

27 Avirulent larvae (genotypes AA and Aa) differentiate into adults with a proportion $m_A(k)$ of
 28 males, and a proportion $1 - m_A(k)$ of females. Virulent larvae (genotype aa) differentiate into
 29 adults with a proportion of $m_a(k)$ of males, and a proportion $1 - m_a(k)$ of females.

30 Sex allocation depends on host resistance during season k :

$$m_A(k) = \begin{cases} m, & \text{if the host is susceptible} \\ 1, & \text{otherwise (if resistance is masculinizing)} \end{cases}, \quad (2a)$$

$$31 \quad m_a(k) = m, \quad (2b)$$

32 in which $0 < m < 1$ is the male fraction when masculinizing resistance does not occur or
 33 is broken. The transition from the larval stage to the adult stage is given by the following
 34 equation:

$$\begin{cases} F_{AA}(t_k + \tau_1) = s(1 - m_A(k))J_{AA}(t_k) \\ F_{Aa}(t_k + \tau_1) = s(1 - m_A(k))J_{Aa}(t_k) \\ F_{aa}(t_k + \tau_1) = s(1 - m_a(k))J_{aa}(t_k) \\ M_{AA}(t_k + \tau_1) = sm_A(k)J_{AA}(t_k) \\ M_{Aa}(t_k + \tau_1) = sm_A(k)J_{Aa}(t_k) \\ M_{aa}(t_k + \tau_1) = sm_a(k)J_{Aa}(t_k) \end{cases}. \quad (3)$$

35 3. We assume that all adults survive until mating occurs (time $t_k + \tau_2$). We assume that males
 36 and females are homogeneously mixed, and that mating occurs at random. The next three
 37 steps lead to the female-to-egg transition from females to eggs.

38 (i) We first derive the frequencies of allele A and a in males and females. The variables
 39 $M = M_{AA} + M_{Aa} + M_{aa}$ and $F = F_{AA} + F_{Aa} + F_{aa}$ are the total densities of males and
 40 females, respectively. The frequency of allele A in females, the frequency of allele a in
 41 females, the frequency of allele A in males, and the frequency of allele a in males, are
 42 respectively:

$$p_f(k) = \frac{F_{AA}(t_k + \tau_1) + \frac{1}{2}F_{Aa}(t_k + \tau_1)}{F(t_k + \tau_1)}, \quad (4a)$$

$$43 \quad q_f(k) = 1 - p_f(k), \quad (4b)$$

$$44 \quad p_m(k) = \frac{M_{AA}(t_k + \tau_1) + \frac{1}{2}M_{Aa}(t_k + \tau_1)}{M(t_k + \tau_1)}, \quad (4c)$$

$$45 \quad q_m(k) = 1 - p_m(k). \quad (4d)$$

46 (ii) The allelic frequencies yield the following next-generation genotypic frequencies:

$$\begin{cases} f_{AA}(k) = p_f(k)p_m(k) \\ f_{Aa}(k) = q_f(k)p_m(k) + p_f(k)q_m(k) \\ f_{aa}(k) = q_m(k)q_f(k) \end{cases}. \quad (5)$$

47 (iii) Egg density is then obtained, for each genotype, as the product of the total female density,
 48 the average number of eggs per female, e , and the next-generation genotype frequency
 49 from equation (5):

$$\begin{cases} E_{AA}(t_k + \tau_2) = eF(t_k + \tau_1)f_{AA}(k) \\ E_{Aa}(t_k + \tau_2) = eF(t_k + \tau_1)f_{Aa}(k) \\ E_{aa}(t_k + \tau_2) = eF(t_k + \tau_1)f_{aa}(k) \end{cases} . \quad (6)$$

50 **Remark.** Equation (6) encompasses polyandry, since males have equal access to females.

51 4. A fraction $(1 - \mu)(1 - h)$ of eggs naturally survives a single period of host absence. This fraction
 52 is the product of the average fraction of viable eggs $(1 - \mu)$ with the fraction of cysts that did
 53 not accidentally hatch $(1 - h)$.

54 In addition to mortality, a fraction $(1 - b)$ of nematodes survives biocontrol application (b is
 55 the biocontrol efficacy fraction). Not using biocontrol amounts to setting $b = 0$.

56 The rotation number r is the number of years without growing potatoes. We assume that
 57 biocontrol is applied every year, regardless of whether potato is grown or not.

58 As the potato growing season is relatively short on an annual scale (16 to 18 weeks), we assume,
 59 for simplicity, that annual cyst mortality is the same regardless of whether the host absence
 60 period is one year or less (one year minus the growing season).

61 At the beginning of the next season, i.e. at time t_{k+1} , we assume that all eggs hatch from
 62 cysts. Therefore, the nematode generations do not overlap.

63 The transition from eggs to larvae is thus given by:

$$\begin{cases} J_{AA}(t_{k+1}) = [(1 - \mu)(1 - h)(1 - b)]^{r+1} E_{AA}(t_k + \tau_2) \\ J_{Aa}(t_{k+1}) = [(1 - \mu)(1 - h)(1 - b)]^{r+1} E_{Aa}(t_k + \tau_2) \\ J_{aa}(t_{k+1}) = [(1 - \mu)(1 - h)(1 - b)]^{r+1} E_{aa}(t_k + \tau_2) \end{cases} . \quad (7)$$

64 To simplify notations, we introduce the reproduction number R' , accounting for biocontrol and
 65 rotation:

$$R' = e[(1 - \mu)(1 - h)(1 - b)]^{r+1} s. \quad (8)$$

66 We also introduce the female fractions

$$\begin{cases} f_A(k) = 1 - m_A(k) \\ f_a(k) = 1 - m_a(k) \end{cases} . \quad (9)$$

67 S1.2 Compact model

68 Renaming the larvae genotype densities $J_{AA}(t_k) = X_k$, $J_{Aa}(t_k) = Y_k$, $J_{aa}(t_k) = Z_k$ for convenience,
 69 we summarize equations (3-7) in the following discrete-time dynamical system:

$$\begin{cases} X_{k+1} = R' \frac{m_A(k)f_A(k)(X_k + \frac{1}{2}Y_k)^2}{m_A(k)(X_k + Y_k) + m_a(k)Z_k} \\ Y_{k+1} = R' \frac{m_A(k)(f_a(k)Z_k + \frac{1}{2}f_A(k)Y_k)(X_k + \frac{1}{2}Y_k) + f_A(k)(X_k + \frac{1}{2}Y_k)(m_a(k)Z_k + \frac{1}{2}m_A(k)Y_k)}{m_A(k)(X_k + Y_k) + m_a(k)Z_k} \\ Z_{k+1} = R' \frac{(f_a(k)Z_k + \frac{1}{2}f_A(k)Y_k)(m_a(k)Z_k + \frac{1}{2}m_A(k)Y_k)}{m_A(k)(X_k + Y_k) + m_a(k)Z_k} \end{cases} \quad (10)$$

70 We modify model (10) to account for intraspecific competition among larvae to host access, in a
71 Beverton-Holt form:

$$\begin{cases} X_{k+1} = \frac{R'}{1 + c(X_k + Y_k + Z_k)} \frac{m_A(k)f_A(k)(X_k + \frac{1}{2}Y_k)^2}{m_A(k)(X_k + Y_k) + m_a(k)Z_k} \\ Y_{k+1} = \frac{R'}{1 + c(X_k + Y_k + Z_k)} \frac{m_A(k)(f_a(k)Z_k + \frac{1}{2}f_A(k)Y_k)(X_k + \frac{1}{2}Y_k) + f_A(k)(X_k + \frac{1}{2}Y_k)(m_a(k)Z_k + \frac{1}{2}m_A(k)Y_k)}{m_A(k)(X_k + Y_k) + m_a(k)Z_k} \\ Z_{k+1} = \frac{R'}{1 + c(X_k + Y_k + Z_k)} \frac{(f_a(k)Z_k + \frac{1}{2}f_A(k)Y_k)(m_a(k)Z_k + \frac{1}{2}m_A(k)Y_k)}{m_A(k)(X_k + Y_k) + m_a(k)Z_k} \end{cases} \quad (11)$$

72 in which c is an intraspecific competition parameter.

73 System (11) defines our demo-genetic model of potato cyst nematode population dynamics.

74 S1.3 Basic demographic model

75 If only susceptible plant hosts are grown over years, the male fractions are $m_A(k) = m_a(k) = m$ and
76 the female fractions are $f_A(k) = f_a(k) = (1 - m)$, for all $k \geq 0$. Model (11) simplifies as:

$$\begin{cases} X_{k+1} = \frac{(1 - m)R'}{1 + c(X_k + Y_k + Z_k)} \frac{(X_k + \frac{1}{2}Y_k)^2}{X_k + Y_k + Z_k} \\ Y_{k+1} = \frac{2(1 - m)R'}{1 + c(X_k + Y_k + Z_k)} \frac{(Z_k + \frac{1}{2}Y_k)(X_k + \frac{1}{2}Y_k)}{X_k + Y_k + Z_k} \\ Z_{k+1} = \frac{(1 - m)R'}{1 + c(X_k + Y_k + Z_k)} \frac{(Z_k + \frac{1}{2}Y_k)^2}{X_k + Y_k + Z_k} \end{cases} \quad (12)$$

77 Let $N_k = X_k + Y_k + Z_k$ be the total nematode density. System (12) yields:

$$N_{k+1} = (1 - m)R' \frac{N_k}{1 + cN_k} \quad (13)$$

78 This is the classical Beverton-Holt model. It has at most two non-negative equilibria: $N = 0$,
79 and $N = ((1 - m)R' - 1)/c$ (if and only if $R' > 1$). The nematode-free equilibrium, $N = 0$, is stable
80 if and only if $(1 - m)R' < 1$. Otherwise (if $(1 - m)R' > 1$), the nematode population grows until
81 reaching its carrying, $K(R', m, c) = ((1 - m)R' - 1)/c$.

82 Next we show that genotype frequencies stabilize at the Hardy-Weinberg equilibrium. Using
 83 equations (12) and (13) yields the following genotype frequencies:

$$\left\{ \begin{array}{l} f_{AA}(k) = \frac{X_{k+1}}{N_{k+1}} = \frac{(X_k + \frac{1}{2}Y_k)^2}{N_k^2} \\ f_{Aa}(k) = \frac{Y_{k+1}}{N_{k+1}} = 2 \frac{(Z_k + \frac{1}{2}Y_k)(X_k + \frac{1}{2}Y_k)}{N_k^2} \\ f_{aa}(k) = \frac{Z_{k+1}}{N_{k+1}} = \frac{(Z_k + \frac{1}{2}Y_k)^2}{N_k^2} \end{array} \right. \quad (14)$$

84 The frequency of allele a on season k is $a_k = (Z_k + \frac{1}{2}Y_k)/N_k$. Using equation (14), one can check
 85 that $a_{k+1}/a_k = 1$ for all $k \geq 0$. Therefore, $a_k = a_0$ for all $k > 0$. The Hardy-Weinberg equilibrium
 86 follows: for all $k > 0$, $f_{AA}(k) = (1 - a_0)^2$, $f_{Aa}(k) = 2a_0(1 - a_0)$, and $f_{aa}(k) = a_0^2$.

87 S1.4 Demo-genetic model with masculinizing resistance

88 In this section, we consider masculinizing resistance to be the only potato variety grown over years.
 89 The male fractions are therefore $m_A(k) = 1$, $m_a(k) = m$ for all growing seasons $k \geq 0$; consequently,
 90 the female fractions are $f_A(k) = 0$ and $f_a(k) = (1 - m)$ for all $k \geq 0$.

91 Equation (11) yields $X_{k+1} = 0$ for all $k \geq 0$, meaning that the homozygous avirulent genotype
 92 (AA) cannot persist. Therefore, $N_k = Y_k + Z_k$ for all $k > 0$. Equation (11) simplifies as:

$$\left\{ \begin{array}{l} Y_{k+1} = (1 - m)R' \frac{Z_k}{1 + cN_k} \frac{\frac{1}{2}Y_k}{Y_k + mZ_k} \\ Z_{k+1} = (1 - m)R' \frac{Z_k}{1 + cN_k} \frac{mZ_k + \frac{1}{2}Y_k}{Y_k + mZ_k} \end{array} \right. \quad (15)$$

93 Summing both equations above yields:

$$N_{k+1} = (1 - m)R' \frac{Z_k}{1 + cN_k}.$$

94 Let the frequency of the virulent genotype (aa) on season k be

$$v_k = \frac{Z_k}{N_k}.$$

95 We obtain

$$v_{k+1} = \frac{Z_{k+1}}{N_{k+1}} = \frac{mZ_k + \frac{1}{2}Y_k}{mZ_k + Y_k} = \frac{mv_k + \frac{1}{2}(1 - v_k)}{mv_k + (1 - v_k)}$$

96 Model (15) can therefore be equivalently expressed as a demo-genetic model coupling population
 97 and virulence dynamics:

$$\left\{ \begin{array}{l} N_{k+1} = (1 - m)R' \frac{N_k}{1 + cN_k} v_k \\ v_{k+1} = \frac{mv_k + \frac{1}{2}(1 - v_k)}{mv_k + (1 - v_k)} \end{array} \right. \quad (16)$$

98 **Non-dimensionalization**

99 We rescale model (16) by letting $n_k = cN_k$, which yields:

$$\begin{cases} n_{k+1} = (1-m)R' \frac{n_k}{1+n_k} v_k \\ v_{k+1} = \frac{mv_k + \frac{1}{2}(1-v_k)}{mv_k + (1-v_k)} \end{cases} . \quad (17)$$

100 **Equilibria**

101 The fixed points of Equation (17) are $(0, 1)$, $((1-m)R' - 1, 1)$, $(0, v^*)$ and (n^*, v^*) , with

$$v^* = \frac{1}{2(1-m)}, \quad (18)$$

102 and

$$n^* = \frac{R'}{2} - 1. \quad (19)$$

103 **Stability of equilibria**

104 The stability of a given fixed point (\bar{n}, \bar{v}) depends on the spectral radius of the following matrix:

$$J(\bar{n}, \bar{v}) = \begin{pmatrix} (1-m)R' \frac{1}{(1+\bar{n})^2} \bar{v} & (1-m)R' \frac{\bar{n}}{1+\bar{n}} \\ 0 & \frac{m}{2(m\bar{v}+1-\bar{v})^2} \end{pmatrix}.$$

105 • We have $J(0, 1) = \begin{bmatrix} (1-m)R' & 0 \\ 0 & \frac{1}{2m} \end{bmatrix}$.

106 The eigenvalues of $J(0, 1)$ are $(1-m)R'$ and $\frac{1}{2m}$. Hence, the fixed point $(0, 1)$ is stable iff
107 $m \geq \frac{1}{2}$ and $(1-m)R' < 1$.

108 • We have $J((1-m)R' - 1, 1) = \begin{bmatrix} \frac{1}{(1-m)R'} & (1-m)R' - 1 \\ 0 & \frac{1}{2m} \end{bmatrix}$.

109 The eigenvalues of $J((1-m)R' - 1, 1)$ are $\frac{1}{(1-m)R'}$ and $\frac{1}{2m}$. Hence, the fixed point $((1-m)R' - 1, 1)$
110 is stable iff $m \geq \frac{1}{2}$ and $\frac{1}{(1-m)R'} < 1$, i.e. $(1-m)R' > 1$.

111 • We have $J(0, v^*) = \begin{bmatrix} \frac{R'}{2} & 0 \\ 0 & 2m \end{bmatrix}$.

112 The eigenvalues of $J(0, v^*)$ are $\frac{R'}{2}$ and $2m$. Hence, the fixed point $(0, v^*)$ is stable iff $m < \frac{1}{2}$
113 and $R' < 2$.

114 • We have $J(n^*, v^*) = \begin{bmatrix} \frac{2}{R'} & -(-1+m)(R'-2) \\ 0 & 2m \end{bmatrix}$.

115 The eigenvalues of $J(n^*, v^*)$ are $\frac{2}{R'}$ and $2m$. Hence, the fixed point (n^*, v^*) is stable iff $m < \frac{1}{2}$
116 and $R' > 2$.

117 Table 1 summarizes the stability of the fixed points.

Fixed Point	Density	Virulence	Stability	
			$m \geq .5$	$m < .5$
$(0, 1)$	Zero	Fixation	stable iff $(1 - m)R' < 1$	unstable
$((1 - m)R' - 1, 1)$	Maximum	Fixation	stable iff $(1 - m)R' > 1$	unstable
$(0, v^*)$	Zero	Polymorphism	unstable	stable iff $R' < 2$
(n^*, v^*)	Intermediate	Polymorphism	unstable	stable iff $R' > 2$

Table 1: Fixed points of the demo-genetic model (17) and their stability.

118 S1.5 Time to effective suppression

119 The nematode pest can be considered as effectively suppressed from growing season k^\dagger if its density,
120 N_k , does not exceed a certain acceptance threshold, τ , for all $k \geq k^\dagger$. We are interested in deriving
121 the length of time required to achieve effective suppression, $k^\dagger(r + 1)$. For simplicity, we consider
122 that the frequency of the virulent genotype, v_k , is initially at equilibrium: i.e., for all $k \geq 0$, $v_k = v^*$,
123 as defined in equation (18). Model (16) simplifies as:

$$N_{k+1} = (1 - m)R' \frac{N_k}{1 + cN_k} v^*. \quad (20)$$

124 We assume $N_0 > \tau$ (the nematode population density is initially above the acceptance threshold).
125 We next focus on dynamics leading the pest to extinction, which occur if and only if $(1 - m)Rv^* < 1$,
126 or equivalently $R < 2$. Using the explicit solution of equation (20), that is

$$N_k = \frac{(1 - \frac{R}{2}) N_0}{(1 - \frac{R}{2} + cN_0) (\frac{R}{2})^{-k} - cN_0},$$

127 we derive the generation k^\dagger from which, for all $k \geq k^\dagger$, $N_k < \tau$: the latter inequality is equivalent to

$$\left(1 - \frac{R}{2}\right) N_0 < \tau \left(\left(1 - \frac{R}{2} + cN_0\right) \left(\frac{R}{2}\right)^{-k} - cN_0 \right) \iff \frac{\left(\frac{1 - \frac{R}{2}}{\tau} N_0 + cN_0\right)}{1 - \frac{R}{2} + cN_0} < \left(\frac{R}{2}\right)^{-k},$$

128 which can be equivalently expressed as

$$\log \left(\frac{\left(\frac{1 - \frac{R}{2}}{\tau} N_0 + cN_0\right)}{1 - \frac{R}{2} + cN_0} \right) < -k \log \left(\frac{R}{2} \right) \iff k > \frac{\log \left(\frac{\left(\frac{1 - \frac{R}{2}}{\tau} N_0 + cN_0\right)}{1 - \frac{R}{2} + cN_0} \right)}{-\log \left(\frac{R}{2} \right)}.$$

129 Hence,

$$k^\dagger = \left\lceil \log \left(\frac{\left(\frac{1 - \frac{R}{2}}{\tau} N_0 + cN_0\right)}{1 - \frac{R}{2} + cN_0} \right) \log \left(\frac{2}{R} \right) \right\rceil. \quad (21)$$

130 Supplementary Material S2. Additional simulations

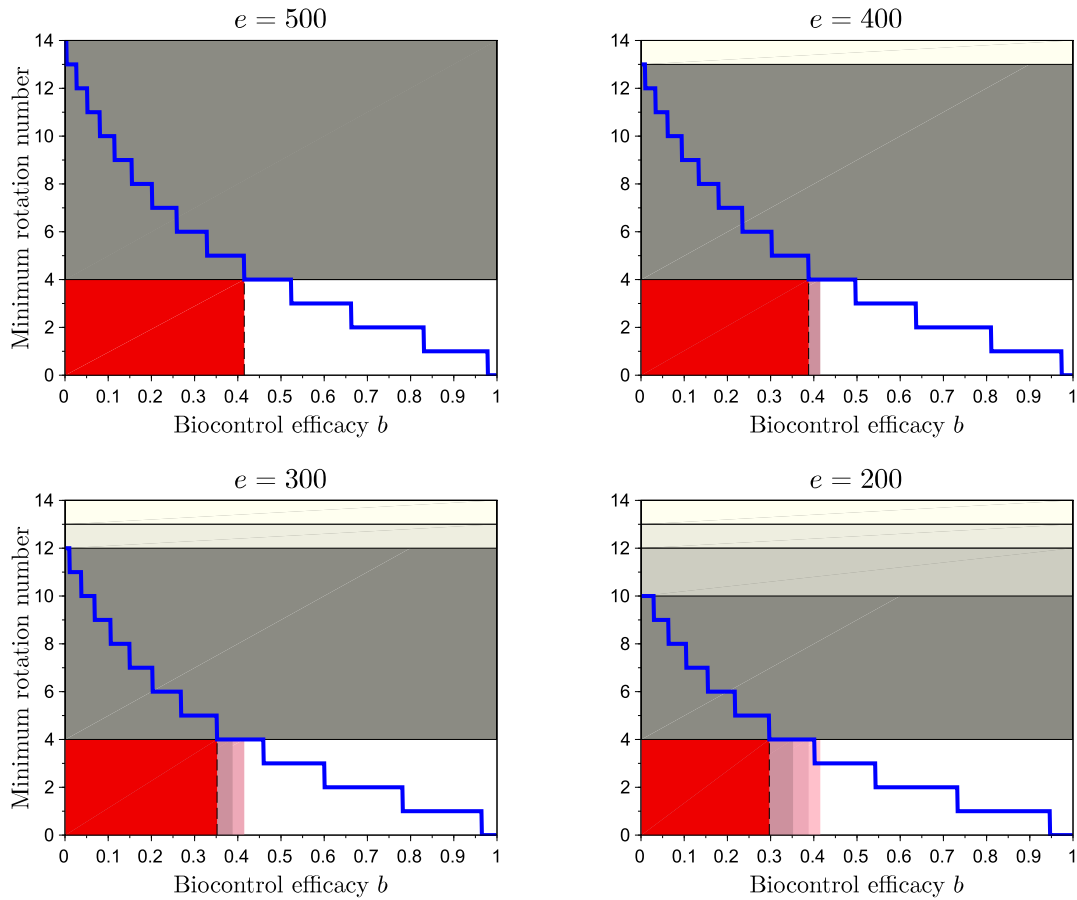


Figure S2: Rotation number required for long-term suppression of *G. pallida* under masculinizing resistance as a function of the biocontrol efficacy b for different values of the number of eggs per cyst, e . The minimum rotation number is maximized for $b = 0$, and is a decreasing function of e . This graphic also illustrates the decrease of biocontrol efficacy needed as e diminishes.

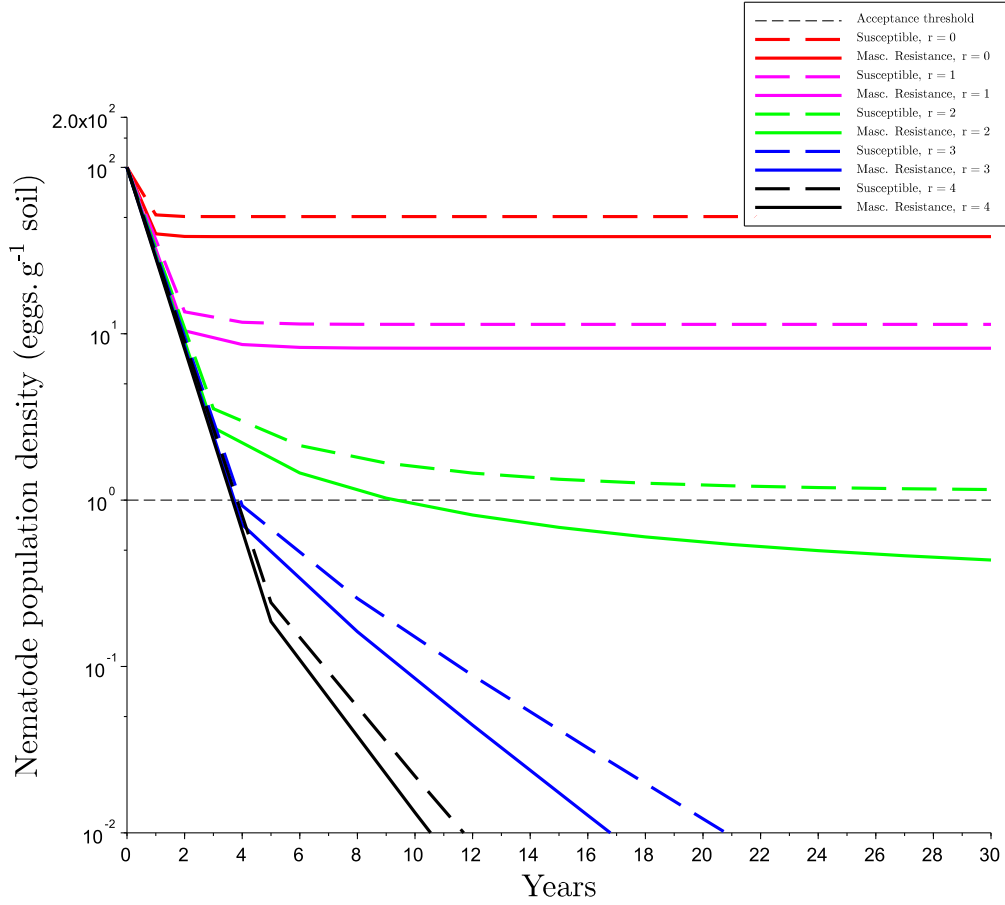


Figure S3: Time required to decrease nematode density under the acceptance threshold $\tau = 1$ nematode per gram of soil, with biocontrol efficacy fraction $b = .65$ and default parameter values, for a range of rotation numbers: $r = 0, 1, 2, 3, 4$. For high rotation numbers ($r = 3, 4$), the masculinizing resistance does not particularly speed up nematode suppression (initially at virulence frequency $v = v^*$). By contrast, for $r = 2$, growing a susceptible variety does not achieve suppression, whereas masculinizing resistance does. Lower rotation numbers ($r = 0, 1$) do not achieve suppression.