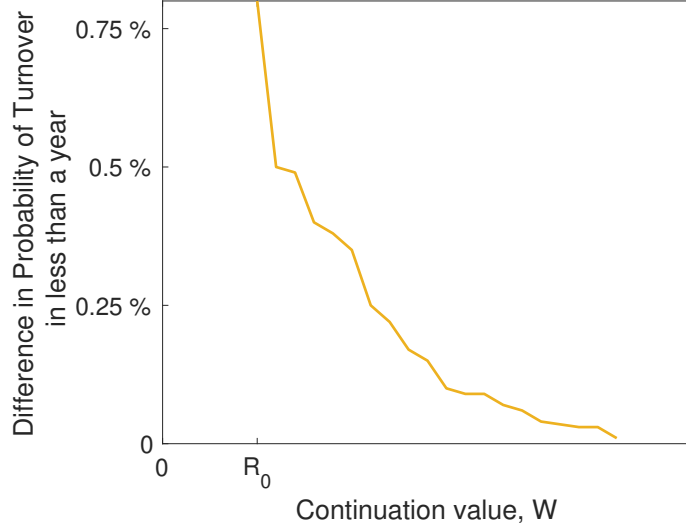


Figure 6: **The competitive labor market mitigates the retention of poor performers.** Here, the change in dismissal is close to zero for any level of continuation value. The parameters value is provided in Table 1, except $\kappa_a = \kappa_p = 1$.

shareholders by improving contract efficiency and reducing payments to top performers, the regulation also inadvertently supports underperformers. Indeed, we show that it may lead to an over-compensation and an over-retention of the underperformers. The underlying mechanism driving this outcome is that, from an ex-ante perspective, it is optimal to design a contract that makes the agent sensitive to the advent of the regulation. As the latter reduces delegation costs, it weakens the effectiveness of contract termination as an incentive mechanism. Consequently, the agent's sensitivity to the shock leads to the continued employment of underperforming agents, even when their performance might have justified termination in the absence of the regulation.

Our analysis also emphasizes the importance of accounting for the interplay between regulatory measures and a competitive labor market for executives when examining the resulting effects on managerial incentives. Indeed, incorporating a competitive labor market with search costs into the model mitigates the undesirable effects of the regulatory shock, such as increased compensation and over-retention of poor performers. The existence of a competitive labor market allows for termination payoffs to become endogenous, enabling both the principal and the agent to match with new counterparts after the agent's contract termination. This dynamic balances

the benefits of regulation while reducing its unintended consequences, highlighting the importance of considering the interaction between regulatory measures and the competitive labor market for executives.

We present testable implications concerning managerial compensation compression and regulation-induced retention, which could prove valuable for future empirical studies. By exploring the dynamic contracting perspective and its implications, this research not only sheds light on the intricate interplay between regulation, managerial incentives, and firm performance, but also lays the groundwork for future research on the optimal design of regulatory policies in the presence of agency conflicts. The findings presented in this paper emphasize the importance of carefully considering the potential consequences of regulatory measures, as they may lead to undesired outcomes unanticipated by policy makers.

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Appendices

A Probabilistic Background

Let Z_t^0 be a Brownian motion on a probability space $(\Omega, \mathcal{F}, \mathbb{P}_0)$, and let $\mathcal{F}(t)$ be a natural filtration for this Brownian motion.

Under the probability measure \mathbb{P}_0 , the firm's cash flows evolves as

$$dX_t = \mu dt + \sigma dZ_t^0 \quad (17)$$

The agent's strategy of cash-flow diversion is defined by the \mathcal{F}_t -adapted process $(a_t)_{t>0}$, with a_t taking value in $\{0, \bar{a}\}$, and $\bar{a} < +\infty$. The agent's decision changes the probability measure and the drift of the cash-flow process. Let us denote by \mathbb{P}_a the probability measure associated to a cash-flow diversion strategy $(a_t)_t$. The dynamics of the firm's cash flows under \mathbb{P}_a is

$$dX_t = (\mu - a_t)dt + \sigma dZ_t \quad (18)$$

Because a_t is bounded, the following Novikov's criterion

$$E \left[\exp\left(\frac{1}{2} \int_0^T \frac{a_s^2}{\sigma} ds\right) \right] < +\infty, \quad (19)$$

is satisfied. Thus, the process

$$\left(\exp \left(\int_0^T -\frac{a_s}{\sigma} dZ_s - \frac{1}{2} \int_0^T \frac{a_s^2}{\sigma} ds \right) \right)_t \quad (20)$$

is a martingale,⁹ and the probability measures \mathbb{P}_a and \mathbb{P}_0 are equivalent and related by the formula

$$\frac{d\mathbb{P}_a}{d\mathbb{P}_0} \Big|_{\mathcal{F}_T} = \exp \left(\int_0^T -\frac{a_s}{\sigma} dZ_s - \frac{1}{2} \int_0^T \frac{a_s^2}{\sigma} ds \right). \quad (21)$$

Following Girsanov theorem, the process $dZ_t = dZ_t^0 + \frac{a_t}{\sigma} dt$ is then a Brownian motion under the probability measure \mathbb{P}_a .

⁹see Proposition 5.12 in Karatzas and Shreve (1991).

B Proofs

B.1 Proof of Lemma 1

First, we remind that $N = \{N_t\}_{t \geq 0}$ is a single-jump process that makes the variable λ jumps from $\bar{\lambda}$ to $\underline{\lambda}$ with intensity p . Specifically, pdt is the probability that the jump occurs during any time interval $(t, t + dt]$. N takes the value 0 when $\lambda = \bar{\lambda}$ and the value 1 when $\lambda = \underline{\lambda}$. We note that $(dN_t - pdt1_{N_t=0})$ is a compensated single-jump process.

Now, let us consider the agent's total expected future compensation from the incentive-compatible contract Π . It is given by

$$\Upsilon_t = \int_0^t e^{-\gamma s} (dU_s + \lambda_s a_s ds) + e^{-\gamma t} W_t \quad (22)$$

$$= E^a \left[\int_0^\tau e^{-\gamma s} (dU_s + \lambda_s a_s ds) \right] \quad (23)$$

By construction, it is a martingale under the probability measure \mathbb{P} introduced in the probabilistic background A.

Thus, we can apply the martingale representation theorem, so and there exists a pair of predictable processes (β, δ) such that

$$\Upsilon_t = \Upsilon_0 + \int_0^t e^{-\gamma s} \beta_s dZ_s + \int_0^t e^{-\gamma s} \delta_s (dN_s - pds1_{N_s=0}) \quad (24)$$

Differentiating (22) and (25) with respect to t we find that

$$dW_t = \gamma W_t dt + \beta_t dZ_t + \delta_t (dN_t - pdt1_{N_t=0}) - \lambda_t a_t dt - dU_t. \quad (25)$$

Thus, a contract Π defines a unique pair of sensitivity processes $(\beta_t(\Pi))_{t \geq 0}$ and $(\delta_t(\Pi))_{t \geq 0}$. Following Décamps and Villeneuve (2019), this means that for any incentive-compatible contract Π , there exists a unique triplet of \mathcal{F}_t adapted process W_t^Π , $\beta = (\beta_t(\Pi))_t$, and $\delta = (\delta_t(\Pi))_t$ such that

$$W_\tau^\Pi = 0, \quad (26)$$

$$dW_t^\Pi = \gamma W_t^\Pi dt + \beta_t(\Pi)(dZ_t - a_t dt) + \delta_t(\Pi)(dN_t - pdt1_{N_t=0}) - dU_t \quad (27)$$

Following Sannikov (2008), the sensitivity processes $\beta_t(\Pi)$, and $\delta_t(\Pi)$ can be seen as a controls. To this end, fix the compensation process $(U_t)_{t \geq 0}$ and consider the

process $(W_t^{\beta,\delta})_{t \geq 0}$ that satisfies the stochastic differential equation, controlled by β and δ , under \mathbb{P}

$$dW_t^{\beta,\delta} = \gamma W_t^{\beta,\delta} dt + \beta_t(\Pi)(dZ_t - a_t dt) + \delta_t(\Pi)(dN_t - p dt 1_{N_t=0}) - dU_t \quad (28)$$

$(W_t^{\beta,\delta})_{t \leq \tau}$ must remain positive to be the agent's continuation value associated to the contract Π . To that purpose, we introduce the stopping-time

$$\tau^{\beta,\delta}(U) = \inf\{t \geq 0, W_t^{\beta,\delta} = 0\}. \quad (29)$$

This implies that

$$\left(e^{-rt} W_t^{\beta,\delta} + \int_0^t e^{-rs} (dU_s + \lambda_s a_s ds) \right)_{t \leq \tau^{\beta,\delta}(U)} \quad (30)$$

is a uniformly integrable martingale under \mathbb{P} . Using Optional sampling Theorem, we have

$$\begin{aligned} 0 \leq W_0^{\beta,\delta} &= E^a \left[\int_0^{\tau^{\beta,\delta}} e^{-rs} (dU_s + \lambda_s a_s ds) \right] \\ &= W_0^\Pi. \end{aligned}$$

Hence, the principal can control the dynamics of the agent's continuation value with the pair of process (β, δ) and terminates the contract the first time the continuation value hits zero.

B.2 Proof of Proposition 2

We follow the proof of Proposition 2 in Hoffmann and Pfeil (2010) that consists in two steps. First, we prove the concavity of the value function V_0 prior to the shock on the agent's benefit of cash-flow diversion. Then, we apply the verification theorem to ensure that V_0 corresponds to the principal's value function.

Concavity

Consider the total surplus $U_0(w) = V_0(w) + w$ generated by the contractual relationship for any level of continuation value w . Its first-derivative satisfies :

$$(r - \gamma)U_0'(w) = (r - \gamma) + (\gamma w - \delta(w)p)U_0''(w) + \frac{1}{2}\bar{\lambda}^2 U_0'''(w), \quad (31)$$

together with $U(0) = L$, $U'(\bar{W}_0) = 0$, and $U''(\bar{W}_0) = 0$. Hence, there exists $\epsilon > 0$ such that

$$U'''(w - \epsilon) > 0, \quad (32)$$

$$\text{and } U'''(w + \epsilon) < 0. \quad (33)$$

If we assume that $\exists \hat{W} := \{\sup w \mid U''(w) > 0\}$, then by continuity, $U''(\hat{W}) = 0$, and from (31), $U'_0(\hat{W}) > 1$.

As δ satisfies $V_0(w) = V_1(w + \delta(w))$, we get by differentiation that $\delta'(\hat{W}) = -1$. Now, consider two points $W_1 < \hat{W} < W_2$ in the neighborhood of \hat{W} , such that $U''_0(W_1) > 0 > U''_0(W_2)$ and $W_1 U'_0(W_1) = W_2 U'_0(W_2)$. We note that

$$\begin{aligned} (r + p)U_0(w) &= \mu + (r - \gamma)w + (\gamma + p)wU'_0(w) + \frac{1}{2}\bar{\lambda}^2 U''_0(w) \\ &\quad + p(U_1(w + \delta(w)) - (w + \delta(w))U'_1(w + \delta(w))) \end{aligned} \quad (34)$$

Hence $U(W_1) > U(W_2)$, which contradicts with $U'(\hat{W}) > 1$. Consequently, U_0 and hence V_0 are concave.

Verification

Let us evaluate the process $(e^{-r(\tau_N \wedge \tau)} V_0(W_{t \wedge \tau}))$ when the shock occurs, i.e., where τ_N is such that $dN_{\tau_N} = 1$. Applying Itô's lemma, we find that :

$$V_0(W_{0-}) = e^{-r(\tau_N \wedge \tau)} V_0(W_{\tau_N \wedge \tau}) - A - B - C, \quad (35)$$

where :

$$\begin{aligned} A &= \int_0^{\tau_N \wedge \tau} e^{-rt} \left[(\gamma W_{t-} - p\delta_t) V'_0(W_{t-}) + \frac{1}{2} \bar{\lambda}^2 V''_0(W_{t-}) \right. \\ &\quad \left. + p(V_1(W_{t-} + \delta_t) - V_0(W_{t-}) - rV_0(W_{t-})) \right], \end{aligned} \quad (36)$$

$$B = - \int_0^{\tau_N \wedge \tau} e^{-rt} V'_0(W_{t-}) dU_t, \quad (37)$$

and

$$C = \int_0^{\tau_N \wedge \tau} e^{-rt} V'_0(W_{t-}) \bar{\lambda} dZ_t + \int_0^{\tau_N \wedge \tau} e^{-rt} (V_1(W_{t-} + \delta_t) - V_0(W_{t-})) (dN_s - pds) \quad (38)$$

From 12, we have that

$$A \leq - \int_0^{\tau_N \wedge \tau} e^{-rt} \mu dt. \quad (39)$$

From the inequality $V'_0(W_{t-} \geq -1$, we have that

$$B \leq \int_0^t e^{-rt} dU_t. \quad (40)$$

Finally, given the martingale property of the processes Z and $(N_t - \int_0^t p dt)_t$, $E[C] = 0$. Consequently, we can rewrite 41 and we get

$$V_0(W_{0-}) \geq E \left[\int_0^{\tau_N \wedge \tau} e^{-rt} (\mu dt - dU_t) + e^{-r(\tau_N \wedge \tau)} V_0(W_{\tau_N \wedge \tau}) \right], \quad (41)$$

and it holds with equality for the contract characterized in Proposition (2).

B.3 Proof of Proposition 3.1

First, we note that the function $\delta(W)$ is continuous and differentiable, where the agent's continuation value $W \in [0, \bar{W}^0]$. To see this, note first that the firm's value functions V_0 and V_1 are continuous and differentiable. From the definition of $\delta(W)$, we have $V'_1(W + \delta(W)) = V'_0(W)$. Taking the derivative of both sides with respect to W , we get $V''_1(W + \delta(W))(1 + \delta'(W)) = V''_0(W)$. Since both V_0 and V_1 are assumed to be continuously differentiable, their second derivatives are also continuous. Therefore, $V''_0(W)$ and $V''_1(W + \delta(W))$ are continuous functions of W . As a result, $\delta'(W)$ is also continuous since it can be written as

$$\delta'(W) = \frac{V''_0(W)}{V''_1(W + \delta(W))} - 1. \quad (42)$$

Now, for $W = 0$, we have $\delta(W)$ that is positive because $V'_1(0) > V'_0(0)$. For $W = \bar{W}^0$, we have δ that is negative because V_1 dominates V_0 , so $\bar{W}^1 \leq \bar{W}^0$. Indeed, these payment barriers are defined such that $V'_1(\bar{W}^1) = V'_0(\bar{W}^0) = -1$, $\delta(\bar{W}^0) = \bar{W}^1 - \bar{W}^0 \leq 0$. The continuity of the function $\delta(W)$ implies that δ takes positive value when W is low (hence for poor performers) and takes negative values when W is high (for good performers).

Therefore, the agent's continuation value of poor performers moves further away from the termination boundary, which is set at $W = 0$, and it translates into a lower risk of dismissal and a larger expected revenue from the contractual relationship. For good performers, a negative delta translates into a drop in their continuation value following the shock, which translates into a lower expected revenue.

B.4 Proof of Proposition 4

To evaluate how in presence of a competitive labor market, the shock impacts the over-retention and over-compensation of poor performers, we have to investigate how it impacts the optimal sensitivity to the shock. The validity of Proposition 4 is based on two conditions: (1) when there is a shock to the agency friction, the principal's termination value increases and the agent's termination value decreases, and (2) there is a negative change in the derivative of the function with respect to the agent's continuation value. The proof consists of analyzing the firm's value before and after the shock, and then discussing the relationship between them. Prior to this point, we have to establish that there exists unique equilibrium values of termination, which are dependent on the search costs.

In Section 5 of the paper, we use simplified notations to make the concepts easier to understand. However, we are using here more specific notations to make it clear that the value of the firm depends on the agent's continuation value, denoted by W , given the termination values. We use R_0 and R_1 to denote the value of the agent to sign a contract with a firm before and after the shock, respectively. Similarly, we use L_0 and L_1 to represent the value for the principal to hire an agent before and after the shock, respectively. Whenever a contract terminates, the firm (respectively the agent) signs a contract with a new and otherwise identical agent (resp. firm).

For now, let us define $V_1(W; R_1, L_1)$ as the firm's value after the shock. Based on Proposition 1 and given the termination values, it follows that the firm's value after the shock is governed by the following equation:

$$rV_1(W; R_1, L_1) = \mu + \gamma W V_1'(W; R_1, L_1) + \frac{1}{2} \lambda^2 V_1''(W; R_1, L_1) \quad \text{for any } W \in [R_1; \bar{W}^1(R_1, L_1)]; \quad (43)$$

together with the lower-boundary condition $V_1(W; R_1, L_1) = L_1$ and the usual smooth-pasting and super-contact conditions at the upper boundary $\bar{W}^1(R_1, L_1)$. As in the baseline model, these conditions collectively determine the behavior of the firm's value after the shock.

Let's now discuss the firm value before the shock, and denoted by $V_0(W; R_0, L_0, R_1, L_1)$. It depends on the agent's continuation value W , given the termination values before the shock, i.e., R_0 and L_0 , and after the shock, i.e., R_1 and L_1 . The firm value before the shock is related to the firm value after the shock, and it follows from Proposition

(2) that the former satisfies the following equation:

$$\begin{aligned}
(p+r)V_0(W; R_0, L_0, R_1, L_1) &= \mu \\
&+ (\gamma W - p\delta(W; R_0, L_0, R_1, L_1)) V'_0(W; R_0, L_0, R_1, L_1) + \frac{1}{2}\bar{\lambda}^2 V''_0(W; R_0, L_0, R_1, L_1) \\
&+ pV_1(W + \delta(W; R_0, L_0, R_1, L_1); R_1, L_1) \quad \forall W \in [R_0; \bar{W}^0],
\end{aligned} \tag{44}$$

together with the lower-boundary condition $V_0(R_0; R_0, L_0, R_1, L_1) = L_0$ and the usual smooth-pasting and super-contact conditions at the upper boundary \bar{W}^0 . The optimal sensitivity to the shock, denoted by $\delta(W; R_0, L_0, R_1, L_1)$, is found using the first order condition, which satisfies $V'_1(Wt^- + \delta(Wt^-; R_0, L_0, R_1, L_1); R_1, L_1) = V'_0(Wt^-; R_0, L_0, R_1, L_1)$. The equation is subject to the lower-boundary condition $V_0(R_0; R_0, L_0, R_1, L_1) = L_0$, and the usual smooth-pasting and super-contact conditions at the upper boundary \bar{W}^0 .

B.4.1 The problem of a labor market equilibrium

Assuming a competitive managerial labor market, both the agent and the principal can instantly find a new contractual relationship upon contract termination. The new contract starts the agent's continuation value at a level that maximizes firm value, but incurs search costs. An equilibrium –where the termination values are impacted by the shock on the agency friction – exists and is unique if the termination values fulfill the following conditions:

$$L_1^* = V_1(W_1(R_1^*, L_1^*); R_1^*, L_1^*) - \kappa_p, \tag{45}$$

$$R_1^* = W_1(R_1^*, L_1^*) - \kappa_a, \tag{46}$$

$$L_0^* = V_0(W_0(R_0^*, L_0^*, R_1^*, L_1^*); R_0^*, L_0^*, R_1^*, L_1^*) - \kappa_p, \tag{47}$$

$$R_0^* = W_0(R_0^*, L_0^*, R_1^*, L_1^*) - \kappa_a, \tag{48}$$

where

$$W_1(R, L) = \arg \max_W V_1(W; R, L)$$

and

$$W_0(R, L, R_1, L_1) = \arg \max_W V_0(W; R, L; R_1, L_1).$$

If we are before the shock, the new contract will be initiated at a value that maximizes the principal's payoff $V_0(W; R_0, L_0, R_1, L_1)$; after the shock, it will be initiated at a value that maximizes $V_1(W; R_1, L_1)$.

To ensure there exists a unique and stationary equilibrium associated with positive termination values, we follow Chemla et al. (2022) and set bounds to the termination costs. Then, we impose

$$0 < \kappa_a \leq \min(W_0(0, 0, 0, 0), W_1(0, 0)), \quad (49)$$

$$\begin{aligned} V_0(\kappa_a; 0, \bar{L}_0, 0, \bar{L}_1) - V_0(0; 0, \bar{L}_0, 0, \bar{L}_1) &\leq \kappa_p \\ &\leq V_0(W_0(0, 0, 0, 0); 0, 0, 0, 0) \\ &\quad - V_0(W_0(0, 0, 0, 0) - \kappa_a; 0, 0, 0, 0), \end{aligned} \quad (50)$$

$$V_1(\kappa_a; 0, \bar{L}_1) - V_1(0; 0, \bar{L}_1) \leq \kappa_p \leq V_1(W_1(0, 0); 0, 0) - V_1(W_1(0, 0) - \kappa_a; 0, 0), \quad (51)$$

where \bar{L}_1 satisfies $W_1(0, \bar{L}_1) = \kappa_a$ and \bar{L}_0 satisfies $W_0(0, \bar{L}_0, 0, \bar{L}_1) = \kappa_a$.

The proof of the existence and the uniqueness of the equilibrium relies on a fixed point argument. We first prove that an equilibrium exists after the shock, and then prove it also exists before the shock.

B.4.2 Existence and uniqueness of the equilibrium after the shock

First, let us fix the termination costs κ_p and κ_a and consider the set $S_1 = [0, \bar{L}_1]$, equipped with the metric $\rho(\hat{L}_x, \hat{L}_y) = |\hat{L}_x - \hat{L}_y|$. We denote by $T_1 : S_1 \rightarrow S_1$ the function

$$T_1(\hat{L}) = L + z_1(\hat{L}), \quad (52)$$

$$\text{where } z_1(\hat{L}) = V_1(W_1(0, \hat{L}); 0, \hat{L}) - \kappa_p - V_1(W_1(0, \hat{L}) - \kappa_a; 0, \hat{L}). \quad (53)$$

We demonstrate in the following Lemma that there exists a unique value \hat{L} that meets the requirements for the presence of equilibrium termination values (45)-(48).

Lemma 2. *Fix the search costs κ_a and κ_p satisfying the inequalities (49)-(51). There exists a unique $\hat{L}_1(\kappa_a, \kappa_p) \in S_1$ such that*

$$z_1(\hat{L}_1) = 0. \quad (54)$$

Proof. For any $\{L_x, L_y\}$ in S_1 , we have :

$$|T_1(L_x) - T_1(L_y)| = |L_x - L_y + z_1(L_x) - z_1(L_y)|. \quad (55)$$

By the Mean Value Theorem for differentiable functions, there exists a point c_1 between L_x and L_y such that

$$z_1(L_x) - z_1(L_y) = z'_1(c_1) * (L_x - L_y). \quad (56)$$

where $z'_1(L)$ is the derivative of $z_1(L)$ with respect to L . Since $z_1(L)$ is a sum of continuous and differentiable functions, it is itself continuous and differentiable. Hence,

$$|T_1(L_x) - T_1(L_y)| = |(1 + z'_1(c_1)) * (L_x - L_y)| \quad (57)$$

$$|T_1(L_x) - T_1(L_y)| \leq \beta_1 |L_x - L_y| \quad (58)$$

Where $\beta_1 = |1 + z'_1(c_1)| < 1$, because¹⁰

$$z'_1(L) = E[e^{-r\tau} | W_0 = W] - E[e^{-r\tau} | W_0 = W - \kappa_a] \quad (59)$$

$$\text{Hence, } z'_1(L) < 0 \quad \text{for any } L \in S_1. \quad (60)$$

By the Contraction Mapping Theorem¹¹, there exists a unique fixed point $\hat{L}_1(\kappa_a, \kappa_p) \in S_1$, such that

$$\hat{L}_1(\kappa_a, \kappa_p) = T_1(\hat{L}_1(\kappa_a, \kappa_p)) \quad (61)$$

Hence the equality (54). \square

Given the search costs κ_a and κ_p , there exists a unique value \hat{L} , which leads to a unique set of termination values R_1^* and L_1^* ¹². This set is determined by the equations:

$$W_1(R_1^*, L_1^*) = W_1(0, \hat{L}_1(\kappa_a, \kappa_p)) \quad (62)$$

$$V_1(W_1, L_1^*); R_1^*, L_1^* = V_1(W_1(0, \hat{L}_1(\kappa_a, \kappa_p)); 0, \hat{L}_1(\kappa_a, \kappa_p)). \quad (63)$$

Using these equations, we can solve for the termination values at the labor market equilibrium:

$$R_1^* = W_1(0, \hat{L}_1(\kappa_a, \kappa_p)) - \kappa_a, \quad (64)$$

$$\text{and : } L_1^* = V_1(W_1(0, \hat{L}_1(\kappa_a, \kappa_p)); 0, \hat{L}_1(\kappa_a, \kappa_p)) - \kappa_p. \quad (65)$$

Therefore, by knowing the search costs κ_a and κ_p , we can determine the unique set of termination values R_1^* and L_1^* based on the existence and uniqueness of the value \hat{L}_1 .

¹⁰This follows Corollary 1 of DeMarzo and Sannikov (2006), page 2718.

¹¹See Theorem 3.2 of Stokey and Lucas (1989), page 50.

¹²For clarity and whenever there is no ambiguity, we omit κ_a and κ_p in the notation.

B.4.3 Existence and uniqueness of the equilibrium before the shock

Having established the existence and uniqueness of the equilibrium termination values after the shock, as given by Equations (64) and (65), we now turn our attention to proving that an equilibrium exists before the shock. To do this, we fix the termination costs κ_p and κ_a , and consider the set $S_0 = [0, \bar{L}_0]$ equipped with the metric $\rho(\hat{L}_x, \hat{L}_y) = |\hat{L}_x - \hat{L}_y|$. We denote by $T_0 : S_0 \rightarrow S_0$ the function

$$T_0(\hat{L}) = \hat{L} + z_0(\hat{L}), \quad (66)$$

with

$$z_0(\hat{L}) = V_0(W_0(0, \hat{L}); 0, \hat{L}, R_1^*, L_1^*) - \kappa_p - V_0(W_0(0, \hat{L}) - \kappa_a; 0, \hat{L}, R_1^*, L_1^*). \quad (67)$$

Note that z_0 is continuous and differentiable, just like z_1 . By applying Lemma 2 to z_0 , we can establish the existence and uniqueness of a fixed point $\hat{L}_0 \in S_0$ that satisfies the following equation:

$$V_0(W_0(0, \hat{L}_0); 0, \hat{L}_0, R_1^*, L_1^*) - \kappa_p = V_0(W_0(0, \hat{L}_0) - \kappa_a; 0, \hat{L}_0, R_1^*, L_1^*). \quad (68)$$

Therefore, by knowing the search costs κ_a and κ_p , the equilibrium values of termination after the jump R_1^* and L_1^* , and based on the existence and uniqueness of the value \hat{L}_0 , we can determine the unique set of termination values R_0^* and L_0^* . These latter jointly determines the equilibrium on the labor market before the shock. The following holds

$$W_0(R_0^*, L_0^*) = W_0(0, \hat{L}_0(\kappa_a, \kappa_p)R_1^*, L_1^*) \quad (69)$$

$$V_0(W_0(R_0^*, L_0^*); R_0^*, L_0^*, R_1^*, L_1^*) = V_0(W_0(0, \hat{L}_0(\kappa_a, \kappa_p)); 0, \hat{L}_0(\kappa_a, \kappa_p), R_1^*, L_1^*). \quad (70)$$

And therefore,

$$R_0^* = W_0(0, \hat{L}_0(\kappa_a, \kappa_p)) - \kappa_a, \quad (71)$$

$$\text{and : } L_0^* = V_0(W_0(0, \hat{L}_0(\kappa_a, \kappa_p)); 0, \hat{L}_0(\kappa_a, \kappa_p)) - \kappa_p. \quad (72)$$

B.4.4 Comparison of R_0^* and R_1^* , and L_0^* and L_1^* .

While we have shown that there exists an equilibrium on the labor market, we still have to compare R_0^* and R_1^* , and L_0^* and L_1^* .

To do so, let us consider the function $F_1^\lambda(W)$ that solves the following stochastic differential equation

$$rF_1^\lambda(W) = \mu + \gamma W F_1^{\prime\lambda}(W) + \frac{1}{2} \lambda^2 F_1^{\prime\prime\lambda}(W) \quad \text{for any } W \in [0; \bar{W}^1]; \quad (73)$$

together with the lower-boundary condition $V_1(0) = \hat{L}_1^\lambda$ and the usual smooth-pasting and super-contact conditions at the upper boundary \bar{W}^1 .

Also, consider the function $F_0^{\lambda_1, \lambda_2}(W)$ that solves the following stochastic differential equation

$$(p+r)F_0(W) = \mu + (\gamma W - p\delta(W)) F_0'(W) + \frac{1}{2}\lambda_1^2 F_0''(W) + pF_1^{\lambda_2}(W + \delta(W)) \quad \forall W \in [0; \bar{W}^0], \quad (74)$$

together with the lower-boundary condition $\hat{F}_0(0) = \hat{L}_0^{\lambda_1, \lambda_2}$ and the usual smooth-pasting and super-contact conditions at the upper boundary \bar{W}^0 .

Clearly, $F_1^\lambda(W)$ is solution of Equation (43) together with $R_1 = 0$, $L_1 = \hat{L}_1^\lambda$ and $\underline{\lambda} = \lambda$; and $F_0^{\bar{\lambda}, \underline{\lambda}}(W)$ is solution of Equation (44) together with $R_0 = R_1 = 0$, $L_0 = \hat{L}_0^{\bar{\lambda}, \underline{\lambda}}$, $L_1 = \hat{L}_1^\lambda$, $\bar{\lambda} = \lambda_1$ and $\underline{\lambda} = \lambda_2$.

As a consequence,

$$F_1^{\bar{\lambda}} \leq F_0^{\bar{\lambda}, \underline{\lambda}} \leq F_1^{\underline{\lambda}}, \quad (75)$$

$$\text{with } \lim_{\underline{\lambda} \rightarrow \bar{\lambda}} F_1^{\bar{\lambda}} = F_1^{\underline{\lambda}} \quad (76)$$

Which leads to

$$\hat{L}_1^{\bar{\lambda}} \leq \hat{L}_0^{\bar{\lambda}, \underline{\lambda}} \leq \hat{L}_1^{\underline{\lambda}}. \quad (77)$$

Using the comparative statics of DeMarzo and Sannikov (2006)¹³,

$$\frac{\partial}{\partial L} \arg \max_W F_1^\lambda(W) < 0 \quad (78)$$

we have

$$W_1^{\underline{\lambda}} \leq W_0^{\bar{\lambda}, \underline{\lambda}} \leq W_1^{\bar{\lambda}}, \quad (79)$$

where $W_1^\lambda = \arg \max_W F_1^\lambda(W)$ and $W_0^{\lambda_1, \lambda_2} = \arg \max_W F_0^{\lambda_1, \lambda_2}(W)$.

As by construction $R_1^* = W_1^\lambda - \kappa_a$ and $R_0^* = W_0^{\bar{\lambda}, \underline{\lambda}} - \kappa_a$ we have that

$$R_1^* \leq R_0^*. \quad (80)$$

¹³See Table AI, page 2719.

Now, we use that

$$F_1^\Delta(W_1^\Delta) \geq F_0^{\bar{\lambda}, \Delta}(W_0^{\bar{\lambda}, \Delta}) \geq F_1^{\bar{\lambda}}(W_1^{\bar{\lambda}}). \quad (81)$$

Because $L_1^* = F_1^\Delta(W_1^\Delta) - \kappa_p$ and $L_0^* = F_0^{\bar{\lambda}, \Delta}(W_0^{\bar{\lambda}, \Delta}) - \kappa_p$, we can conclude that

$$L_1^* \geq L_0^*. \quad (82)$$

Now, to evaluate how the jump from R_0^* to R_1^* and from L_0^* to L_1^* at the shock impacts the over-retention and over-compensation of poor performers, we investigate how it impacts the agent's sensitivity to the shock. To do so, we compare first the firm's value after the shock with what it would have been if the termination values remain unchanged. Using the notations introduced at the beginning of the proof, we compare $V_1(\cdot; R_1^*, L_1^*)$ and $V_1(\cdot; R_0^*, L_0^*)$. First, as

$$V_1(W; R_1^*, L_1^*) \geq V_1(W; R_0^*, L_0^*), \quad \forall W \in [R_0^*, \bar{W}_1(R_1^*, L_1^*)]. \quad (83)$$

It is clear that $\bar{W}_1(R_1^*, L_1^*) \leq \bar{W}_1(R_0, L_0)$, which are the payment barriers that are characterized by being the smallest continuation value where the slope of the firm value is -1 . Second, $W_1(R_1^*, L_1^*) \leq W_1(R_0, L_0)^{14}$, which is the supremum of the respective firm's value, so where the slope is 0. Finally, differentiating the boundary condition in equation (43) with respect to R , we have:

$$\frac{\partial V_1(R; R, L)}{\partial R} = -V_1'(R; R, L) \quad (84)$$

$$\text{so, } \frac{\partial^2 V_1(R; R, L)}{\partial R^2} = -V_1''(R; R, L). \quad (85)$$

Because the value function is concave, we have that

$$V_1'(R_0^*; R_0^*, L_0^*) > V_1'(R_1^*; R_1^*, L_1^*). \quad (86)$$

Recall that the optimal sensitivity to the shock denoted by δ is such that $V_1'(W_{t-} + \delta(W_{t-}; R_1, L_1)) = V_0'(W_{t-}; R_0^*, L_0^*, R_1, L_1)$; with $R_1 = R_1^*$ and $L_1 = L_1^*$ if the termination values change after the shock, and $R_1 = R_0^*$ and $L_1 = L_0^*$ otherwise. Consequently, as the firm values are concave, the sensitivity to the shock $\delta(W)$ is smaller $\forall W \geq R_0^*$ when the termination values jump from R_0^* to R_1^* and from L_0^* to L_1^* rather than when it remains unchanged.

¹⁴Again, see Table AI, page 2719 of DeMarzo and Sannikov (2006)