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JOINT ESTIMATION OF WATER COLUMN PARAMETERS AND SEABED REFLECTANCE COMBINING MAXIMUM LIKELIHOOD AND UNMIXING ALGORITHM

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ABSTRACT

Bathymetry and Water column constituent estimation is a challenging task for the study of coastal zones. Although most extensively used methods are based on the pixel wise inversion of semi empirical models, we have recently proposed a Maximum Likelihood (ML) method to retrieve such parameters. One limitation of the method is that the seabed reflectance is supposed to be known and homogeneous in a sample zone. We propose here to go beyond this limitation by jointly estimating the bathymetry, the concentrations of the constituents, and the reflectance spectra in an inhomogeneous seabed zone. To do so we introduce a non stationary Likelihood for the sample zone, and we also exploit a triple non-negative matrix factorization. We propose an Estimation-Unmixing (E-U) recursive algorithm to solve the problem. The water column parameters are estimated within the ML step, while the unmixing step allows to recover the bottom reflectance in each pixel. When tested on real hyperspectral data acquired in the Quiberon peninsula on French West coast, the method leads to consistent estimated maps of bathymetry and seabed reflectance.

1. INTRODUCTION

For the last decades, hyperspectral airborne remote-sensing has been widely used for mapping of water composition and bathymetry. Various models have been developed to describe the relation between the bottom reflectance and remote-sensing reflectance by taking into account water attenuation [1], [2]. These models usually consider that four parameters affect the water-leaving radiance: depth and concentrations of optically active constituents, i.e., phytoplankton (*phy*), colored dissolved organic matter (*cdom*) and non-algal particles (*nap*). Because each parameter has specific effects on spectra, it is thus possible to retrieve such water column properties from spectral shape under some conditions. In [3], [4] we have proposed a maximum Likelihood (ML) based method to

Thanks to French Defense Agency for data campaign funding within the exploratory research and innovation project named HypLitt, and to Actimar agency which carried out the field measurement campaign. ActiMar is a company specialized in operational oceanography and high resolution remote sensing, based in Brest, France (www.actimar.fr).

retrieve the concentrations of constituents and the bathymetry from samples of subsurface reflectance pixels, the bottom reflectance being supposed known and homogeneous in the sample zone. In this method the estimation is performed on a sample which contains, typically, 400 pixels (see [5]), which are supposed to be different realizations of a single statistical law. However in real scenarios the bottom reflectance can be both unknown and inhomogeneous, because many materials are present in a single sample zone. It is then necessary to modify the Likelihood function in order to take into account this situation, as well as estimating the bottom reflectance.

2. PROPOSED METHOD

In this section, we first recall the principle of ML estimation for the bathymetry and the concentrations of water constituents [3], [5]. Then we present the triple NMF we have recently developed for underwater material unmixing [6], and finally we propose a new method, combining these two approaches, to estimate conjointly the bathymetry, the water composition, and the reflectance of the sea bottom in each pixel.

2.1. Statistical estimation of bathymetry and water constituents

In shallow water, the simplified subsurface reflectance model of [1] can be used to describe the radiative transfer within the water column:

$$\mathbf{r}_t = \mathbf{r}_\infty + (\mathbf{r}_b - \mathbf{r}_\infty) \mathbf{K}, \quad (1)$$

where \mathbf{r}_t is the subsurface reflectance vector of one pixel, \mathbf{r}_∞ the reflectance of an infinite water column, \mathbf{r}_b the bottom reflectance, and \mathbf{K} a diagonal matrix composed of the water attenuation coefficients $\exp^{-k(\theta,\lambda)H}$, which depend on the constituent concentrations $\theta = [C_{phy}, C_{cdom}, C_{nap}]$, and the bathymetry H .

The precise relationship between the global attenuation coefficient and the parameters θ has been established using Lee's model [2] and can be found in [4].

From a statistical point of view, it is generally accepted that hyperspectral data can be described by a Gaussian law, so the likelihood associated to a statistical sample χ containing N spectral vectors is :

$$P(\chi|\Theta) = \prod_{i=1}^N \frac{1}{\sqrt{(2\pi)^L |\Gamma|}} e^{-\frac{1}{2}(\mathbf{r}_i - \boldsymbol{\mu})^t \Gamma^{-1} (\mathbf{r}_i - \boldsymbol{\mu})}. \quad (2)$$

Here, the mean vector $\boldsymbol{\mu}$ and the covariance matrix Γ depend on $\Theta = \{\theta, H\}$, via the radiative transfer model of eq. 1. More precisely, the mean $\boldsymbol{\mu}$ is parameterized as a function of Θ with the same expression as \mathbf{r}_t in equation 1.

If the covariance matrix is unknown, it can be estimated and consequently the ML estimation of the parameters $\{\theta, H\}$ is given by

$$\begin{aligned} \Theta_{MV}(\chi) &= \arg \max(P(\chi|\Theta, \hat{\Gamma}_{ML})) \\ &= \arg \max \det \left| \frac{1}{N} \sum_i (\mathbf{r}_i - \boldsymbol{\mu}(\Theta)) (\mathbf{r}_i - \boldsymbol{\mu}(\Theta))^t \right|^{-1} \end{aligned} \quad (3)$$

which has been developed in [5]. However, although this method has been shown to allow a better estimation accuracy than the classical inversion method that minimizes a quadratic error function at each pixel, it suffers from a limitation due to the size of the statistical sample. Indeed, it assumes that the sea bottom and the water composition is the same in the whole sample, which is not a very hard limitation in general, as marine environment do not vary much in small areas. But in some cases this assumption does not hold, for example in some coastal environments, where there can be man made materials, rocks or algae for example.

The second limitation of the method is that the value of the bottom reflectance is supposed to be known, which is not always the case in real situations.

We want here to go beyond these two limitations, i.e. we propose a method to conjointly estimate $\{\Theta, \mathbf{R}_b\}$, where \mathbf{R}_b is the matrix of bottom reflectance spectra \mathbf{r}_b .

To this end, we re-formulate the Likelihood function associated with the sample, assuming that each pixel follows a different Gaussian law in each pixel, each mean vector $\boldsymbol{\mu}_i, i = \{1 \dots N\}$, being associated with a different bottom reflectance vector \mathbf{r}_{bi} . Let note that we also allow the bathymetry H to vary in each pixel, while the water composition is supposed fixed.

It was not possible to estimate so many parameters in a single ML estimation procedure. We then propose to combine ML estimation with an unmixing technique, generalized to underwater data. Indeed, nonnegative matrix factorization was able to model the data by a triple factorization.

2.2. Triple NMF for unmixing in shallow water

The basic NMF formulation consists in approximating a non-negative matrix \mathbf{R} by a product of two matrices [7]

$$\mathbf{R} \simeq \hat{\mathbf{S}}\hat{\mathbf{A}}. \quad (4)$$

These matrices can be found by solving the following optimisation problem

$$\min_{\mathbf{S}, \mathbf{A}} \|\mathbf{R} - \mathbf{S}\mathbf{A}\|_F^2 \text{ s. c. } \mathbf{S} \geq 0, \mathbf{A} \geq 0 \quad (5)$$

We use here an extension of the classic NMF problem to a product of three matrices. Indeed, the so-called method 'Tri-NMF' [7], [6] can be applied to the bathymetric model that allows one to jointly estimate the bottom endmember spectra \mathbf{S} , the abundances \mathbf{A} and the attenuation matrix \mathbf{K} . By exploiting the sum-to-one constraint on the abundances, the bathymetric model of eq.1 can be rewritten as

$$\mathbf{R}_t - \mathbf{r}_\infty \mathbf{1}_{1N} = \mathbf{K}(\mathbf{S} - \mathbf{r}_\infty \mathbf{1}_{1J})\mathbf{A}, \quad (6)$$

in which $\mathbf{r}_\infty \mathbf{1}_{1N}$ is a matrix composed of N vectors of the infinite water column reflectance, supposed to be known.

By defining $\boldsymbol{\rho} = \mathbf{R}_t - \mathbf{r}_\infty \mathbf{1}_{1I}$ and $\boldsymbol{\rho}_0 = \mathbf{S} - \mathbf{r}_\infty \mathbf{1}_{1J}$, the following optimisation problem is obtained

$$\min_{\mathbf{K}, \mathbf{A}, \mathbf{S}} \|\boldsymbol{\rho} - \mathbf{K}\boldsymbol{\rho}_0\mathbf{A}\|^2 \text{ s. c. } \mathbf{K} \geq 0, \mathbf{A} \geq 0, \mathbf{S} \geq 0. \quad (7)$$

Strictly speaking, this is not a NMF optimization problem since the positivity constraint is on the spectra \mathbf{S} which is actually indirectly present in the divergence to minimize. Nevertheless, the constraints are sufficiently strong that this problem can still be considered as a NMF problem.

The bottom reflectance is then estimated by

$$\mathbf{R}_b = \mathbf{S}\mathbf{A},$$

which is afterwards used in the ML method for the estimation of the set of parameters Θ .

2.3. Proposed method

We want to estimate θ, H, \mathbf{r}_b in each pixel. We apply a grid pattern to extract zones of constant size, making a compromise between a small size to reduce the parameters variability, and a sufficient size to perform good estimation. For each zone, we make the following assumptions:

- The bottom is inhomogeneous and follows a linear mixing model
- The corresponding endmembers are fixed in one zone and are initialized with chosen representative spectra
- The constituent concentrations do not vary in the zone
- The depth can vary and can be calculated in each pixel by a Lagrange polynomial used for interpolating between the four corner's depth, H_1, H_2, H_3, H_4 .
- Two adjacent zones have same depth values on the adjacent corners

The ML estimation method allows to find a good estimation of the water column parameters Θ , with as fixed entry the bottom reflectance \mathbf{R}_b . Reversely, the Tri-NMF output gives an estimation of three matrices $\mathbf{K}, \mathbf{S}, \mathbf{A}$. In fact, the estimation of the water attenuation \mathbf{K} is not as much performing as the one of the ML estimation, and will not be used in the algorithm, but the product $\mathbf{S}\mathbf{A}$ gives the estimation of the bottom reflectance \mathbf{R}_b , which is used for the ML algorithm input.

We try to make use of both methods in a recursive procedure, and for each zone, we apply the following algorithm 1,

Algorithm 1 Estimation-Unmixing (E-U) algorithm

while $\epsilon > th$ **do**
 $\{\Theta^{(n)}\} \leftarrow$ Maximum Likelihood (\mathbf{R}_b fixed)
 $\mathbf{A}^{(n)}, \mathbf{S}^{(n)}, \mathbf{K}^{(n)} \leftarrow$ Tri-NMF (θ, H fixed)
 $\mathbf{R}_b^{(n)} = \mathbf{A}^{(n)}\mathbf{S}^{(n)}$
 $\epsilon = \max_p(|\mathbf{r}_p^{(n)} - \mathbf{r}_p^{(n-1)}|/|\mathbf{r}_p^{(n)}|)$
 $n = n + 1$
end while

In this algorithm, to take into consideration the hypothesis, the Likelihood function takes the form:

$$P(\chi|\Theta, \mathbf{R}_b) = \prod_{i=1}^N \frac{1}{\sqrt{(2\pi)^L |\Gamma_i|}} e^{-\frac{1}{2}(\mathbf{r}_i - \boldsymbol{\mu}_i)^t \Gamma_i^{-1} (\mathbf{r}_i - \boldsymbol{\mu}_i)} \quad (8)$$

The bathymetry within the sample zone is modeled by a Lagrangian polynomial, that interpolates between the four corner's depth H_1, H_2, H_3, H_4 .

Here, the mean vector $\boldsymbol{\mu}_i$ and the covariance matrix Γ_i depend on the pixel, as H and the bottom reflectance can vary from one pixel to the other. However only four values of H at the corners are calculated, while the other values are obtained using the Lagrange polynomial for interpolation. Then $\Theta = \{C_{phy}, C_{dom}, C_{nap}, H_1, H_2, H_3, H_4\}$.

We also adapt the Tri-NMF algorithm to the considered problem. First, it is well known that the NMF is sensitive to the initialization step. Here we initialize the attenuation matrix with $\mathbf{K}^{(0)} = \frac{1}{N} \sum_i \mathbf{K}_i$, VCA algorithm [8] for the endmembers, and pseudo-inversion for abundances.

Second, we develop a specific regularization scheme, applying a constraint on the endmembers and on the attenuation matrix. Indeed, \mathbf{K} being a diagonal matrix, there can be a confusion with the matrix \mathbf{S} during the unmixing, so we encourage \mathbf{K} to remain close to the estimation given by the ML. In order to reduce the domain of the endmember estimation, we construct a spectral base \mathbf{S}_0 , containing representative spectra that we usually find in benthic environments, and we encourage the matrix \mathbf{S} to remain close to the spectral base \mathbf{S}_0 .

Let note that during the unmixing by the Tri-NMF, the attenuation matrix is supposed to be the same for each pixel,

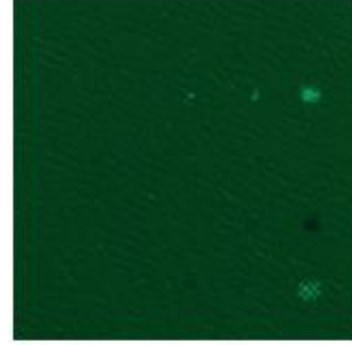


Fig. 1. A coastal zone of the Quiberon peninsula, France, with sand and targets in the bottom. The water column depth is 2.83 meters at the center

and is initialized with the mean depth obtained with previous ML. We then consider a rougher approximation of the effect of the water column than in the ML step. The objective function is then:

$$f = \|\rho - \mathbf{K}\rho_0\mathbf{A}\|^2 + \beta_1 \|\mathbf{K} - \mathbf{K}_0\|^2 + \beta_2 \|\mathbf{S} - \mathbf{S}_0\|^2,$$

where β_1 and β_2 are the weights of the regularization, experimentally fixed.

Finally, only the product $\mathbf{R}_b = \mathbf{A}\mathbf{S}$ is used from the output of the Tri-NMF, as an input parameter for the ML estimation in the algorithm 1.

3. EXPERIMENTS AND RESULTS

We show the results obtained for hyperspectral data from the HypLitt campaign. A colored composition of the image is shown in Fig.1. Twelve tarpaulins have been placed on the sandy bottom, of three different sizes and six different colors: three white, three gray, three blacks, a big checked pattern, and small green, red and blue ones. The three white tarpaulins, a black one and the checked one can be distinguished under the water column in Fig 1.

The hyperspectral image has been corrected in reflectance and geometrically referenced by the company Actimar, which also constructed a reference library with the reflectance spectra of the tarpaulins and of several kinds of sand, acquired in the seashore with a spectrometer. We made a reference endmember matrix \mathbf{S}_0 with six spectra extracted from the spectral library; here the red, blue, and green tarpaulins, light sand, dark sand and granite were selected (not the white and black ones, in order to lower the size of the endmember matrix).

Let recall that the estimated endmembers are initialized with \mathbf{S}_0 and are constrained to remain close to this matrix, so the choice of the reference \mathbf{S}_0 needs a previous knowledge of the possible composition of the sea bottom. Nevertheless, in these environments are usually made of few materials, such as sand, coral, rocks, algae, or specific on-shore equipments, the constitution of such a reference matrix is generally possible. Let emphasize that it is more important to have different types of spectra than to have precise ones, because of the natural variability of the materials, and because the endmembers are re-estimated by the NMF.

We applied a grid with eight zones of 40×40 pixels and ran the Estimation-Unmixing (E-U) algorithm in each zone with the constraint of keeping the same depth in the adjacent corners. However, due to the intermediate depth (2.8 m) we used the transfer model of Lee [2] given by following equation (eq. 9), more reliable for intermediate depths, instead of the one of eq. 1, in the Likelihood function.

$$\mathbf{r}_t = \mathbf{r}_\infty + (\mathbf{r}_b \mathbf{K}_b - \mathbf{r}_\infty \mathbf{K}_c) \mathbf{K}_d \quad (6)$$

In this model, \mathbf{K}_b , \mathbf{K}_c , \mathbf{K}_d are diagonal matrices whose diagonal elements are respectively $(e^{-k_c(\lambda)H}, e^{-k_b(\lambda)H}, e^{-k_d(\lambda)H})$, with k_b, k_c, k_d depending on θ . This model can be seen as a refinement of the simple model of eq. 1, and is used to improve the ML estimation part of the E-U algorithm.

The results are presented in Fig. 2. In Fig. 2(1), the bottom obtained by the product $\mathbf{R}_b = \mathbf{S}\mathbf{A}$ is given, while Fig. 2(2) shows the results of the inversion of eq. 3, which is affected by noise due to the inversion. Fig. 2(3) shows the grid for the analysis, and Fig. 2(4) the obtained bathymetry map. We only had available the groundtruth for the center point, which is $H = 2.83m$, while the estimated value is $2.61m$, giving 7.7% error.

We can see that the tarpaulins and the background sand are better recovered with the product $\mathbf{S}\mathbf{A}$ than with the simple inversion of the radiative equation 3. The bathymetry is quite well estimated, with an error of 7.7% in the middle of the scene, and is not affected by the estimation on separate grid patterns, due to the Lagrangian polynomial interpolation.

4. CONCLUSION

We propose a new estimation-unmixing (E-U) method, for the retrieval of many parameters in underwater environment, using both physical modelling for ML estimation and unmixing method. We have shown on real data the efficacy of such an approach, which allows to obtain good spatial resolution for the estimation, with the accuracy of ML estimation for the water parameters. In order to estimate many parameters such as the sea bottom endmembers and abundances, and the water parameters, we have previously constructed a representative data base, containing initial spectral vectors.

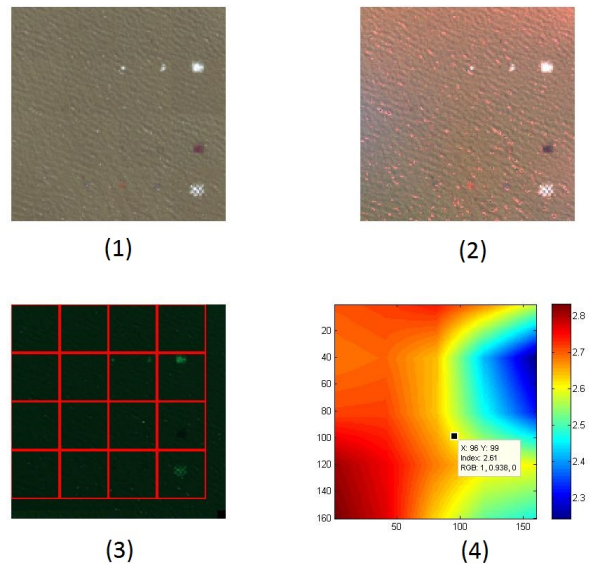


Fig. 2. Results of the bottom estimation (1) with E-U algorithm and $\mathbf{R}_b = \mathbf{S}\mathbf{A}$, (2) with E-U algorithm and \mathbf{r}_b given by the inversion of eq. 9. (3) shows the grid used, and (4) gives the bathymetry map

In future works we shall compare our approach with a pixel-wise mean square inversion and unknown background. Furthermore the analysis could be improved with the use of overlapping zones instead of separate ones.

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