



HAL
open science

Quaternion based registration method in medical imaging

J. Flifla, J.L. Dillenseger, P. Haigron, J.L. Coatrieux

► **To cite this version:**

J. Flifla, J.L. Dillenseger, P. Haigron, J.L. Coatrieux. Quaternion based registration method in medical imaging. 1992 International Biomedical Engineering Days, 1992, Istanbul, Turkey. pp.161-165, 10.1109/ibed.1992.247105 . hal-04720988

HAL Id: hal-04720988

<https://hal.science/hal-04720988v1>

Submitted on 4 Oct 2024

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

AN INTEGRATED MULTI-PURPOSE RAY TRACING FRAMEWORK FOR THE VISUALIZATION OF MEDICAL IMAGES

J. Fifla, J.-L. Dillenseger, P. Haigron and J.-L. Coatrieux

Laboratoire Traitement du Signal et de l'Image, Inserm U335,
Université de Rennes 1, Campus de Beaulieu, Rennes, France

ABSTRACT

This paper studies the determination of the rigid body transformation allowing the automatic correlation of the 3-D information inherent in different imaging devices. It addresses a part of the overall methodology, namely the global registration problem (e.g translation and rotation estimation). Several approaches have been reported according to the representations of geometric data (point and line segment sets, curves, surfaces, volumes). Two original schemes have been derived when the registration operates directly on the 3-D sampled volume. The first one is based on 2-D registration of multiple 2-D binary projections of the objects. The second one makes use of quaternion theory.

Introduction

3-D imaging techniques offer an attractive framework for the display, the manipulation and the characterization of anatomical organs or lesions but also the means to achieve accurate surgery or radiation therapy planning. The next generation of 3-D workstations in this field will be directed to 3-D multiple modalities management for which methodological attempts (theoretical foundation) and technological developments (higher size of the data sets and interactivity constraint) are needed. We address in this paper the automatic correlation of structures imaged from the same or different devices. Temporal comparative studies for patient following (same modality) but also complementary information reports (bone or soft tissues in Computerized Tomography and Magnetic Resonance Imaging, or physiological activity from Positron Emitted Tomography for example) could be achieved. The problem at hand is the estimation of the geometrical transformations (scaling, translation, rotation) linking several sets of imaging data of the same organ (assumed to be a rigid one). The 3-D anatomy description is assumed to be provided from stacks of 2-D slices at any orientation. Other techniques such as Digital Subtraction radiography, producing 2-D projection information, are not considered here.

A classical solution to this problem is the use of an external frame of a priori known geometrical features (head

mounted for example) and the computation, through its intersection points with the slices, of their orientation. A more original and interesting way to handle this problem is based on the use of intrinsic data set information. A characterization of several approaches can be done according to the entities used for the registration and the space in which this registration is performed [1]. Different situations can be pointed out :

(1) the location of robust landmarks within the volume of the two 3-D databases. Two point sets with or without correspondences have to be matched. Such landmarks can be found on the organs (anterior and posterior commissures in the brain for example) but noise and image resolution can lead to imprecise location.

(2) the correlation of 3-D lines. Noticeable curves can be found within the anatomy and detected by means of appropriate image processing.

(3) the correlation of 3-D surfaces. This scheme must be considered when the structures of interest are described from different imaging modalities (different physical description). Then, surfaces can be correlated such as the skin for CT and MRI.

(4) the correlation of the whole volume image sets. If we assume that the same organ is imaged (without significant pathological change), from the same modality, the physical values attributed to the voxels of the entire volume can be used. The 3-D volume registration can also be performed between binary 3-D shapes extracted from different modalities.

Some important considerations that have an impact on the reliability of the methods further described are not taken into account (slice spacing and thickness, scaling, partial matching, etc...).

1. THE MATCHING PROBLEM

The databases are represented by finite sized two-dimensional or three-dimensional discrete signals, denoted $g_i(\mathbf{r})$. These discrete functions are non-negative definite functions, vanishing out of the definition domain, and provide positive valued integration. The images are related to the reference object by the following equation:

$$g_2(\mathbf{r}) = g_1(\mathbf{R} \cdot \mathbf{r} + \mathbf{T} + \mathbf{N}) + \mathbf{N}' \quad (1)$$

where \mathbf{R} represents a rotation around the center of mass of the structure, \mathbf{T} is a translation vector, \mathbf{N} and \mathbf{N}' are additional noises and \mathbf{r} the points coordinates describing the organ. In the following sections, only the 3-D case will be considered.

2. GLOBAL SHAPE REGISTRATION

2.1. Landmark using

Characteristic points (landmarks) are selected manually or automatically from the object. After transformation this set of points P_{1i} becomes a new set P_{2i} with or without explicit correspondence between points. In both cases, the translation can be estimated by matching the center of mass of the two sets of points. The coordinates of a point relative to the center of mass are given by:

$$q_{1i} = P_{1i} - P_{1c} \text{ and } q_{2i} = P_{2i} - P_{2c} \quad (2)$$

where P_{1c} and P_{2c} are the respective centers of mass coordinates.

(i) In the correspondence case (a priori knowledge of pairing points), the problem is classic and the rotation matrix has to minimize the following quantity:

$$\Sigma = \sum_{i=1}^N |q_{2i} - \mathbf{R} \cdot q_{1i}|^2 \quad (3)$$

(ii) In the no-correspondence case, the problem becomes more complex. Two functions $g_1(q)$ and $g_2(q)$ are defined :

$$g_1(q) = \sum_{i=1}^N \delta(q - q_{1i}) \text{ and } g_2(q) = \sum_{i=1}^N \delta(q - q_{2i}) \quad (4)$$

where $\delta(q)$ is a Dirac delta function.

If $G_1(f)$ and $G_2(f)$ are their Fourier transforms, they satisfy the equation [2]:

$$G_2(f) = G_1(\mathbf{R}^T(f)) \quad (5)$$

The rotation is estimated in two steps. The rotation axis Δ is first determined by maximizing a cost function:

$$C_1(a, b, c) = \frac{G_{12}(a, b, c)}{[G_{11}(a, b, c) \cdot G_{22}(a, b, c)]^{1/2}} \quad (6)$$

with $G_{mn}(a, b, c) = \int_{-A}^{+A} G_n(f)G_m^*(f)dr$ and $f = r|a, b, c|^T$

where a, b, c are the directional cosines of the rotation axis Δ and A is a constant. Then the angle of rotation about Δ is evaluated by maximizing a second cost function (see [2] for details).

2.2. Point / surface matching [3]

In this approach, one of the surfaces is modeled by polygons after triangulation of sampling point projections upon a plane (a bijective correspondence between 3-D points and 2-D projection is assumed). A second set of sampling points B_i is extracted from the surface to be matched (this set can be less numerous than the first one). Then the rotation \mathbf{R} and the translation \mathbf{T} have to minimize the distance d :

$$d = \sum_{i=1}^p |z(B_i) - z_a(B_i)|^2 \quad (7)$$

where $z(B_i)$ and $z_a(B_i)$ are the third coordinate of B_i and the z value on the corresponding polygon for the same $x - y$ coordinates respectively.

2.3. Volume matching

Direct 3D registration can be performed for gray scale objects imaged in the same modality and for binary objects in multi-modal data. Substructures appearing in both images have to be extracted. Translation and rotation could be successively estimated from relation (1) where $g_i(r)$ is a tridimensional volumic function.

2.3.1. Use of cross-correlation function for translation estimation

Translation vector components u, v, w can be computed by maximizing the cross-correlation function C of the two images $g_1(i, j, k)$ and $g_2(i, j, k)$:

$$C(u, v, w) = \sum_{i,j,k} g_1(i, j, k) \cdot g_2(i + u, j + v, k + w) \quad (8)$$

This function is computed by means of Fourier transforms :

$$C(u, v, w) = \text{FFT}^{-1} (G_1(x, y, z) \cdot G_2^*(x, y, z)) \quad (9)$$

where : $G_1(x, y, z)$ is the Fourier Transform of $g_1(i, j, k)$; $G_2^*(x, y, z)$ the conjugate of $G_2(x, y, z)$; FFT^{-1} the Inverse Fourier Transform.

2.3.2. 3D moment methods

This class of methods is aimed at the definition of an intrinsic reference for each of the objects. Registration consists in finding the matching of the two references.

Object modelling by means of moment or tensor of inertia is a powerful tool for deriving the geometric transformations (see [4] for an application in medical image registration). Translation is estimated from the centers of mass (moment of order 1). Rotation is evaluated from the principal axis

(the eigenvectors of the moment matrix). This class of methods can be applied to the registration of binary 3D shapes as well as to volume data.

3. SILHOUETTE AND QUATERNION BASED VOLUME REGISTRATION METHODS

3.1. Silhouette method

Direct volume registration is quite difficult. Our approach uses well known 2D matching methods for the registration in 3D space.

The basic idea of the algorithm is based on the binary reprojection on three orthogonal planes of the "reference" object $f(x, y, z)$ and the "data" object $g(x, y, z)$ (see figure 1).

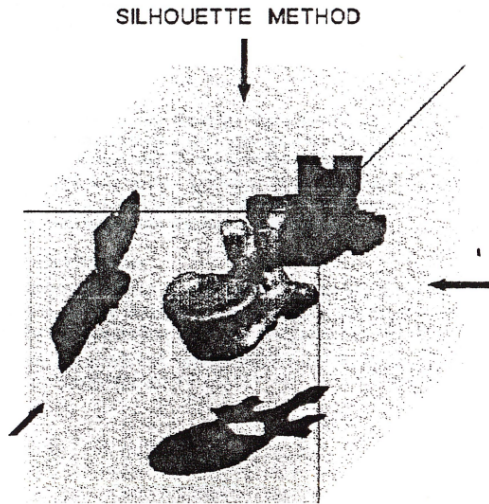


Fig. 1. Binary projections of the 3-D object.

The registration is iteratively performed in 2D on the three planes. From a coordinate space (O, x, y, z) , a rotation is defined by $\mathbf{R}: Rz(\gamma) \cdot Ry(\beta) \cdot Rx(\alpha)$

\mathbf{R} is the concatenation of three rotations of angles α, β and γ respectively about the axis.

The translation is defined by a 3D vector $\mathbf{T}(x_0, y_0, z_0)$.

In a first step the reference object is projected along each axis: $f(x, y, z) \rightarrow f(x, y), f(x, z), f(y, z)$

Translation estimation :

- $g(x, y, z)$ is projected along the scene axis: $g(x, y, z) \rightarrow g(x, y), g(x, z), g(y, z)$;
- the translation is computed on each plane using 2D cross-correlation. The estimation with $f(x, y)$ and $g(x, y)$ provides x_0 and y_0 . Each coordinate is evaluated twice and we keep the mean value of the two evaluations;
- $g(x, y, z)$ and the projections are matched relative to \mathbf{T} .

Rotation estimation:

The 2D estimation of a rotation is reduced to a 2D translation estimation after transformation of the coordinate system into polar representation. Each plane estimates one angle (α, β or γ). The angles are computed successively and independently. The whole iterative process is depicted in Figure 2.

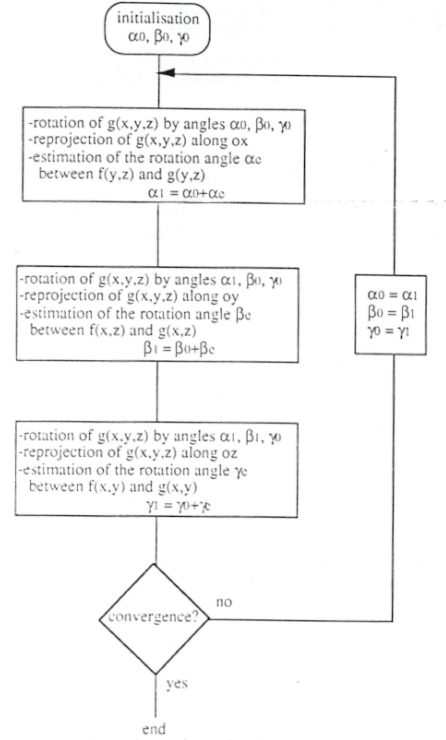


Fig. 2. Rotation registration algorithm.

Translation and rotation are estimated iteratively until convergence.

This approach can be applied to multimodal images.

4. QUATERNION BASED VOLUME REGISTRATION METHOD

In 1848 Hamilton extended the isomorphism between the unit complex number and the rotations in \mathbb{R}^2 to the rotations in \mathbb{R}^3 by use of quaternion theory. Quaternion, the \mathbb{R}^4 extension of the complex numbers $\{1, i, j, k\}$, can be defined by:

$$q = q_0 + i \cdot q_1 + j \cdot q_2 + k \cdot q_3 \text{ or by: } q = (r, \mathbf{p}) \quad (10)$$

where $r = q_0 \in \mathbb{R}$ real part of q and $\mathbf{p} = [q_1, q_2, q_3]^T \in \mathbb{R}^3$ being the imaginary part of q . Each vector in \mathbb{R}^3 can be described by a pure imaginary quaternion.

Any rotation in \mathbb{R}^3 is characterized by a unit quaternion ($q_0 \geq 0$ and $q_0^2 + q_1^2 + q_2^2 + q_3^2 = 1$). The generation of a rotation matrix by this unit quaternion can be found in [1]. If λ is this

unit quaternion, the rotation in \mathbb{R}^3 of a 3-D vector, described by the pure imaginary μ , is defined by:

$$\lambda \cdot \mu \cdot \lambda^{-1} \quad (11)$$

with:

$$\lambda = \left\{ \cos \frac{\theta}{2}, \mathbf{n} \cdot \sin \frac{\theta}{2} \right\} \quad (12)$$

where $\mathbf{n} \in \mathbb{R}^3$ is the unit vector of the rotation axis and θ the angle of rotation about \mathbf{n} .

The interest of the quaternion framework has been emphasized by Hebert and Faugeras [5] for robotics applications. This theory has led to explicit solutions for the minimization of some usual error criterions (least square registration of corresponding 3-D points or plane sets for example). It specifies that any rotation can be uniquely described through a rotation about an axis to be determined.

We assume that the translation is estimated independently first (e.g. the data are centered). The 3-D rotation registration is then reduced to the determination of : (1) the rotation axis and (2) the corresponding rotation. This registration scheme is limited to images within the same modality, and the voxel values of the entire volume can be used.

Rotation axis estimation

The rotation axis (with \mathbf{n} its directional vector) is stationary during the rotation. If q is the associated quaternion of a vector along the rotation axis, $q = (0, a \cdot \mathbf{n})$, and λ the unit quaternion describing the rotation, then :

$$\lambda \cdot q \cdot \lambda^{-1} = q \quad (13)$$

The axis is estimated through the unchanged voxel values obtained by means of subtraction (classical or logarithmical) of the two data sets. These voxels are clustered along the axis we are looking for. The axis is then extracted from the cluster by use of a combination of a simplified 3-D Hough Transform (the rotation center is known) and a line regression algorithm.

Rotation angle estimation

An orthogonal vector to \mathbf{n} is rotated by an angle θ in a plane orthogonal to \mathbf{n} . The rotation angle estimation can be reduced to a 3-D translation estimation after transformation of the coordinate system into a cylindrical representation (r, ϕ, z) in which z is colinear to \mathbf{n} (see figure 3).

The 3-D translation can be estimated by means of 3-D cross-correlation, for example (2.3.1).

One of the particularities of our approach is that it can be applied on data with only partial correspondence.

5. RESULTS

The evaluation of several methods has been done using simulation and CT data.

The criterion used for this evaluation is the normalized cross-correlation coefficient. This coefficient is computed

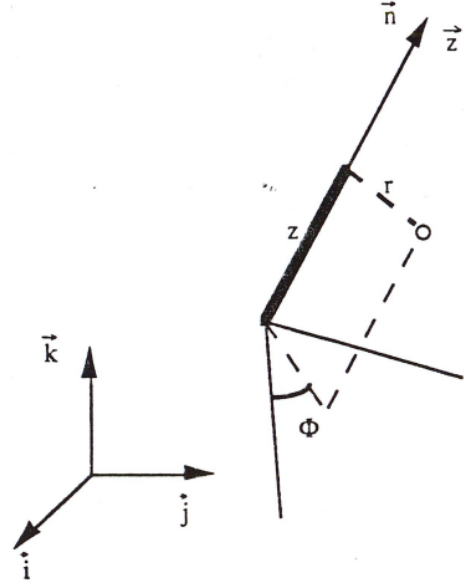


Fig. 3. Cylindrical representation of a 3-D point.

from the "reference" object and the transformed "data" object:

$$Cf g = \frac{\sum \sum \sum f(r) \cdot g'(r)}{[\sum \sum \sum f^2(r) \cdot \sum \sum \sum g'^2(r)]^{1/2}} \quad (14)$$

with: $f(r)$ the "reference" object ; $g'(r) = g(R \cdot r + T)$; $g(r)$ the "data" object ; R and T the estimated rotation and translation.

This method needs a resampling (by tri-linear interpolation) of the "data" object.

Each method is first estimated on a simulated 3-D smooth and assymmetric surface. Its objective is to evaluate both the potential accuracy and the robustness to noise.

Figure 4 shows that all these methods provide good accuracy. For $Cf g$ values higher than 0.95, the mean errors in translation and rotation are less than 0.5 voxels and 0.5 degrees respectively. The comparative study performed in noisy situations shows that the methods requiring no preprocessing or feature extraction are more efficient (e.g. those based on the raw volume information). Conversely, the use of a limited number of intrinsic points leads to very low time computation for the registration procedure. A compromise between these criterions can be derived according to the application at hand.

The methods have also been compared on two sets of CT images. For each object two acquisitions with two known positions were performed. The first object is an extracted vertebra (see Figure 1)).

This object presents a complex shape and is appropriate for landmark designation. The second one is an extracted top part of a skull. This object is smooth without relief and quasi symmetrical. Table 1 gives the characteristics of the data.

name	acq.	reconst.	resolution	slic.thick.	slic. spac.
vert 1	$256^2 \times 51$	$256^2 \times 102$	1 mm	1 mm	2 mm
vert 2	$256^2 \times 50$	$256^2 \times 102$	1 mm	1 mm	2 mm
skull 1	$256^2 \times 45$	$256^2 \times 102$	1 mm	2 mm	3 mm
skull 2	$256^2 \times 47$	$256^2 \times 102$	1 mm	2 mm	3 mm

acq : aquisition size
reconst : reconstituted size
resolution : slice resolution
slic. thick : slice thickness
slic. spac. : inter-slice spacing

Table 1. Database characteristics.

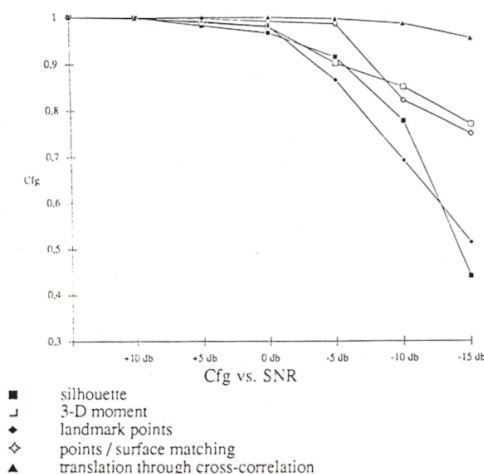


Fig. 4. Results on a simulated surface

We studied the different methods on a multi-scale approach. Reduction coefficients of 2, 4, 8 and 16 were performed on the data. The point / surface matching method could not be applied to the vertebra databases (lack of bijection correspondence between 3-D points and 2-D projections).

Figures 5 and 6 show the comparative studies in multi-scale situations for the vertebra and the skull cap respectively. Here also the methods requiring no preprocessing or feature extraction seem to be more efficient.

The quaternion based method has been tested on a CT database rotated and resampled by trilinear interpolation. The two objects used for the previous evaluation were too homogeneous (e.g. bone) to be relevant. This method provides good accuracy within less than one degree for the axis orientation and the rotation angle estimation. The results demonstrate the feasibility of the method. Our effort is now directed to the quantification of errors in noisy and multi-scale situations on real data.

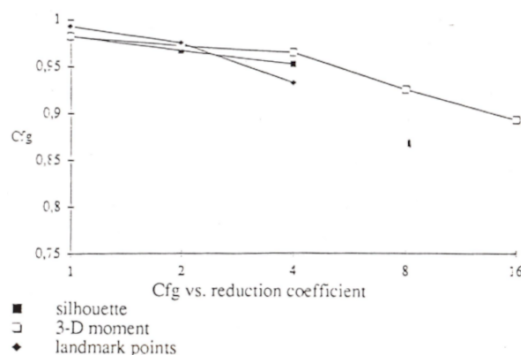


Fig. 5. Results on the vertebra database

6. CONCLUSION

We have discussed different ways to correlate multiple mono and multimodality data in medical imaging when the rigid body assumption holds. The choice between them depends on the nature of the data that can be defined : anatomical landmarks, surface or entire volume sets. Accuracy, preprocessing requirements and computation time are some of the criterions which allow the selection of the best suited ones. Further studies are needed to evaluate their efficiency and the effects of critical parameters which have been pointed out. It must be emphasized that they can be combined to offer a good compromise between the above mentioned constraints.

7. REFERENCES

- [1] P. J. Besl and N. D. McKay, "A method for registration of 3-D shapes," *IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol. 14, no. 2, pp. 239–256, Feb. 1992.
- [2] Z. C. Lin, T. S. Huang, S. D. Blostein, H. Lee, and E. A. Margerum, "Motion estimation from 3-D point sets with and without correspondences." in *Proc. of IEEE Com-*

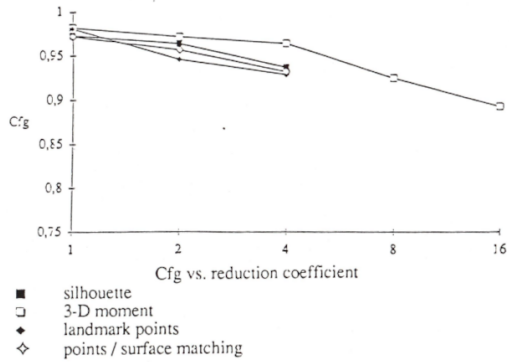


Fig. 6. Results on the skull cap database

puter Vision and Pattern Recognition, Miami, 1986, pp. 194–197.

- [3] J. Flifla, R. Collorec, A. Bouliou, and J. L. Coatrieux, “Three-dimensional positioning from multimodality medical imaging,” in *Proceedings of the Annual International Conference of the IEEE Engineering in Medicine and Biology Society*, New Orleans, 1988, pp. 422–423.
- [4] T. L. Faber and E. M. Stokely, “Orientation of 3-D structures in medical images,” *IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol. 10, no. 5, pp. 626–633, Sep. 1988.
- [5] O. D. Faugeras and M. Hebert, “The representation, recognition, and locating of 3-D objects,” *The International Journal of Robotics Research*, vol. 5, no. 3, pp. 27–52, Sep. 1986.