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Mergers, Firm Size, and Volatility in a Granular Economy*

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Abstract

We study the firm dynamics associated with mergers and acquisitions (M&A) and their implications at the micro and macro levels. Our paper presents three main findings: (i) mergers generate a more fat-tailed firm-size distribution, thereby amplifying granular fluctuations and increasing aggregate volatility; (ii) the impact of mergers depends on strategic market power and endogenous markups; and (iii) under endogenous markups, we provide a novel characterization of the firm size-volatility relationship in which volatility declines disproportionately with size. We build a quantitative model of domestic horizontal mergers and find a sizeable impact of mergers on aggregate volatility using counterfactual analysis.

JEL classification codes: E30, E32, F12, G34, L11

Keywords: firm-size distribution, mergers and acquisitions, granularity, size-volatility relationship, variable markups

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1 Introduction

Firm dynamics influence the composition of firms and the overall economy. In particular, mergers and acquisitions (M&A) play a key role in driving such market dynamics, and this has important implications for industry structure and the macroeconomy. For example, productivity gains through mergers directly increase aggregate productivity, and the reallocation of resources from less to more productive firms also promotes growth (e.g., Dimopoulos and Sacchetto, 2017; Xu, 2017; David, 2020). At the same time, the firm dynamics associated with mergers may affect the volatility of an economy and the variance of growth at the macro level.¹ In this paper, we theoretically and empirically investigate the impact of mergers on this second moment of growth.

Our paper studies domestic horizontal M&A and makes three main contributions. First, we show that mergers generate a more fat-tailed firm-size distribution, thereby amplifying granular fluctuations and increasing aggregate volatility. To derive this result, we focus on the role of firm size. As demonstrated by Gabaix (2011), in a granular economy with a fat-tailed firm-size distribution that follows Zipf's law, the idiosyncratic shocks of large firms contribute significantly to the macroeconomic fluctuations observed and aggregate volatility is a function of market concentration. Because mergers increase the size of acquirer firms, the firm-size distribution shifts to become more fat tailed. This in turn leads to higher market concentration and as a result, aggregate volatility in the economy rises.

Second, we find that the impact of mergers on aggregate volatility depends crucially on the degree of firms' market power and the role of endogenous markups. In a multi-sector setting with oligopolistic competition, firms command strategic market power and internalize the effects of their pricing decisions on the sectoral price index (e.g., Bernard et al., 2018; Parenti, 2018). Hence, they charge variable markups and this results in the incomplete pass-through of shocks to prices (Atkeson and Burstein, 2008; Burstein et al., 2020). A negative firm size-volatility relationship is generated as larger firms have lower pass-through and their sales are less sensitive to the shocks that they face. Thus, at the firm level, the acquirer's volatility declines after the merger and this mitigates the overall impact of mergers. While mergers create larger firms and a more concentrated economy, the variable markups that they charge serve to dampen the effect of mergers on aggregate fluctuations.

Third, as a corollary to our second main finding, we show that under variable markups, the negative firm size-volatility relationship is characterized by the linear combination of the log-log and log-linear functions. This novel result contrasts with the log-log relationship that has been traditionally used and studied in the prior literature (e.g., Stanley et al., 1996; Sutton, 2002). Whereas the log-log function implies that volatility falls proportionally with firm size, the log-linear function gives the alternative prediction that volatility decreases disproportionately with size, especially for firms at the right tail of the distribution. This result is critical for quantifying the impact of M&A since large firms are often involved.

¹Firm dynamics from entry and exit have been shown to amplify productivity shocks and play an important role in shaping aggregate fluctuations. For example, see Clementi and Palazzo (2016), Lee and Mukoyama (2018), and Carvalho and Grassi (2019).

Large firms dominate economic activity around the world and their proliferation may in part be attributed to M&A deals. Globally, the number of M&A transactions has risen steadily, reaching record levels in 2015 with so-called “mega-deals” between some of the world’s biggest companies.² In this paper, we rely on rich register data from Denmark for our empirical and quantitative analysis. Examining our sample of horizontal mergers in Denmark reveals that consolidation usually occurs between very large acquirer and target firms. Over 50% of acquirers belong to the top decile of the firm-size distribution, and the targets matched to these firms are on average at the 74th percentile of the size distribution (see Appendix Figure B.3). In general, targets are by no means small, suggesting that mergers have the potential to induce sizable shifts in the right tail of the firm-size distribution. The dominance of large firms in merger markets also underscores the importance of taking into account firms’ strategic market power.

In this paper, we build a quantitative multi-sector model of horizontal mergers in a granular economy. The model employs the nested constant-elasticity-of-substitution (CES) demand structure, where within each sector, there is a discrete number of heterogeneous firms producing differentiated varieties (Atkeson and Burstein, 2008). Entrants pay a sunk entry cost to make productivity draws from an initial distribution. We add to this model a merger market with endogenous search decisions and sorting of acquirer and target firms (David, 2020). To model the growth in firm size, we assume a merger technology function that determines the productivity and size of the merged entity. Firms also face idiosyncratic shocks after the merger market closes, which generate fluctuations in aggregate output growth. Aggregate volatility in this model, measured by the standard deviation of total output growth, is increasing in market concentration. Mergers raise market concentration by driving less productive firms out of the market (i.e., a selection effect) and by shifting the firm-size distribution toward a fatter tail. Thus, compared to the initial pre-merger productivity distribution, the post-merger distribution has more mass in its right tail.

Empirically, we employ the universe of firms in Denmark to conduct a thorough investigation of the firm size-volatility relationship. Our regressions confirm the negative relationship from the literature (e.g., Cooley and Quadrini, 2001; Sutton, 2002; Koren and Tenreyro, 2013; Yeh, 2023). However, we also present novel, robust evidence to show that, in line with our prediction, volatility decays more rapidly for large firms than the rate suggested by the traditional log-log function. Specifically, the slope of the relationship becomes monotonically steeper as firm size increases, indicating that volatility declines disproportionately with size. This non-linearity in the size-volatility relationship is precisely what the presence of strategic market power and the implied log-linear component predict. We further provide corroborating evidence that merger-induced growth is associated with a post-merger decline in volatility for acquirer firms.

We calibrate the model’s parameters by targeting key moments in the Danish data with respect to the firm-size distribution and domestic M&A activity. In the model, a stationary

²For example, see <https://archive.annual-report.thomsonreuters.com/2015/articles/2015-year-of-the-mega-deal.html>. In Denmark, the biggest companies are also extremely large, with the top 50 and 100 firms accounting for roughly one-quarter and one-third of total domestic sales, respectively (see Appendix Figure B.1). These statistics are very similar to those of a large country like the US (Gabaix, 2011). Furthermore, Appendix Figure B.2 shows the rise in M&A deals in Denmark.

condition is imposed that links the pre-merger and post-merger productivity distributions. By assuming that the post-merger distribution is Pareto, this allows us to back out the counterfactual pre-merger distribution. Comparing the benchmark economy with mergers to a counterfactual economy without mergers, we find that mergers increase the granularity of the economy and thus, aggregate volatility. In the multi-sector economy with endogenous markups and incomplete pass-through, mergers increase volatility by 9.3 to 12.4%. To quantify the contribution of endogenous markups, we consider a hypothetical scenario where shocks have complete pass-through to prices. Shutting down the channel of endogenous markups overestimates aggregate volatility by almost 50%, suggesting that their quantitative effects are economically significant.

Related literature: This paper builds upon several lines of research in the prior literature. Most broadly, our work relates to a large literature that studies business cycles and macroeconomic fluctuations. Fluctuations may arise from aggregate shocks to supply or demand, as well as sectoral shocks that propagate through the economy.³ A growing strand of literature examines the contribution of shocks at the even more disaggregate firm level. In seminal work, Gabaix (2011) establishes the key result that in a granular economy with a fat-tailed firm-size distribution that follows Zipf’s law, the law of large numbers does not apply and firm-level shocks do not cancel out. Studies show that these micro-level shocks contribute substantially to macro-level fluctuations (e.g., Gabaix, 2011; di Giovanni et al., 2014; Magerman et al., 2016).⁴

Our paper examines an important channel for the creation of large firms and the rise in granular fluctuations, namely, mergers and acquisitions. By investigating the origins of large firms, our research contributes to a further understanding of the fundamental driving forces behind the volatility of economies. Related work by di Giovanni and Levchenko (2012) examines international trade as an alternative channel for generating a more fat-tailed firm-size distribution and increasing aggregate volatility. In particular, they find that the country of Denmark is 16% more volatile with trade, and this increase in the variance of output growth is amongst the highest found across countries. We find effects of a similar magnitude from mergers, even after accounting for endogenous markups. Carvalho and Grassi (2019) further study the role of large firm dynamics in driving aggregate fluctuations. Given that the largest firms in the world are typically multinationals, their shocks may propagate at an international scale to give rise to the comovement of GDP growth between countries (di Giovanni et al., 2018).

Moreover, our paper adds to the literature that investigates the economic implications of M&A. The efficiency gains of horizontal mergers often come at the expense of lower competition and a rise in market concentration (e.g., Williamson, 1968).⁵ The Herfindahl-Hirschman index

³For instance, economies face aggregate shocks to technology, total factor productivity, and fiscal shocks from changes in government policy (e.g., Kydland and Prescott, 1982; Long and Plosser, 1983; Greenwood et al., 1988; Rotemberg and Woodford, 1992). Sectoral linkages imply that sector-specific shocks can play an important role in generating aggregate fluctuations (e.g., Stockman, 1988; Horvath, 1998, 2000; Acemoglu et al., 2012; Grazzini and Massaro, 2022).

⁴The role of granular forces has also been studied in relation to comparative advantage in international trade (Gaubert and Itskhoki, 2021) and the banking sector (Bremus et al., 2018). Furthermore, Gaubert et al. (2021) examine welfare and antitrust policy implications from a merger between two firms in oligopoly.

⁵At the micro level, mergers may be viewed as a capital reallocation process that increases productive efficiency

(HHI) is commonly used as an indicator of market concentration to screen M&A deals (e.g., Whinston, 2007; Asker and Nocke, 2021). Here, we show that in individual sectors and the aggregate economy, the rise in market concentration from mergers is also associated with higher volatility. This suggests that macroeconomic fluctuations may be linked to the periodic waves of mergers that are observed.⁶

Our work is also closely related to recent studies on the aggregate implications of M&A. For example, David (2020) demonstrates that output rises not only due to productivity gains achieved by the consolidation of acquirers and targets, but also from the reallocation of resources across firms.⁷ Similar to our model, Cavenaile et al. (2021) present a framework of oligopoly in a discrete setting, where firms are either price-setters or price-takers. They study the role of antitrust policies for firm innovation. In contrast to this prior literature, we focus on the dynamic implications of mergers in terms of the aggregate volatility of the economy. Our multi-sector model also takes into account the strategic interactions of firms to allow for endogenous markups. De Loecker and Eeckhout (2021) document a strong correlation over time between global M&A activity and aggregate markups, which suggests that incorporating the channel of endogenous markups is important for understanding the impact of mergers.⁸

The paper is organized as follows. In Section 2, we present our general equilibrium model of heterogeneous firms with mergers and derive the theoretical results. Section 3 describes the data and performs the empirical analysis. In Section 4, we discuss how the model is quantified and present the results from our counterfactual analysis. Lastly, Section 5 concludes.

2 Theoretical model and analysis

2.1 Model setup

2.1.1 Preferences

The economy is set in continuous time and has an infinite horizon. The representative household maximizes utility given labor income and dividends from firms' profits:

$$U(Y, \mathcal{L}) = \log Y - \psi \mathcal{L} \quad s.t. \quad w\mathcal{L} + \Pi = PY, \quad (1)$$

where Y is the final good, \mathcal{L} is labor supply, w is the wage, Π is aggregate firm profits, and P is the aggregate price index. By assuming linear disutility of work, this implies that total market

(e.g., Jovanovic and Rousseau, 2008; Maksimovic et al., 2011). As a result, pro-competitive outcomes may be generated (e.g., Eckbo, 2014; Sheen, 2014; Ashenfelter et al., 2015; Braguinsky et al., 2015).

⁶For example, merger waves are well-documented in the finance literature, see among others, Harford (2005), Maksimovic et al. (2013), and Eckbo (2014).

⁷See also Dimopoulos and Sacchetto (2017), Levine (2017), and Xu (2017) for recent contributions in this line of research. Similar models of heterogeneous firms with mergers are examined in the context of cross-border M&A (e.g., Nocke and Yeaple, 2007; Blonigen et al., 2014; Brakman et al., 2018).

⁸Using data from the Worldscope database, De Loecker and Eeckhout (2021) find that between 1985 and 2016, global M&A increased more than tenfold and the aggregate markup rose by around 30 percentage points.

size is a constant, i.e., solving the household's maximization problem gives

$$PY = \frac{w}{\psi}. \quad (2)$$

We make this assumption for tractability and to simplify the quantification of our model.⁹ The rate of time discount is denoted by ρ , equal to the real interest rate. Total output in the economy and the price index are defined, respectively, by:

$$Y = \left[\sum_{k=1}^N D_k^{\frac{1}{\phi}} Y_k^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}}, \quad P = \left[\sum_{k=1}^N D_k P_k^{1-\phi} \right]^{\frac{1}{1-\phi}}. \quad (3)$$

The final good aggregates sectoral output Y_k from N sectors using a CES aggregator with the elasticity of substitution $\phi \geq 1$, and D_k is the demand shifter. Sectoral price indices are denoted by P_k . Meanwhile, within each sector, there is a discrete number M_k of heterogeneous firms producing differentiated varieties. Denote the output and price of firm i as y_{ki} and p_{ki} , respectively. Sectoral output and sectoral price indices are:

$$Y_k = \left[\sum_{i=1}^{M_k} y_{ki}^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}, \quad P_k = \left[\sum_{i=1}^{M_k} p_{ki}^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}. \quad (4)$$

Here, ε denotes the elasticity of substitution within a sector. Goods are assumed to be imperfect substitutes, and more substitutable within than across sectors, i.e., $\varepsilon > \phi$ (Atkeson and Burstein, 2008). Solving the household's maximization problem yields the following demand function:

$$y_{ki} = \left(\frac{p_{ki}}{P_k} \right)^{-\varepsilon} Y_k = \left(\frac{p_{ki}}{P_k} \right)^{-\varepsilon} \left(\frac{P_k}{P} \right)^{-\phi} D_k Y. \quad (5)$$

2.1.2 Firms, markups, and profits

New entrant firms pay a sunk entry cost wc_k^e to obtain a random productivity draw z_{ki} from the initial productivity distribution $F_k(z_{ki})$ with range $\mathbf{z}_k = \{z_{k1}, z_{k2}, \dots\}$, where $z_{k1} < z_{k2} < \dots$. The probability mass function (pmf) is defined accordingly as $f_k(z_{k1}) \equiv F_k(z_{k1})$ and $f_k(z_{kj}) \equiv F_k(z_{kj}) - F_k(z_{k,j-1}) \forall j \geq 2$. All costs are measured in labor units. Production requires both a fixed cost of production, wc_k^d , and variable costs. The latter consist of wage payments to labor ℓ_{ki} , which, for simplicity, is assumed to be the only factor of production. Firms produce with the technology of $y_{ki} = z_{ki} \ell_{ki}$. Then, the marginal cost of a firm with productivity z_{ki} is equal

⁹Burstein et al. (2020) derive their main theoretical results under the same assumption of linear disutility of work (i.e., the Frisch elasticity of labor supply is infinity), though generalization to any arbitrary value of the Frisch elasticity is also derived. In our setting, there is a discrete number of firms and the model must be simulated with a large number of samples when analyzing the multi-sector model. Firms' merger decisions depend on the economy's market size through the profit function, and the market size may also respond to mergers through changes in the firm-size distribution. Hence, both the market size and merger market are potentially endogenous. Here, we eliminate the response of market size to the merger market by assuming linear disutility, holding the former constant, while allowing the latter to be endogenous. This is a practically more feasible approach than assuming other values of the labor elasticity (including inelastic labor, due to non-zero aggregate profits, see Appendix Section A.1), which would require a joint solution of endogenous PY and the merger market.

to w/z_{ki} , and its markup is

$$m_{ki} = \frac{p_{ki}}{w/z_{ki}} = \frac{p_{ki}y_{ki}}{w\ell_{ki}}. \quad (6)$$

For notational convenience, $m_{ki} \equiv m_k(z_{ki})$. We use the two notations interchangeably for variables that depend on the firm's productivity.

After the merger market clears (discussed below), every firm in the economy receives an independent shock that affects its size. Following di Giovanni and Levchenko (2012), we assume that this shock is transitory and influences firm size through changes in productivity. It should be noted, however, that in this setup, firm-level productivity and demand shocks are isomorphic. In the granular economy that we are interested in studying, these idiosyncratic shocks are the source of aggregate fluctuations. Denote the shock as ϵ_{ki} , and normalize the mean of the shocks such that $\mathbb{E}_\epsilon[(z_{ki}\epsilon_{ki})^{\epsilon-1}] = z_{ki}^{\epsilon-1}$. Then, by Eq. (5), expected profits are:

$$\pi_k^E(z_{ki}) = \mathbb{E}_\epsilon \left[\left(p_{ki} - \frac{w}{z_{ki}} \right) y_{ki} \right] - wc_k^d = \mathbb{E}_\epsilon \left[\left(\frac{m_{ki}w}{z_{ki}} \right)^{1-\epsilon} \left(1 - \frac{1}{m_{ki}} \right) P_k^\epsilon Y_k \right] - wc_k^d. \quad (7)$$

2.1.3 Mergers

The merger market is modeled following David (2020). Incumbent firms participate and search randomly on both sides, i.e., for targets as acquirers and vice versa. Due to search and matching frictions, firms incur a cost of effort (e.g., time) which is increasing in their search intensity.¹⁰ In this paper, we consider only horizontal mergers of firms in the same sector k , which constitute the vast majority of M&A deals in Denmark (see Smeets et al. (2016) and Section 3 below).

A merger involves a one-to-one match between an acquirer and target firm. Mergers may occur for various reasons; for instance, firms may seek synergies with other companies or aim to capture a larger market share. Generally, the acquirer grows bigger after consolidation with its target. Thus, to capture this increase in firm size that the acquirer firm experiences after a merger, we rely on a merger technology function that depends on the acquirer's and target's productivity. Specifically, for an acquirer with productivity z_{ki}^a purchasing a target with productivity z_{ki}^t , we define the productivity of the merged entity by:

$$s[z_{ki}^a, z_{ki}^t] = A(z_{ki}^a)^\gamma (z_{ki}^t)^\nu. \quad (8)$$

Provided the parameters in this function are sufficiently large, then the productivity of the merged entity will be greater than that of the acquirer, i.e., $s[z_{ki}^a, z_{ki}^t] > z_{ki}^a$ and mergers will induce growth in the size of acquirer firms. This Cobb-Douglas function is also able to generate the positive assortative matching pattern observed in the data, for example, given sufficient

¹⁰Search frictions, price negotiation, and bargaining are important aspects of the M&A process (e.g., DePamphilis, 2018). Empirical evidence also suggests that deals are initiated not only by acquirers searching for targets, but often by targets seeking prospective buyers (e.g., Masulis and Simsir, 2018; Fidrmuc and Xia, 2019). Moreover, we follow previous literature (e.g., di Giovanni and Levchenko, 2012; Burstein et al., 2020) to assume that firms produce and sell one good. After the deal is completed, the merged entity (i.e., post-merger acquirer) continues to produce the same variety as the pre-merger acquirer. Meanwhile, the target exits and its variety is no longer sold on the market. For a study of multi-product firms and mergers, see, e.g., Chan et al. (2024).

complementaries between the acquirer and target (David, 2020). The acquirer obtains a share of the merger gains equal to $\Sigma_k^a(z_{ki}^a, z_{ki}^t) = \beta \Sigma_k(z_{ki}^a, z_{ki}^t)$, where β is the acquirer's bargaining power under Nash bargaining. Likewise, for the target, $\Sigma_k^t(z_{ki}^a, z_{ki}^t) = (1 - \beta) \Sigma_k(z_{ki}^a, z_{ki}^t)$. Merger gains are defined as the difference between the value of the merged entity and the values of the pre-merger acquirer and target firms:

$$\Sigma_k(z_{ki}^a, z_{ki}^t) = V_k(s[z_{ki}^a, z_{ki}^t]) - V_k(z_{ki}^a) - V_k(z_{ki}^t). \quad (9)$$

To complete the deal, the acquirer must offer the target an acquisition price equal to the target's value plus its share of the merger gains: $V_k(z_{ki}^t) + (1 - \beta) \Sigma_k(z_{ki}^a, z_{ki}^t)$.¹¹ The acquirer makes the purchase if $\beta \Sigma_k(z_{ki}^a, z_{ki}^t) \geq 0$, or equivalently, if merger gains are positive.

As an acquirer, firms search with intensity $\lambda_k(z_{ki})$ for meeting a potential target, and this requires search costs (in labor units) of

$$C(\lambda_k(z_{ki})) = \frac{B}{\eta} (\lambda_k(z_{ki}))^\eta. \quad (10)$$

Targets search with intensity $\mu_k(z_{ki})$ to find a potential acquirer and face the same convex search cost function. The meeting rate in the merger market of sector k depends on the minimum of the total search intensities of acquirers and targets, given by $\sum_{z_{ki}=\bar{z}_k}^{z_k^m} \lambda_k(z_{ki}) g_k(z_{ki})$ and $\sum_{z_{ki}=\bar{z}_k}^{z_k^m} \mu_k(z_{ki}) g_k(z_{ki})$, respectively, where z_k^m denotes the productivity of the most productive firm.¹² For instance, when the total search intensity of targets is greater than that of acquirers, the meeting rate depends on the latter, and targets are on the long side of the market. While firms draw from the *pre-merger* (i.e., initial) productivity distribution $F_k(z_{ki})$, the distribution evolves as a result of mergers. Total search intensities depend on the productivity distribution in the *post-merger* equilibrium $G_k(z_{ki})$, with pmf $g_k(z_{ki})$ defined analogously to $f_k(z_{ki})$.

The rate at which an acquirer with type z_{ki}^a meets a target with type z_{ki}^t is equal to

$$\lambda_k(z_{ki}^a) \min \left\{ \underbrace{\frac{\sum_{z_{ki}'=\bar{z}_k}^{z_k^m} \mu_k(z_{ki}') g_k(z_{ki}')}{\sum_{z_{ki}'=\bar{z}_k}^{z_k^m} \lambda_k(z_{ki}') g_k(z_{ki}')}}_{\theta_k^a}, 1 \right\} \underbrace{\frac{\mu_k(z_{ki}^t) g_k(z_{ki}^t)}{\sum_{z_{ki}'=\bar{z}_k}^{z_k^m} \mu_k(z_{ki}') g_k(z_{ki}')}}_{\Gamma_k(z_{ki}^t)}.$$

Market tightness on the acquirer side is given by θ_k^a , and the relative search intensity of targets with productivity z_{ki}^t by $\Gamma_k(z_{ki}^t)$. We have analogous expressions on the target side with θ_k^t as the market tightness and $\Lambda_k(z_{ki}^a)$ as the relative search intensity of acquirers.¹³

We can now define the value of a firm, which is equal to the present discounted value of its

¹¹Using the data available from Zephyr for Danish domestic mergers, the deal price and target size (as measured by its total revenues) are highly correlated at 0.67.

¹²Note that we switch between indexing firms by $i \in \{1, \dots, M_k\}$ and by their productivity $z_{ki} \in \{\bar{z}_k, \dots, z_k^m\}$.

¹³Specifically, $\theta_k^t \equiv \min \left\{ \frac{\sum_{z_{ki}'=\bar{z}_k}^{z_k^m} \lambda_k(z_{ki}') g_k(z_{ki}')}{\sum_{z_{ki}'=\bar{z}_k}^{z_k^m} \mu_k(z_{ki}') g_k(z_{ki}')}, 1 \right\}$ and $\Lambda_k(z_{ki}^a) \equiv \frac{\lambda_k(z_{ki}^a) g_k(z_{ki}^a)}{\sum_{z_{ki}'=\bar{z}_k}^{z_k^m} \lambda_k(z_{ki}') g_k(z_{ki}')}$. The first-order condition for optimal search of targets is $C'(\mu_k(z_{ki})) = B \cdot (\mu_k(z_{ki}))^{\eta-1} = \theta_k^t \mathbb{E}_{z_{ki}^a} [\Sigma_k^t(z_{ki}^a, z_{ki})]$.

expected profits plus expected merger gains as acquirers and targets net of search costs:

$$rV_k(z_{ki}) = \max_{\lambda_k(z_{ki}), \mu_k(z_{ki})} \pi_k^E(z_{ki}) - wC(\lambda_k(z_{ki})) - wC(\mu_k(z_{ki})) + \lambda_k(z_{ki})\theta_k^a \mathbb{E}_{z_{ki}^t} [\Sigma_k^a(z_{ki}, z_{ki}^t)] + \mu_k(z_{ki})\theta_k^t \mathbb{E}_{z_{ki}^a} [\Sigma_k^t(z_{ki}^a, z_{ki})], \quad (11)$$

where r is the discount rate. Moreover, the first-order condition for the optimal search intensity of acquirers (and analogously for targets) is:

$$C'(\lambda_k(z_{ki})) = B \cdot (\lambda_k(z_{ki}))^{\eta-1} = \theta_k^a \mathbb{E}_{z_{ki}^t} [\Sigma_k^a(z_{ki}, z_{ki}^t)]. \quad (12)$$

2.1.4 General equilibrium

For entrants that draw productivity below the threshold \bar{z}_k , the fixed cost of production is too high and they choose to exit the market. The cutoff firm must have a value of zero (cf. zero profit condition in Melitz (2003)):

$$V_k(\bar{z}_k) = 0. \quad (13)$$

The cutoff \bar{z}_k also defines the lower bound of the productivity distribution $G_k(z_{ki})$ for active firms in the market. Denote M_k^e as the number of potential entrants in sector k . In a stationary equilibrium, the number of firms that enter and exit the market must be equal at any point of the productivity distribution. The stationary condition for every type $z_{ki} \geq \bar{z}_k$ is:

$$\begin{aligned} & \underbrace{M_k^e f_k(z_{ki})}_{\text{Entrants}} + \underbrace{M_k \sum_{z_{ki}^a = \bar{z}_k}^{z_k^m} \lambda_k(z_{ki}^a) \theta_k^a \mathbf{1}[\Sigma_k(z_{ki}^a, s^{-1}[z_{ki}, z_{ki}^a]) \geq 0] \Gamma_k(s^{-1}[z_{ki}, z_{ki}^a]) g_k(z_{ki}^a)}_{\text{Merged firms}} \\ &= \underbrace{\delta_k M_k g_k(z_{ki})}_{\text{Exiting firms from exit shock}} + \underbrace{M_k \lambda_k(z_{ki}) \theta_k^a g_k(z_{ki}) \sum_{z_{ki}^t = \bar{z}_k}^{z_k^m} \mathbf{1}[\Sigma_k(z_{ki}, z_{ki}^t) \geq 0] \Gamma_k(z_{ki}^t)}_{\text{Exiting acquirers in mergers}} \\ &+ \underbrace{M_k \mu_k(z_{ki}) \theta_k^t g_k(z_{ki}) \sum_{z_{ki}^a = \bar{z}_k}^{z_k^m} \mathbf{1}[\Sigma_k(z_{ki}^a, z_{ki}) \geq 0] \Lambda_k(z_{ki}^a)}_{\text{Exiting targets in mergers}}, \end{aligned} \quad (14)$$

where $s^{-1}[z_{ki}, z_{ki}^a] = \{z_{ki}^t : s[z_{ki}^a, z_{ki}^t] = z_{ki}\}$ is a function that determines the productivity of the target merging with an acquirer with productivity z_{ki}^a to create a new firm with productivity z_{ki} , and $\mathbf{1}[\cdot]$ is the indicator function. Incumbent firms face an exogenous probability of exit δ_k . To close the model, the number of entrants is determined by the free entry condition:

$$\sum_{z_{ki}=z_{k1}}^{z_k^m} V_k(z_{ki}) f_k(z_{ki}) = w c_k^e. \quad (15)$$

That is, the expected value of entry is equal to the sunk cost of entry.

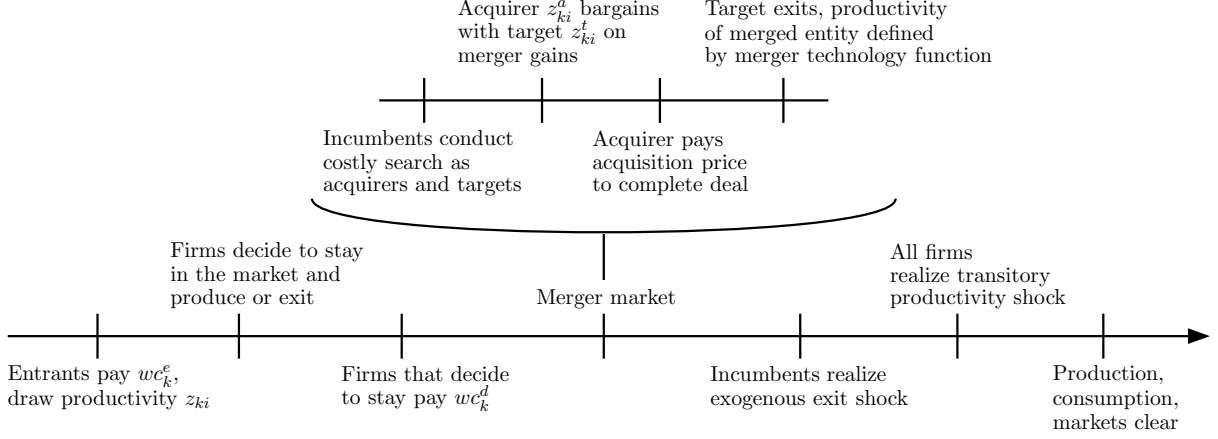


Figure 1: The timing of the economy.

Following di Giovanni and Levchenko (2012), transitory shocks are realized after firms' decision to produce and do not affect the number of firms M_k . Denote the merger rate in sector k as Υ_k , then the number of mergers is $\Upsilon_k M_k$.¹⁴ Aggregate variables must satisfy the goods and labor market clearing conditions in equilibrium. Thus, total output is equal to total consumption and the household's budget constraint holds. Market size PY is determined by Eq. (2). Lastly, aggregating across all sectors, labor supply must be equal to the sum of production labor, fixed costs of production, sunk costs of entry, and merger market search costs.¹⁵

The timing of the economy is summarized in Figure 1. Conditional on the set of parameters $\{\psi, c_k^d, c_k^e, \delta_k, \gamma, \nu, A, B, \eta\}$ and the initial productivity distribution $F_k(z_{ki})$, a *stationary equilibrium* consists of expected firm profits $\pi_k^E(z_{ki})$, value function $V_k(z_{ki})$, entry threshold \bar{z}_k , wage rate w , sectoral prices P_k , sectoral output Y_k , the mass of active firms M_k , the mass of entrants M_k^e , and the post-merger productivity distribution $G_k(z_{ki})$ such that: (i) the household maximizes its utility, (ii) firms maximize their value, (iii) the goods and labor markets clear, and (iv) the evolution of firm types follows the stationary condition in Eq. (14).

2.2 Theoretical analysis

2.2.1 Aggregate volatility in the single-sector economy

To derive our first main result that mergers increase aggregate volatility, we consider the simpler setting of a single-sector economy with $N = 1$. In this case, the separation of sectors disappears, and there must be a unique elasticity of substitution across firm varieties, which we denote as

¹⁴The merger rate is $\Upsilon_k \equiv \sum_{z_{ki}^t = \bar{z}_k}^{z_{ki}^m} \sum_{z_{ki}^a = \bar{z}_k}^{z_{ki}^m} \mu_k(z_{ki}^t) \theta_k^t \mathbf{1}[\Sigma_k(z_{ki}^a, z_{ki}^t) \geq 0] \Lambda_k(z_{ki}^a) g_k(z_{ki}^t)$, or equivalently, $\sum_{z_{ki}^a = \bar{z}_k}^{z_{ki}^m} \sum_{z_{ki}^t = \bar{z}_k}^{z_{ki}^m} \lambda_k(z_{ki}^a) \theta_k^a \mathbf{1}[\Sigma_k(z_{ki}^a, z_{ki}^t) \geq 0] \Gamma_k(z_{ki}^t) g_k(z_{ki}^a)$. Note that aggregating the stationary condition gives $M_k^e (1 - F_k(\bar{z}_k)) = M_k (\delta_k + \Upsilon_k)$.

¹⁵Note that all costs (i.e., variable and fixed costs of production, sunk costs of entry, and search costs) are subtracted to obtain realized aggregate profits Π (i.e., after the transitory shocks are revealed). Because merger gains are realized in the same period as profits (see Eq. (11)), aggregate profits are zero in this model except for the difference between expected and realized profits (Appendix Section A.1). Denoting L_k as production labor, labor market clearing implies $\mathcal{L} = \sum_{k=1}^N \left(L_k + M_k c_k^d + M_k^e c_k^e + M_k \sum_{z_{ki} = \bar{z}_k}^{z_{ki}^m} \left[C(\lambda_k(z_{ki})) + C(\mu_k(z_{ki})) \right] g_k(z_{ki}) \right)$.

$\tilde{\varepsilon}$. The model reduces to standard monopolistic competition, where all firms are “small” and have no strategic market power. Hence, all firms charge the same constant markup of $\frac{\tilde{\varepsilon}}{\tilde{\varepsilon}-1}$.

We define aggregate volatility as the standard deviation of output growth $\sigma[\hat{Y}] \equiv \sqrt{\text{Var}[\hat{Y}]}$, with \hat{Y} approximated by the log change, i.e., $\hat{Y} \equiv \frac{\Delta Y}{Y} \approx \Delta \log Y$. Given constant markups, the power law for firm size follows immediately from the assumptions of CES preferences and the Pareto distribution of firm productivity, $G(z_i) = 1 - \bar{z}^\xi z_i^{-\xi}$ (di Giovanni and Levchenko, 2012):

$$\Pr[\text{Sales}_i > c] = 1 - G\left(\frac{\tilde{\varepsilon}}{\tilde{\varepsilon}-1} w(P^{\tilde{\varepsilon}} Y)^{\frac{1}{1-\tilde{\varepsilon}}} c^{\frac{1}{\tilde{\varepsilon}-1}}\right) = \kappa c^{-\xi/(\tilde{\varepsilon}-1)}, \quad (16)$$

where κ is a constant. The economy is granular if the firm-size distribution follows a power law in which the absolute value of the power law exponent is sufficiently close to one, i.e., $\xi/(\tilde{\varepsilon}-1) \approx 1$. Under such conditions where Zipf’s law holds, Gabaix (2011) shows that the law of large numbers does not apply and aggregate volatility decays at the slower rate of $\log M$ instead of \sqrt{M} . For now, we assume that the idiosyncratic shocks that firms face are i.i.d. with constant variance $[v_z(s_i)]^2 = \bar{v}_z^2 \forall i$, i.e., they do not depend on firms’ market shares s_i . By Eq. (2), PY is a constant. Therefore, the variance of output growth \hat{Y} depends on fluctuations in the growth rate of the aggregate price index \hat{P} . A first-order Taylor approximation gives:

$$\hat{P} \approx \sum_{i=1}^M \frac{p_i^{1-\tilde{\varepsilon}}}{\sum_{i'=1}^M p_{i'}^{1-\tilde{\varepsilon}}} \hat{p}_i = \sum_{i=1}^M s_i \hat{p}_i. \quad (17)$$

Because $\text{Var}[\hat{p}_i] = \text{Var}[\hat{z}_i] = \bar{v}_z^2$, we further obtain

$$\text{Var}[\hat{Y}] = \bar{v}_z^2 HHI, \quad (18)$$

where $HHI \equiv \sum_i s_i^2$ is the Herfindahl-Hirschman index. In other words, aggregate volatility is an increasing function of market concentration.

2.2.2 Extensive and intensive margins

Intuitively, mergers create larger, more productive firms which drives up market concentration and aggregate volatility. The net effect of mergers can be decomposed into the extensive and intensive margins. Starting with the extensive margin, denote \bar{z} (\bar{z}^F) as the cutoff productivity in the economy with (without) mergers. Because the market becomes more competitive with mergers, less productive entrants must exit, which implies that $\bar{z}^F < \bar{z}$. Changes at the extensive margin are driven by selection effects, which we examine by holding fixed the initial productivity distribution $F(z_i)$ and raising the threshold for market entry. In a discrete setting, the market shares of firms with productivity between \bar{z}^F and \bar{z} that were active in the economy without mergers must be distributed among the surviving firms with productivity $z_i \geq \bar{z}$. From Eq. (17), a firm’s market share is proportional to $p_i^{1-\tilde{\varepsilon}}$ and to $z_i^{\tilde{\varepsilon}-1}$. Therefore, given that we hold fixed the productivity distribution and only shift up the entry threshold, it must be that the market

shares of all surviving firms increase proportionally. In other words, the market shares of all surviving firms must grow by the same factor to maintain the same proportions within the set of surviving firms as before. This further implies that HHI also increases by the same factor, and by Eq. (18), the economy is more volatile.¹⁶

Next, by holding the cutoff productivity level fixed at \bar{z} , we isolate the intensive margin accounted for by the shift in the productivity distribution. As smaller target firms are consolidated by larger acquirers, the initial pre-merger productivity distribution $F(z_i)$ evolves into the post-merger distribution $G(z_i)$. The creation of larger merged entities generates a more fat-tailed firm-size distribution. A thicker tail leads to a more concentrated market and implies a more volatile economy. Thus, mergers amplify granular fluctuations in the macroeconomy. While we cannot derive further analytical results, a comparison of the pre and post-merger firm-size distributions is illustrated in Figure 7 of Section 4.3 below where the model is quantified.

2.2.3 Aggregate volatility in the multi-sector economy

We now analyze the multi-sector economy with oligopolistic competition to show our second main result that the impact of mergers on aggregate volatility depends on the role of endogenous markups. Following Burstein et al. (2020), we consider the markup adjustments of firms under Cournot competition. By Eq. (6), the market share of firm i in sector k under oligopoly is now:

$$s_{ki} = \frac{p_{ki}y_{ki}}{P_k Y_k} = \frac{z_{ki}^{\varepsilon-1} m_{ki}^{1-\varepsilon}}{\sum_{i'=1}^{M_k} z_{ki'}^{\varepsilon-1} m_{ki'}^{1-\varepsilon}}. \quad (19)$$

With Cournot competition, the firm's markup is an increasing function of its market share:

$$m_{ki} = \frac{\varepsilon}{\varepsilon - 1} \left[1 - \left(\frac{\varepsilon/\phi - 1}{\varepsilon - 1} \right) s_{ki} \right]^{-1}. \quad (20)$$

Furthermore, define the pass-through rate as:

$$\alpha_{ki} \equiv \frac{1}{1 + (\varepsilon - 1) \frac{\partial \log m_{ki}}{\partial \log s_{ki}}} = \frac{\varepsilon - 1 - \left(\frac{\varepsilon}{\phi} - 1 \right) s_{ki}}{\varepsilon - 1 + (\varepsilon - 2) \left(\frac{\varepsilon}{\phi} - 1 \right) s_{ki}} \leq 1 \text{ if } \varepsilon \geq 1, \quad (21)$$

where the markup elasticity is $\frac{\partial \log m_{ki}}{\partial \log s_{ki}} = \left(\frac{\varepsilon}{\phi} - 1 \right) s_{ki} \left[\varepsilon - 1 - \left(\frac{\varepsilon}{\phi} - 1 \right) s_{ki} \right]^{-1}$. In the single-sector economy, all firms charge the same constant markup of $m_i = \frac{\varepsilon}{\varepsilon - 1}$, the markup elasticity

¹⁶The extensive margin described here encompasses both the entry and selection effects in David (2020), where the former accounts for the change in the number of firms (or mass in a continuous setting) and the latter for the change in the cutoff productivity. While a change in the cutoff productivity holding the number of firms fixed affects average productivity in David (2020), we are interested in volatility, which depends on HHI and the allocation of market shares. If we hold the number of firms and the productivity distribution fixed and increase the cutoff productivity from \bar{z}^F to \bar{z} , then the price index and sales of all firms would adjust proportionally by $\frac{1-F(\bar{z}^F)}{1-F(\bar{z})}$. Average productivity increases, but the distribution of market shares and HHI remain unaffected. The number of firms must change to release market shares that are distributed among the surviving firms. Then, the market shares of surviving firms increase by the factor $\left[\sum_{z_{i'}=\bar{z}^F}^{z^m} z_{i'}^{\varepsilon-1} f(z_{i'}) \right] / \left[\sum_{z_{i'}=\bar{z}}^{z^m} z_{i'}^{\varepsilon-1} f(z_{i'}) \right] > 1$, where the summation in the numerator (denominator) has the lower limit \bar{z}^F (\bar{z}). This factor is exactly equal to the inverse of the sum of surviving firms' market shares under the economy without mergers.

is zero, and pass-through is complete. By contrast, pass-through is incomplete in the multi-sector economy where firms charge variable markups, i.e., $\alpha_{ki} < 1$. By Eqs. (6), (19), and (20), the log change in price is now approximated as:

$$\widehat{p}_{ki} \approx -\alpha_{ki}\widehat{z}_{ki} + (1 - \alpha_{ki})\widehat{P}_k. \quad (22)$$

We further relax the assumption of i.i.d. shocks, allowing the variance of shocks to depend on firm size. In particular, we follow Sutton (2002) and di Giovanni and Levchenko (2012) to assume a power law relationship between the standard deviation of shocks and firm size:

$$v_z(\mathbf{s}_{ki}) = \bar{v}_z \mathbf{s}_{ki}^{-\chi}. \quad (23)$$

This may capture, for instance, the diversification of firms as they grow larger. Again, the variance of \widehat{Y} depends on fluctuations in \widehat{P} and now in turn on the growth rates of the sectoral price indices \widehat{P}_k . Performing the same first-order approximation as before, but taking into account the effects of strategic market power through Eq. (22), we obtain:

$$\text{Var}[\widehat{Y}] = \bar{v}_z^2 \sum_{k=1}^N \mathcal{S}_k^2 \sum_{i=1}^{M_k} \left(\frac{\alpha_{ki} \mathbf{s}_{ki}^{1-\chi}}{\sum_{i'=1}^{M_k} \alpha_{ki'} \mathbf{s}_{ki'}} \right)^2, \quad (24)$$

where $\mathcal{S}_k \equiv \frac{P_k Y_k}{PY}$ is the share of sector k 's sales.¹⁷

Aside from the exponent $1 - \chi$, which appears because we had relaxed the assumption on the variance of shocks, the major difference between this equation and Eq. (18) in the single-sector economy with constant markups is the presence of the pass-through rates α_{ki} . Eq. (24) demonstrates that in the multi-sector framework, volatility is not exactly a function of the Herfindahl-Hirschman index. Rather, market shares are weighted by pass-through rates α_{ki} . From Eq. (21), α_{ki} is decreasing in \mathbf{s}_{ki} , which means that the weight of large firms in the sectoral and aggregate price index is smaller *vis-à-vis* the single-sector model. As shown by Burstein et al. (2020), the impact of large firms on volatility is mitigated by incomplete pass-through. Because mergers are a driving force behind a more granular economy, the response of aggregate volatility depends on endogenous markups and the relationship between α_{ki} and \mathbf{s}_{ki} . For example, setting $\phi = 1$ and $\varepsilon = 5$, Figure 2 indicates that the product $\alpha_{ki} \mathbf{s}_{ki}$ is increasing (at a decreasing rate) for market shares below 0.33. Hence, we can generally expect mergers to increase the unweighted and weighted market shares (i.e., \mathbf{s}_{ki} and $\alpha_{ki} \mathbf{s}_{ki}$) of acquirers, thereby generating greater variance in total output growth. However, the impact of the large firms created through mergers is mitigated due to incomplete pass-through, as α_{ki} declines with \mathbf{s}_{ki} .

¹⁷An alternative to Eq. (23) is to assume that the standard deviation of shocks is a function of the firm's absolute, as opposed to relative, size. This does not affect our empirical analysis, because the inclusion of sector-period fixed effects controls for sector market size in a given period. For our quantitative assessment, with $\phi = 1$, Eq. (24) becomes $\text{Var}[\widehat{Y}] = \bar{v}_z^2 \left(\frac{w}{\psi}\right)^2 \sum_{k=1}^N \mathcal{S}_k^{2(1-\chi)} \sum_{i=1}^{M_k} \left(\frac{\alpha_{ki} \mathbf{s}_{ki}^{1-\chi}}{\sum_{i'=1}^{M_k} \alpha_{ki'} \mathbf{s}_{ki'}} \right)^2$ if the variance of shocks depends on the firm's absolute size. We estimate χ in Section 3 to be roughly 0.14, which suggests that our quantitative results are generally robust to this alternative assumption.

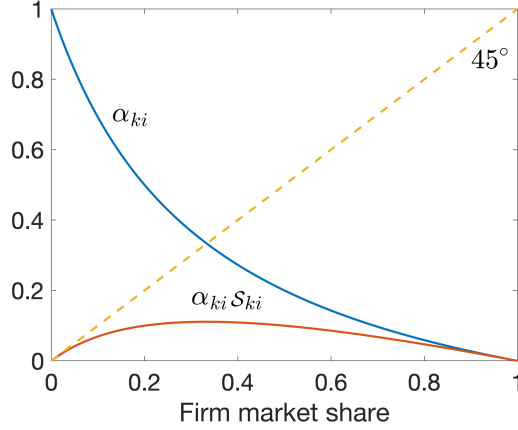


Figure 2: This figure plots the pass-through rate α_{ki} and the pass-through rate multiplied by the market share $\alpha_{ki}s_{ki}$ against the market share, assuming $\phi = 1$ and $\varepsilon = 5$.

Finally, Eq. (24) reveals that the variance of shocks also affects aggregate volatility through χ . *A priori*, there is no clear-cut prediction on how aggregate volatility changes, since the variance of large firms is lower but there is an opposite effect on small firms. di Giovanni and Levchenko (2012) refer to this as a “double-edged sword.” To summarize, mergers create larger firms, increasing market concentration and aggregate volatility. With endogenous markups, this effect is mitigated by the decrease in volatility from merger-induced growth at the firm level.¹⁸

2.2.4 Firm-level volatility

As we have seen, the degree to which mergers affect aggregate volatility depends crucially on the firm size-volatility relationship. We now provide a characterization of this relationship at the micro level to understand the rate at which volatility declines with size. In the multi-sector economy, firms are large within their respective sectors, which gives rise to strategic market power and variable markups. As a result, the sectoral price index is not a constant from the firm’s perspective (e.g., Bernard et al., 2018; Parenti, 2018). Here, we set $\phi = 1$ for simplicity, which removes the effects from the other sectors k' ; Appendix Section A.2 derives the analogous equation without imposing this restriction. Denote firm-level sales or revenue as $r_{ki} \equiv p_{ki}y_{ki}$. Then, by Eqs. (5) and (22), the variance of firm-level sales growth is:

$$\text{Var}[\hat{r}_{ki}] = (\varepsilon - 1)^2 \alpha_{ki}^2 \underbrace{\left(1 + \frac{\sum_{i'=1}^{M_k} (\alpha_{ki'} s_{ki'})^2 \frac{[v_z(s_{ki'})]^2}{[v_z(s_{ki})]^2}}{\left(\sum_{i'=1}^{M_k} \alpha_{ki'} s_{ki'}\right)^2} - \frac{2\alpha_{ki} s_{ki}}{\sum_{i'=1}^{M_k} \alpha_{ki'} s_{ki'}} \right)}_{\text{Strategic market power effect on volatility}} [v_z(s_{ki})]^2. \quad (25)$$

¹⁸The analysis in Section 2.2.2 on the extensive and intensive margins for the single-sector economy generally applies for each of the individual sectors in the multi-sector model. Analysis of the extensive margin differs slightly, as the market shares of firms that exit due to selection effects are no longer proportionally distributed. Instead, with variable markups, the biggest firms charge the highest markups. Thus, when the effective market size increases for the surviving firms, the largest firms benefit the most and their markups and market shares rise disproportionately more than the smaller firms.

We refer to the term before $[v_z(s_{ki})]^2$ as the strategic market power (SMP) effect on volatility, to distinguish this channel from the size-variance relationship associated with the shocks themselves. The SMP effect shows up because firms internalize the impact of their prices on the sectoral price index. This results in incomplete pass-through, as firms respond to their shocks by adjustments through markups. Thus, Eq. (25) gives the analytical relationship between firm-level volatility and the size of the firm.

Taking logs of Eq. (25), we immediately obtain the log-log component of the size-volatility relationship due to the variance of shocks. What is the relationship between log volatility and the log SMP effect? We first demonstrate that the log SMP effect falls with firm size, then show that the rate is approximately linear. Since $\log(1+x) \approx x$ for small x , the term in parentheses within the SMP effect already suggests that a linear function may serve as a good approximation. To derive analytical results on how the SMP effect varies with size, we set $\chi = 0$ and further assume for simplicity that all aggregate sums (e.g., $\sum_{i'=1}^{M_k} \alpha_{ki'} s_{ki'}$) are constant. Appendix Section A.2 provides an extensive explanation for why these simplifications are of second order and do not affect our main findings. Taking the derivative of the log SMP effect with respect to s_{ki} gives:

$$-2 \left[- \left(1 - \frac{\alpha_{ki} s_{ki}}{\sum_{i'} \alpha_{ki'} s_{ki'}} \right) \frac{\partial \log \alpha_{ki}}{\partial s_{ki}} + \frac{\alpha_{ki} s_{ki}}{\sum_{i'} \alpha_{ki'} s_{ki'}} \frac{1}{s_{ki}} \right] < 0. \quad (26)$$

This is a weighted average in which the semi-elasticity is negative. Again, from Figure 2, we know that there is an inverse relationship between α_{ki} and s_{ki} . When market shares are large, shocks are absorbed by changes in markups such that fluctuations in the growth rate of sales are suppressed. Thus, we can conclude that merger-induced firm growth is associated with a decline in post-merger firm-level volatility, even if the variance of shocks were held fixed.

Moreover, it can be shown that the derivative in Eq. (26) is roughly constant; see Appendix Section A.2. By contrast, the derivative of the log SMP effect with respect to the logarithm of market shares, $\log s_{ki}$, becomes more negative as s_{ki} increases. Figure 3(a) plots the log SMP effect against market share in levels and logs. While the relationship with market share in levels is not exactly linear, the slope is close to constant, especially in comparison to the relationship with log market share. Hence, we rely on the log-linear functional form as an approximation for the effect of variable markups on volatility. Additionally, from Eq. (23), we have $\log v_z(s_{ki}) = \log \bar{v}_z - \chi \log s_{ki}$ with regards to the variance of shocks. The firm size-volatility relationship can then be characterized by the linear combination of the log-log and log-linear functions. The volatility of firm-level sales growth in Eq. (25) can be rewritten in the form:

$$\log \sigma[\hat{r}_{ki}] = c_0 - c_1 \log s_{ki} - c_2 s_{ki}. \quad (27)$$

How does each component affect the rate at which volatility declines with size? In Figure 3(b), we plot the log-log and log-linear components separately, as well as their linear combination on a logarithmic x -axis. We set c_1 and c_2 in Eq. (27) to 0.138 and 3.304, respectively (see Section 3 below). When market shares are small, $\log s_{ki}$ is much larger than the linear

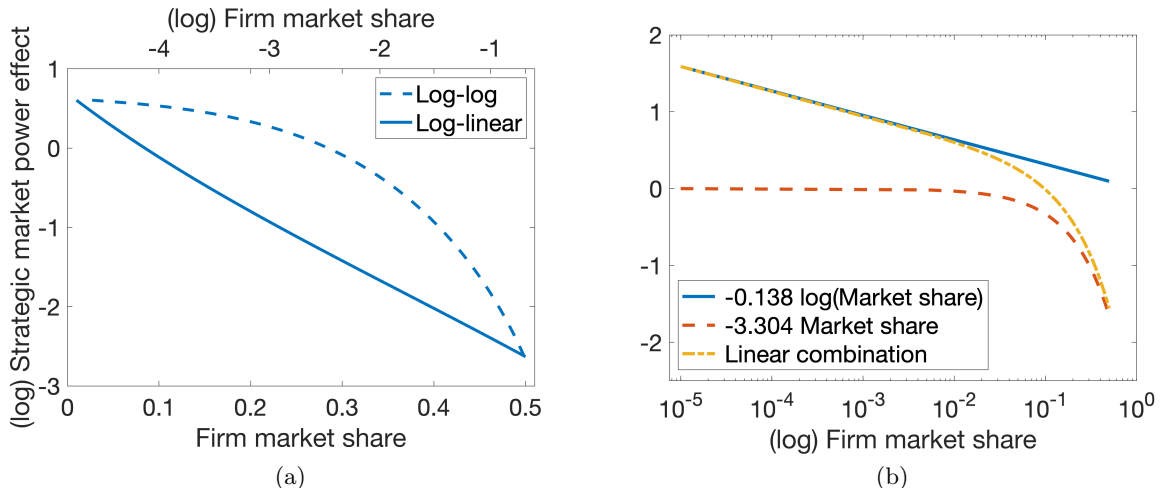


Figure 3: This figure plots (a) the (log) strategic market power effect on volatility in Eq. (25) against firm market share in levels and logarithms, and (b) three equations with (log) firm-level volatility, $\log \sigma[\hat{r}_{ki}]$, on the vertical axis: (i) $-0.138 \log(s_{ki})$, (ii) $-3.304 s_{ki}$, and (iii) their linear combination. For purposes of illustration, in panel (a), we set $\phi = 1$, $\varepsilon = 5$, $\chi = 0$, $\sum_{i'=1}^{M_k} (\alpha_{ki'} s_{ki'})^2 = 1$, and $\sum_{i'=1}^{M_k} \alpha_{ki'} s_{ki'} = 1$.

term s_{ki} , and the log-linear function dominates. In other words, the log-log function describes the firm size-volatility relationship well for small firms. However, the result is quite different for large firms, with the log-linear function dominating. The steep slope indicates that volatility decays much more rapidly compared to the log-log function. Because acquirers tend to be large, even before the merger, this implies that variable markups may play an important role in contributing to lower firm-level volatility. In the next section, we empirically examine our characterization of the size-volatility relationship.

3 Data and empirical analysis

3.1 Description of data

We employ detailed register data from Statistics Denmark for our empirical analysis and quantification of the model. The dataset contains the universe of firms, and we follow the methodology in Smeets et al. (2016) to identify domestic merger deals as well as the acquirer and target firms involved. The establishment register *IDAS* provides a unique establishment identifier that can be followed over time, along with a firm identifier in both the current and following year. This allows for changes in the firm identifier to be tracked. A change of this identifier to that of an existing firm indicates a change of ownership, which we classify as a merger deal. The pre-merger firm identifier of the establishments before the switch determines the target firm, while the post-merger firm identifier gives the acquirer firm.¹⁹

¹⁹Following Smeets et al. (2016), spurious changes to non-existing firm identifiers, for instance, when headquarters move to a different location, are excluded. Moreover, we also remove all partial mergers from our sample. This eliminates scenarios where: (i) the acquirer receives a fraction of the target's establishments, and the target

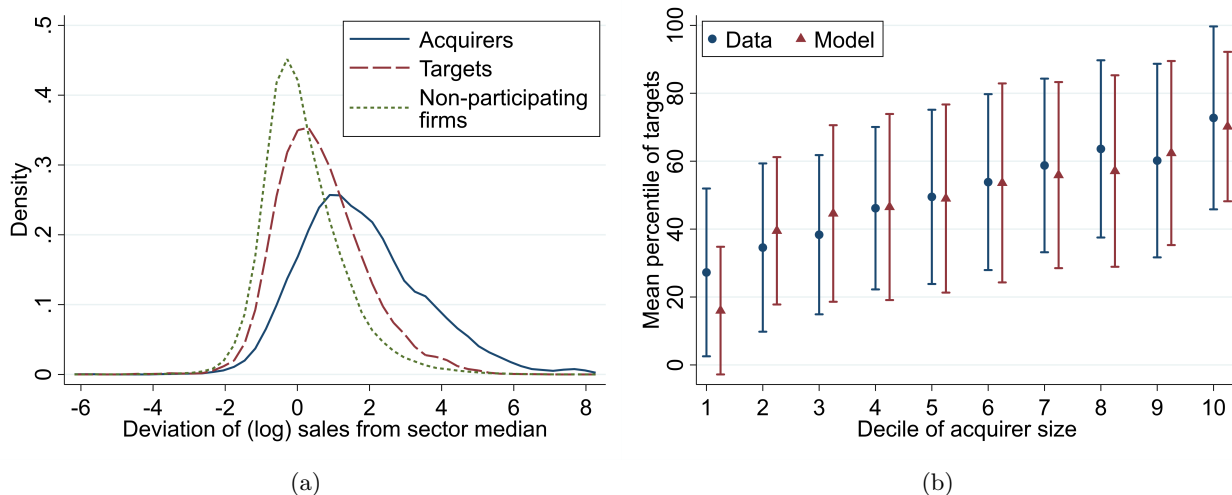


Figure 4: This figures plots: (a) the kernel densities of firm size for acquirers, targets, and non-participating firms; and (b) the mean percentile rank of targets (relative to all other targets) across deciles of acquirer size in the data and from model simulation.

From the data registers *FIRM*, *UHDI*, *IDAN*, and *BEF*, we obtain information on firms' annual revenues, exports, number of employees, and their 4-digit Danish industry codes. These industry codes correspond to the NACE Rev. 2 classification in the year 2007. We restrict the sample to firms in private industries (i.e., excluding utilities, public administration and defense, education, health services, culture and entertainment) with at least five full-time employees between the ages of 18 and 65. We also exclude firms from agriculture, mining, and finance and insurance due to a lack of information. To ensure that sectors have a sufficiently large number of firms, we employ a broad classification to group firms into 17 related sectors, listed in Appendix Table B.1. For example, the manufacture of food, beverages, and tobacco products, which have different 2-digit industry codes, are combined into one category. In line with our model, we focus on horizontal mergers of firms within the same sector. Using our broad sector definitions, 80% of merger deals are classified as horizontal. Even at the 2-digit level, 70% of deals are completed within the same sector.

Our sample contains 3,575 horizontal mergers for the period from 1993 to 2015. In Figure 4(a), we first compare the size of acquirers, targets, and non-participating firms. Firm size is measured by domestic sales, which are computed by subtracting annual exports from total sales. For each group, we plot kernel densities of the firms' (log) domestic sales normalized by the sector median. Overall, acquirers tend to be the biggest firms. Targets are on average also larger than the firms not engaged in M&A activity. Similar patterns can be obtained with other measures such as employment. To complement this graph, we follow di Giovanni et al.

remains in the market as an independent firm (i.e., with its original firm identifier) after the merger, and (ii) the target's establishments are purchased and controlled by multiple acquirers. Hence, we study the M&A deals in which the acquirer gains full ownership of the target. Lastly, as in Smeets et al. (2016), we exclude joint mergers where two or more firms merge to create a firm with a new (i.e., non-existing) identifier. In this case, the acquirer and target firms cannot be differentiated. We focus on domestic as opposed to foreign acquisitions of Danish firms since we do not observe the foreign acquirers nor their characteristics.

(2011) to estimate the firm-size distributions (i.e., Eq. (16)); the distribution of acquirers is most fat-tailed, followed by targets, and then non-participating firms (Appendix Figure B.4).

Second, we show in Figure 4(b) that there is a positive sorting pattern between acquirers and targets based on firm size. Earlier literature (e.g., Xu, 2017; David, 2020) documents positive assortative matching in terms of profits and productivity (e.g., sales over total assets) in the merger market using M&A transaction-level data. Here, we split the sample of acquirers into deciles by their domestic sales (again, measured by deviations from the sector median). Next, for each target firm, we compute their percentile rank within the sample of targets. For each decile of acquirer size, we then compute the mean percentile of targets purchased by acquirers in that bin. Figure 4(b) displays strong positive assortative matching on domestic sales between acquirers and targets. Together with Appendix Figure B.3, these graphs suggest that merger-induced firm growth may be substantial, as the largest acquirers are most likely affected and targets are generally bigger than the average firm.

3.2 Empirical analysis of the size-volatility relationship

3.2.1 Cross-sectional and panel estimates

As demonstrated in Section 2.2.4, the relation between firm size and volatility can be approximated by the linear combination of the log-log and log-linear functions. We now examine the fit of these functional forms. As a first step, we estimate the log-log specification that has been commonly used in the prior literature, setting $c_2 = 0$ in Eq. (27) (e.g., Sutton, 2002; Koren and Tenreyro, 2013; Yeh, 2023). The dependent variable is the (log) volatility of domestic sales growth over a 5-year period (i.e., 1995-9, 2000-4, 2005-9, and 2010-4), and the regressor of interest is the average size of the firm in that period, measured by its market share within the sector in logarithms. Sales are deflated using the GDP deflator.²⁰ Table 1 presents estimates from the cross-sectional and panel regressions. The former employs sector-period fixed effects, while the latter also includes firm fixed effects. Note that sector-period fixed effects control for market size, so we would obtain numerically identical coefficient estimates using absolute sales in place of market shares. The sample is restricted to firm-period observations where all five years within the period are observed, but results are similar with the entire sample.

Consistent with the prior literature, we find a negative size-volatility relationship at the firm level in Table 1 columns 1 and 4. The coefficient estimates in the cross-sectional and panel regressions are -0.039 and -0.142 , respectively. The latter is much closer in magnitude to previous estimates in the literature. For example, studying the sales of US firms, Koren and Tenreyro (2013) and Yeh (2023) obtain estimates of -0.157 and -0.149 , respectively. The disparity between our cross-sectional and panel estimates may be explained in part by omitted variable bias, as firm size is correlated with other firm characteristics such as productivity, the propensity to export and enter M&A deals, etc. It can be shown that, controlling for these

²⁰Coefficient estimates are very close but not identical without deflating sales. The dependent variable is a non-linear function (i.e., logarithm of the standard deviation) of sales growth, which implies that price deflators are not absorbed by sector-period fixed effects.

Table 1: Firm Size-Volatility Relationship

Dep. var.	Cross-sectional estimates			Panel estimates		
	(log) Volatility		(log) Adjusted volatility	(log) Volatility		(log) Adjusted volatility
	(1)	(2)	(3)	(4)	(5)	(6)
(log) Market share	-0.039*** (0.004)	-0.038*** (0.005)	-0.032*** (0.004)	-0.142*** (0.013)	-0.138*** (0.013)	-0.137*** (0.013)
Market share		-0.613 (1.105)			-3.304* (1.934)	
Sector-period FE	Y	Y	Y	Y	Y	Y
Firm FE				Y	Y	Y
N	49,064	49,064	49,064	49,064	49,064	49,064
R ²	0.13	0.13	0.13	0.63	0.63	0.63

Notes: Volatility is measured by the standard deviation of domestic sales growth over a 5-year period (i.e., 1995-9, 2000-4, 2005-9, and 2010-4). “Adjusted volatility” in columns 3 and 6 is computed as volatility minus the strategic market power effect on volatility assuming $\phi = 1$ and $\varepsilon = 5$. Standard errors in parentheses clustered at the firm level. ***, **, and * indicate statistical significance at the 1%, 5%, and 10% level.

other covariates, the coefficient magnitude on (log) market share increases. On average, large firms also tend to have bigger, more positive changes in their market shares over time. Taken together, this suggests that empirically, firm fixed effects play an important role in controlling for unobserved heterogeneity in the panel regressions.²¹

In columns 2 and 5, we estimate Eq. (27) without imposing $c_2 = 0$, allowing for both the log-log and log-linear components. We find that the coefficients of market share in logs and levels are both negative. The linear term is less precisely estimated, though this does not necessarily imply that strategic market power has no quantitative significance. The weak statistical significance of market share in levels can be attributed in part to its collinearity with the log term; the raw correlation between the two is 0.38. Appendix Table B.4 shows that when we regress volatility on the linear term alone without its log counterpart, the coefficients are statistically significant. Furthermore, from Figure 3(b), we know that the log-log relationship dominates for small firms and the contribution of variable markups to volatility becomes relevant only when market shares are large. In the data, the firm-size distribution roughly follows Zipf’s law, with many small firms and few large firms (e.g., Gabaix, 2011). However, the OLS regressions in columns 2 and 5 minimize the sum of squared errors across *all* observations. This may also explain why we obtain a strong fit for the log-log function while the log-linear function is imprecisely estimated.

Finally, in columns 3 and 6, we obtain a model-based estimate of χ in the log-log relationship, taking into account the variable markup channel. We impose $\phi = 1$ and $\varepsilon = 5$ to compute the SMP effect in the register data and subtract it from volatility to obtain what we call “adjusted volatility” as the dependent variable (in logs). The magnitudes of the coefficient estimates are slightly smaller than the baseline estimates without the adjustment in columns 1 and 4. Appendix Table B.5 and Appendix Figure B.8 further reveal that our results in this section are qualitatively similar using alternative definitions of sectors at the 2 and 4-digit levels.

²¹Moreover, we find that the cross-sectional regression estimate varies depending on the sample period of interest. In particular, restricting our sample of Danish firms to the same years of 2003–2007 as in Koren and Tenreyro (2013), we obtain larger coefficient magnitudes in the cross-section of around 0.09.

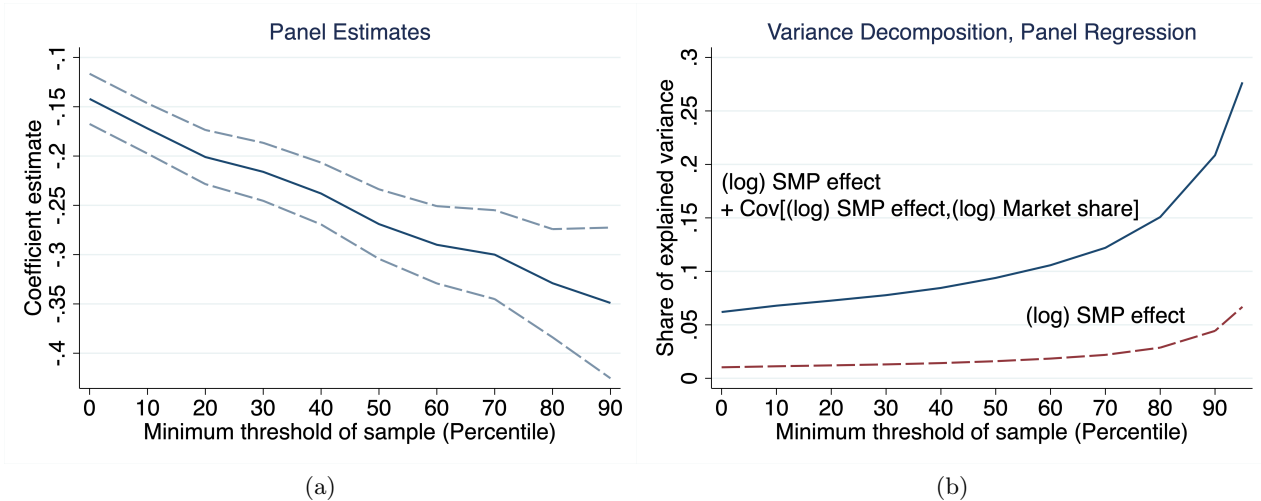


Figure 5: In panel (a), we plot coefficient estimates and 95% confidence intervals from panel regressions of (log) volatility on (log) firm size across samples restricted to a minimum size threshold at the 0th, 10th, ..., 80th, and 90th percentiles. In panel (b), we plot the share of explained variance in (log) volatility accounted for by the log SMP effect and the covariance between the log SMP effect and log market share. This is shown for samples restricted to a minimum size threshold at the 0th, 10th, ..., 90th, and 95th percentiles. The variance decomposition is based on the panel regression with firm and sector-period fixed effects partialled out.

3.2.2 Evidence for a non-linear size-volatility relationship

In the remainder of this section, we focus on the panel regressions; results in the cross-section are generally similar (see Appendix Section B.5). Recall that the log-linear component predicts volatility to fall disproportionately for large firms, and this implies a non-linear relationship between log volatility and log size. We provide empirical evidence consistent with this hypothesis in Figure 5(a). Specifically, we continue to estimate the log-log relationship corresponding to Table 1 column 4, while progressively restricting the sample to bigger and bigger firms. The horizontal axis in Figure 5(a) indicates the minimum size thresholds of the samples. For example, a value of 0 means that all firms are included; a value of 10 means that only firms above the 10th percentile are included; and so on.

Figure 5(a) plots the coefficient estimates obtained from regressions with the log-log specification separately for each sample. A striking pattern is observed. As the size threshold of the sample rises, the coefficient estimate becomes more negative and increases in magnitude. The regression coefficient magnitudes vary substantially, ranging from 0.142 in the entire sample to 0.349 for the top decile of firms. This increase in the slope is precisely what Figure 3(b) predicts based on the linear combination of the log-log and log-linear functions. The log-log function alone would instead predict a constant slope (i.e., a flat line). Hence, the evidence presented strongly supports our hypothesis of a non-linear relationship between firm size and volatility.

We also provide corroborating evidence by dividing the firm-size distribution into deciles and estimating the log-log model separately for each bin. To fully exploit the within-firm variation,

we construct the bins based on firms' sector and either their initial or final period in the sample (see Appendix Figures B.10 and B.11). While the estimates are more noisy, we find results that are consistent with Figure 5(a), as the coefficient estimates are also generally more negative as firm size increases. Additional supportive evidence is presented in Appendix Figure B.12, where we use fractional polynomials to fit the size-volatility relationship across all firms (Royston and Altman, 1994; Royston and Sauerbrei, 2008). We estimate the relation between log volatility and log market share as a second-degree fractional polynomial, and include the same set of fixed effects as above. The results confirm that volatility declines disproportionately with size.

3.2.3 Variance decomposition

Next, we perform another test of our hypothesis by performing a variance decomposition analysis to investigate the importance of the SMP effect in explaining overall firm-level volatility. The specification of interest is Table 1 column 5. For this exercise, we replace market share in levels with the log SMP effect to more closely align with our model, but the results are very similar using either variable. To begin, we partial out the fixed effects from the dependent and independent variables to isolate the two channels of interest. Thus, we separately regress log volatility, log market share, and the log SMP effect on firm and sector-period fixed effects. Using the residuals, we then decompose the variance of log volatility into the variance of log market share, the variance of the log SMP effect, and their covariance.

The share of explained variance accounted for by the log SMP effect and its cumulative share with the covariance term are shown in Figure 5(b). The remaining fraction is the contribution from log market share. Again, we repeat this calculation for different samples to test whether the SMP effect is more important for large firms. In the entire sample, the vast majority (93.8%) of the variance in firm-level volatility is explained solely by the log-log component. Recall that in our theory, this is interpreted as a catch-all variable that captures the decreasing relationship between the variance of shocks and firm size. The SMP effect alone accounts for only 1.1% of the variance in volatility, although the contribution from the covariance term (5.2%) is non-trivial.

Figure 5(b) also shows that as the minimum threshold of firm size in the sample rises, the share explained by the SMP effect monotonically increases. This is precisely what we would expect given Figure 5(a). The share of variance explained by the log SMP effect and its covariance with log market share rises quickly at the right tail of the firm-size distribution. Above the 90th percentile, the two terms account for 4.8% and 16.8% of explained variance, respectively. Above the 95th percentile, these further increase to 7.3% and 21.5%. Because most mergers involve large acquirer and target firms, we can see that this is exactly the region of the firm-size distribution for which the effects of granularity are the strongest and the SMP effect matters the most. A sensitivity analysis is further performed in Appendix Figure B.13.

3.3 Empirical analysis of mergers and firm-level volatility

So far, our analysis in Section 3.2 has not distinguished the source of firm-level growth. One channel through which firm size increases is the acquisition of a target firm. Does merger-

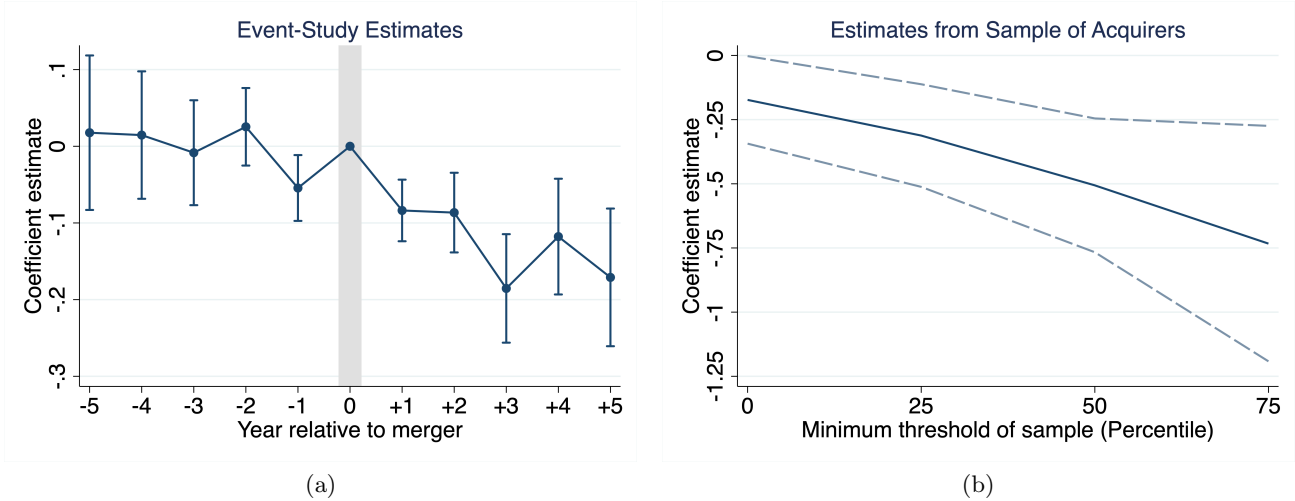


Figure 6: In panel (a), we plot coefficient estimates and 95% confidence intervals over the event window from estimating Eq. (28) to show the impact of mergers on (log) volatility. In panel (b), we plot coefficient estimates and 95% confidence intervals from regressing (log) volatility on (log) firm size for acquirers around the merger across samples restricted to a minimum size threshold at the 0th, 25th, 50th, and 75th percentiles.

induced growth also lead to a decline in volatility at the firm level? Next, we provide additional empirical evidence for the inverse size-volatility relationship by investigating the impact of M&A on firm-level volatility. We employ two complementary approaches. First, we estimate the average decline in the acquirer’s volatility after the merger using a simple event-study design:

$$\log(\text{Volatility}_{kit}) = c_i + c_{kt} + \sum_{\tau=-5, \neq 0}^{+5} \beta_{\tau} \mathbf{1}[t - \text{MergerYear}_{ki} = \tau] + e_{kit}. \quad (28)$$

To keep the sample clean, we drop firms that have multiple years with acquisition deals. As before, volatility is measured by the standard deviation of domestic sales growth over a five-year interval. As an example, two years before the merger year (i.e., $t - \text{MergerYear}_{ki} = -2$), we compute the standard deviation of sales growth between $\tau = -4$ and $\tau = 0$. Because consolidation of the target firm inflates the growth rate of the acquirer in the year after the merger, we exclude it from the calculation of volatility for $t - \text{MergerYear}_{ki} \in \{-1, 0, +1, +2, +3\}$. The event window is 11 years and the coefficients of interest are β_{τ} , which can be interpreted (approximately) as the percentage change in volatility in year τ relative to the merger year. Figure 6(a) plots the estimated coefficients and it shows no obvious pre-trends before the merger. Importantly, we find that volatility clearly falls after the merger, on average by around 9–19%.²²

²²The specification in Eq. (28) is a two-way fixed effects estimator that includes both firm (c_i) and sector-year (c_{kt}) fixed effects to control for unobserved heterogeneity. Thus, we estimate changes over time within a firm. For the years $t - \text{MergerYear}_{ki} \in \{-1, 0, +1, +2, +3\}$, volatility is computed over four years instead of five. We also bin distant relative years (i.e., $\tau < -5$ and $\tau > +5$), and all coefficient estimates are reported in Appendix Table B.6. Appendix Table B.6 further demonstrates that the results are quantitatively similar when we include non-participating firms in the sample as a control group and also when we use the imputation estimator from Borusyak et al. (2024) to address the issues of staggered treatment and heterogeneous treatment effects.

Second, we employ a difference-in-differences approach to test the degree to which the acquirer’s volatility falls given the observed change in firm size. Using the sample of acquirers, we estimate the log-log specification from Table 1 column 4 with periods defined now either as the 5-year period before or after the M&A deal. The regression continues to include both firm and sector-period fixed effects. Thus, we exploit the within-firm over-time variation around the merger using the subset of firms that enter M&A as the sample. The changes in firm size that are associated with M&A tend to be more positive and greater than the organic growth of firms that occurs without acquisition activity. To avoid the complication described above, we again omit the year after the merger and define the post-merger period between $\tau = +2$ and $\tau = +6$.

For the entire sample of acquirers, we obtain a coefficient magnitude for the size-volatility relationship of 0.173, see Figure 6(b). Moreover, in the rest of Figure 6(b), we repeat the same exercise to progressively increase the sample’s size threshold. Because we rely only on acquirers here, the sample is smaller and we divide the size distribution of these firms into quartiles. The coefficient again becomes more negative as the sample is restricted to larger and larger acquirers. These results suggest that changes in volatility from merger-induced growth for acquirers can be described by a similar size-volatility relationship that we have established for all firms.

4 Quantitative assessment

4.1 Estimation

We now perform a quantitative assessment of our model to evaluate the impact of mergers on aggregate volatility. To estimate the model parameters, we match key moments in our model to the Danish data. Appendix Section B.2 provides details of the estimation process. We begin by parameterizing the multi-sector model, as corresponding parameters in the single-sector model can be computed by aggregating across sectors. First, the elasticity of substitution across sectors ϕ is set to 1, which implies Cobb-Douglas preferences over sectors (e.g., Atkeson and Burstein, 2008). Thus, D_k is exactly equal to the sales share of each sector \mathcal{S}_k . Moreover, we follow Gaubert and Itskhoki (2021) and choose $\varepsilon = 5$ for the elasticity of substitution within a sector. The market size PY is normalized to 100 (i.e., marginal disutility of labor $\psi = 0.01$).

We take the post-merger productivity distribution to be the observed distribution in the data and assume that it follows a Pareto distribution with shape parameter ξ_k . Following Burstein et al. (2020), ξ_k is set to match the model-implied market concentration as measured by the Herfindahl-Hirschman index to its observed values in the data (Appendix Table B.1 column 1). To do so, we need to solve for the equilibrium market shares of active firms in the multi-sector economy. We normalize wage $w = 1$ as the numéraire and set the number of firms $M = 100,000$, which is roughly the average number of firms with five or more full-time employees in any given year. The share of firms in each sector M_k/M is presented in Appendix Table B.1 column 2. Now, for a guess of the shape parameter ξ_k , we randomly draw M_k productivity levels from the Pareto distribution with scale parameter 1. This allows us to solve firm market shares s_{ki} as a fixed point problem using Eqs. (19) and (20) and to calculate HHI. We repeat this process for

1,001 random samples and take the median value. The guess of ξ_k is updated until the market concentration in each sector matches the data. Estimates of ξ_k and the ratios $\frac{\xi_k}{\varepsilon-1}$ are shown in Appendix Table B.1 columns 3 and 4, respectively. The latter ranges from 1.08 to 1.45, which suggests that the firm-size distributions are indeed fat tailed.

For the single-sector economy, we aggregate the sector-specific parameters up to the economy-wide level. Using the estimates of ξ_k from above, we back out a value of the elasticity of substitution $\tilde{\varepsilon}$ that is consistent with the multi-sector model. To do so, we first compute firm-level markups m_{ki} in the multi-sector economy using Eq. (20) and aggregate up to the sector level (\mathcal{M}_k). We then use the sectoral market shares \mathcal{S}_k (Appendix Table B.2 column 1) to compute the aggregate markup \mathcal{M} . Given that all firms charge the same markup in the single-sector economy, we set $\mathcal{M} = \frac{\tilde{\varepsilon}}{\tilde{\varepsilon}-1}$ to solve for $\tilde{\varepsilon} = 4.85$. Employing the same method as above, we set the Pareto shape parameter of the productivity distribution $\xi = 4.48$ to match the observed economy-wide HHI. Lastly, from the sectoral exit rates δ_k (Appendix Table B.2 column 2), we compute the aggregate exogenous exit rate, i.e., $\delta = \sum_k M_k \delta_k = 0.113$. Firms are considered to leave the market when they have no production for three years straight. Because this includes the scenario where the firm is acquired as a target, we subtract raw exit rates by the merger rates of each sector when calculating δ_k .

4.2 Merger market parameters

We follow the approach in David (2020) and use simulated method of moments (SMM) to estimate the merger market parameters, $\Theta = \{\gamma, \nu, A, B, \eta\}$. In this paper, we do not exploit the heterogeneity of the merger market across sectors. Therefore, we simply use the single-sector economy to estimate the parameters. Due to the lack of data on transaction values, we are also unable to estimate the merger premium as in David (2020). Instead, we rely on his estimate of the acquirer's bargaining power $\beta = 0.51$. Evidence from the literature suggests that merger premia are in fact similar in Denmark and the US (e.g., Rossi and Volpin, 2004; Weitzel and Berns, 2006). Moreover, results from a sensitivity analysis indicate that our findings are robust to other choices of β . The discount rate is $\rho = r = 0.05$.

Given candidate values of γ , ν , and A , the merger matrix is defined by computing the productivity of the merged firm $s[z_i^a, z_i^t]$ for every pair of acquirer and target firms using Eq. (8). We perform value function iteration, where, for candidate values of B and η in the search cost function (i.e., Eq. (10)), the value of the firm is updated according to Eq. (11). The cutoff firm has a normalized productivity level of $\bar{z} = 1$, and we use the operating profits of this cutoff firm to determine the fixed cost of production (see Eq. (13)).²³ We then simulate the economy to obtain the model-implied moments. Following David (2020), the five moments chosen are: (i) the median acquirer size, (ii) median target size, (iii) share of targets in the bottom decile of the

²³We normalize the cutoff productivity \bar{z} (or $\bar{z}_k \forall k$ in the variable-markup economy) to 1, instead of the sunk cost of entry c^e (or c_k^e). Hence, in the multi-sector economy, we allow sunk entry costs to vary across sectors. In other words, rather than fixing c_k^e and guessing M_k (or equivalently, the price index P_k), we fix M_k and solve for c_k^e . This is equivalent to finding a sunk cost of entry that satisfies the free entry condition and generates a model-implied survival rate that matches the data.

Table 2: Merger Market Moments and Parameter Estimates

Parameter	Estimate	Target moment	Data	Model
γ	0.90 (0.01)	Median of $\log(r_{ki}^A)$	1.72	1.71
ν	0.42 (0.01)	Median of $\log(r_{ki}^T)$	0.67	0.67
A	1.01 (0.001)	Share of targets in bottom decile	0.05	0.05
B	1.00×10^{11} (0.28×10^{11})	Aggregate merger rate	0.005	0.005
η	13.00 (0.14)	Coefficient of variation of r_{ki}^T	2.47	2.47

Notes: Log sales, $\log(r_{ki})$, are measured by deviations from the median firm. Annual averages of the data moments are computed over the sample period. Per data confidentiality requirements, the median is approximated by the mean of the five observations centered around it. Bootstrapped standard errors in parentheses account for sampling error and simulation error following the method in Eaton et al. (2011).

firm-size distribution, (iv) aggregate merger rate, and (v) coefficient of variation of target size. Here, firm size is again measured by domestic sales, and the first two moments are measured as deviations from the median firm in logarithms. Because our register data contains the universe of firms, the merger rate is one order of magnitude smaller than David (2020), who uses a sample of Compustat firms. This procedure is iterated to minimize the difference between the moments constructed with the simulated economy and the data.²⁴

Table 2 presents the parameter estimates. Their magnitudes are similar to those from David (2020). In particular, with regards to the merger technology function, we obtain $\gamma = 0.90$ and $\nu = 0.42$. Their sum is greater than 1, which suggests that there are strong complementarities in the mergers. Next, the estimated value of A is 1.01. This parameter captures merger-induced growth that is independent of the acquirer's and target's characteristics. Furthermore, we obtain $B = 1.00 \times 10^{11}$ and $\eta = 13.0$, indicating a very convex search cost function.

To gauge the model fit, we find that all of the targeted moments in Table 2 can be matched by the model. This includes the average size of the merger partners, the dispersion of targets, and the merger rate. Moreover, Figure 4(b) shows that the model can replicate the positive assortative matching pattern from the data fairly well. Using the sample of acquirers and targets generated by the model, we repeat the same set of calculations. Consistent with the data, the average size of targets is monotonically increasing in the size of acquirers. Furthermore, we find that the model also performs well in matching the average size of targets for any decile in the distribution of acquirers.

²⁴We iterate on our procedure to minimize the loss function $(m - \hat{m}(\Theta))' W (m - \hat{m}(\Theta))$, where m is the vector of the five target moments from the data, $\hat{m}(\Theta)$ is the vector of corresponding moments constructed using the simulated economy with parameters $\Theta = \{\gamma, \nu, A, B, \eta\}$, and W is a matrix of weights. We use the generalized inverse of the estimated variance-covariance matrix of the moments computed from the data. As in David (2020), the parameters are jointly estimated, which implies that the individual parameters and moments do not have a one-to-one mapping. Appendix Figure B.5 demonstrates that the moments display sensitivity to the parameters.

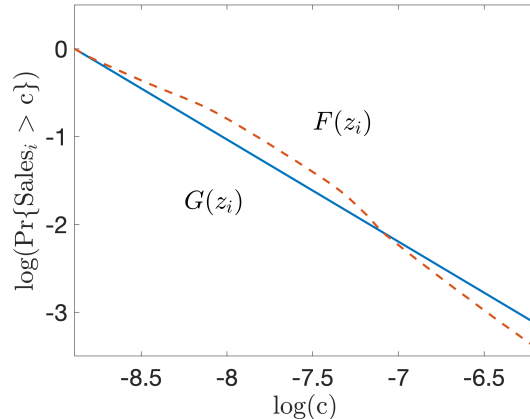


Figure 7: This figure plots the firm-size distributions of the benchmark economy under $G(z_i)$ and the counterfactual economy under $F(z_i)$ for $1 \leq z_i \leq 10^{0.3}$. Firm size is measured by sales.

4.3 Counterfactuals in the single-sector economy

4.3.1 Solving the equilibrium

We begin by conducting counterfactuals in the single-sector economy to understand the main channels through which mergers raise aggregate volatility. The role of endogenous markups will be examined in the following section using the multi-sector economy. With the merger market parameters estimated, we employ the stationary condition in Eq. (14) to construct the pre-merger productivity distribution $F(z_i)$. The aggregate survival rate is $\frac{M}{M^e} = \left[\sum_{k=1}^N \frac{M_k^e}{M_k} \frac{M_k}{M} \right]^{-1} = 0.54$, where the sectoral survival rates are taken from Appendix Table B.2 column 3. These rates are defined as successful entry after five years, and are similar to those found in, for example, the US (e.g., Bartelsman et al., 2013). Hence, we derive the pmf $f(z_i)$ for all $z_i \geq \bar{z}$. This also allows us to compute the entry cost c^e using the free entry condition in Eq. (15). By definition, firms with productivity below \bar{z} exit the market after discovering their productivity draw.

Figure 7 plots the firm-size distributions in the benchmark economy with $G(z_i)$ and the counterfactual economy with $F(z_i)$ for $z_i \geq \bar{z}$. We zoom into the section with lower values of (log) sales to demonstrate how the two distributions differ. Note that in the benchmark economy with mergers, $G(z_i)$ follows the Pareto distribution. Thus, the firm-size distribution follows a power law and has the constant slope of $\xi/(\tilde{\varepsilon} - 1)$. The figure shows that at the lowest levels of productivity and sales, the (log) fraction of firms with sales greater than c , $\Pr\{Sales_i > c\}$, is higher in the counterfactual economy without mergers compared to the benchmark economy with mergers. However, at the right tail where firms and their sales are large, the reverse is true. Hence, mergers generate a more fat-tailed firm-size distribution. As the economy becomes more granular, firm-level idiosyncratic shocks result in greater aggregate fluctuations.

We can also empirically estimate the post-merger increase in firm size using our data to examine the potential strength of granular forces. We employ a similar specification to Eq. (28) with (log) sales as the dependent variable and a post-merger indicator variable as the regressor (i.e., in place of the individual years in the event window). The results in Appendix Table B.7

indicate that acquirer firms in the top decile of the firm-size distribution grow by over 10% after a merger. This growth is substantial, especially given the initial size of these large firms, and is consistent with the idea that mergers contribute to a fatter tail in the firm-size distribution.

The counterfactual economy without mergers is less competitive, as there are fewer large firms and smaller firms can survive more easily (e.g., David, 2020). Denote the counterfactual cutoff productivity level as $\bar{z}^F < \bar{z}$. The stationary condition does not directly provide information on the shape of the pre-merger productivity distribution for $\bar{z}^F \leq z_i < \bar{z}$. Following David (2020), we extrapolate the function $f(z_i)$ for $z_i < \bar{z}$ to generate a reasonable estimate of the productivity distribution. Details are provided in Appendix Section B.3.²⁵ Without mergers, the economy collapses to the Melitz (2003) model without trade, with $F(z_i)$ as the productivity distribution. Thus, we can directly determine \bar{z}^F by solving for the point that satisfies the free entry condition, and compute the number of firms, M^F , in the counterfactual equilibrium.

4.3.2 Results

We now compare aggregate volatility in the benchmark economy under $G(z_i)$ against the counterfactual economy under $F(z_i)$. First, we consider the case of i.i.d. productivity shocks with constant variance (i.e., $\chi = 0$ in Eq. (23)). The median volatility of each economy is calculated over 1,001 samples. Taking the ratio gives

$$\frac{\sigma[\hat{Y}^G(\bar{z}|\chi = 0)]}{\sigma[\hat{Y}^F(\bar{z}^F|\chi = 0)]} = 1.044 = \underbrace{\frac{\sigma[\hat{Y}^F(\bar{z}|\chi = 0)]}{\sigma[\hat{Y}^F(\bar{z}^F|\chi = 0)]}}_{\text{Extensive margin}} \times \underbrace{\frac{\sigma[\hat{Y}^G(\bar{z}|\chi = 0)]}{\sigma[\hat{Y}^F(\bar{z}|\chi = 0)]}}_{\text{Intensive margin}} = 1.011 \times 1.032.$$

This indicates that in the single-sector model, the benchmark economy with mergers is around 4.4% more volatile than the counterfactual economy without mergers.

Importantly, we can decompose the total effect of mergers in the calculation above into the contributions of the extensive and intensive margins. As discussed in Section 2.2.2, the extensive margin is driven by selection effects that raise the cutoff productivity, i.e., $\bar{z}^F < \bar{z}$, and the intensive margin by a shift in the productivity distribution from $F(z_i)$ to $G(z_i)$. Because there is a discrete number of firms, the market shares of small firms that exit due to selection effects are divided among the surviving firms, increasing market concentration. To isolate the extensive margin, we compute aggregate volatility in a hypothetical scenario where firms with $z_i < \bar{z}$ are dropped from the counterfactual economy, holding fixed the productivity distribution at $F(z_i)$ for $z_i \geq \bar{z}$. This is denoted as $\sigma[\hat{Y}^F(\bar{z}|\chi = 0)]$. Its ratio to the counterfactual

²⁵ To extrapolate $f(z_i)$, we run a regression of $\log[f(z_i)]$ on $\log(z_i) \forall z_i \geq \bar{z}$. Then, we extend the productivity grid space for $z_i < \bar{z}$ (also log-spaced) and compute $\log[f(z_i)]$ for these additional grid points given the value of $\log[f(\bar{z})]$ and the slope coefficient obtained from the regression. For comparison, Appendix Figure B.6 also shows the productivity distribution from performing a linear extrapolation. The distributions do not deviate much for z_i close to \bar{z} , and the ratio of $\sigma[\hat{Y}^G(\bar{z}|\chi = 0)]/\sigma[\hat{Y}^F(\bar{z}^F|\chi = 0)]$ in the single-sector economy computed using this method is 1.040. The reason for not using linear extrapolation is that in the multi-sector economy, the number of firms in each sector M_k is much smaller than M . The randomness of the productivity draws implies that there is no guarantee for $f_k(\bar{z}_k)$ to be greater than the next point (i.e., to the right) in the pmf. If $f_k(\bar{z}_k)$ were less than the next point in the pmf, simple linear extrapolation would predict a pmf that is *increasing* for $z_{ki} < \bar{z}_k$.

Table 3: Results from Counterfactual Exercises in the Single-Sector Economy

Variance of shocks		Constant	Decreasing in size
		$\chi = 0$	$\chi = 0.14$
		(1)	(2)
Total effect of mergers	$\frac{\sigma[\hat{Y}^G(\bar{z} \chi)]}{\sigma[\hat{Y}^F(\bar{z}^F \chi)]}$	1.044	1.037
Extensive margin	$\frac{\sigma[\hat{Y}^F(\bar{z} \chi)]}{\sigma[\hat{Y}^F(\bar{z}^F \chi)]}$	1.011	1.010
Intensive margin	$\frac{\sigma[\hat{Y}^G(\bar{z} \chi)]}{\sigma[\hat{Y}^F(\bar{z} \chi)]}$	1.032	1.027

Notes: The standard deviation of shocks is defined by $v_z(s_{ki}) = \bar{v}_z s_{ki}^{-\chi}$ for $\chi = 0$ or 0.14.

volatility $\sigma[\hat{Y}^F(\bar{z}^F|\chi = 0)]$ is equal to 1.011. Hence, the extensive margin accounts for one-quarter (i.e., 1.1/4.4) of the rise in aggregate fluctuations from mergers. The remaining 75% is accounted for by the intensive margin, indicated by the ratio of $\sigma[\hat{Y}^G(\bar{z}|\chi = 0)]$ in the benchmark economy to $\sigma[\hat{Y}^F(\bar{z}|\chi = 0)]$. This ratio compares two productivity distributions holding fixed the productivity threshold. Thus, the major driving force behind the increase in macroeconomic volatility is the shift in the productivity distribution, which generates a more fat-tailed firm-size distribution. The thicker tail raises the contribution of granular forces, amplifying the effects from the fluctuations of large firms on the aggregate economy.²⁶

Next, we relax the assumption of i.i.d. shocks, and suppose that there is a negative relationship between size and volatility because larger firms face shocks with lower variance. In particular, we set $\chi = 0.14$ in Eq. (23). Recall from the discussion in Section 2.2.3 that this gives an ambiguous prediction with regards to whether volatility is amplified or mitigated. Nonetheless, performing the same calculations as above gives:

$$\frac{\sigma[\hat{Y}^G(\bar{z}|\chi = 0.14)]}{\sigma[\hat{Y}^F(\bar{z}^F|\chi = 0.14)]} = 1.037 = \frac{\sigma[\hat{Y}^F(\bar{z}|\chi = 0.14)]}{\sigma[\hat{Y}^F(\bar{z}^F|\chi = 0.14)]} \times \frac{\sigma[\hat{Y}^G(\bar{z}|\chi = 0.14)]}{\sigma[\hat{Y}^F(\bar{z}|\chi = 0.14)]} = 1.010 \times 1.027.$$

Compared to the case of i.i.d. shocks, the impact of mergers on aggregate volatility is relatively smaller under $\chi = 0.14$. However, the shift in the productivity distribution again accounts for the majority of the total effect. Although smaller firms have higher variance in sales growth, selection effects imply that the smallest, most volatile firms exit the market. Moreover, mergers create larger, less volatile firms. Table 3 summarizes results for the single-sector economy.

4.4 Counterfactuals in the multi-sector economy

4.4.1 Solving the equilibrium

From the previous steps in Section 4.1, we obtain the Pareto shape parameters ξ_k , productivity distributions $G_k(z_{ki})$, and market shares s_{ki} for each of 1,001 random samples. With

²⁶The alternative decomposition is $\frac{\sigma[\hat{Y}^G(\bar{z}^F|\chi=0)]}{\sigma[\hat{Y}^F(\bar{z}^F|\chi=0)]} \times \frac{\sigma[\hat{Y}^G(\bar{z}|\chi=0)]}{\sigma[\hat{Y}^G(\bar{z}^F|\chi=0)]}$, where mergers shift the productivity distribution holding fixed the counterfactual economy's cutoff productivity, and selection effects change the cutoff productivity while maintaining the Pareto distribution. In this case, we obtain 1.023×1.020 , and the intensive margin continues to account for the majority of the total effect.

i.i.d. shocks, we can approximate firms' expected profits using a Taylor expansion: $\pi_k^E(z_{ki}) = \pi_k(z_{ki}) + \frac{1}{2} \frac{\partial^2 \pi_k(z_{ki})}{\partial z_{ki}^2} \bar{v}_z^2$. The second-order partial derivative is calculated numerically, and we set $\bar{v}_z = 0.1$ (e.g., di Giovanni and Levchenko, 2012). The computation of markups m_{ki} , pass-through rates α_{ki} , and profits $\pi_k(z_{ki})$ follows directly from Eqs. (20), (21), and (7).

The value of a firm is determined by current expected profits $\pi_k^E(z_{ki})$ and expected merger gains net of search costs. We use the same method as the single-sector economy to solve the merger market in every sector k for each of the random samples. Because market shares in the Cournot equilibrium are computed for a specific set of firms, when defining the merger matrix $s[z_{ki}^a, z_{ki}^t]$, we restrict merged entities to the same grid points as the initial draw of firm productivities.²⁷ For each sector, the counterfactual productivity distribution $F_k(z_{ki})$ is constructed for $z_{ki} \geq \bar{z}_k$. Fixed costs of production c_k^d and sunk costs of entry c_k^e are derived accordingly from Eqs. (13) and (15). Their (median) values are shown in Appendix Table B.3.

As in the single-sector economy, we extrapolate the productivity distribution $F_k(z_{ki})$ for $z_{ki} < \bar{z}_k$. However, because firms charge variable markups, we can no longer use the free entry condition to directly solve for the cutoff productivity. Instead, we must simulate and solve the equilibrium in each sector; see Appendix Section B.3.2 for details. By doing so, we obtain the values of the endogenous variables for the counterfactual economy.

4.4.2 Results

The steps taken above also provide us with market shares s_{ki} and pass-through rates α_{ki} for the counterfactual economy. Using Eq. (24), we compute aggregate volatility $\sigma[\hat{Y}]$, where sectoral market shares \mathcal{S}_k are fixed because of Cobb-Douglas preferences. Again, we first consider the case of idiosyncratic shocks with constant variance. We allow for the incomplete pass-through of shocks to prices with $\alpha_{ki} < 1$. The ratio of aggregate volatilities in the benchmark economy under $G_k(z_{ki})$ and the counterfactual economy under $F_k(z_{ki})$ is:

$$\frac{\sigma[\hat{Y}^G(\bar{z}_k | \alpha_{ki} < 1, \chi = 0)]}{\sigma[\hat{Y}^F(\bar{z}_k^F | \alpha_{ki} < 1, \chi = 0)]} = 1.124.$$

Thus, in the multi-sector economy, mergers increase aggregate fluctuations by around 12.4%.²⁸

Next, we investigate the quantitative significance of endogenous markups in mitigating the impact of mergers. In the multi-sector economy, strategic market power and incomplete pass-

²⁷If the value of $s[z_{ki}^a, z_{ki}^t]$ between two firms is not an element of the set of drawn productivities in the random sample, the next highest value is used instead.

²⁸In general, we are interested in comparing the volatility of the economy under different scenarios conditional on the model setup, i.e., either the single-sector or multi-sector model. The effects of mergers on aggregate volatility are quantitatively larger in the multi-sector economy compared to the single-sector economy. This is because mergers have a greater impact when the number of firms is smaller. While Eq. (24) takes into account the size of each sector in computing aggregate volatility, the impact of mergers is magnified within each individual sector. For example, if M is lowered to 19900, the largest M_k observed across sectors, then aggregate volatility in the single-sector economy rises by 8.5% with mergers. Given that the multi-sector economy has the same number of firms in total as the single-sector economy, $\sum_k M_k = M$, volatility in the former responds more to M&A than the latter. Furthermore, in the multi-sector economy, we can also decompose the overall effect into the extensive and intensive margins; the latter accounts for around 81% of the rise in aggregate volatility.

Table 4: Results from Counterfactual Exercises in the Multi-Sector Economy

Variance of shocks		Constant	Decreasing in size
		$\chi = 0$	$\chi = 0.14$
		(1)	(2)
Total effect of mergers			
Incomplete pass-through	$\frac{\sigma[\hat{Y}^G(\bar{z}_k \alpha_{ki}<1,\chi)]}{\sigma[\hat{Y}^F(\bar{z}_k^F \alpha_{ki}<1,\chi)]}$	1.124	1.093
Complete pass-through	$\frac{\sigma[\hat{Y}^G(\bar{z}_k \alpha_{ki}=1,\chi)]}{\sigma[\hat{Y}^F(\bar{z}_k^F \alpha_{ki}=1,\chi)]}$	1.184	1.138

Notes: The standard deviation of shocks is defined by $v_z(s_{ki}) = \bar{v}_z s_{ki}^{-\chi}$ for $\chi = 0$ or 0.14 .

through imply that larger firms experience smaller fluctuations in their sales growth. We quantify the importance of this channel by comparing the baseline result above to a hypothetical scenario of the multi-sector economy where firms are small with no market power. In this case, pass-through is complete, i.e., $\alpha_{ki} = 1$ and firms charge constant markups. To assess the effect of mergers in this hypothetical scenario, we compute the ratio of volatilities in the economies with and without mergers:

$$\frac{\sigma[\hat{Y}^G(\bar{z}_k|\alpha_{ki} = 1, \chi = 0)]}{\sigma[\hat{Y}^F(\bar{z}_k^F|\alpha_{ki} = 1, \chi = 0)]} = 1.184.$$

This is substantially higher than our baseline result with incomplete pass-through, and can be explained by two factors. First, for given market shares, increasing α_{ki} to 1 directly removes the dampening effect of endogenous markups (i.e., Eq. (22)). Second, for a given function α_{ki} , the market shares of large firms are higher when setting constant markups at $\varepsilon/(\varepsilon-1)$, as opposed to variable markups, so the market tends to be more concentrated. Shutting down the channel of endogenous markups, we find that the increase in aggregate volatility is overestimated by almost 50% (i.e., $18.4/12.4 - 1 = 0.484$). Our result indicates that even when shocks have constant variance, the market power of large firms formed through mergers limits the fluctuations of their sales and, by extension, movements in the sectoral and aggregate price indices. Therefore, the impact of mergers on aggregate volatility depends crucially on the role of endogenous markups, and our findings suggest that their effects are economically significant.

Furthermore, we repeat the exercise above by assuming that the variance of shocks decreases with firm size, setting $\chi = 0.14$ instead of zero. Similar results are obtained. The effects of mergers are further dampened, as aggregate volatility rises by around 9.3% with incomplete pass-through. Again, shutting down the channel of endogenous markups increases fluctuations by nearly 50% (i.e., $13.8/9.3 - 1 = 0.484$). Table 4 provides a summary of our quantitative findings in the multi-sector economy.

Our result for the importance of endogenous markups in quantifying aggregate volatility contrasts with what di Giovanni and Levchenko (2012) find in their setting. They show that by approximating the firm's markup over marginal cost with the constant markup of $\frac{\tilde{\varepsilon}}{\tilde{\varepsilon}-1}$, a firm's price can be decomposed into the constant-markup price multiplied by an adjustment factor

arising from general equilibrium effects:

$$p_i = \frac{\tilde{\varepsilon}}{\tilde{\varepsilon} - 1} \frac{w}{z_i} \times \frac{\tilde{\varepsilon} - 1}{\tilde{\varepsilon} - 1 - s_i}.$$

di Giovanni and Levchenko (2012) make this derivation in a single-sector economy under monopolistic competition. In our multi-sector economy with oligopoly and Cournot competition, by Eq. (20), the analogous equation is:

$$p_{ki} = \frac{\varepsilon}{\varepsilon - 1} \frac{w}{z_{ki}} \times \frac{\varepsilon - 1}{\varepsilon - 1 - (\varepsilon/\phi - 1)s_{ki}}.$$

The latter component is the same general equilibrium contribution now under oligopoly, which appears due to strategic market power. For comparability, suppose as before that $\phi = 1$ and $\varepsilon = 5$, and the elasticity of substitution in the single-sector economy is also $\tilde{\varepsilon} = 5$. Then, at a market share of 30%, the adjustment under monopolistic competition is only 1.081, while it is 1.429 under oligopoly. Thus, endogenous markups have a much bigger quantitative impact in our setting with oligopolistic competition, and they play a critical role to diminish the contribution of large firms created through M&A on aggregate fluctuations in the economy.²⁹

5 Conclusion

In this paper, we theoretically and empirically study the implications of domestic horizontal M&A for volatility. We show that the firm dynamics associated with mergers have a significant impact on the second moment of growth. In order to quantify the effects of mergers, we build a multi-sector model of horizontal mergers that features a discrete number of firms in oligopoly. Our model demonstrates that at the macro level, mergers generate a more fail-tailed firm-size distribution, thereby increasing aggregate fluctuations in a granular economy. Our analysis also reveals that the impact of mergers depends crucially on firms' market power and endogenous markups. Incomplete pass-through mitigates the granular fluctuations of large firms created through M&A. At the micro level, we characterize the firm size-volatility relationship by the linear combination of the log-log and log-linear functions and show that volatility declines disproportionately with size.

Using Danish register data from 1993 to 2015, we provide empirical evidence for the effect of endogenous markups on volatility and perform a quantitative assessment of our model. Comparing our multi-sector benchmark economy with mergers to a counterfactual economy without mergers, we find that mergers increase aggregate volatility by around 9 to 12%. The negative firm size-volatility relationship considerably dampens the impact of mergers. Our results demonstrate that mergers contribute to greater macroeconomic fluctuations, as the economy becomes more sensitive to the shocks of large firms created through merger deals.

²⁹In Appendix Section B.4, we further investigate the heterogeneous impact of mergers on volatility across sectors. In particular, we show that the dampening effect from endogenous markups is stronger in sectors with higher market concentration.

References

- Acemoglu, D., V. M. Carvalho, A. Ozdaglar, and A. Tahbaz-Salehi (2012). The network origins of aggregate fluctuations. *Econometrica* 80(5), 1977–2016.
- Ashenfelter, O. C., D. S. Hosken, and M. C. Weinberg (2015). Efficiencies brewed: Pricing and consolidation in the US beer industry. *RAND Journal of Economics* 46(2), 328–361.
- Asker, J. and V. Nocke (2021). Chapter 12 - Collusion, mergers, and related antitrust issues. In K. Ho, A. Hortaçsu, and A. Lizzeri (Eds.), *Handbook of Industrial Organization*, Volume 5, pp. 177–279. Elsevier.
- Atkeson, A. and A. Burstein (2008). Pricing-to-market, trade costs, and international relative prices. *American Economic Review* 98(5), 1998–2031.
- Bartelsman, E., J. Haltiwanger, and S. Scarpetta (2013). Cross-country differences in productivity: The role of allocation and selection. *American Economic Review* 103(1), 305–334.
- Bernard, A. B., J. B. Jensen, S. J. Redding, and P. K. Schott (2018). Global firms. *Journal of Economic Literature* 56(2), 565–619.
- Blonigen, B. A., L. Fontagné, N. Sly, and F. Toubal (2014). Cherries for sale: The incidence and timing of cross-border M&A. *Journal of International Economics* 94, 341–357.
- Borusyak, K., X. Jaravel, and J. Spiess (2024). Revisiting event study designs: Robust and efficient estimation. *Review of Economic Studies*, rdae007.
- Braguinsky, S., A. Ohyama, T. Okazaki, and C. Syverson (2015). Acquisitions, productivity, and profitability: Evidence from the Japanese cotton spinning industry. *American Economic Review* 105(7), 2086–2119.
- Brakman, S., H. Garretsen, M. Gerritse, and C. van Marrewijk (2018). A model of heterogeneous firm matches in cross-border mergers & acquisitions. *CESifo Working Paper No. 7083*.
- Bremus, F., C. M. Buch, K. N. Russ, and M. Schmitzer (2018). Big banks and macroeconomic outcomes: Theory and cross-country evidence of granularity. *Journal of Money, Credit and Banking* 50(8), 1785–1825.
- Burstein, A., V. M. Carvalho, and B. Grassi (2020). Bottom-up markup fluctuations. *NBER Working Paper No. 27958*.
- Carvalho, V. M. and B. Grassi (2019). Large firm dynamics and the business cycle. *American Economic Review* 109(4), 1375–1425.
- Cavenaile, L., M. A. Celik, and X. Tian (2021). The dynamic effects of antitrust policy on growth and welfare. *Journal of Monetary Economics* 121, 42–59.

- Chan, J. M., M. Irlacher, and M. Koch (2024). Multiproduct firms, horizontal mergers, and international trade. *Aarhus University mimeo*.
- Clementi, G. L. and B. Palazzo (2016). Entry, exit, firm dynamics, and aggregate fluctuations. *American Economic Journal: Macroeconomics* 8(3), 1–41.
- Cooley, T. F. and V. Quadrini (2001). Financial markets and firm dynamics. *American Economic Review* 91(5), 1286–1310.
- David, J. M. (2020). The aggregate implications of mergers and acquisitions. *Review of Economic Studies* 88(4), 1796–1830.
- De Loecker, J. and J. Eeckhout (2021). Global market power. *NBER Working Paper No. 24768*.
- DePamphilis, D. M. (2018). *Mergers, Acquisitions, and Other Restructuring Activities: An Integrated Approach to Process, Tools, Cases, and Solutions* (Ninth ed.). Academic Press, London, UK.
- di Giovanni, J. and A. A. Levchenko (2012). Country size, international trade, and aggregate fluctuations in granular economies. *Journal of Political Economy* 120(6), 1083–1132.
- di Giovanni, J., A. A. Levchenko, and I. Mejean (2014). Firms, destinations, and aggregate fluctuations. *Econometrica* 82(4), 1303–1340.
- di Giovanni, J., A. A. Levchenko, and I. Mejean (2018). The micro origins of international business-cycle comovement. *American Economic Review* 108(1), 82–108.
- di Giovanni, J., A. A. Levchenko, and R. Rancière (2011). Power laws in firm size and openness to trade: Measurement and implications. *Journal of International Economics* 85, 42–52.
- Dimopoulos, T. and S. Sacchetto (2017). Merger activity in industry equilibrium. *Journal of Financial Economics* 126(1), 200–226.
- Eaton, J., S. Kortum, and F. Kramarz (2011). An anatomy of international trade: Evidence from French firms. *Econometrica* 79(5), 1453–1498.
- Eckbo, B. E. (2014). Corporate takeovers and economic efficiency. *Annual Review of Financial Economics* 6, 51–74.
- Fidrmuc, J. P. and C. Xia (2019). M&A deal initiation and managerial motivation. *Journal of Corporate Finance* 59, 320–343.
- Gabaix, X. (2011). The granular origins of aggregate fluctuations. *Econometrica* 179(3), 733–772.
- Gabaix, X. and R. Ibragimov (2011). Rank $-1/2$: A simple way to improve the OLS estimation of tail exponents. *Journal of Business & Economic Statistics* 29(1), 24–39.

- Gaubert, C. and O. Itskhoki (2021). Granular comparative advantage. *Journal of Political Economy* 129(3), 871–939.
- Gaubert, C., O. Itskhoki, and M. Vogler (2021). Government policies in a granular global economy. *Journal of Monetary Economics* 121, 95–112.
- Grazzini, J. and D. Massaro (2022). Great volatility, great moderation and great moderation again. *Review of Economic Dynamics* 44, 269–283.
- Greenwood, J., Z. Hercowitz, and G. W. Huffman (1988). Investment, capacity utilization, and the real business cycle. *American Economic Review* 78(3), 402–417.
- Harford, J. (2005). What drives merger waves? *Journal of Financial Economics* 77(3), 529–560.
- Horvath, M. (1998). Cyclicalities and sectoral linkages: Aggregate fluctuations from independent sectoral shocks. *Review of Economic Dynamics* 1(4), 781–808.
- Horvath, M. (2000). Sectoral shocks and aggregate fluctuations. *Journal of Monetary Economics* 45(1), 69–106.
- Jovanovic, B. and P. L. Rousseau (2008). Mergers as reallocation. *Review of Economics and Statistics* 90(4), 765–776.
- Koren, M. and S. Tenreyro (2013). Technological diversification. *American Economic Review* 103(1), 378–414.
- Kydland, F. E. and E. C. Prescott (1982). Time to build and aggregate fluctuations. *Econometrica* 50(6), 1345–1370.
- Lee, Y. and T. Mukoyama (2018). A model of entry, exit, and plant-level dynamics over the business cycle. *Journal of Economic Dynamics and Control* 96, 1–25.
- Levine, O. (2017). Acquiring growth. *Journal of Financial Economics* 126(2), 300–319.
- Long, J. B. and C. I. Plosser (1983). Real business cycles. *Journal of Political Economy* 91(1), 39–69.
- Magerman, G., K. D. Bruyne, E. Dhyne, and J. V. Hove (2016). Heterogeneous firms and the micro origins of aggregate fluctuations. *National Bank of Belgium Working Paper No. 312*.
- Maksimovic, V., G. Phillips, and N. Prabhala (2011). Post-merger restructuring and the boundaries of the firm. *Journal of Financial Economics* 102(2), 317–343.
- Maksimovic, V., G. Phillips, and L. Yang (2013). Private and public merger waves. *Journal of Finance* 68(5), 2177–2217.
- Masulis, R. W. and S. A. Simsir (2018). Deal initiation in mergers and acquisitions. *Journal of Financial and Quantitative Analysis* 53(6), pp. 2389–2430.

- Melitz, M. J. (2003). The impact of trade on intra-industry reallocations and aggregate industry productivity. *Econometrica* 71, 1695–1725.
- Nocke, V. and S. Yeaple (2007). Cross-border mergers and acquisitions vs. greenfield foreign direct investment: The role of firm heterogeneity. *Journal of International Economics* 72, 336–365.
- Parenti, M. (2018). Large and small firms in a global market: David vs. Goliath. *Journal of International Economics* 110, 103–118.
- Rossi, S. and P. F. Volpin (2004). Cross-country determinants of mergers and acquisitions. *Journal of Financial Economics* 74(2), 277–304.
- Rotemberg, J. J. and M. Woodford (1992). Oligopolistic pricing and the effects of aggregate demand on economic activity. *Journal of Political Economy* 100(6), 1153–1207.
- Royston, P. and D. G. Altman (1994). Regression using fractional polynomials of continuous covariates: Parsimonious parametric modelling. *Journal of the Royal Statistical Society. Series C (Applied Statistics)* 43(3), 429–467.
- Royston, P. and W. Sauerbrei (2008). *Multivariable Model - Building: A Pragmatic Approach to Regression Analysis based on Fractional Polynomials for Modelling Continuous Variables*. Wiley, Chichester, UK.
- Sheen, A. (2014). The real product market impact of mergers. *Journal of Finance* 69(6), 2651–2688.
- Smeets, V., K. Ierulli, and M. Gibbs (2016). An empirical analysis of post-merger organizational integration. *Scandinavian Journal of Economics* 118(3), 463–493.
- Stanley, M. H. R., L. A. N. Amaral, S. V. Buldyrev, S. Havlin, H. Leschhorn, P. Maass, M. A. Salinger, and H. E. Stanley (1996). Scaling behaviour in the growth of companies. *Nature* 379, 804–806.
- Stockman, A. C. (1988). Sectoral and national aggregate disturbances to industrial output in seven European countries. *Journal of Monetary Economics* 21(2), 387–409.
- Sutton, J. (2002). The variance of firm growth rates: The ‘scaling’ puzzle. *Physica A* 312, 577–590.
- Weitzel, U. and S. Berns (2006). Cross-border takeovers, corruption, and related aspects of governance. *Journal of International Business Studies* 37, 786–806.
- Whinston, M. D. (2007). Chapter 36 - Antitrust policy toward horizontal mergers. In M. Armstrong and R. Porter (Eds.), *Handbook of Industrial Organization*, Volume 3, pp. 2369–2440. Elsevier.

- Williamson, O. E. (1968). Economies as an antitrust defense: The welfare tradeoffs. *American Economic Review* 58(1), 18–36.
- Xu, J. (2017). Growing through the merger and acquisition. *Journal of Economic Dynamics and Control* 80, 54–74.
- Yeh, C. (2023). Revisiting the origins of business cycles with the size-variance relationship. *Review of Economics and Statistics*, 1–28.

Online Appendix

Mergers, Firm Size, and Volatility in a Granular Economy

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A Theoretical Appendix

A.1 Firm profits

We derive sectoral firm profits Π_k here; the computation of aggregate profits is straightforward by adding up profits across all sectors. The firm's realized productivity after learning its transitory shock is $z_{ki}\epsilon_{ki}$. We use $\pi_k(z_{ki}|\epsilon_{ki})$ to denote the firm's realized profits. Below, we minimize notation and use $\sum_{z_{ki}}$ to denote the summation $\sum_{z_{ki}=\bar{z}_k}^{z_k^m}$. Thus,

$$\Pi_k \equiv M_k \sum_{z_{ki}} \pi_k(z_{ki}|\epsilon_{ki}) g_k(z_{ki}) - M_k^e c_k^e - M \sum_{z_{ki}} [C(\lambda_k(z_{ki})) + C(\mu_k(z_{ki}))] g_k(z_{ki}).$$

Define the difference between realized and expected profits of a firm as $\Delta\pi_k(z_{ki}) \equiv \pi_k(z_{ki}|\epsilon_{ki}) - \pi_k^E(z_{ki})$. Then, by Eqs. (11) and (15):

$$\begin{aligned} \Pi_k &= M_k \sum_{z_{ki}} [\Delta\pi_k(z_{ki}) + \pi_k^E(z_{ki})] g_k(z_{ki}) - M_k^e c_k^e - M \sum_{z_{ki}} [C(\lambda_k(z_{ki})) + C(\mu_k(z_{ki}))] g_k(z_{ki}) \\ &= M_k \sum_{z_{ki}} \Delta\pi_k(z_{ki}) g_k(z_{ki}) + M_k \sum_{z_{ki}} \left\{ \delta_k V_k(z_{ki}) - \lambda_k(z_{ki}) \theta_k^a \mathbb{E}_{z_{ki}^t} [\Sigma_k^a(z_{ki}, z_{ki}^t)] \right. \\ &\quad \left. - \mu_k(z_{ki}) \theta_k^t \mathbb{E}_{z_{ki}^a} [\Sigma_k^t(z_{ki}^a, z_{ki})] \right\} g_k(z_{ki}) - M_k \sum_{z_{ki}} V_k(z_{ki}) \frac{M_k^e}{M_k} f_k(z_{ki}). \end{aligned}$$

From the stationary condition in Eq. (14), we have

$$\begin{aligned} \sum_{z_{ki}} V_k(z_{ki}) \frac{M_k^e}{M_k} f_k(z_{ki}) &= \sum_{z_{ki}} V_k(z_{ki}) \left\{ \lambda_k(z_{ki}) \theta_k^a g_k(z_{ki}) \sum_{z_{ki}^t} \mathbf{1} [\Sigma_k(z_{ki}, z_{ki}^t) \geq 0] \Gamma_k(z_{ki}^t) \right. \\ &\quad \left. + \mu_k(z_{ki}) \theta_k^t g_k(z_{ki}) \sum_{z_{ki}^a} \mathbf{1} [\Sigma_k(z_{ki}^a, z_{ki}) \geq 0] \Lambda_k(z_{ki}^a) + \delta_k g_k(z_{ki}) \right. \\ &\quad \left. - \sum_{z_{ki}^a} \lambda_k(z_{ki}^a) \theta_k^a \mathbf{1} [\Sigma_k(z_{ki}^a, s^{-1}[z_{ki}, z_{ki}^a]) \geq 0] \Gamma_k(s^{-1}[z_{ki}, z_{ki}^a]) g_k(z_{ki}^a) \right\}. \end{aligned}$$

Without loss of generality, suppose acquirers are on the short side of the market (i.e., $\sum_{z_{ki}} \mu_k(z_{ki})g_k(z_{ki}) > \sum_{z_{ki}} \lambda_k(z_{ki})g_k(z_{ki})$), which implies $\theta_k^a = 1$. By the definition of merger gains, we have

$$\begin{aligned}
& \sum_{z_{ki}} \left\{ \lambda_k(z_{ki})\theta_k^a\beta \sum_{z_{ki}^t} \Sigma_k(z_{ki}, z_{ki}^t)\Gamma_k(z_{ki}^t) + \mu_k(z_{ki})\theta_k^t \sum_{z_{ki}^a} (1-\beta)\Sigma_k(z_{ki}^a, z_{ki})\Lambda_k(z_{ki}^a) \right\} g_k(z_{ki}) \\
&= \beta \sum_{z_{ki}^a} \sum_{z_{ki}^t} \Sigma_k(z_{ki}^a, z_{ki}^t) \frac{\lambda_k(z_{ki}^a)\mu_k(z_{ki}^t)}{\sum_{z_{ki}'} \mu_k(z_{ki}')g_k(z_{ki}')} g_k(z_{ki}^t)g_k(z_{ki}^a) \\
&\quad + (1-\beta) \sum_{z_{ki}^a} \sum_{z_{ki}^t} \Sigma_k(z_{ki}^a, z_{ki}^t) \frac{\lambda_k(z_{ki}^a)\mu_k(z_{ki}^t)}{\sum_{z_{ki}'} \mu_k(z_{ki}')g_k(z_{ki}')} g_k(z_{ki}^t)g_k(z_{ki}^a) \\
&= \sum_{z_{ki}^a} \sum_{z_{ki}^t} \Sigma_k(z_{ki}^a, z_{ki}^t) \frac{\lambda_k(z_{ki}^a)\mu_k(z_{ki}^t)}{\sum_{z_{ki}'} \mu_k(z_{ki}')g_k(z_{ki}')} g_k(z_{ki}^t)g_k(z_{ki}^a).
\end{aligned}$$

Excluding the difference between realized and expected profits, Π_k is therefore equal to

$$\begin{aligned}
& -M_k \sum_{z_{ki}^a} \sum_{z_{ki}^t} \left[V_k(s[z_{ki}^a, z_{ki}^t]) - V_k(z_{ki}^a) - V_k(z_{ki}^t) \right] \frac{\lambda_k(z_{ki}^a)\mu_k(z_{ki}^t)}{\sum_{z_{ki}'} \mu_k(z_{ki}')g_k(z_{ki}')} g_k(z_{ki}^t)g_k(z_{ki}^a) \\
& -M_k \sum_{z_{ki}} V_k(z_{ki}) \left\{ \sum_{z_{ki}^t=\bar{z}_k}^{z_k^m} \frac{\lambda_k(z_{ki})\mu_k(z_{ki}^t)}{\sum_{z_{ki}'} \mu_k(z_{ki}')g_k(z_{ki}')} \mathbf{1}[\Sigma_k(z_{ki}, z_{ki}^t) \geq 0] g_k(z_{ki}^t)g_k(z_{ki}) \right. \\
& + \sum_{z_{ki}^a} \frac{\lambda_k(z_{ki}^a)\mu_k(z_{ki})}{\sum_{z_{ki}'} \mu_k(z_{ki}')g_k(z_{ki}')} \mathbf{1}[\Sigma_k(z_{ki}^a, z_{ki}) \geq 0] g_k(z_{ki}^a)g_k(z_{ki}) \\
& \left. - \sum_{z_{ki}^a} \frac{\lambda_k(z_{ki}^a)\mu_k(s^{-1}[z_{ki}, z_{ki}^a])}{\sum_{z_{ki}'} \mu_k(z_{ki}')g_k(z_{ki}')} \mathbf{1}[\Sigma_k(z_{ki}^a, s^{-1}[z_{ki}, z_{ki}^a]) \geq 0] g_k(s^{-1}[z_{ki}, z_{ki}^a])g_k(z_{ki}^a) \right\}.
\end{aligned}$$

The terms exactly cancel out, which means

$$\Pi_k = M_k \sum_{z_{ki}} \Delta\pi_k(z_{ki})g_k(z_{ki}).$$

Remark: In the derivation above, we have used a discount factor of δ_k . If the discount factor is instead $r + \delta_k$, then aggregate firm profits are simply the aggregate value of firms captured as a result of the difference in the perceived discount factor (i.e., $r + \delta_k$) and the actual (exogenous) exit rate (i.e., δ_k). This is equal to $M_k \sum_{z_{ki}} rV_k(z_{ki})$, and is added on top of the difference in expected and realized profits.¹ However, this does not affect our main results.

A.2 Characterizing the size-volatility relationship

Here, we provide further details on the characterization of the firm size-volatility relationship. First, we note that in Eq. (16), the constant $\kappa \equiv \left(\bar{z}^{\tilde{\varepsilon}-1} \left(\frac{\tilde{\varepsilon}}{\tilde{\varepsilon}-1} w \right)^{1-\tilde{\varepsilon}} P^{\tilde{\varepsilon}} Y \right)^{\frac{\varepsilon}{\tilde{\varepsilon}-1}}$. Next, we derive

¹To understand this point, suppose firms perceive the exogenous rate to be δ'_k , but the actual exit rate is δ_k . Then, the number of firms will be smaller than that implied under a stationary equilibrium, and the remaining firms that do not exit will capture a portion of the firms' aggregate value as profits today.

Eqs. (25) to (27). We start by log-linearizing the price indices:

$$\hat{P} = \sum_{k=1}^N \mathcal{S}_k \hat{P}_k, \text{ and } \hat{P}_k = -\frac{\sum_{i=1}^{M_k} \alpha_{ki} \mathcal{S}_{ki} \hat{z}_{ki}}{\sum_{i'=1}^{M_k} \alpha_{ki'} \mathcal{S}_{ki'}}.$$

Eq. (24) can be derived accordingly:

$$\mathbb{V}ar[\hat{Y}] = \mathbb{V}ar[\hat{P}] = \sum_{k=1}^N \mathcal{S}_k^2 \mathbb{V}ar[\hat{P}_k] = \sum_{k=1}^N \mathcal{S}_k^2 \sum_{i=1}^{M_k} \frac{\alpha_{ki}^2 \mathcal{S}_{ki}^2 \mathbb{V}ar[\hat{z}_{ki}]}{\left(\sum_{i'=1}^{M_k} \alpha_{ki'} \mathcal{S}_{ki'}\right)^2} = \bar{v}_z^2 \sum_{k=1}^N \mathcal{S}_k^2 \sum_{i=1}^{M_k} \left(\frac{\alpha_{ki} \mathcal{S}_{ki}^{1-\chi}}{\sum_{i'=1}^{M_k} \alpha_{ki'} \mathcal{S}_{ki'}}\right)^2.$$

At the firm level, the derivation of Eq. (22) for the log change in firm-level prices is:

$$\begin{aligned} \hat{p}_{ki} &= -\hat{z}_{ki} + \hat{m}_{ki} \\ &= -\hat{z}_{ki} + \frac{1}{\varepsilon - 1} (\alpha_{ki}^{-1} - 1) \hat{S}_{ki} \\ &= -\hat{z}_{ki} + (1 - \alpha_{ki}^{-1}) (\hat{p}_{ki} - \hat{P}_k) \\ &= -\alpha_{ki} \hat{z}_{ki} + (1 - \alpha_{ki}) \hat{P}_k. \end{aligned}$$

Firm revenues are defined by:

$$r_{ki} = p_{ki}^{1-\varepsilon} P_k^\varepsilon Y_k = p_{ki}^{1-\varepsilon} P_k^{\varepsilon-\phi} P^{\phi-1} D_k P Y,$$

so the first-order approximation is

$$\begin{aligned} \hat{r}_{ki} &\approx (1 - \varepsilon) \hat{p}_{ki} + (\varepsilon - \phi) \hat{P}_k + (\phi - 1) \hat{P} \\ &\approx (1 - \varepsilon) \hat{p}_{ki} + (\varepsilon - \phi) \hat{P}_k + (\phi - 1) \sum_k \mathcal{S}_k \hat{P}_k \\ &= (1 - \varepsilon) \hat{p}_{ki} + (\varepsilon - \phi + (\phi - 1) \mathcal{S}_k) \hat{P}_k + (\phi - 1) \sum_{k' \neq k} \mathcal{S}_{k'} \hat{P}_{k'}. \end{aligned}$$

To build intuition, we can first examine the case of the single-sector economy. Then, there is a unique elasticity of substitution and the equation reduces to $\hat{r}_i \approx (1 - \varepsilon) \hat{p}_i + (\tilde{\varepsilon} - 1) \hat{P}$. If general equilibrium effects are not taken into account, then P is a constant from the individual firm's perspective, and we simply have $\hat{r}_i = (1 - \varepsilon) \hat{p}_i = (\tilde{\varepsilon} - 1) \hat{z}_i$ and $\mathbb{V}ar[\hat{r}_i] = (\tilde{\varepsilon} - 1)^2 [v_z(\mathcal{S}_i)]^2$. We can build intuition for the variance of sales growth in the multi-sector economy by analyzing the case of the single-sector economy taking into account general equilibrium effects:

$$\begin{aligned} \hat{r}_i &\approx (1 - \tilde{\varepsilon}) \hat{p}_i + (\tilde{\varepsilon} - 1) \hat{P} \\ &= (\tilde{\varepsilon} - 1) (\hat{z}_i + \hat{P}), \end{aligned}$$

and the variance of the sales growth is

$$\begin{aligned}
\text{Var}[\hat{r}_i] &= (\tilde{\varepsilon} - 1)^2 \left(\text{Var}[\hat{z}_i] + \text{Var}[\hat{P}] + 2\text{Cov}[\hat{z}_i, \hat{P}] \right) \\
&= (\tilde{\varepsilon} - 1)^2 \left([v_z(\mathbf{s}_i)]^2 + \sum_{i'} s_{i'}^2 [v_z(\mathbf{s}_{i'})]^2 - 2s_i [v_z(\mathbf{s}_i)]^2 \right) \\
&= (\tilde{\varepsilon} - 1)^2 \left(1 + \sum_{i'} s_{i'}^2 \frac{[v_z(\mathbf{s}_{i'})]^2}{[v_z(\mathbf{s}_i)]^2} - 2s_i \right) [v_z(\mathbf{s}_i)]^2.
\end{aligned}$$

Taking the logarithm of this equation, we can approximate the middle term by $-2s_i$ if market shares are small. Therefore, we can characterize the firm size-volatility relationship by the linear combination of the log-log and log-linear functions. The log-linear term appears only if firms take into account the effects of their pricing decisions on the price index.

We now generalize this result for the multi-sector economy, where firms have strategic market power and internalize the effects of their pricing decisions on the sectoral price index P_k :

$$\begin{aligned}
\hat{r}_{ki} &\approx (1 - \varepsilon) \left(-\alpha_{ki} \hat{z}_{ki} + (1 - \alpha_{ki}) \hat{P}_k \right) + (\varepsilon - \phi + (\phi - 1) \mathcal{S}_k) \hat{P}_k + (\phi - 1) \sum_{k' \neq k} \mathcal{S}_{k'} \hat{P}_{k'} \\
&= (\varepsilon - 1) \alpha_{ki} \hat{z}_{ki} + \tilde{\beta}_{ki} \hat{P}_k + (\phi - 1) \sum_{k' \neq k} \mathcal{S}_{k'} \hat{P}_{k'}
\end{aligned}$$

where $\tilde{\beta}_{ki} \equiv (1 - \varepsilon)(1 - \alpha_{ki}) + (\varepsilon - \phi + (\phi - 1) \mathcal{S}_k)$. Note, if $\phi = 1$, then $\tilde{\beta}_{ki} = (\varepsilon - 1) \alpha_{ki}$. This implies that

$$\begin{aligned}
\text{Var}[\hat{r}_{ki}] &\approx (\varepsilon - 1)^2 \alpha_{ki}^2 \text{Var}[\hat{z}_{ki}] + \tilde{\beta}_{ki}^2 \text{Var}[\hat{P}_k] + 2(\varepsilon - 1) \alpha_{ki} \tilde{\beta}_{ki} \text{Cov}[\hat{z}_{ki}, \hat{P}_k] + (\phi - 1)^2 \sum_{k' \neq k} \mathcal{S}_{k'}^2 \text{Var}[\hat{P}_{k'}] \\
&= (\varepsilon - 1)^2 \alpha_{ki}^2 [v_z(\mathbf{s}_{ki})]^2 + \tilde{\beta}_{ki}^2 \frac{\sum_{i'} (\alpha_{ki'} \mathcal{S}_{ki'})^2 [v_z(\mathbf{s}_{ki'})]^2}{(\sum_{i'} \alpha_{ki'} \mathcal{S}_{ki'})^2} - 2(\varepsilon - 1) \alpha_{ki} \tilde{\beta}_{ki} \frac{\alpha_{ki} \mathcal{S}_{ki} [v_z(\mathbf{s}_{ki})]^2}{\sum_{i'} \alpha_{ki'} \mathcal{S}_{ki'}} \\
&\quad + (\phi - 1)^2 \sum_{k' \neq k} \mathcal{S}_{k'}^2 \frac{\sum_{i'} (\alpha_{k'i'} \mathcal{S}_{k'i'})^2 [v_z(\mathbf{s}_{k'i'})]^2}{(\sum_{i'} \alpha_{k'i'} \mathcal{S}_{k'i'})^2} \\
&= (\varepsilon - 1)^2 \alpha_{ki}^2 \left(1 + \frac{\tilde{\beta}_{ki}^2}{(\varepsilon - 1)^2 \alpha_{ki}^2} \frac{\sum_{i'} (\alpha_{ki'} \mathcal{S}_{ki'})^2 \frac{[v_z(\mathbf{s}_{ki'})]^2}{[v_z(\mathbf{s}_{ki})]^2}}{(\sum_{i'} \alpha_{ki'} \mathcal{S}_{ki'})^2} - \frac{2\tilde{\beta}_{ki}}{(\varepsilon - 1) \alpha_{ki}} \frac{\alpha_{ki} \mathcal{S}_{ki}}{\sum_{i'} \alpha_{ki'} \mathcal{S}_{ki'}} \right. \\
&\quad \left. + \frac{(\phi - 1)^2}{(\varepsilon - 1)^2 \alpha_{ki}^2} \sum_{k' \neq k} \mathcal{S}_{k'}^2 \frac{\sum_{i'} (\alpha_{k'i'} \mathcal{S}_{k'i'})^2 \frac{[v_z(\mathbf{s}_{k'i'})]^2}{[v_z(\mathbf{s}_{ki})]^2}}{(\sum_{i'} \alpha_{k'i'} \mathcal{S}_{k'i'})^2} \right) [v_z(\mathbf{s}_{ki})]^2.
\end{aligned}$$

The term $\alpha_{ki}^2 (1 + \dots)$ is what we refer to as the strategic market power effect on volatility. In the case of $\phi = 1$, this reduces to Eq. (25), which we repeat here:

$$\text{Var}[\hat{r}_{ki}] = (\varepsilon - 1)^2 \alpha_{ki}^2 \underbrace{\left(1 + \frac{\sum_{i'=1}^{M_k} (\alpha_{ki'} \mathcal{S}_{ki'})^2 \frac{[v_z(\mathbf{s}_{ki'})]^2}{[v_z(\mathbf{s}_{ki})]^2}}{(\sum_{i'=1}^{M_k} \alpha_{ki'} \mathcal{S}_{ki'})^2} - \frac{2\alpha_{ki} \mathcal{S}_{ki}}{\sum_{i'=1}^{M_k} \alpha_{ki'} \mathcal{S}_{ki'}} \right)}_{\text{Strategic market power effect on volatility}} [v_z(\mathbf{s}_{ki})]^2.$$

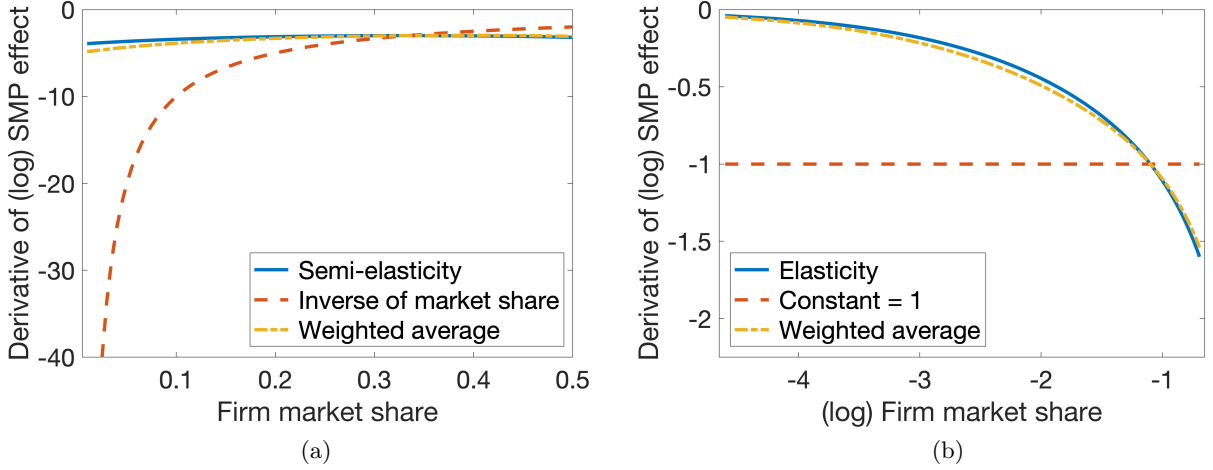


Figure A.1: This figure plots the derivative of the (log) strategic market power (SMP) effect on volatility in Eq. (25) against firm market share in (a) levels, and (b) logarithms. This is denoted by the weighted average. We also decompose each derivative into two components, the semi-elasticity and inverse of the market share in (a), and the elasticity and the constant 1 in (b). For purposes of illustration, we assume $\phi = 1$, $\varepsilon = 5$, $\chi = 0$, $\sum_{i'=1}^{M_k} (\alpha_{ki'} s_{ki'})^2 = 1$, and $\sum_{i'=1}^{M_k} \alpha_{ki'} s_{ki'} = 1$.

Using the approximation $\log(1+x) \approx x$, the derivative of the log SMP effect w.r.t. s_{ki} is given by Eq. (26), which we repeat here:

$$-2 \left[- \left(1 - \frac{\alpha_{ki} s_{ki}}{\sum_{i'} \alpha_{ki'} s_{ki'}} \right) \frac{\partial \log \alpha_{ki}}{\partial s_{ki}} + \frac{\alpha_{ki} s_{ki}}{\sum_{i'} \alpha_{ki'} s_{ki'}} \frac{1}{s_{ki}} \right] < 0.$$

This derivative is a weighted average between the semi-elasticity $\frac{\partial \log \alpha_{ki}}{\partial s_{ki}}$ and the inverse of market share. The semi-elasticity is equal to

$$\frac{\partial \log \alpha_{ki}}{\partial s_{ki}} = \frac{-(\varepsilon - 1)^2 \left(\frac{\varepsilon}{\phi} - 1 \right)}{\left(\varepsilon - 1 + (\varepsilon - 2) \left(\frac{\varepsilon}{\phi} - 1 \right) s_{ki} \right) \left(\varepsilon - 1 - \left(\frac{\varepsilon}{\phi} - 1 \right) s_{ki} \right)} < 0.$$

The semi-elasticity is very flat because the two terms in the denominator move in opposite directions when s_{ki} changes. Meanwhile, when s_{ki} is small, the weight $\frac{\alpha_{ki} s_{ki}}{\sum_{i'} \alpha_{ki'} s_{ki'}}$ is small while the inverse of the market share $1/s_{ki}$ is large. Hence, the derivative of the log SMP effect on volatility w.r.t. s_{ki} puts less weight on the term $1/s_{ki}$ in Eq. (26) precisely when s_{ki} is small, so the two roughly balance out. Appendix Figure A.1(a) provides an illustration by plotting each of the components and their weighted average. The semi-elasticity is very flat, which means that the derivative of the log SMP effect is roughly constant.

Meanwhile, the derivative of the log SMP effect w.r.t. $\log s_{ki}$ is given by:

$$-2 \left[- \left(1 - \frac{\alpha_{ki} s_{ki}}{\sum_{i'} \alpha_{ki'} s_{ki'}} \right) \frac{\partial \log \alpha_{ki}}{\partial \log s_{ki}} + \frac{\alpha_{ki} s_{ki}}{\sum_{i'} \alpha_{ki'} s_{ki'}} \right].$$

This is a weighted average of the elasticity $\frac{\partial \log \alpha_{ki}}{\partial \log s_{ki}}$ and the constant of 1. The elasticity is equal to

$$\frac{\partial \log \alpha_{ki}}{\partial \log s_{ki}} = \frac{-(\varepsilon - 1)^2 \left(\frac{\varepsilon}{\phi} - 1\right) s_{ki}}{\left(\varepsilon - 1 + (\varepsilon - 2) \left(\frac{\varepsilon}{\phi} - 1\right) s_{ki}\right) \left(\varepsilon - 1 - \left(\frac{\varepsilon}{\phi} - 1\right) s_{ki}\right)},$$

This term must become more negative as s_{ki} increases. Hence, the slope of the derivative of the log SMP effect w.r.t. $\log s_{ki}$ becomes steeper and more negative as market share increases. See Appendix Figure A.1(b) for an illustration.

Lastly, we show that changes to the other terms (e.g., the summations and $v_z(s_{ki})$) are of second order in the derivative of the log SMP effect w.r.t. s_{ki} and they do not affect our main results. First, we examine (the negative of) the last term in the large parentheses of Eq. (25), $\frac{2\alpha_{ki}s_{ki}}{\sum_{i'=1}^{M_k} \alpha_{ki'}s_{ki'}}$. Its derivative w.r.t. s_{ki} is:

$$\begin{aligned} & \frac{\left(2\alpha_{ki} + 2\alpha_{ki}s_{ki} \frac{\partial \log \alpha_{ki}}{\partial s_{ki}}\right) \sum_{i'=1}^{M_k} \alpha_{ki'}s_{ki'} - 2\alpha_{ki}s_{ki} \left(\alpha_{ki} + \alpha_{ki}s_{ki} \frac{\partial \log \alpha_{ki}}{\partial s_{ki}}\right)}{\left(\sum_{i'=1}^{M_k} \alpha_{ki'}s_{ki'}\right)^2} \\ &= \frac{2 \left(\alpha_{ki} + \alpha_{ki}s_{ki} \frac{\partial \log \alpha_{ki}}{\partial s_{ki}}\right) \left(\sum_{i'=1}^{M_k} \alpha_{ki'}s_{ki'} - \alpha_{ki}s_{ki}\right)}{\left(\sum_{i'=1}^{M_k} \alpha_{ki'}s_{ki'}\right)^2} \end{aligned} \quad (\text{A.1})$$

In the numerator, it is clear that $\sum_{i'=1}^{M_k} \alpha_{ki'}s_{ki'}$ is the first-order effect while $\alpha_{ki}s_{ki}$ is only of second order (since it is only one term, as opposed to the summation). Then this can be approximated as

$$\frac{2 \left(\alpha_{ki} + \alpha_{ki}s_{ki} \frac{\partial \log \alpha_{ki}}{\partial s_{ki}}\right)}{\sum_{i'=1}^{M_k} \alpha_{ki'}s_{ki'}} = \frac{2\alpha_{ki}s_{ki}}{\sum_{i'=1}^{M_k} \alpha_{ki'}s_{ki'}} \frac{1}{s_{ki}} + \frac{\alpha_{ki}s_{ki}}{\sum_{i'=1}^{M_k} \alpha_{ki'}s_{ki'}} \frac{\partial \log \alpha_{ki}}{\partial s_{ki}}.$$

The difference between this expression and Eq. (26) is $2 \frac{\partial \log \alpha_{ki}}{\partial s_{ki}}$, which is simply the derivative of α_{ki}^2 w.r.t. s_{ki} (see Eq. (25)). Likewise, we set $\chi = 0$ to examine the contribution of changes to the term $\frac{\sum_{i'=1}^{M_k} (\alpha_{ki'}s_{ki'})^2}{\left(\sum_{i'=1}^{M_k} \alpha_{ki'}s_{ki'}\right)^2}$. The derivative is:

$$\begin{aligned} & \frac{\left(2\alpha_{ki}^2s_{ki} + 2\alpha_{ki}^2s_{ki}^2 \frac{\partial \log \alpha_{ki}}{\partial s_{ki}}\right) \left(\sum_{i'=1}^{M_k} \alpha_{ki'}s_{ki'}\right)^2 - 2\left(\sum_{i'=1}^{M_k} \alpha_{ki'}s_{ki'}\right) \left(\alpha_{ki} + \alpha_{ki}s_{ki} \frac{\partial \log \alpha_{ki}}{\partial s_{ki}}\right) \sum_{i'=1}^{M_k} (\alpha_{ki'}s_{ki'})^2}{\left(\sum_{i'=1}^{M_k} \alpha_{ki'}s_{ki'}\right)^4} \\ &= \frac{2 \left(\alpha_{ki} + \alpha_{ki}s_{ki} \frac{\partial \log \alpha_{ki}}{\partial s_{ki}}\right) \left(\alpha_{ki}s_{ki} - \frac{\sum_{i'=1}^{M_k} (\alpha_{ki'}s_{ki'})^2}{\sum_{i'=1}^{M_k} \alpha_{ki'}s_{ki'}}\right)}{\left(\sum_{i'=1}^{M_k} \alpha_{ki'}s_{ki'}\right)^2} \end{aligned} \quad (\text{A.2})$$

By comparing this to Eq. (A.1), it is clear that these effects are of second order since $\left(\sum_{i'=1}^{M_k} \alpha_{ki'}s_{ki'}\right)^2 > \sum_{i'=1}^{M_k} \alpha_{ki'}^2s_{ki'}^2$. Finally, we have the effects arising from the variance of shocks within the SMP effect. For simplicity, we treat summations as constants and consider the

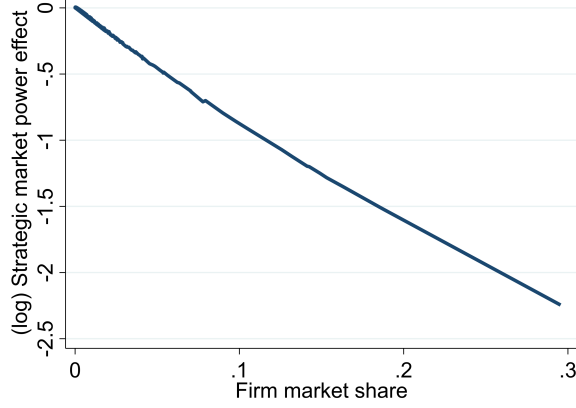


Figure A.2: This figure plots the (log) strategic market power effect on volatility against firm market share in levels using Danish data for the year 2010 with $\phi = 1$, $\varepsilon = 5$, and $\chi = 0.14$.

derivative of $\frac{\sum_{i'=1}^{M_k} (\alpha_{ki'} s_{ki'} v_z(s_{ki'}))^2}{\left(\sum_{i'=1}^{M_k} \alpha_{ki'} s_{ki'}\right)^2} \frac{1}{[v_z(s_{ki})]^2}$ w.r.t. s_{ki} :

$$\frac{\sum_{i'=1}^{M_k} (\alpha_{ki'} s_{ki'} v_z(s_{ki'}))^2}{\left(\sum_{i'=1}^{M_k} \alpha_{ki'} s_{ki'}\right)^2} 2\bar{v}_z \chi s_{ki}^{2\chi-1} \quad (\text{A.3})$$

How does $\sum_{i'=1}^{M_k} (\alpha_{ki'} s_{ki'} v_z(s_{ki'}))^2 \bar{v}_z \chi s_{ki}^{2\chi-1}$ compare to $\left(\alpha_{ki} + \alpha_{ki} s_{ki} \frac{\partial \log \alpha_{ki}}{\partial s_{ki}}\right) \sum_{i'=1}^{M_k} \alpha_{ki'} s_{ki'}$? Notice that in the former, the squares of $s_{ki'}$ and $\alpha_{ki'}$ (along with $v_z(s_{ki'})$) are summed, while in the latter they are summed in levels. Also, it turns out that $s_{ki}^{2\chi-1}$ is only large for very small values of s_{ki} . These two results imply that the combined effects are also of second order, and they do not influence the overall relationship between the (log) SMP effect and market share. In Appendix Figure A.2, we use data for Danish firms in the year 2010 to show the shape of the relationship between the log SMP effect and market share in levels.² Instead of setting $\chi = 0$ and assuming arbitrary numbers for the values of the summations (e.g., $\sum_{i'=1}^{M_k} \alpha_{ki'} s_{ki'}$), we set $\chi = 0.14$ and use the data to calculate the values of the summations. The shape of the graph is very similar to Figure 3(a), supporting our claim that the simplifications used in the main text are of second order.

²We use a line graph instead of a scatter plot due to data confidentiality requirements.

B Empirical Appendix

B.1 Large firms and mergers

In Appendix Figure B.1, we plot the fraction of total domestic sales in Denmark accounted for by the top 50 and 100 firms in each year from 1993 to 2015. The figure demonstrates that the biggest companies are extremely large, with the top 50 and 100 firms accounting for roughly one-quarter and one-third of total domestic sales, respectively. These statistics are very similar to those of a large country like the US (Gabaix, 2011). Meanwhile, Appendix Figure B.2 shows the rise in M&A deals in Denmark. Because the register data does not contain information about foreign (i.e., non-Danish) acquirer firms, we rely on data from Zephyr, Bureau van Dijk to compare the number of domestic and foreign horizontal acquisitions of Danish target firms.

Furthermore, we show in Appendix Figure B.3 that the merger market is dominated by large firms. The figure employs the Danish register data. We first divide the distribution of firm size (as measured by sales) into ten bins and count the number of acquirer firms in our sample within each decile. For the targets matched to the acquirers in each bin, we also compute their average size in the firm-size distribution. The figure shows that over 50% of acquirers belong to the top decile and the targets matched to these firms are on average at the 74th percentile of the size distribution. Meanwhile, the typical target matched in the second highest decile lies at the 63rd percentile.

Lastly, in Appendix Figure B.4, we further follow di Giovanni et al. (2011) to estimate and plot the firm-size distributions for different groups of firms. As shown in Section 2.2.1 (i.e., Eq. (16)), firm size follows a power law under the assumption of a Pareto productivity distribution for active firms. For each group of firms, Appendix Figures B.4(a) and (b) estimate the cdf and pdf of the power law, respectively, in their linear forms. We measure firm size with (log) deflated domestic sales. Following standard practice, we impose a minimum size cutoff, which we choose to be the bottom 10th percentile. Note that for the pdf method, the power law coefficient is equal to the absolute value of the slope coefficient minus one. Consistent with Figure 4(a), the firm-size distribution is most fat-tailed (i.e., the slope coefficient is smallest in magnitude) for acquirers, followed by targets, and then non-participating firms. Similar patterns are obtained using a log-log rank-size regression (Gabaix and Ibragimov, 2011). These findings are also robust to using the deviation of sales from the sector median as the measure of firm size and separately dropping the bottom 10th percentile of each category of firms; these results are available upon request.



Figure B.1: This figure plots the fraction of total domestic sales accounted for by the top 50 and 100 Danish firms in each year from 1993 to 2015.

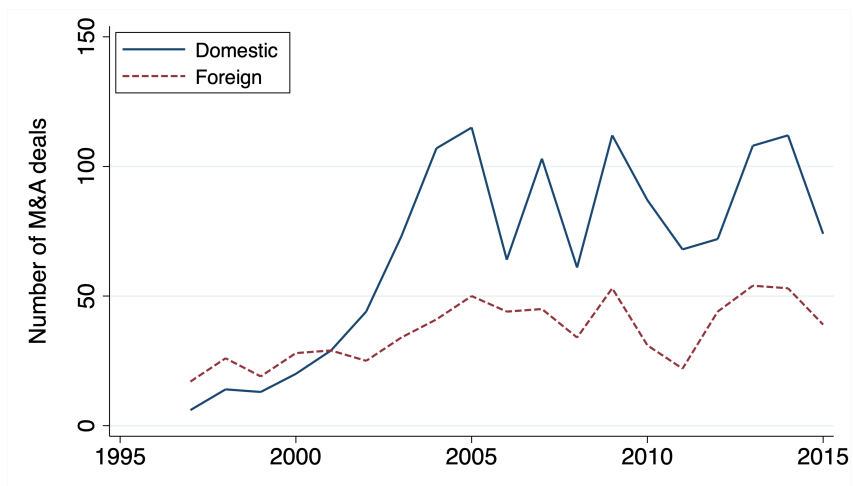


Figure B.2: This figure plots the number of domestic and foreign horizontal acquisitions of Danish target firms from 1997 to 2015 using data from Zephyr, Bureau van Dijk. This includes both “completed-confirmed” and “completed-assumed” deals, and we employ the same sector classifications as the main text to classify horizontal M&A deals.

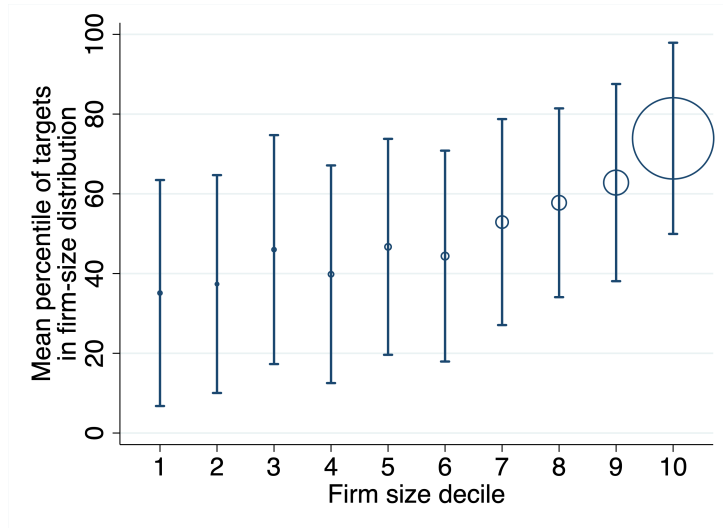


Figure B.3: Using the sample of domestic horizontal M&A in Denmark from 1993 to 2015, this figure plots the mean percentile (\pm SD) of targets in the firm-size distribution matched to the acquirers of each decile of firm size. Firm size is measured by domestic sales. From the lowest to highest decile, $N = 77, 60, 78, 104, 128, 156, 267, 315, 550,$ and $1840,$ respectively.

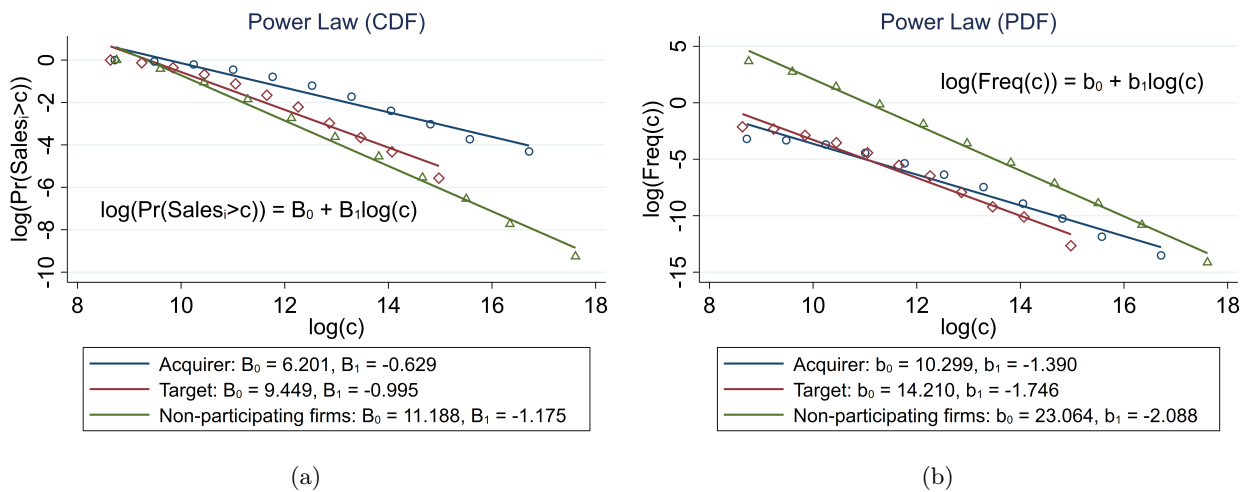


Figure B.4: This figures plots the estimated power law in firm size using the (a) cdf and (b) pdf methods. Firm size is measured by deflated domestic sales.

B.2 Model estimation

We use a simulated method of moments (SMM) estimator to estimate our model. In the first step, the Pareto shape parameters ξ_k of the observed productivity distributions are estimated for the multi-sector model. For each sector k , we discretize the productivity distribution of z_{ki} over a (log) grid space with 500 points in the interval between \bar{z}_k , normalized to 1, and $z_k^m = 10^3$. This implies that the largest firm is around 10^7 times bigger in sales than the smallest, which is in the same order of magnitude as in the data. Given M_k and $\mathcal{S}_k = D_k$ from the data (see Appendix Tables B.1 and B.2), $\phi = 1$, and $\varepsilon = 5$, we use the following algorithm to estimate ξ_k for each sector and each of the random 1,001 samples drawn:

1. For candidate ξ_k , construct the productivity distribution from the Pareto distribution $G_k(z_{ki}) = 1 - z_{ki}^{-\xi_k}$.
2. Draw M_k random numbers and compute the simulated productivity distribution, $G_k^{sim}(z_{ki})$, and the corresponding pmf $g_k^{sim}(z_{ki})$.
3. Solve for market shares as a fixed point problem:
 - (a) Guess market shares s_{ki} , and update using Eqs. (19) and (20).
 - (b) Normalize such that the updated market shares add up to 1; iterate on s_{ki} .
4. Compute the Herfindahl-Hirschman index $HHI_k = M_k \sum_{z_{ki}=\bar{z}_k}^{z_k^m} s_{ki}^2 g_k^{sim}(z_{ki})$.

The process is iterated until the model-implied market concentration (given by the median over the random samples) matches the data. The estimated Pareto shape parameters are shown in Appendix Table B.1.

From Eq. (20), firm-level markups are determined. Sector and aggregate level markups are defined by:

$$\mathcal{M}_k = \left[\sum_{z_{ki}=\bar{z}_k}^{z_k^m} s_{ki} m_{ki}^{-1} \right]^{-1}, \quad \mathcal{M} = \left[\sum_{k=1}^N \mathcal{S}_k \mathcal{M}_k^{-1} \right]^{-1}.$$

In the single-sector economy, firms charge a constant markup equal to $\frac{\tilde{\varepsilon}}{\tilde{\varepsilon}-1}$, which we set equal to \mathcal{M} . Thus, $\tilde{\varepsilon} = \frac{\mathcal{M}}{\mathcal{M}-1} = 4.85$. The algorithm above is also utilized to set the Pareto shape parameter ξ of the aggregate productivity distribution. Then, we construct the productivity distribution by drawing 1 million firms from $G(z_i) = 1 - z_i^{-\xi}$ for the simulation. We calculate firm-level prices, the price index, and firm-level profits and sales (note $\pi(z_i) = \pi^E(z_i)$). From this random sample, we also compute the size of the median firm and the productivity level at the 10th percentile, which are used to compute the model-implied moments.

Next, we estimate the merger market parameters $\Theta = \{\gamma, \nu, A, B, \eta\}$ with the following algorithm in the single-sector economy:

1. Guess candidate γ , ν , and A from the merger technology function Eq. (8) and B and η from the merger search cost function Eq. (10).

Table B.1: Market Concentration, Share of Firms, and Pareto Shape Parameters Across Sectors

Codes	Sector	HHI (1)	Share of firms (M_k/M) (2)	ξ_k (3)	$\frac{\xi_k}{(\varepsilon-1)}$ (4)
10-12	Food, beverages, and tobacco	0.0092	0.026	4.73	1.18
13-15	Textiles and apparel	0.0156	0.010	4.59	1.15
16-18	Wood and paper	0.0093	0.028	4.71	1.18
19-23	Chemicals, plastics, and non-metallic mineral products	0.0089	0.023	4.78	1.20
24-25	Metal and fabricated metal products	0.0055	0.041	5.04	1.26
26-27	Computers, electronics, and electrical equipment	0.0175	0.014	4.33	1.08
28	Machinery	0.0105	0.027	4.61	1.15
29-33	Transport equipment, furniture, and other manufacturing	0.0106	0.030	4.65	1.16
41-43	Construction	0.0019	0.194	5.32	1.33
45-46	Wholesale trade	0.0009	0.199	5.80	1.45
47	Retail trade	0.0029	0.115	5.18	1.29
49-53	Transportation and storage	0.0047	0.064	4.97	1.24
55-56	Accommodation and food service activities	0.0070	0.050	4.81	1.20
58-63	Information and communication	0.0074	0.040	4.77	1.19
68	Real estate	0.0122	0.016	4.60	1.15
69-75	Professional, scientific, and technical activities	0.0048	0.084	4.92	1.23
77-82	Administrative and support service activities	0.0095	0.039	4.58	1.15

Notes: Authors' calculations using Danish register data. In columns 1 and 2, averages over the sample period are computed. 2-digit industry codes follow the NACE Rev. 2 statistical classification of economic activities. The Herfindahl-Hirschman index (HHI) is defined as the sum of market shares squared.

Table B.2: Share of Domestic Sales, Exit Rates, and Survival Rates Across Sectors

Codes	Sector	Share of sales (\mathcal{S}_k) (1)	Exit rate (δ_k) (2)	Survival rate (M_k/M_k^e) (3)
10-12	Food, beverages, and tobacco	0.047	0.109	0.593
13-15	Textiles and apparel	0.005	0.119	0.658
16-18	Wood and paper	0.024	0.105	0.316
19-23	Chemicals, plastics, and non-metallic mineral products	0.039	0.067	0.540
24-25	Metal and fabricated metal products	0.028	0.082	0.600
26-27	Computers, electronics, and electrical equipment	0.017	0.078	0.510
28	Machinery	0.025	0.069	0.628
29-33	Transport equipment, furniture, and other manufacturing	0.022	0.103	0.596
41-43	Construction	0.097	0.118	0.532
45-46	Wholesale trade	0.355	0.089	0.611
47	Retail trade	0.095	0.127	0.568
49-53	Transportation and storage	0.072	0.117	0.473
55-56	Accommodation and food service activities	0.018	0.170	0.496
58-63	Information and communication	0.052	0.120	0.518
68	Real estate	0.010	0.172	0.531
69-75	Professional, scientific, and technical activities	0.061	0.120	0.581
77-82	Administrative and support service activities	0.033	0.158	0.476

Notes: Averages over the sample period are computed. For a given year, exit rates are defined as the share of firms with no production for three years straight. Survival rates are defined as the share of new firms that survive for at least five years. Data on new firms is retrieved from the register *IVNV*, beginning in year 2001.

2. Construct the merger matrix $s[z_i^a, z_i^t]$.
3. Guess candidate $V(z_i)$ and evaluate the merger matrix (i.e., determine whether $\Sigma(z_i^a, z_i^t)$ in Eq. (9) is positive).
4. Guess candidate $\mu(z_i)$ and θ^a . Given $\Sigma(z_i^a, z_i^t)$, solve for $\lambda(z_i)$ and θ^t . Iterate on θ^a and $\mu(z_i)$ until convergence.
5. Compute $V(z_i)$ from Eq. (11).
6. Solve for c^d such that $V(\bar{z}) = 0$ (i.e., Eq. (13)). Iterate on $V(z_i)$.

After this procedure, the merger market is simulated with random meeting rates between acquirer and target firms. A merger is successful if merger gains are positive. Then, we construct the five target moments and compute the objective function $(m - \hat{m}(\Theta))' W (m - \hat{m}(\Theta))$, where m is the vector of five target moments from the data, $\hat{m}(\Theta)$ is the vector of corresponding moments constructed using the simulated economy with parameters $\Theta = \{\gamma, \nu, A, B, \eta\}$, and W is a matrix of weights. We use the generalized inverse of the estimated variance-covariance matrix of the moments computed from the data. This process is iterated until convergence.

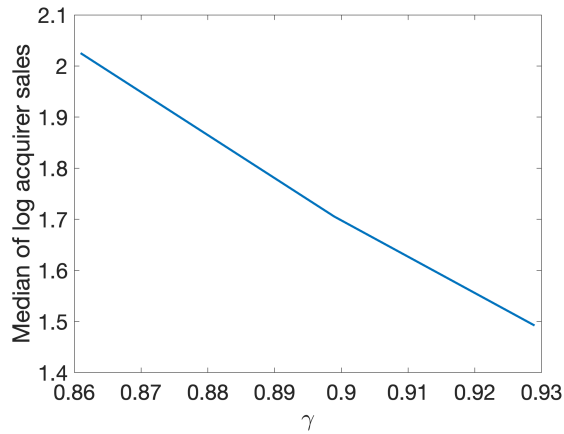
Appendix Figure B.5 demonstrates that the moments display sensitivity to the parameters. For example, an increase in γ , all else equal, raises merger gains and therefore lowers the median size of acquirers. A rise in B discourages mergers and the merger rate declines, while an increase in η raises the search costs especially for larger firms. Thus, smaller firms search more, and the dispersion of targets increases.

B.3 Counterfactuals

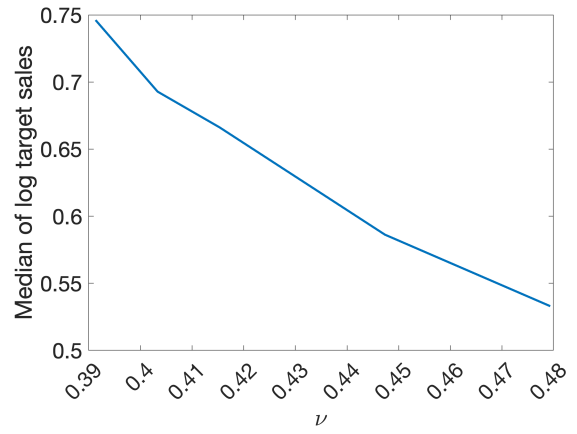
B.3.1 Single-sector economy

By the stationary condition in Eq. (14), we construct $F(z_i)$ for $z_i \geq \bar{z}$. From Eqs. (13) and (15), we compute the values of c^d and c^e . In order to extrapolate $f(z_i)$ for $z_i < \bar{z}$, we regress $\log[f(z_i)]$ on $\log(z_i)$. Note that if the productivity distribution follows the Pareto distribution, then we have a perfect fit in the regression (i.e., $R^2 = 1$).³ The fit will not be perfect because $F(z_i)$ does not follow the Pareto distribution. Nonetheless, this regression provides an approximation for the shape of the productivity distribution in which it is (by definition) unobserved. The slope coefficient is $\hat{\beta}_1 = -4.46$, which is slightly lower in magnitude than ξ , with an R^2 of 0.9997. In order to guarantee that $F(z_i)$ is strictly increasing, we do not use the coefficient of the constant term from the regression. Instead, for the grid point below \bar{z} , we compute $\exp\{\log[f(z_i)] + \hat{\beta}_1 \times \text{Stepsize}\}$, where Stepsize is the step size between grid points in logarithms, and so on. This gives us $F(z_i) \forall z_i < \bar{z}$. The extrapolation is displayed in Appendix Figure B.6, along with a comparison to a linear extrapolation.

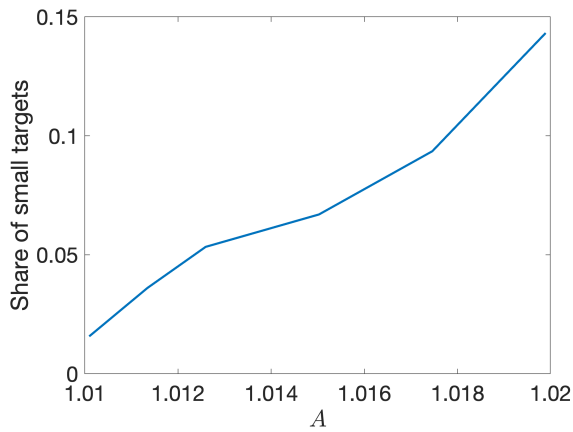
³For example, $G(z_i)$ follows the Pareto distribution with shape parameter ξ . In a continuous setting, $\log[g(z)] \propto -(\xi + 1) \log(z)$. Given our definition of the pmf (i.e., $g(z_j) = G(z_j) - G(z_{j-1})$), in a discrete setting with evenly log-spaced grid points, we have $\log[g(z_i)] \propto -\xi \log(z_i)$. This means that the slope is equal to $-\xi$.



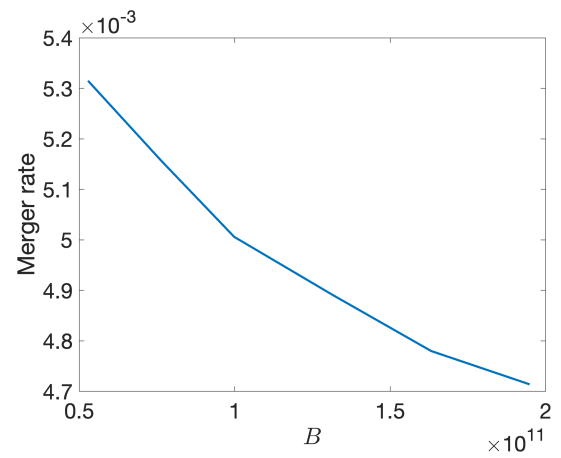
(a)



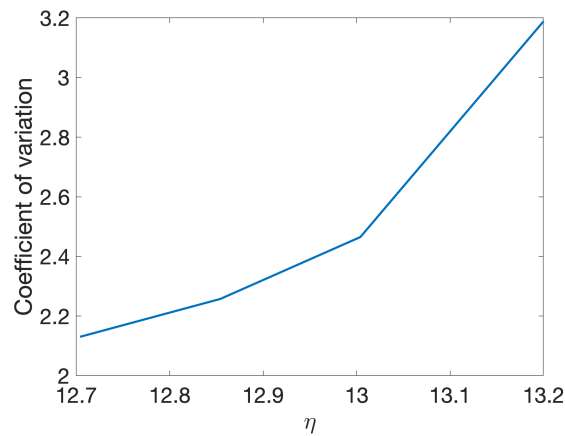
(b)



(c)



(d)



(e)

Figure B.5: This figure plots, for various values of the parameters: (a) the median deviation of (log) acquirer sales from the sector median against γ in the merger technology function; (b) the median deviation of (log) target sales from the sector median against ν in the merger technology function; (c) the share of targets in the bottom decile of the firm-size distribution against A in the merger technology function; (d) the aggregate merger rate against B in the search cost function; and (e) the coefficient of variation of target sales against η in the search cost function.

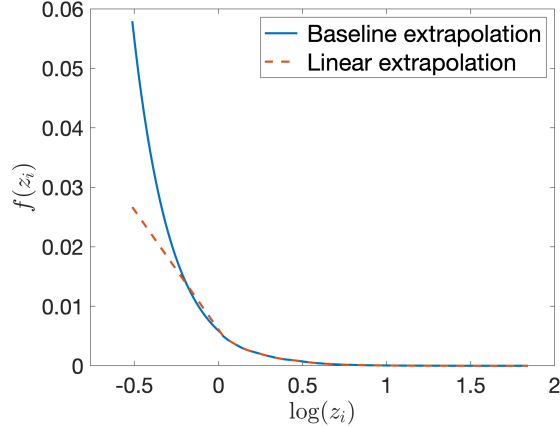


Figure B.6: This figure plots the productivity distribution in the counterfactual economy without mergers $f(z_i)$ for $10^{-0.2} \leq z_i \leq 10^{0.8}$. Two different extrapolations are shown. Above $\log(\bar{z}) = 0$, the two curves are identical. Below $\log(\bar{z})$, $f(z_i)$ is extrapolated. The baseline extrapolation is described in Footnote 25 and Appendix Section B.3, while linear extrapolation uses the slope from the neighboring grid points.

In the single-sector economy without mergers, we have the zero profit condition and free entry condition, respectively:

$$\begin{aligned} \pi(\bar{z}^F) &= 0, \\ \sum_{z_i=\bar{z}^F}^{z^m} \frac{\pi(z_i)}{\delta} f(z_i) &= wc^e. \end{aligned}$$

Given c^d and c^e , we search for the point \bar{z}^F that satisfies both conditions, and compute the equilibrium number of firms M^F accordingly. Analogous to the benchmark economy with mergers, the counterfactual economy without mergers is simulated.

B.3.2 Multi-sector economy

From Appendix Section B.2, we obtain ξ_k , $G_k(z_{ki})$ (or equivalently, $G_k^{sim}(z_{ki})$), and s_{ki} . Pass-through rates α_{ki} , markups m_{ki} , prices p_{ki} , operating profits π_k^o follow immediately from Eqs. (21), (20), (6), and (7). The value function depends on expected profits, which we approximate with a Taylor expansion under the assumption of i.i.d. productivity shocks. Because we normalize the mean of the shocks to 1 (i.e., $\mathbb{E}_\epsilon[(z_{ki}\epsilon_{ki})^{\epsilon-1}] = z_{ki}^{\epsilon-1}$), and $\bar{v}_z^2 = 0.01$, expected operating profits are approximated by:

$$\pi_k^{o,E}(z_{ki}) = \pi_k^o(z_{ki}) + \frac{1}{2} \frac{\partial^2 \pi_k^o(z_{ki})}{\partial z_{ki}^2} \bar{v}_z^2.$$

The second-order partial derivative is computed numerically.

Next, we follow the same steps as the single-sector economy to solve the merger market, back out $F_k(z_{ki})$ for $z_{ki} \geq \bar{z}_k$, and compute c_k^d and c_k^e . Likewise, for each sector and random

sample, we extrapolate $f_k(z_{ki})$ for $z_{ki} < \bar{z}_k$. However, because markups are not constant, the number of firms M_k^F cannot be determined by simply combining the zero profit and free entry conditions. Instead we employ the following algorithm to solve the counterfactual economy without mergers:

1. Guess candidate sectoral price index P_k^F .
2. Guess candidate cutoff productivity for the counterfactual economy, \bar{z}_k^F . The pmf $f_k(z_{ki}) \forall z_{ki} \geq \bar{z}_k^F$ is obtained from the extrapolation above.
3. Guess candidate M_k^F .
4. Construct the simulated counterfactual productivity distribution $F_k^{sim}(z_{ki})$ (i.e., by simulating the economy with random productivity draws).
5. As in the benchmark economy, solve for firm-level market shares as a fixed point problem:
 - (a) Guess market shares s_{ki}^F , and update using Eq. (19) and (20).
 - (b) Normalize such that the updated market shares add up to 1; iterate on s_{ki}^F .
6. Compute markups, prices, and based on the guess of P_k^F , update M_k^F from Eq. (4); iterate on M_k^F until convergence.
7. Using c_k^d , compute expected profits at the cutoff productivity level $\pi_k^{F,E}(\bar{z}_k^F)$ using the zero profit condition, i.e., Eq. (13). Iterate on \bar{z}_k^F until convergence. For example, if $\pi_k^{F,E}(\bar{z}_k^F)$ is negative, move \bar{z}_k^F up.
8. Using $F_k(\bar{z}_k^F)$, $f_k^{sim}(z_{ki})$, c_k^d , c_k^e , and expected profits $\pi_k^{F,E}(z_{ki})$, check the free entry condition in Eq. (15). Update P_k^F using the free entry condition, and iterate on P_k^F until convergence.

This gives us the endogenous variables P_k^F , \bar{z}_k^F , M_k^F , and $F_k(z_{ki}) \forall z_{ki} \geq \bar{z}_k^F$ for every sector in the counterfactual economy. It is then straightforward to compute pass-through rates α_{ki}^F from Eq. (21) and subsequently, aggregate volatility $\sigma[\hat{Y}^F]$ from Eq. (24). The estimated values of c_k^d and c_k^e are presented in Appendix Table B.3.

Lastly, our results are also robust to other choices of the acquirer's bargaining power β . In general, the increase in volatility (relative to the counterfactual economy) is greater for larger values of β . The acquirer must offer the target an acquisition price equal to the target's value plus its share of the merger gains: $V_k(z_{ki}^t) + (1-\beta)\Sigma_k(z_{ki}^a, z_{ki}^t)$. Thus, as β increases, all else equal, the acquisition price falls. This makes M&A deals cheaper for the acquirer and the merger rate rises, thereby the firm-size distribution associated with the productivity distribution G becomes more fat-tailed relative to F . The relationship is non-monotonic, especially for intermediate values of β , because the change in β also induces an endogenous response in the merger market, e.g., firms' search intensities as acquirers and targets. At $\beta = 0.1$ and 0.9 , the total effect of mergers in the single-sector economy under $\chi = 0$ is 1.037 and 1.044, respectively. In the

Table B.3: Estimated Costs Across Sectors

Codes	Sector	Fixed cost of production ($c_k^d, \times 10^{-5}$)	Sunk cost of entry ($c_k^e, \times 10^{-4}$)
10-12	Food, beverages, and tobacco	7.51	1.83
13-15	Textiles and apparel	2.02	0.58
16-18	Wood and paper	3.49	0.47
19-23	Chemicals, plastics, and non-metallic mineral products	7.33	1.57
24-25	Metal and fabricated metal products	3.28	0.66
26-27	Computers, electronics, and electrical equipment	4.05	1.16
28	Machinery	3.56	1.02
29-33	Transport equipment, furniture, and other manufacturing	2.84	0.76
41-43	Construction	2.63	0.41
45-46	Wholesale trade	11.4	1.56
47	Retail trade	4.10	0.74
49-53	Transportation and storage	5.14	0.87
55-56	Accommodation and food service activities	1.51	0.30
58-63	Information and communication	5.40	1.14
68	Real estate	2.50	0.58
69-75	Professional, scientific, and technical activities	3.16	0.70
77-82	Administrative and support service activities	3.14	0.70

Notes: Estimated fixed costs of production and sunk costs of entry from model estimation. The median value over 1,001 random samples is computed and shown.

multi-sector economy with incomplete pass-through, the corresponding results are 1.116 and 1.126.

B.4 Sector heterogeneity

The results presented in Table 4 for the multi-sector economy provide estimates of the aggregate effect of mergers as the sales-weighted average over sectors. However, the outcomes also differ across sectors, and exploring this heterogeneity can be helpful in further understanding the mechanisms involved. In particular, some sectors are more concentrated than others. These differences in market concentration are driven by the variation of the underlying productivity distributions. In Appendix Table B.1, the correlation coefficient between the Herfindahl-Hirschman indices of the 17 sectors and the estimated Pareto shape parameters ξ_k is -0.89 , indicating a strong negative relationship.

Our analysis in Section 2.2.3 demonstrated that the reduction in volatility from endogenous markups is increasing in firm size. In Figure 3(b), the curve representing the linear combination of the log-log and log-linear functions becomes steeper as market shares rise. In sectors with higher market concentration, there are relatively more and/or more dominant large players in the market. This suggests that the dampening effect from endogenous markups is relatively more important. In other words, we predict firm-level volatility to decline more for the acquirers in sectors with a high HHI, which in turn implies a greater mitigation of the rise in aggregate fluctuations.

To test this hypothesis, we take the ratios of volatilities between the benchmark and counterfactual economies by sector. We do this separately for the cases of incomplete pass-through

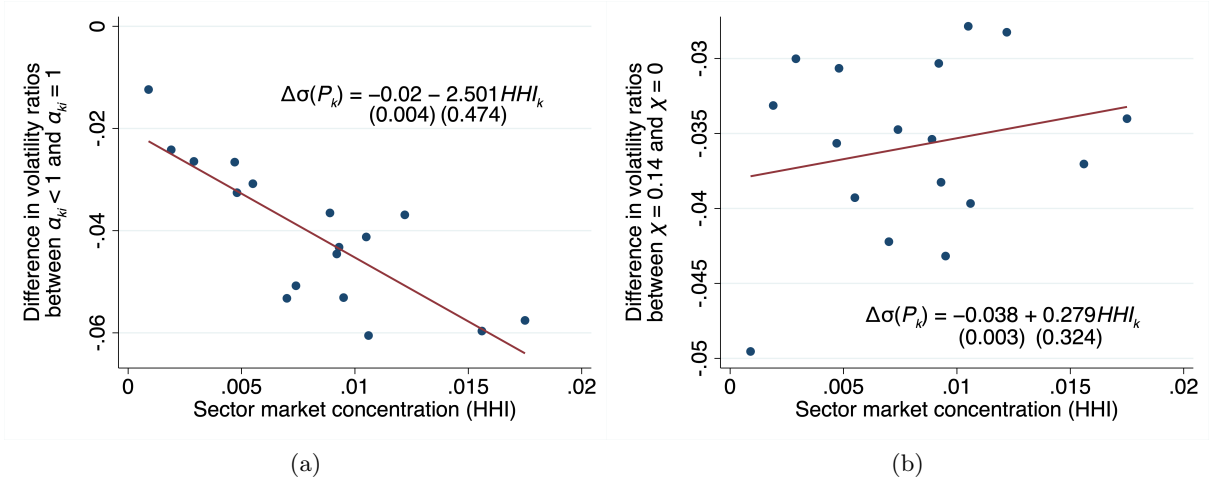


Figure B.7: This figure plots the model-implied difference in volatility ratios between (a) the cases of incomplete and complete pass-through, and (b) the cases in which the variance of shocks declines with size (i.e., $\chi = 0.14$) and in which shocks are i.i.d., against sector market concentration as measured by the Herfindahl-Hirschman index in the data.

$\alpha_{ki} < 1$ and complete pass-through $\alpha_{ki} = 1$. Then, for each sector, we subtract the ratio computed under incomplete pass-through by the ratio obtained under complete pass-through. That is, we calculate:⁴

$$\frac{\sigma[\hat{P}_k^G(\bar{z}_k|\alpha_{ki} < 1, \chi = 0.14)]}{\sigma[\hat{P}_k^F(\bar{z}_k^F|\alpha_{ki} < 1, \chi = 0.14)]} - \frac{\sigma[\hat{P}_k^G(\bar{z}_k|\alpha_{ki} = 1, \chi = 0.14)]}{\sigma[\hat{P}_k^F(\bar{z}_k^F|\alpha_{ki} = 1, \chi = 0.14)]}$$

and multiply by 100 to obtain the percentage point difference. At the aggregate level with $\sigma[\hat{Y}]$, we know from Table 4 that this difference in outcomes between the cases of incomplete and complete pass-through is negative. This is also true with $\sigma[\hat{P}_k]$. Importantly, we expect sectors with higher market concentration to have a more negative difference. Appendix Figure B.7(a) plots the difference in ratios against HHI for the 17 sectors, and we find a strong negative correlation as predicted. This provides further evidence consistent with our theory that firm-level volatility falls disproportionately by size, due to firms' strategic market power and endogenous markups.

We can repeat this exercise to examine whether differences in sector market concentration can also be explained by the heterogeneity in the variance of shocks. In Appendix Figure B.7(b), we plot

$$\frac{\sigma[\hat{P}_k^G(\bar{z}_k|\alpha_{ki} < 1, \chi = 0.14)]}{\sigma[\hat{P}_k^F(\bar{z}_k^F|\alpha_{ki} < 1, \chi = 0.14)]} - \frac{\sigma[\hat{P}_k^G(\bar{z}_k|\alpha_{ki} < 1, \chi = 0)]}{\sigma[\hat{P}_k^F(\bar{z}_k^F|\alpha_{ki} < 1, \chi = 0)]}$$

against market concentration. Firm-level volatility is a weighted average of the log-log and log-linear components, and the former dominates when \mathfrak{s}_{ki} is small. Thus, in contrast to the variable markup channel, mergers between small firms play a more important role in mitigating

⁴Note that with $\phi = 1$, sectoral market sizes \mathcal{S}_kPY are also fixed and $\hat{Y}_k = -\hat{P}_k$. We assume $\chi = 0.14$, but the results are quantitatively similar with $\chi = 0$.

aggregate volatility when market concentration is low. The expression above is expected to be more negative in this case. Conversely, the contribution of χ as a dampening force is smaller when there are many large firms, so we predict the expression to be closer to zero. Indeed, Appendix Figure B.7(b) shows a positive correlation, albeit weak and statistically insignificant. The assumed log-log (i.e., power law) relationship implies that volatility declines proportionally with firm size, which may explain why strong differences are not observed across sectors.

B.5 Additional regression results

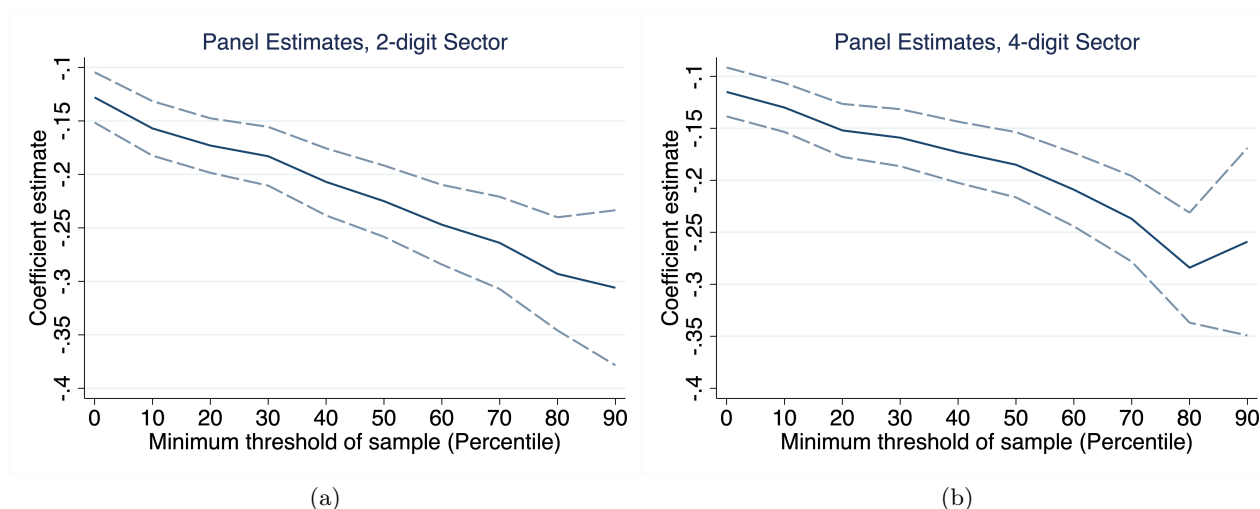


Figure B.8: This figure plots coefficient estimates and 95% confidence intervals from panel regressions of (log) volatility on (log) firm size, defining sectors at the NACE Rev. 2 (a) 2-digit and (b) 4-digit levels. This is shown for samples restricted to a minimum size threshold at the 0th, 10th, ..., 80th, and 90th percentiles.

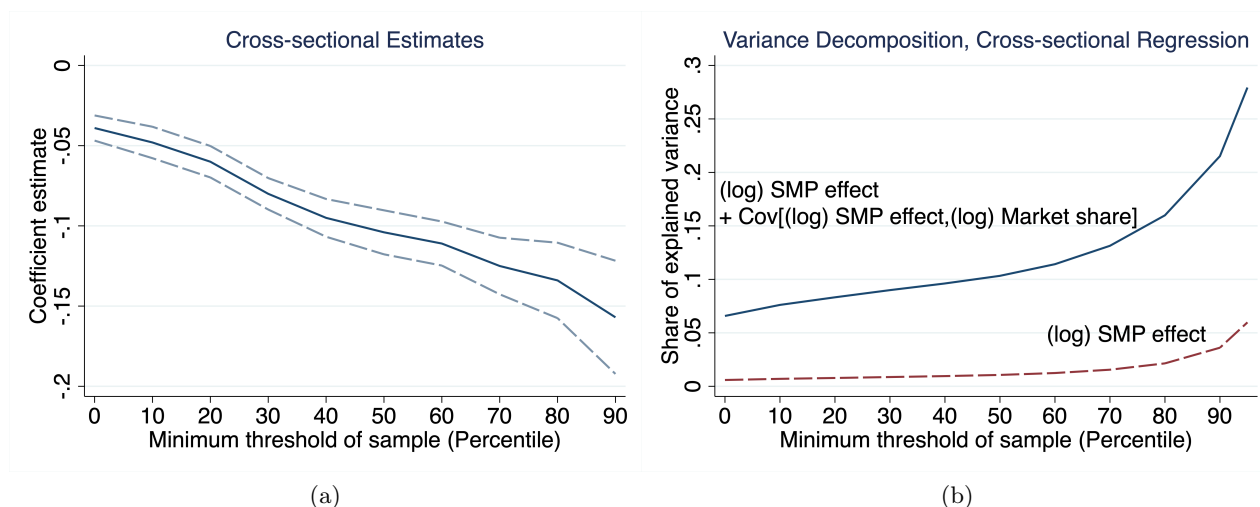


Figure B.9: In panel (a), we plot coefficient estimates and 95% confidence intervals from cross-sectional regressions of (log) volatility on (log) firm size across samples restricted to a minimum size threshold at the 0th, 10th, ..., 80th, and 90th percentiles. In panel (b), we plot the share of explained variance in (log) volatility accounted for by the log SMP effect and the covariance between the log SMP effect and log market share. This is shown for samples restricted to a minimum size threshold at the 0th, 10th, ..., 90th, and 95th percentiles. The variance decomposition is based on the cross-sectional regression with sector-period fixed effects partialled out.

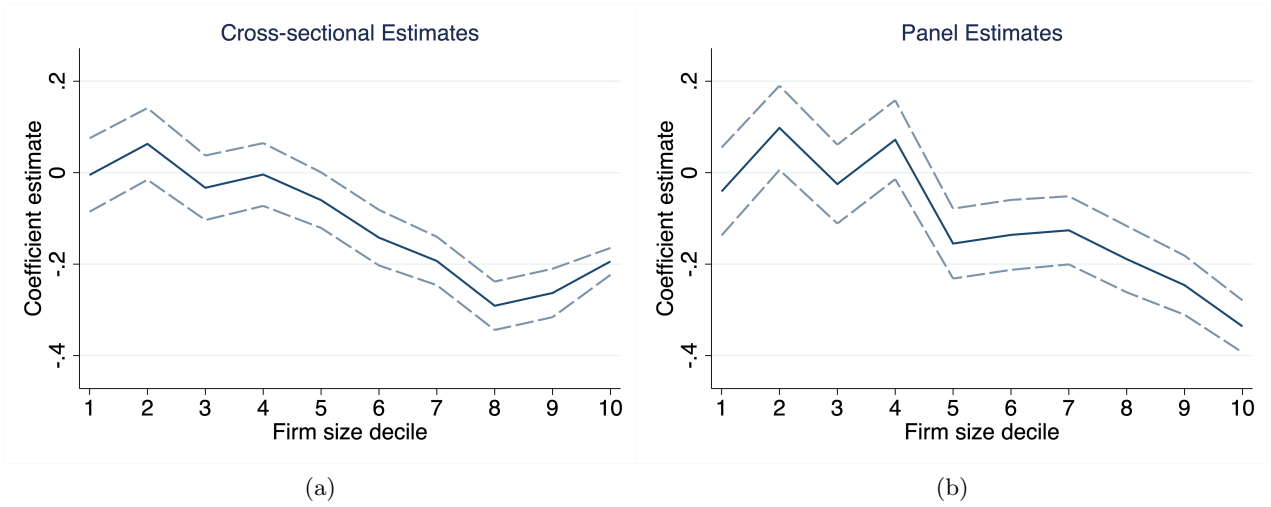


Figure B.10: This figure plots coefficient estimates and 95% confidence intervals from (a) cross-sectional and (b) panel regressions of (log) volatility on (log) firm size separately by decile. Deciles are constructed based on firms' sector and *initial* period in the sample.

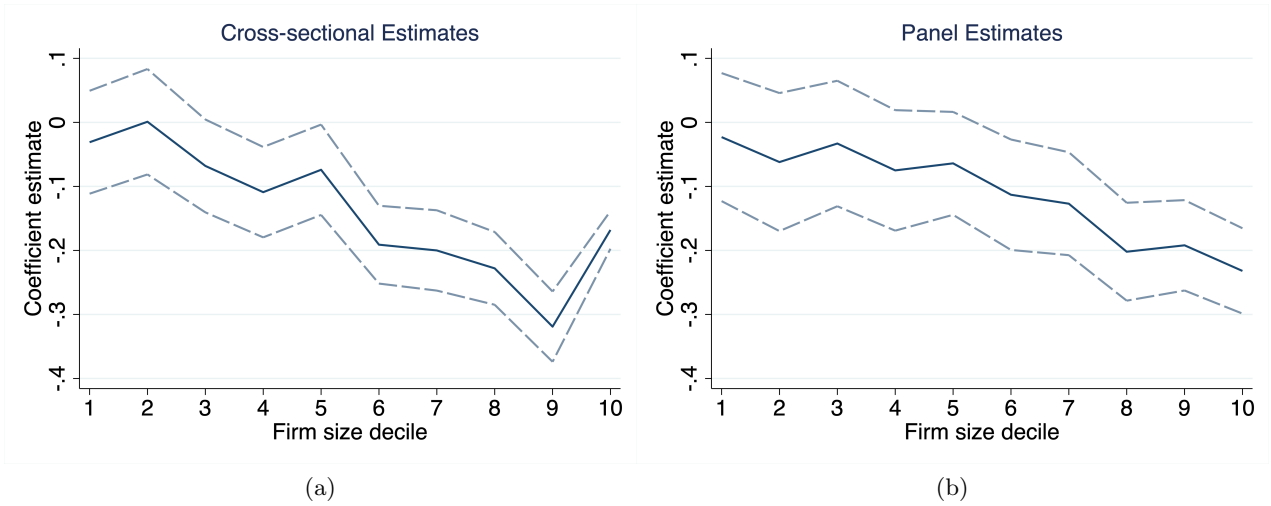


Figure B.11: This figure plots coefficient estimates and 95% confidence intervals from (a) cross-sectional and (b) panel regressions of (log) volatility on (log) firm size separately by decile. Deciles are constructed based on firms' sector and *final* period in the sample.

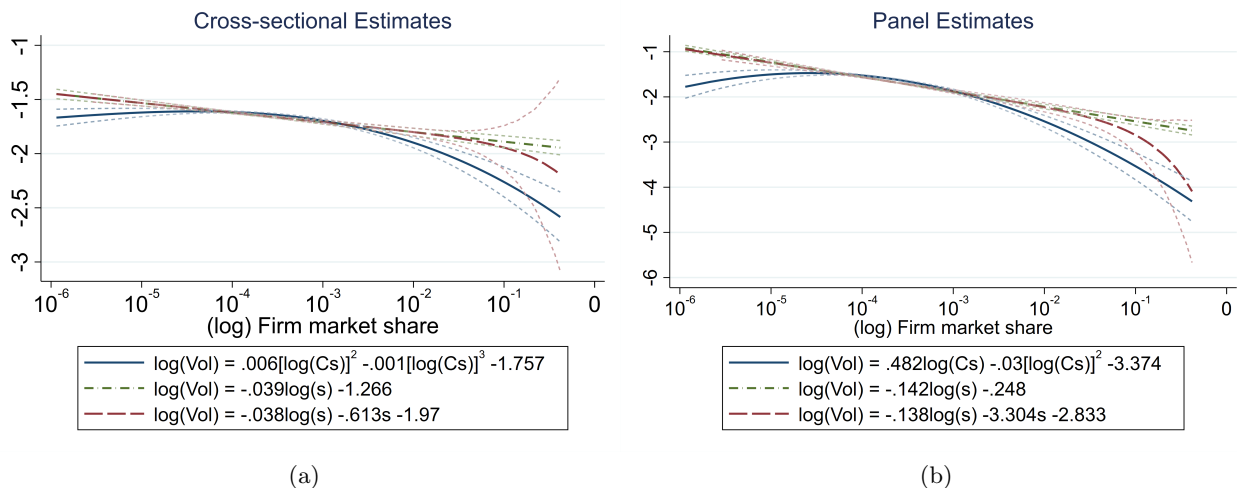


Figure B.12: This figure plots the predicted size-volatility relationship using (a) cross-sectional and (b) panel regressions with (i) fractional polynomials, (ii) the log-log function, and (iii) the linear combination of the log-log and log-linear functions. To estimate the fractional polynomials, market shares are scaled by the constant $C = 10^8$ such that their logarithm is positive. We employ a second-degree fractional polynomial with powers among the set $\{-2, -1, -0.5, 0, 0.5, 1, 2, 3\}$, where a power of 0 is defined as the logarithm, and select the best-fitting polynomial based on the lowest model deviance.

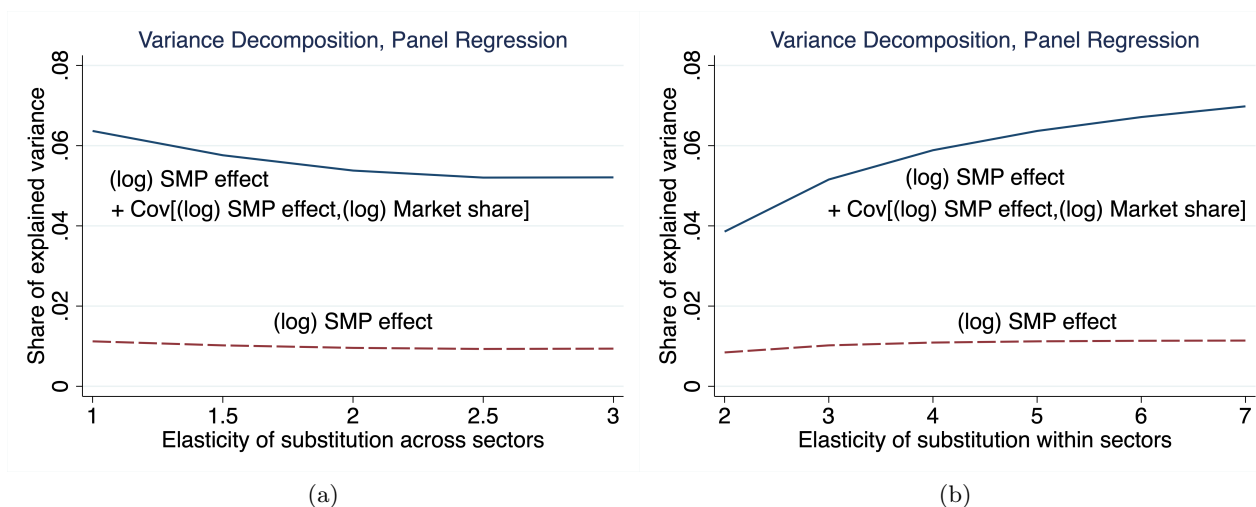


Figure B.13: This figure plots the share of explained variance in (log) volatility accounted for by the log SMP effect and the covariance between the log SMP effect and log market share for varying values of (a) ϕ and (b) ϵ . The variance decomposition is based on panel regressions with firm and sector-period fixed effects partialled out. From Eq. (21), pass-through rates are increasing in ϕ and decreasing in ϵ , i.e., $\frac{\partial \alpha_{ki}}{\partial \phi} > 0$ and $\frac{\partial \alpha_{ki}}{\partial \epsilon} < 0$. Thus, as ϕ rises, α_{ki} also increases, and we see from Eq. (22) that price fluctuations depend more on the firm's own shock and less on strategic market power (i.e., through the sectoral price index). The contribution of the SMP effect generally declines as a result. Conversely, as α_{ki} falls with higher ϵ , the SMP effect becomes more important in explaining firm-level volatility.

Table B.4: Estimates from Log-Linear Relationship

Dep. var.	(log) Volatility	
	(1)	(2)
Market share	-3.955** (1.640)	-10.007*** (2.293)
Sector-period FE	Y	Y
Firm FE		Y
N	49,064	49,064
R ²	0.13	0.63

Notes: Volatility is measured by the standard deviation of domestic sales growth over a 5-year period (i.e., 1995-9, 2000-4, 2005-9, and 2010-4). Standard errors in parentheses clustered at the firm level. ***, **, and * indicate statistical significance at the 1%, 5%, and 10% level.

Table B.5: Alternative Sector Definitions

Sector	2-digit			4-digit		
	(log) Volatility		(log) Adjusted volatility	(log) Volatility		(log) Adjusted volatility
Dep. var.	(1)	(2)	(3)	(4)	(5)	(6)
(log) Market share	-0.128*** (0.012)	-0.127*** (0.012)	-0.113*** (0.012)	-0.115*** (0.012)	-0.110*** (0.013)	-0.032*** (0.012)
Market share		-0.293 (0.654)			-0.225 (0.206)	
Sector-period FE	Y	Y	Y	Y	Y	Y
Firm FE	Y	Y	Y	Y	Y	Y
N	48,848	48,848	48,848	48,848	48,848	48,848
R ²	0.63	0.63	0.63	0.65	0.65	0.67

Notes: Volatility is measured by the standard deviation of domestic sales growth over a 5-year period (i.e., 1995-9, 2000-4, 2005-9, and 2010-4). “Adjusted volatility” in columns 3 and 6 is computed as volatility minus the strategic market power effect on volatility assuming $\phi = 1$ and $\varepsilon = 5$. Standard errors in parentheses clustered at the firm level. ***, **, and * indicate statistical significance at the 1%, 5%, and 10% level.

Table B.6: Event-Study Regression Estimates

Estimation method	OLS	OLS	Imputation estimator
	(1)	(2)	(3)
Merger >+5	-0.154*** (0.058)	-0.150*** (0.035)	
Merger +5	-0.171*** (0.046)	-0.166*** (0.032)	-0.176*** (0.032)
Merger +4	-0.118*** (0.039)	-0.115*** (0.028)	-0.117*** (0.030)
Merger +3	-0.185*** (0.036)	-0.183*** (0.030)	-0.188*** (0.032)
Merger +2	-0.087*** (0.027)	-0.084*** (0.023)	-0.071*** (0.028)
Merger +1	-0.084*** (0.021)	-0.084*** (0.019)	-0.043* (0.025)
Merger year	–	–	-0.008 (0.042)
Merger –1	-0.054** (0.022)	-0.054*** (0.021)	-0.060 (0.041)
Merger –2	0.025 (0.026)	0.024 (0.022)	0.015 (0.039)
Merger –3	-0.008 (0.035)	-0.016 (0.028)	-0.026 (0.038)
Merger –4	0.015 (0.042)	0.005 (0.032)	-0.001 (0.035)
Merger –5	0.018 (0.051)	0.007 (0.038)	0.003 (0.033)
Merger <–5	0.037 (0.064)	0.009 (0.041)	
Sector-year FE	Y	Y	Y
Firm FE	Y	Y	Y
N	21,033	691,872	685,863
R ²	0.49	0.61	

Notes: The dependent variable is (log) volatility. The computation of volatility excludes the year after the merger for $t - MergerYear_{ki} \in \{-1, 0, +1, +2, +3\}$. Column 3 employs the imputation estimator from Borusyak et al. (2024). Standard errors in parentheses clustered at the firm level. ***, **, and * indicate statistical significance at the 1%, 5%, and 10% level.

Table B.7: Post-Merger Increase in Firm Size

Dep. var.	(log) Domestic sales		(log) Total sales	
	90	99	90	99
Threshold percentile	(1)	(2)	(3)	(4)
PostMerger	0.107*** (0.023)	0.171** (0.065)	0.133*** (0.022)	0.182*** (0.066)
Sector-Year FE	Y	Y	Y	Y
Firm FE	Y	Y	Y	Y
N	3,971	414	3,971	414
R ²	0.91	0.96	0.93	0.96

Notes: The dependent variable is (log) deflated domestic or total sales. Acquirers above the threshold percentile of firm size across all firms are included in the sample. Standard errors in parentheses are clustered at the firm level. ***, **, * denote significance level at 1%, 5% and 10% respectively.