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# Use of Cramer-Rao bounds approach to predict the minimum uncertainties in the inversion of ocean color optical properties

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## ► To cite this version:

Malik Chami, Sylvain Jay, Mireille Guillaume, Yannick Deville, Bruno Lafrance, et al.. Use of Cramer-Rao bounds approach to predict the minimum uncertainties in the inversion of ocean color optical properties. Oceans Optics XXIV, The Oceanography Society (TOS)., Oct 2024, Dubrovnik, Croatia. hal-04720962

**HAL Id: hal-04720962**

**<https://hal.science/hal-04720962v1>**

Submitted on 4 Oct 2024

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ABSTRACT

An analytical approach based on Cramer-Rao Bounds (CRBs) is proposed to investigate the uncertainties in ocean color optical properties. Here, CRBs are explicitly developed using a water reflectance model corresponding to shallow waters. CRBs are shown to be consistent with the experimental estimation variances obtained using the well known least square optimization technique. CRBs does not require to perform any inversion of the reflectance. It is shown that *a priori* knowledge of bathymetry greatly improves the retrieval of bottom cover for shallow waters. CRBs are used to quantify the minimum uncertainties in the inversion of ocean color parameters for the MSI, HICO, OLCI and MODIS configurations for a variety of oceanic and coastal waters [Jay et al., 2018]. CRBs are shown to provide valuable information about the best inversion performances that may be expected for a given sensor.

1. REFLECTANCE MODEL

The subsurface remote-sensing reflectance  $\mu$  is given by the following semi-analytical model [Lee et al., 1999] (1)

$$\mu = r_{\infty} (1 - e^{-(k_a + k_u)H}) + (1/\pi) [B\rho_{b,1} + (1 - B)\rho_{b,2}] e^{-(k_a + k_u)H} \quad (1)$$

$r_{\infty}$  is the deep water reflectance, H is the depth,  $\rho_{b,1}$  and  $\rho_{b,2}$  are two substratum albedos, B is the fractional cover of substratum,  $k_d$  and  $k_u$  are the attenuation coefficients.

2. DEVELOPMENT OF CRAMER-RAO BOUNDS (CRBs)

$\Delta$  = vector of unknown ocean color parameters (i.e., H, IOPs, B). The variance of any unbiased estimator  $\hat{\Delta}_i$  of any element  $\Delta_i$  of  $\Delta$  is bounded [Garthwaite, 2002] (2). The inverse of CRB( $\Delta$ ) is the Fisher information matrix  $IF(\Delta)$  (3).

$$\mathbb{E} [(\hat{\Delta}_i(r_{rs}) - \Delta_i)^2] \geq [CRB(\Delta)]_{i,i} \quad (2)$$

$$[IF(\Delta)]_{i,j} = \frac{\partial \mu}{\partial \Delta_i} \Gamma_s^{-1} \frac{\partial \mu}{\partial \Delta_j} \quad (3)$$

$\Gamma_s$  is the covariance matrix of the environmental noise. CRB( $\Delta$ ) thus represents the minimum uncertainty in  $\hat{\Delta}_i$ . CRB includes the uncertainties related to the reflectance model, environmental noise and sensor radiometric specifications (not those related to ground truth measurements, atmospheric correction or inverse methods).

3. RESULTS

The square roots of CRBs, denoted  $CRB^{0.5}$ , are actually shown since they directly provide the minimum estimation standard deviation that can be possibly attained by any unbiased estimator  $\hat{\Delta}_i$ . Estimate of the environmental noise matrix  $\Gamma_s$  has been derived from images acquired by Sentinel 2-MSI, HICO, Sentinel 3-OLCI and MODIS (figure 1).

3.A Evaluation of the performances of ocean color remote-sensing methods using CRBs

The minimum uncertainties in the inversion of ocean color parameters, as provided by the CRBs, are compared to the experimental estimation standard deviations obtained using the LS (Least-Square) [Lee et al., 1999; Werdell et al., 2013] and MILE (MaxImum Likelihood estimation including Environmental noise) methods [Jay et al., 2017] (figure 2).

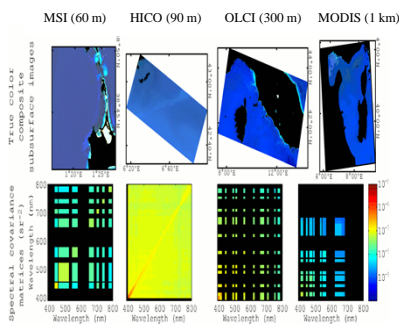


Figure 1: satellite images (top) and noise matrix  $\Gamma_s$  (down)

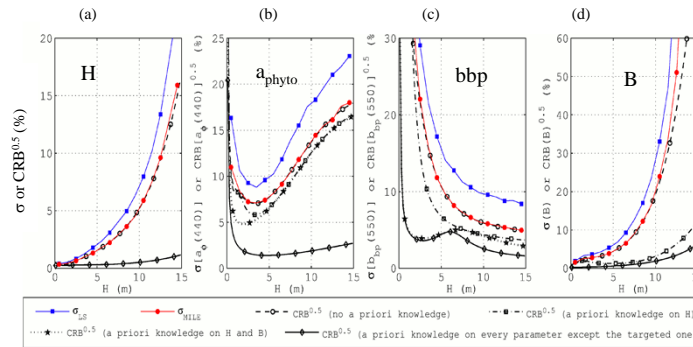


Figure 2:  $CRBs^{0.5}$  (in %) of (a) H, (b)  $a_{phyto}(440)$ , (c)  $bbp(550)$  and (d) B versus depth for shallow waters and HICO configuration

- ✓ CRBs obtained without *a priori* knowledge vary similarly to LS and MILE inversion uncertainties.
- ✓ CRBs of H and B reach 17 and 85%, respectively, when H = 15 m.
- ✓ Estimation of IOPs becomes unreliable for very shallow waters.

3.B Evaluation of the influence of a priori knowledge on the model inversion

CRBs are used to study how an *a priori* knowledge on given ocean color parameters could improve the estimation of unknown parameters (figure 2).

- ✓ Greatest gain in performance is obtained for B estimates (fig. 2d) when H is known *a priori* : CRB is seven-fold lower at 15 m.
- ✓ *A priori* knowledge on both H and B significantly improves IOPs retrieval : gain by a factor 1.6 for  $a_{phyto}$  (fig.2b) and 7 for  $bbp$  (fig. 2c) when H=1m.

3.C Minimum inversion uncertainties for MSI, HICO, OLCI and MODIS radiometric configurations

CRBs are used to predict the minimum uncertainties in the inversion of ocean color parameters for various satellite sensors (figure 3, table 1).

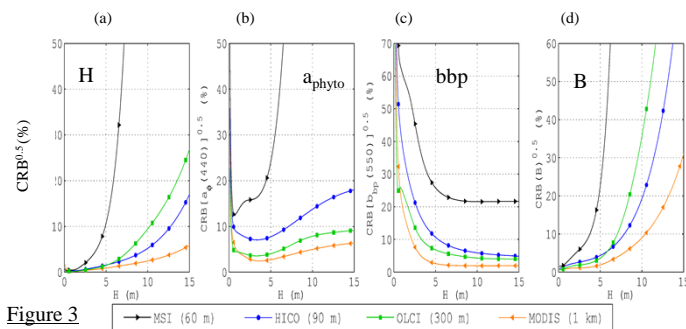


Figure 3

- ✓ MODIS shows lower CRBs than the other sensors.
- ✓ Great potential of HICO-like sensors for accurately characterizing coastal areas.
- ✓ MSI provides accurate inversion performances for H, B and, to a lesser extent,  $a_{phyto}$ , for H < 5 m with CRBs lower than 25%.

Table 1

	Clear waters (CRBs in %)		Coastal waters (CRBs in %)	
	$a_{phyto}(440)$	$bbp(550)$	$a_{phyto}(440)$	$bbp(550)$
MSI (60 m)	27-216	4-16	3-45	1-27
HICO (90 m)	7-23	2-11	1-22	0-17
OLCI (300 m)	5-15	2-10	1-10	0-10
MODIS (1 km)	2-8	1-3	1-12	0-10

Acknowledgments: NASA (MODIS), ESA (OLCI/MSI), Oregon State University (HICO)

Funding: Direction Générale Armement (DGA), ANR (project #ANR-15-ASTR-0019)