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Regular Article

Formal insurance and altruism networks[☆]

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ABSTRACT

We study how altruism networks affect the demand for formal insurance. Agents with CARA utilities are connected through a network of altruistic relationships. Incomes are subject to a common shock and to a large individual shock, generating heterogeneous damages. Agents can buy formal insurance to cover the common shock, up to a coverage cap. We find that *ex-post* altruistic transfers induce interdependence in *ex-ante* formal insurance decisions. We characterize the Nash equilibria of the insurance game and show that agents act as if they are trying to maximize the expected utility of a representative agent with average damages. Altruism thus tends to increase demand of low-damage agents and to decrease demand of high-damage agents. Its aggregate impact depends on the interplay between demand homogenization, the zero lower bound and the coverage cap. We find that aggregate demand is higher with altruism than without altruism at low prices and lower at high prices. Nash equilibria are constrained Pareto efficient.

1. Introduction

The poor in poor countries generally face large risks, especially when it comes to health (illnesses, accidents) and livelihood (climate events), see [Banerjee and Duflo \(2011\)](#). These risks are a major source of stress and reduced well-being, as well as a likely cause of poverty traps. Many such risks could, in principle, be covered by formal insurance, like public universal health coverage and crop and livestock insurances offered by insurance companies. In the past 40 years, governments and development institutions have worked hard to make formal insurance accessible to households in need. Disappointingly, however, these efforts have often encountered low take-up, e.g., [Cole et al. \(2013\)](#). A recent industry report estimates that just 7% of the value of the microinsurance market in developing countries is currently captured, see [Merry and Rozo Calderon \(2022\)](#).

One likely explanation for such limited adoption of formal insurance in high-risk contexts is informal safety nets, which may act as barriers to formal insurance.¹ There is widespread evidence that social networks help individuals and households cope with negative shocks through informal financial transfers and help in kind. These transfers and assistance are motivated, to a large extent, by altruism, as individuals give

to others they care about.² How do effective altruism networks, then, affect the demand for formal insurance? Does altruism always reduce the adoption of formal insurance? These questions have, so far, been neglected; we review the scant literature on the interaction between formal and informal insurance below.

This paper provides the first analysis of how altruism networks affect the demand for formal insurance. We consider a community of agents who care about each other. Agents face both a common and an individual shock and can buy formal insurance to cover the common shock. Once shocks and insurance claims are realized, agents make private transfers to each other to support friends in need. We find that altruism networks have a profound impact on demand for formal insurance. Under altruism, an agent anticipates that her own insurance decision will affect the outcomes of others she cares about. *Ex-post* altruistic transfers thus induce interdependence in *ex-ante* decisions to buy formal insurance. In our benchmark model, agents have utilities with Constant Absolute Risk Aversion, heterogeneous damages, and can buy any amount of formal insurance up to a coverage cap. We find that altruism tends to homogenize the demands for formal insurance. It increases demand of low-damage agents and decreases

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¹ Other explanations include price and income effects, liquidity constraints, mistrust, and lack of experience with insurance products, see [Platteau et al. \(2017\)](#). These explanations are not mutually exclusive. We analyze how price effects interact with altruism networks in determining the demand for formal insurance.

² Informal transfers may notably be motivated by altruism, social pressure, and informal insurance. We review the literature documenting evidence on altruism below.

demand of high-damage agents. The overall impact then depends on the interplay between damage heterogeneity, the zero lower bound and the coverage cap. We find that the demand for formal insurance is higher with altruism than without altruism at relatively low prices and lower at relatively high prices. Altruism networks and formal insurance are thus complements at low prices and substitutes at high prices. Overall, our analysis shows that an appropriate description of the way informal safety nets operate is key to understanding the determinants and impacts of formal insurance adoption.

We introduce formal insurance into the model of altruism in networks studied in [Bourlès et al. \(2017, 2021\)](#). Agents are embedded in a fixed altruism network, describing the structure of social preferences in the community. An agent's altruistic utility is a linear combination of her private CARA utility and the private utilities of others she cares about. We consider a connected altruism network: any agent can be reached from any other agent through a directed path of caring relationships. We assume that the common and the idiosyncratic shocks are binary and independent, generate heterogeneous damages, and that the idiosyncratic shock is large and only affects one agent at a time. This guarantees that a directed path of transfers flows from any other agent to the affected agent in equilibrium, a key simplifying assumption ([Assumption 1](#)). We develop our analysis in several stages.

We first obtain an explicit characterization of the Nash equilibria of the insurance game ([Theorem 1](#)). Under altruism, the insurance game displays strategic substitutes: an agent's demand for formal insurance decreases when others buy more formal insurance. We show that all agents act as if they are trying to maximize the utility of a representative agent with average damages and average demand. In equilibrium, the average demand of altruistic agents is thus equal to the demand of a selfish agent with average damages. Even though this average demand is well-defined, Nash equilibria and individual demands are generally indeterminate.

Second, we introduce a natural selection criterion to address equilibrium indeterminacy. We say that a Nash equilibrium is robust to conformism when it remains an equilibrium when adding vanishingly small conformist pressures. We then show that there is a unique Nash equilibrium robust to conformism, which minimizes variance over all equilibria ([Proposition 2](#)). In this equilibrium, demand for formal insurance of low-damage agents is larger than without altruism while demand for formal insurance of high-damage agents is lower. Altruism thus tends to homogenize the demands for formal insurance.

Third, we compare the aggregate demand for formal insurance with and without altruism, both in the absence of a cap ([Theorem 2](#)) and when the cap is binding for all ([Theorem 3](#)). In the absence of a cap, altruism is neutral at low prices and reduces demand at high prices. When the price is relatively high, low-damage agents are constrained by the zero lower bound on the amount of formal insurance they can buy while high-damage agents are unconstrained. This reduces the increases in demand for low-damage agents induced by altruism, and leads to a negative aggregate impact. When the cap is binding for all, altruism increases demand at low prices and reduces demand at high prices. When the price is relatively low, high-damage agents are now constrained by the coverage cap while low-damage agents are unconstrained. This lowers the reductions in demand for high-damage agents induced by altruism, and leads to a positive aggregate impact. Overall, we find that altruistic transfers and formal insurance are complements at low prices and substitutes at high prices.

Fourth, we analyze welfare and show that the Nash equilibria of the insurance game are constrained Pareto efficient ([Proposition 3](#)). Conditional on the constraint that informal transfers are obtained as a Nash equilibrium, individual incentives to adopt formal insurance are thus aligned with social welfare. This remarkable feature relies on the representative agent's property, which guarantees that payoffs all move in the same direction. This provides a new context where a counterpart to the first welfare theorem holds in the presence of strategic interactions.

Our analysis contributes, first, to the literature on the economics of altruism, initiated by [Barro \(1974\)](#) and [Becker \(1974\)](#).³ Altruism appears to be a main motive behind informal transfers. Studies finding evidence that some transfers are altruistically motivated include [Foster and Rosenzweig \(2001\)](#), [Leider et al. \(2009\)](#), [De Weerd and Fafchamps \(2011\)](#), [Ligon and Schechter \(2012\)](#) and [Fafchamps and Heß \(2021\)](#). For instance in a study on rural Tanzania, [De Weerd and Fafchamps \(2011\)](#) find that people with persistent health shocks and chronic disabilities receive net support from family and friends. This indicates that altruism, rather than a reciprocated insurance arrangement, is operative. Altruism likely explains a large proportion of family remittances, a main source of income for many poor households, e.g., [Yang \(2011\)](#). We focus on this motive in this paper, and provide the first analysis of the impact of altruism networks on formal insurance.

Our analysis contributes, second, to the literature on the interactions between informal transfers and formal insurance.⁴ [Arnott and Stiglitz \(1991\)](#) showed early on that informal risk-sharing can crowd out demand for formal insurance.⁵ In their framework, informal risk-sharing takes place within pairs of symmetrical agents and under moral hazard. By contrast, we consider altruism networks connecting heterogeneous agents and without moral hazard, and find that altruistic transfers can be a complement to formal insurance at low prices. In an empirical study on rural India, [Rosenzweig \(1988\)](#), finds that private transfers in networks of families and friends play a central role in risk-sharing, and often crowd out formal loans. [Kinnan and Townsend \(2012\)](#) analyze data on formal and informal loans in rural Thailand. They find evidence of large network spillovers: having an indirect connection to a household with a formal loan has the same, strong impact on consumption smoothing than having a formal loan. These findings are consistent with our theoretical results. In our setup when one agent adopts formal insurance, every agent indirectly connected in the altruism network benefits. [De Janvry et al. \(2014\)](#) analyze the demand for formal insurance against common shocks, when individual utility depends on individual and aggregate wealth. They highlight the strategic interactions and free-riding in individual decisions to adopt formal insurance. While our setup differs in important ways, our analysis confirms the key insight that in the presence of informal transfer arrangements, individual decisions to adopt formal insurance are interdependent.⁶ We show that these strategic interactions do not necessarily lead to free-riding, however. In our context, even though individual decisions to adopt formal insurance are strategic substitutes, Nash equilibria are constrained Pareto efficient. Overall, we provide the first analysis of demand for formal insurance when agents make informal transfers through networks.

A recent branch of the literature on formal and informal insurance studies index insurance, an innovative financial product where transfers received by agents depend on an objective index, such as the amount of rainfall measured at a weather station. A key feature of index insurance, however, is that it carries basis risk. Agents have a risk of paying for the insurance and suffering losses without being indemnified.⁷ Several studies find empirical evidence that the demand for index insurance rises with increased informal insurance, see [Mobarak and Rosenzweig](#)

³ See [Galperti and Strulovici \(2017\)](#) and [Ray and Vohra \(2020\)](#) for recent theoretical studies of economic models of altruism.

⁴ One branch of this literature looks at how the introduction of formal insurance affects existing informal arrangements, see, e.g., [Attanasio and Ríos-Rull \(2000\)](#), [Boucher et al. \(2016\)](#) and [Takahashi et al. \(2019\)](#).

⁵ [Ehrlich and Becker \(1972\)](#) show that formal insurance and self-insurance are substitutes, where self-insurance is defined as costly actions that an agent can take to reduce damages from the shock.

⁶ In our setup, formal insurance only covers the common shock rather than overall wealth fluctuations and individual utility does not depend on individual and aggregate wealth.

⁷ Strictly speaking, index insurance should thus be classified as a derivative contract rather than an insurance contract, see [Clarke \(2016\)](#).

(2012, 2013), Dercon et al. (2014) and Berg et al. (2020). These studies develop models where individuals informally share risk in a group, and the complementarity between index insurance and informal risk-sharing arises because informal insurance helps cover the basis risk. By contrast, we consider a standard indemnity insurance in our setup, without basis risk, and networks of altruistic relationships. We show that the demand for formal insurance can be higher with altruism than without, in the presence of a coverage cap and damage heterogeneity.

Our analysis contributes, third, to a literature on informal transfers and networks.⁸ Ambrus et al. (2014) characterize Pareto-constrained risk-sharing arrangements when transfers flow through networks and links can be used as social collateral. Ambrus et al. (2022) analyze Pareto-constrained risk-sharing arrangements under local informational constraints. Bourlès et al. (2017) consider a network of altruistic relationships and characterize the Nash equilibria of the game of transfers for non-stochastic incomes. Bourlès et al. (2021) look at altruism networks when incomes are stochastic. None of these studies consider formal insurance, however. We introduce formal insurance into this literature, and provide the first analysis of the interplay between formal insurance and informal transfers through networks.⁹ We find that altruism networks have a first-order impact on demand for formal insurance.

The remainder of the paper is organized as follows. We introduce our framework in Section 2. We analyze the insurance game and the impact of altruism networks on the demand for formal insurance in Section 3. We provide a concluding discussion in Section 4.

2. Framework

We introduce formal insurance into the model of altruism in networks studied by Bourlès et al. (2017, 2021). Consider a community of $n \geq 2$ altruistic agents. Incomes are stochastic, and subject to a common shock and to an individual shock. The common shock affects all agents and both shocks generate heterogeneous damages. An external institution sells a formal insurance covering damages from the common shock. Each agent decides, *ex-ante*, how much formal insurance to buy, up to a coverage cap. Once incomes and insurance claims are realized, altruistic agents make informal transfers to each other. We assume that agents act non-cooperatively in their formal insurance and informal transfer decisions. The model thus has 3 stages. In stage 1, agents decide how much formal insurance to buy. In stage 2, income shocks and insurance claims are realized. In stage 3, agents make private transfers, conditional on realized incomes.

Stochastic Incomes. Agent i has baseline wealth w_i and faces a common and an individual shock. We consider binary shocks for simplicity.¹⁰ Denote by \bar{I}_c a binary random variable indicating whether the common shock occurs: $\bar{I}_c = 1$ with probability q_c and 0 with probability $1 - q_c$. This common shock yields an income loss of $\mu_i > 0$ for agent i . This shock could represent a problematic weather event, such as heavy rainfalls or a drought, affecting farmers' crops or pastoralists' livestock. It could also represent a natural catastrophe, like a flood or an earthquake. While all agents in the community are affected, some agents may suffer higher losses than others due to a higher risk exposure.

⁸ One branch of the literature analyzes the stability of risk-sharing networks, see, e.g., Bloch et al. (2008) and Bramoullé and Kranton (2007).

⁹ Gagnon and Goyal (2017) develop a model where agents choose a network and a market binary action. They assume that the two actions are either substitutes or complements, and analyze equilibria, welfare, and inequality. By contrast, market and network actions are not binary in our setup and whether the two actions are substitutes or complements is not assumed, but rather a main outcome of the analysis.

¹⁰ The assumption that shocks are binary is common in the literature on insurance, see, e.g., Arnott and Stiglitz (1991), Berg et al. (2020), Clarke (2016), De Janvry et al. (2014) and Mobarak and Rosenzweig (2012).

Agents may also suffer from an individual shock, independent from the common shock. Denote by $\tilde{\lambda}_i$ the random variable capturing agent i 's stochastic income loss from the individual shock: $\tilde{\lambda}_i = \lambda_i > 0$ with probability q_i and 0 with probability $1 - q_i$. We assume that one agent, and only one, is affected. This means that $\sum_i q_i = 1$ and $\tilde{\lambda}_i > 0 \Rightarrow \tilde{\lambda}_j = 0$ for $j \neq i$. We will assume below that these individual shocks are large, representing a serious adverse event such as an incapacitating accident or illness, or the death of a household member.

Taken together, the facts that the individual shock is large and only affects one agent help structure the way informal transfers flow through the altruism network. It leads all agents to make direct or indirect transfers eventually reaching the agent hit by the shock, as expressed in Assumption 1 and Lemma 1 below. These assumptions thus provide a natural benchmark allowing us to manage the complexity inherent to networks.¹¹ We discuss the implications of relaxing these assumptions, and exploring richer stochastic structures and network patterns, in Section 4.

To sum up, agent i faces stochastic income

$$w_i - \mu_i \bar{I}_c - \tilde{\lambda}_i. \quad (1)$$

Note that this formalization captures a large range of heterogeneities: in baseline wealth, w_i , damage from the common shock, μ_i , damage from the individual shock, λ_i , and in probability to be affected by the individual shock, q_i .

Formal insurance. An external institution offers insurance contracts that cover damages from the common shock. This could represent an agricultural microinsurance covering crops or livestock. This could also be a flood or earthquake insurance. By contrast, we assume that the individual shock cannot be formally insured. This is consistent with the fact that formal insurance schemes are expanding in a very uneven way in poor countries, depending on idiosyncratic factors such as targeted government interventions and specific business initiatives, see Merry and Rozo Calderon (2022). At this stage, it thus not uncommon for farmers in poor villages to have to decide whether to adopt some crop microinsurance even when they do not have access to formal health and funeral insurances.¹²

Formal insurance has unit price $p \geq 0$ and coverage cap $D \geq 0$. Agent i may buy a quantity x_i of formal insurance, where $0 \leq x_i \leq \min(D, \mu_i)$, at cost $p x_i$. This quantity is bounded from above by the coverage cap and by the individual-specific damage. Let $D_i = \min(D, \mu_i)$ represent the maximal amount of insurance that agent i can buy. Caps on insurable damages are common features of insurance contracts, see Cummins and Mahul (2004). They help insurers limit moral hazard and fraud, two major concerns with microinsurance. Caps are also prevalent in disaster insurance, when the financial capacity of the insurance institution may be limited by the extent of global losses.

To facilitate exposition and algebra, we derive some of our results below in the two benchmark cases of no cap, when all agents can be fully covered and $\forall i, D_i = \mu_i$, and a binding cap, when no agent can be fully covered and $\forall i, D_i = D$. Our analysis extends to the general case where the cap is binding for agents with high damages, but not for agents with low damages.

If the common shock occurs, a part x_i of the damages is then covered by the formal insurance contract, and the effective income loss is equal to the uninsured part $\mu_i - x_i$. Agent i 's stochastic income at the end

¹¹ The assumption that one, and only one, agent is affected by a shock is also used for tractability in the literature on financial networks, see, e.g., Babus (2016) and Cabrales et al. (2017). We show that that our results extend to situations where several agents are affected by a shock when the network is complete in Section 4.

¹² Similarly, Attanasio and Ríos-Rull (2000) assume that agents face an idiosyncratic and an aggregate shock and that the idiosyncratic shock is not formally insured while the aggregate shock is insured by the government.

of stage 1, after formal insurance decisions but before realizations of shocks and informal transfers, is thus equal to

$$\tilde{y}_i = w_i - px_i - (\mu_i - x_i)\tilde{l}_c - \tilde{\lambda}_i. \tag{2}$$

We focus on the demand for formal insurance in our analysis, and take the characteristics of the insurance contract, its price and coverage limit, as given. Both characteristics of course reflect features of the supply side. On prices, an important benchmark is provided by the actuarial price $p = q_c$, equal to the equilibrium price in a competitive insurance market with free entry, no frictions and no administrative costs. These assumptions are very unlikely to hold in the emerging insurance markets of poor countries, characterized by high barriers to entry, frictions of all kinds, and high operational costs. All these factors push the price upwards.¹³ And indeed, a main private actor of the industry concludes that, compared to standard insurance contracts in rich countries, microinsurance contracts in poor countries often display higher premiums, see [Lloyd's \(2009\)](#) p.8.¹⁴ Thus, our working assumption is that the price of formal insurance is higher than the actuarial price, $p > q_c$, and that this price captures, in a reduced-form way, the extent of frictions in the local insurance market.

Informal transfers. In stage 3, once shocks and insurance claims are realized, altruistic agents make informal transfers to each other. We next describe how these transfers are determined. We adopt the framework of [Bourlès et al. \(2017\)](#). We consider a simultaneous-move game with complete information. Preferences and realized incomes are thus common knowledge among agents.

Let c_i denote consumption of agent i after informal transfers are realized. Let \mathbf{c}_{-i} denote the consumption profile of the other agents. Agents may care about each other. Preferences have a private and a social component. Agent i 's private preferences are represented by utility function $u_i : \mathbb{R} \rightarrow \mathbb{R}$ with Constant Absolute Risk Aversion (CARA):

$$u_i(c) = -e^{-Ac}. \tag{3}$$

Agent i may be altruistic towards others and her preferences are represented by the altruistic utility function $v_i : \mathbb{R}^n \rightarrow \mathbb{R}$ such that

$$v_i(c_i, \mathbf{c}_{-i}) = u_i(c_i) + \sum_{j \neq i} \alpha_{ij} u_j(c_j) \tag{4}$$

where $\alpha_{ij} \in [0, 1]$ represents the strength of the altruistic relationship between i and j . By convention, $\alpha_{ii} = 0$. The *altruism network* is represented by the matrix $\alpha = (\alpha_{ij})_{i,j=1}^n$, describing the structure of social preferences in the community.

Agent i can give $t_{ij} \geq 0$ to agent j . By convention, $t_{ii} = 0$. The collection of bilateral transfers defines a *network of transfers*, represented by the matrix $\mathbf{t} \in \mathbb{R}_+^{n \times n}$. Agent i 's consumption is equal to

$$c_i = y_i + \sum_{j \neq i} (t_{ji} - t_{ij}) \tag{5}$$

Since there is no transfer cost, informal transfers redistribute aggregate income among agents: $\sum_i c_i = \sum_i y_i$.

In this third stage, agents play a non-cooperative game. Agents make informal transfers to others in order to maximize their altruistic utility, conditional on transfers made by others. We assume that *the network of informal transfers is a Nash equilibrium of this transfer game*. The transfer network is therefore characterized by the following conditions,

¹³ By contrast, donors' and governments' subsidies would push the insurance price downwards.

¹⁴ One important source of price differences is that microinsurance contracts in poor countries typically feature common pricing and broad eligibility. Heterogeneity in risk is thus incorporated into higher premiums. By contrast, insurers in rich countries often practice screening and price discrimination based on individual attributes, see [Lloyd's \(2009\)](#) p. 7–8.

see [Bourlès et al. \(2017\)](#) for details. If $\alpha_{ij} > 0$, define $\kappa_{ij} = -\ln(\alpha_{ij})$ as a virtual cost associated with the link between i and j . Stronger links have lower virtual cost. Then, \mathbf{t} is a Nash equilibrium of the transfer game if and only if

$$\forall i, j, c_i \leq c_j + \frac{\kappa_{ij}}{A} \text{ and } t_{ij} > 0 \Rightarrow c_i = c_j + \frac{\kappa_{ij}}{A} \tag{6}$$

where, recall, A is the degree of absolute risk aversion. An agent does not let the consumption of someone she cares about fall too much below her own consumption.

For all profiles of incomes before transfers, a Nash equilibrium exists and the profile of equilibrium incomes after transfers is unique. This yields a well-defined mapping from incomes before transfers \mathbf{y} to incomes after transfers \mathbf{c} . With CARA utilities, this mapping has a complex piecewise linear shape which generally depends on details of income realizations and of the structure of the altruism network.

Since we are interested here in how operative informal transfers affect formal insurance take-up, we make the following simplifying assumption. Say that agent i_0 receives indirect support from the full community if for any $i \neq i_0$, there exists a path of informal transfers connecting i to i_0 , i.e., a set of distinct agents $j_1 = i, j_2, \dots, j_l = i_0$ such that for any $s < l, t_{j_s, j_{s+1}} > 0$. While amounts transferred are not necessarily large, every other agent in the community is involved in transfers eventually reaching agent i_0 .

Assumption 1. For any realization of income shocks and any profile of formal insurance decisions, the agent hit by the individual shock receives indirect support from the full community.

We show in the [Appendix](#) that for any connected altruism network α , there exists a threshold level on the magnitude of the idiosyncratic shock, $\tilde{\lambda}$, such that [Assumption 1](#) holds if $\forall i, \lambda_i \geq \tilde{\lambda}$. This threshold may be quite low when altruistic ties are strong and with homogeneous wealth and damages. It may be quite high, by contrast, when ties are weak or under high wealth and damage heterogeneity. To sum up, [Assumption 1](#) holds when the altruism network is connected and when the magnitude of the individual shock is high.

Note that this assumption does not mean that agents have aligned incentives. In sparse connected networks, like the star or the line, most agents only care about a small set of other agents. When i cares about j , j cares about k , and i does not care about k , the altruistic utility of i drops when j transfers money to k . Still, the interplay of altruistic behavior means that shocks propagate in the network. A shock on one agent induces support from her direct friends. If the shock is large, direct friends are, in turn, supported by their own friends, and so on, leading to indirect support from the full community. We further discuss [Assumption 1](#), and what happens when it does not hold, in [Section 4](#).

A key implication of [Assumption 1](#) is that we can simply express how consumption depends on incomes before transfers. Let $\hat{\kappa}_{ij}$ denote the virtual cost of a least-cost path connecting i to j in α and let $\hat{\kappa}_{ij} = -\ln(\hat{\alpha}_{ij})$. Transfers must flow through such least-cost paths in a Nash equilibrium. Note that when the altruism network is connected, there is a path connecting any two agents in it and these least costs are well defined for any pair of agents. Let $\bar{y} = \frac{1}{n} \sum_i y_i$ denote the average income before transfers in the community.

Lemma 1. Suppose that agent i_0 suffers from the individual shock and receives informal support from the full community. Then, for all i including i_0 ,

$$c_i = \bar{y} + \frac{\hat{\kappa}_{ii_0}}{A} - \frac{1}{n} \sum_j \frac{\hat{\kappa}_{ji_0}}{A}$$

We provide the proof of [Lemma 1](#) and of all other results in [Appendix](#). [Lemma 1](#) shows that income after transfers is equal to the sum of two terms: average income before transfers and a network term that depends on who is hit by the individual shock and of relative positions in the altruism network with respect to this agent. Income

after transfers tends to be lower for agents who are “closer” to i_0 in the altruism network. To see why, consider a binary altruism network where all links have the same strength, $\alpha_{ij} \in \{0, \alpha\}$. Then $\hat{\kappa}_{i_0}$ is simply proportional to the network distance between i and i_0 in the altruism network, i.e., the number of links in a shortest path between them. In that case, c_i is higher when i is more distant from i_0 . More generally, $\hat{\kappa}_{i_0}$ is a measure of distance between i and i_0 that extends the usual notion of network distance to account for the strength in altruistic relationships - a stronger link being associated with a lower distance.

From an *ex-ante* point of view, consumption \tilde{c}_i is then the sum of two random variables: average income and a network-based stochastic term. When the altruism network is connected and $\alpha_{ij} > 0 \Rightarrow \alpha_{ij} = 1$, then $\hat{\kappa}_{ij} = 0$ and the second term disappears. Altruistic transfers then yield equal income sharing and efficient insurance, see Proposition 1 in [Bourlès et al. \(2021\)](#). In general, however, altruistic links have strength lower than 1 and *altruistic transfers do not yield efficient insurance*. Agents still bear some idiosyncratic risk, captured by the network-based stochastic term identified in [Lemma 1](#). This term generally depends on q_i 's, the probabilities to be hit by the individual shock, and on the altruism network α .

The insurance game and equilibrium selection. In the first stage, each agent decides how much formal insurance to buy, anticipating how informal transfers will operate *ex-post*. We consider, again and consistently, a simultaneous-move game with complete information. A profile of insurance decisions \mathbf{x}^* is a Nash equilibrium of the insurance game if $\mathbb{E}v_i(x_i^*, \mathbf{x}_{-i}^*) \geq \mathbb{E}v_i(x_i, \mathbf{x}_{-i}^*), \forall i$ and $\forall x_i \in [0, D_i]$. The expected utility is computed over all possible realizations of common and individual shocks.

We will show below that Nash equilibria of the insurance game are generally indeterminate, which naturally raises the question of selection. To make progress on this issue, we propose a natural selection criterion based on conformism. Say that a Nash equilibrium is robust to conformism if it remains an equilibrium when adding vanishingly small conformist pressures. There is widespread evidence that conformism matters in many contexts.¹⁵ It likely plays a role, in addition to altruism, in explaining the decisions to buy unfamiliar insurance products for individuals and households in poor countries' communities.

Formally, let $\varepsilon > 0$ and define payoff functions of a perturbed game, π_ε , as follows:

$$-\ln(-\pi_\varepsilon(x_i, \mathbf{x}_{-i})) = -\ln(-\mathbb{E}v_i(x_i, \mathbf{x}_{-i})) - \frac{1}{2}\varepsilon(x_i - \bar{x})^2. \tag{7}$$

The average community choice, \bar{x} , defines a community norm, and agents incur a small log-additive cost of deviating from the norm.¹⁶ Formally, we say that a Nash equilibrium \mathbf{x}^* of the original game is robust to conformism if \mathbf{x}^* is the limit of a sequence of Nash equilibria of the perturbed game, \mathbf{x}_ε^* , for a sequence of $\varepsilon > 0$ converging to 0.

Note in particular that if the insurance game has a symmetric equilibrium, where all agents buy the same amount of insurance, then this symmetric equilibrium is also a Nash equilibrium of the perturbed game for any $\varepsilon > 0$, and hence is robust to conformism. In a symmetric equilibrium, all agents play the same action and hence incur no cost of deviating from the community norm.

In what follows, our main objectives are to characterize the Nash equilibria of the insurance game and the equilibria robust to conformism, and to analyze their main properties (comparative statics, welfare).

¹⁵ See, e.g., [Boucher \(2016\)](#), [Goeree and Yariv \(2015\)](#) and [Munshi and Myaux \(2006\)](#).

¹⁶ There are different ways to introduce conformism into our setup. Payoffs (7) capture two main features. First, altruistic agents care about others' private well-being and do not internalize others' social preferences. Second, a given distance to the community norm induces a proportional reduction in agents' utilities.

3. Analysis

We develop our analysis in four stages. First, we compute the demand for formal insurance in the absence of altruism. Second, we characterize the Nash equilibria of the insurance game under altruism, and the equilibria robust to conformism. Third, we study how the demand for insurance with altruism compares to the demand without altruism. Fourth, we analyze welfare.

3.1. Insurance demand without altruism

We start by characterizing the demand for formal insurance when agents are not altruistic. Agents are then not affected by others' decisions and can only rely on their own formal insurance as a protection against the common shock. This provides a key benchmark with which to compare insurance demand under altruism. In addition, our equilibrium analysis below uncovers deep connections between both kinds of demands.

Considering all the possible realizations of shocks, the expected utility of agent i as a function of the quantity of formal insurance x_i is equal to

$$\mathbb{E}u_i(x_i) = -[(1 - q_c)(1 - q_i)e^{-A(w_i - px_i)} + (1 - q_c)q_i e^{-A(w_i - px_i - \lambda_i)}] + q_c(1 - q_i)e^{-A(w_i - px_i - \mu_i + x_i)} + q_c q_i e^{-A(w_i - px_i - \mu_i + x_i - \lambda_i)},$$

which simplifies into

$$\mathbb{E}u_i(x_i) = -U_i(1 - q_c + q_c e^{A(\mu_i - x_i)})e^{Apx_i} \tag{8}$$

with $U_i > 0$. Taking derivatives, we see that $\frac{\partial^2 \mathbb{E}u_i}{\partial x_i^2} < 0$ and hence $\mathbb{E}u_i$ is strictly concave in x_i . In addition,

$$\frac{\partial \mathbb{E}u_i}{\partial x_i} = 0 \Leftrightarrow x_i = \mu_i - \frac{1}{A} \ln\left(\frac{1 - q_c}{q_c} \frac{p}{1 - p}\right). \tag{9}$$

Given that the amount of insurance bought, x_i^S , is greater than or equal to 0 and lower than or equal to D_i , this yields¹⁷:

Proposition 1. *In the absence of altruism, individual demand for formal insurance is equal to*

$$x_i^S = \min(\max(\mu_i - \frac{1}{A} \ln(\frac{1 - q_c}{q_c} \frac{p}{1 - p}), 0), D_i) \tag{10}$$

The comparative statics of insurance demand in the absence of altruism follow directly from [Proposition 1](#). We see that individual demand for formal insurance x_i^S is weakly decreasing in price p and weakly increasing with damage μ_i , risk aversion A , and probability of the shock q_c . Individual demand for formal insurance is also unaffected by wealth w_i and by features of the individual shock λ_i and q_i . This reflects the well-known property that choices of agents with CARA preferences do not depend on wealth nor on the presence of an independent shock.

From [Proposition 1](#) and some simple algebra, we can also see precisely when demand is interior, and this will play an important role in the equilibrium analysis. When there is no cap, $\forall i, \mu_i \leq D$, an agent demands full coverage, $x_i^S = \mu_i$ if and only if the price is lower than or equal to the actuarial price, $p \leq q_c$. At the other extreme, let $\bar{p}(\mu_i)$ denote the threshold price level above which individual demand is equal to zero. We have: $\bar{p}(\mu_i) = \frac{q_c e^{A\mu_i}}{1 - q_c + q_c e^{A\mu_i}}$. Then individual demand is interior, $0 < x_i^S < \mu_i$, if and only if $q_c < p < \bar{p}(\mu_i)$. Denote by μ_{min} and μ_{max} the lowest and highest damages in the population. All demands are interior if and only if $q_c < p < \bar{p}(\mu_{min})$. When $\bar{p}(\mu_{min}) \leq p \leq \bar{p}(\mu_{max})$, demand of high-damage agents is interior while low-damage agents do not buy any insurance.

¹⁷ We further show in [Appendix](#) that the profile of individual demands identified in [Proposition 1](#) is also the only profile robust to conformism. Adding a vanishingly small amount of conformism has no impact in the absence of altruism.

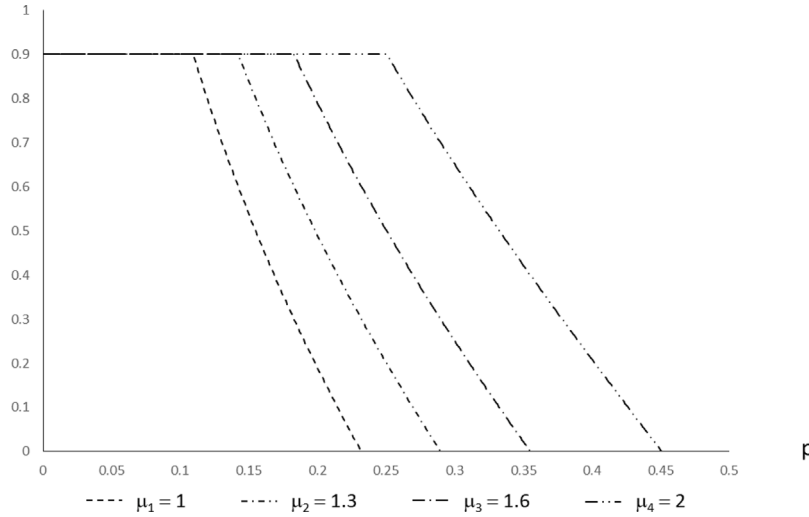


Fig. 1. Individual demands for formal insurance without altruism and with coverage cap $D = 0.9$.

By contrast when the cap is binding, $D \leq \mu_i$, agents may demand the maximal amount of insurance for price levels above the actuarial price. Let $\underline{p}(\mu_i) > q_c$ denote the threshold price level below which individual demand is equal to the coverage cap, $x_i^S = D \Leftrightarrow p \leq \underline{p}(\mu_i)$. Proposition 1 implies that $\underline{p}(\mu_i) = \frac{q_c e^{A(\mu_i - D)}}{1 - q_c + q_c e^{A(\mu_i - D)}}$. Then individual demand is interior, $0 < x_i^S < D$ if and only if $\underline{p}(\mu_i) < p < \bar{p}(\mu_i)$. Moreover, the two threshold prices $\underline{p}(\mu_i)$ and $\bar{p}(\mu_i)$ are both increasing in μ_i .

One implication is that there is no price for which all demands are interior when heterogeneity in damages is large enough. Then, there is no price for which all demands are interior if and only if $\bar{p}(\mu_{min}) \leq \underline{p}(\mu_{max})$, which is equivalent to $\mu_{max} - \mu_{min} \geq D$. In this case, the demand of the agent with largest damage is still at the coverage cap when the demand of the agent with lowest damage becomes equal to zero. By contrast if $\mu_{max} - \mu_{min} < D$, then $\underline{p}(\mu_{max}) < \bar{p}(\mu_{min})$ and all individual demands are interior when $\underline{p}(\mu_{max}) < p < \bar{p}(\mu_{min})$. This range of prices tends to shrink with an increase in damage heterogeneity, through an increase in the largest damage or a decrease in the lowest damage, and with a decrease in the coverage cap.

We illustrate in Fig. 1. We consider a community of $n = 4$ agents with $A = 1$, $q_c = 0.1$, heterogeneous damages $\mu_1 = 1, \mu_2 = 1.3, \mu_3 = 1.6, \mu_4 = 2$, and binding cap $D = 0.9$. We see that individual demand decreases with price over its interior domain and that demands are ordered by increasing damages. Fig. 1 illustrates a situation where demands are never all interior. The price at which the lowest demand becomes equal to zero is lower than the price at which the highest demand becomes lower than the coverage cap.

3.2. Equilibrium characterization

Assume now that agents are altruistic. They decide how much formal insurance to buy conditional on formal insurance decisions of others, and anticipating how informal transfers will operate at the last stage. Our key result here is to show that all agents act as if they are trying to maximize the expected utility of a representative agent with average damages and average demand. Formally, the insurance game has a weighted potential function, equal to the expected utility of this representative agent, see Monderer and Shapley (1996).

To show this, we first compute agents' expected utility under altruism and Assumption 1. From Lemma 1, income after transfers is equal to the sum of average income before transfers and of a network term. This network term depends on agents' relative positions with respect to the agent hit by the individual shock. Therefore, it is stochastically independent from the common shock. Next, let $\bar{\mu} = \frac{1}{n} \sum_i \mu_i$ denote the average damage from the common shock and $\bar{x} = \frac{1}{n} \sum_i x_i$ the average

demand for formal insurance. In our next result, we show that we can reformulate agents' expected utilities under altruism as the product of two terms.

Lemma 2. *When agents are altruistic and Assumption 1 holds, there exist $V_i > 0$ such that*

$$\mathbb{E}v_i(x_i, \mathbf{x}_{-i}) = -V_i(1 - q_c + q_c e^{A(\bar{\mu} - \bar{x})})e^{Ap\bar{x}}$$

The expected utility of any agent under both shocks is proportional to the expected utility of a representative agent with average damage and average insurance demand facing the common shock only. The network position of agent i affects her expected utility only through its impact on the positive proportionality constant V_i . Formally, introduce

$$v(\mu, x) = -(1 - q_c + q_c e^{A(\mu - x)})e^{Ap\bar{x}} \quad (11)$$

In the absence of altruism, the demand of agent i solves the problem of maximizing $v(\mu_i, x_i)$ under the constraint that $x_i \in [0, D_i]$. Under altruism, by contrast, Lemma 2 shows that any agent is trying to maximize the function $v(\bar{\mu}, \bar{x})$ under the same constraint.

This means that $v(\bar{\mu}, \bar{x})$ is a weighted potential of the insurance game, see Monderer and Shapley (1996). Moreover, this function is strictly concave in \bar{x} , and hence concave in \mathbf{x} , which guarantees that Nash equilibria coincide with potential maxima. We provide a detailed proof in Appendix. Let $x^S(\bar{\mu}, \bar{D})$ denote the solution to the problem of maximizing $v(\bar{\mu}, x)$ under the constraint $x \in [0, \bar{D}]$. By Proposition 1, $x^S(\bar{\mu}, \bar{D}) = \min(\max(\bar{\mu} - \frac{1}{A} \ln(\frac{1 - q_c}{q_c} \frac{p}{1 - p}), 0), \bar{D})$. This is the individual demand of an agent with average damages and facing a cap \bar{D} in the absence of altruism.

Theorem 1. *Suppose that agents are altruistic and that Assumption 1 holds. A profile of insurance decisions, \mathbf{x}^* , is a Nash equilibrium of the insurance game if and only if $\bar{x}^* = x^S(\bar{\mu}, \bar{D})$ and $\forall i, x_i^* \in [0, D_i]$. Nash equilibria are the feasible profiles for which average demand is equal to the demand of an agent with average damages in the absence of altruism.*

Theorem 1 shows that in equilibrium, agents act as if they are following the program of a social planner who maximizes a representative agent's expected utility. We emphasize that in our setup, agents typically have misaligned incentives, i.e., they may care about different people and informal transfers yield inefficient risk-sharing. Still, Theorem 1 shows that agents' incentives end up being aligned for formal insurance decisions.

This results has several noteworthy implications. It shows, first, that the insurance game is a game of strategic substitutes. More precisely, the best-response of agent i to insurance decisions \mathbf{x}_{-i} is $x_i =$

$\min(\max(nx^S(\bar{\mu}) - \sum_{j \neq i} x_j, 0), D_i)$. Any agent tends to decrease their demand of formal insurance when others increase their demands. Moreover, this decrease is one-to-one in the domain where individual demand is interior. When agent j adopts one unit of formal insurance, this replaces stochastic $-\frac{1}{n}\mu_j \bar{I}_c$ by non-stochastic $-\frac{1}{n}p$ in the income after transfers of agent i . This reduces income variability and the incentives to also adopt formal insurance.

Second, as long as the altruism network is connected and the size of the individual shock is sufficiently high, *Nash equilibria are unaffected by the network's structure* and by agents' network positions. Any feasible profile where the average demand is equal to the equilibrium value is a Nash equilibrium, regardless of who buys precisely which amount. And these Nash equilibria do not change following changes in the altruism network that respect [Assumption 1](#). In particular, we can show that if [Assumption 1](#) holds for α , it also holds for α' where $\alpha'_{ij} \geq \alpha_{ij}$. Nash equilibria thus do not change following increases in the strength of altruistic ties. This unexpected neutrality is a consequence of the multiplicative separability uncovered in [Lemma 2](#).

Third, *the aggregate demand for formal insurance under altruism is well-defined*, and does not depend on equilibrium selection. For any price of formal insurance p , there is a unique level of equilibrium demand $\sum_i x_i^*(p)$. This demand inherits intuitive comparative statics. It decreases weakly with price p and increases weakly following an increase in damages μ_i , risk aversion A , and probability of the common shock q_c . Moreover, aggregate demand increases linearly with n if damage mean and cap mean are preserved when population size increases. Even though aggregate demand is well-defined, however, individual demands are generally indeterminate. This motivates the analysis of Nash equilibria robust to conformism.

Proposition 2. *There is a unique Nash equilibrium robust to conformism, equal to the equilibrium with lowest variance among all Nash equilibria. When $x^S(\bar{\mu}, \bar{D}) \leq \min(\mu_{\min}, D)$, this equilibrium is symmetric and the demand of all agents is equal to $x_i^* = x^S(\bar{\mu}, \bar{D})$. In this case, $\mu_i \leq \bar{\mu} \Rightarrow x_i^* \geq x_i^S$ and $\mu_i \geq \bar{\mu} \Rightarrow x_i^* \leq x_i^S$.*

When $x^S(\bar{\mu}, \bar{D}) > \min(\mu_{\min}, D)$, there exist two threshold damage levels μ^1, μ^2 such that $\mu_{\min} < \mu^1 < \mu_{\max}$ and $\bar{\mu} < \mu^2 < \mu_{\max}$. Agents with damages below μ^1 demand full insurance, $\mu_i < \mu^1 \Rightarrow x_i^ = \mu_i$, while all agents with damages above μ^1 demand the same amount. In addition, $\mu_i < \mu^2 \Rightarrow x_i^* \geq x_i^S$ and $\mu_i > \mu^2 \Rightarrow x_i^* \leq x_i^S$.*

[Proposition 2](#) shows that adding a vanishingly small amount of conformism is sufficient to break equilibrium indeterminacy. When a symmetric equilibrium exists, it is the unique equilibrium robust to conformism and the demand of all agents is simply equal to the demand of the representative agent. Existence of a symmetric equilibrium is guaranteed, in particular, when the cap is binding for all, $\forall i, D \leq \mu_i$. In the absence of a cap or when the cap is not binding for low-damage agents, a symmetric equilibrium may not exist. In that case, the unique equilibrium robust to conformism has lowest variance among all Nash equilibria. It displays a natural form of constrained symmetry: agents with low damages demand full insurance, while agents with high damages all demand the same amount of insurance. If there are k agents such that $\mu_i > \mu^1$, then this amount is equal to $(nx^S(\bar{\mu}, \bar{D}) - \sum_{\mu_i < \mu^1} \mu_i)/k$.

[Proposition 2](#) further identifies a key implication of altruistic transfers under damage heterogeneity. Altruism tends to homogenize the demands for formal insurance: it increases demand of agents with low damages and reduces demand of agents with high damages. *Ex-post* altruistic transfers leads to align *ex-ante* incentives to buy insurance. All agents act as if they are trying to maximize the utility of a representative agent with average damages. This leads low-damage agents to demand more insurance than without altruism, and high-damage agents to demand less. To analyze the impact of altruism on aggregate demand, we then need to check whether the increases dominate the decreases.

We illustrate [Proposition 2](#) in [Fig. 2](#). Parameter values are the same as in [Fig. 1](#), except that now $\mu_{\max} < D$ and all agents can

obtain full coverage. Individual demands with altruism and in the unique equilibrium robust to conformism are depicted with plain lines. Demands without altruism are depicted with dotted lines. Here, $\bar{\mu} = 1.475$ and the threshold price level above which $x^S(\bar{\mu}) \leq \mu_{\min}$ and a symmetric equilibrium exists is $p = 0.152$. Above this threshold, individual demands with altruism are all equal to the demand of the representative agent. Over the full price range, we see that demand with altruism is larger than without altruism for agents with damages $\mu_1 = 1$ and $\mu_2 = 1.3$ and lower for agents with damages $\mu_4 = 2$. For agent with damage $\mu_3 = 1.6$, demand with altruism is (slightly) higher than without altruism at relatively low prices and lower than without altruism at higher prices.

3.3. Impact of altruism on aggregate demand: no cap

We now analyze how altruism affects the aggregate demand for formal insurance. We consider the situation with no cap in this Section and analyze a binding cap in the next Section. Assume, then, that all agents can be fully covered, $\forall i, \mu_i \leq D$. We show that two different domains emerge. When all selfish demands are interior, increases in demand for low-damage agents induced by altruism exactly compensate the decreases in demand for high-damage agents. Altruism has no impact on aggregate demand. By contrast when low-damage agents are constrained by the zero lower bound in the absence of altruism, they do not buy insurance. This reduces how much their demand increases under altruism. High-damage agents are not constrained, however. Decreases in demand for high-damage agents now dominate increases in demand for low-damage agents, and the overall impact of altruism is to reduce demand for formal insurance.

Denote by x^S aggregate demand in the absence of altruism and x^A aggregate demand under altruism. By [Theorem 1](#), $x^S = \sum_i x^S(\mu_i)$ while $x^A = nx^S(\bar{\mu})$. Moreover, all agents obtain full coverage at low prices, $x_S = x_A = n\bar{\mu}$ if $p \leq q_c$, while no agent buys formal insurance at high prices, $x^S = x^A = 0$ if $p \geq \bar{p}(\mu_{\max})$. Our next result characterizes what happens for intermediate levels of prices.

Theorem 2. *Suppose that all agents can be fully covered, $\forall i, \mu_i \leq D$. Depending on insurance price p , aggregate demand in the absence of altruism, x^S , is equal to, or lower than, aggregate demand under altruism, x^A .*

$$p \leq \bar{p}(\mu_{\min}) \Rightarrow x^A = x^S \text{ and } \bar{p}(\mu_{\min}) < p < \bar{p}(\mu_{\max}) \Rightarrow x^A < x^S.$$

[Theorem 2](#) uncovers the existence of two qualitatively different domains. When $q < p < \bar{p}(\mu_{\min})$, all selfish demands are interior. We know from [Proposition 1](#) that interior demand is a linear function of damage μ_i . The average of the selfish demands is then equal to the demand of a representative agent with average damages. And this neutrality range shrinks following a decrease of the lowest damage. By contrast when $\bar{p}(\mu_{\min}) < p < \bar{p}(\mu_{\max})$, selfish agents with relatively low damages are constrained by the zero lower bound, and do not buy any formal insurance. Demand of the representative agent is then lower than the average of the selfish demands. This substitution range expands following an increase in damage heterogeneity, through an increase in highest damage or a decrease in lowest damage.

3.4. Impact of altruism on aggregate demand: binding cap

We now analyze how altruism affects the aggregate demand for formal insurance under a binding cap, $\forall i, D \leq \mu_i$. The cap gives rise to a range of relatively low prices where demand of high-damage agents is constrained, while demand of low-damage agents is not. On this range, the altruism-induced reductions in insurance demand for high-damage agents are then lower in magnitude while the increases in insurance demand for low-damage agents are unaffected, as compared to the no cap situation. Overall, the increases dominate the decreases, and altruism now leads to a *higher* aggregate demand at relatively low

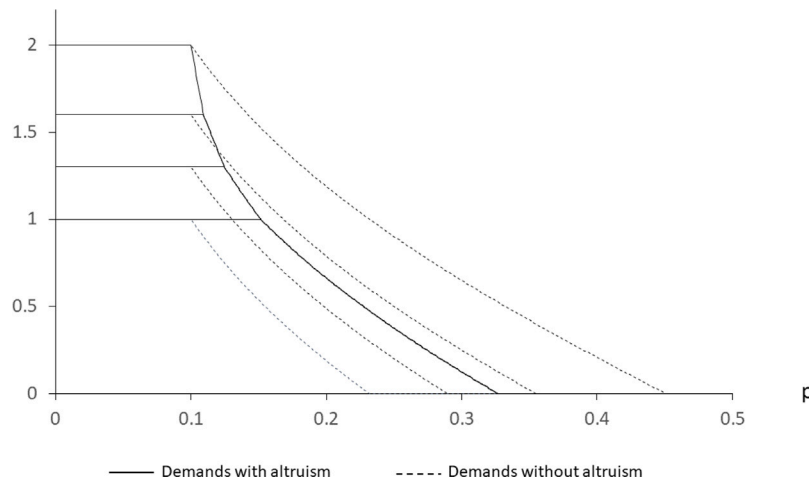


Fig. 2. Demands for formal insurance with and without altruism in the absence of a cap, for $\mu_1 = 1, \mu_2 = 1.3, \mu_3 = 1.6, \mu_4 = 2$.

prices. By contrast, what happens at relatively high prices is similar to the no cap case, and altruism leads to a lower aggregate demand. Whether there exists an intermediate price range where altruism is neutral now depends on damage heterogeneity.

Under a binding cap, demand is maximal at low prices, $x^S = x^A = nD$ if $p \leq \underline{p}(\mu_{min})$, and equal to zero at high prices, $x^S = x^A = 0$ if $p \geq \bar{p}(\mu_{max})$. Our next result characterizes what happens for intermediate levels of prices.

Theorem 3. Suppose that no agent can be fully covered, $\forall i, D \leq \mu_i$. The comparison between aggregate demands in the absence of altruism, x^S , and under altruism, x^A , depends on damage heterogeneity and on insurance price, p .

Under high damage heterogeneity if $\mu_{max} - \mu_{min} \geq D$, then there exists p^* such that $\underline{p}(\mu_{min}) \leq p^* \leq \underline{p}(\mu_{max})$ and $\underline{p}(\mu_{min}) < p < p^* \Rightarrow x^S < x^A$ and $p^* < p < \bar{p}(\mu_{max}) \Rightarrow x^S > x^A$.

Under low damage heterogeneity if $\mu_{max} - \mu_{min} < D$, then $\underline{p}(\mu_{min}) < p < \underline{p}(\mu_{max}) \Rightarrow x^S < x^A$, $\underline{p}(\mu_{max}) \leq p \leq \bar{p}(\mu_{min}) \Rightarrow x^S = x^A$, and $\bar{p}(\mu_{min}) < p < \bar{p}(\mu_{max}) \Rightarrow x^S > x^A$.

Theorem 3 shows that there are two qualitatively different cases. When damage heterogeneity is large and $\mu_{max} - \mu_{min} \geq D$, demands in the absence of altruism are never all interior. In this case, there is no price range over which altruism is neutral. Aggregate demand with altruism is first higher, and then lower than aggregate demand without altruism as insurance price increases. By contrast when damage heterogeneity is low, there is an intermediate price range for which all selfish demands are interior. On this price range, altruism has no impact on aggregate demand. Below this price range high-damage agents are constrained by the cap, while above it low-damage agents are constrained by the zero lower bound. Overall, this shows that with a coverage cap, formal insurance and altruistic transfers are complements at relatively low prices and substitutes at relatively high prices.

We illustrate Theorem 3 in Figs. 3 and 4. The Figures depict the average demand for formal insurance with and without altruism. Fig. 3 represents a situation of high heterogeneity, for the same parameters as in Fig. 1. Fig. 4 represents a situation of low heterogeneity, with damages equal to $\mu_1 = 1, \mu_2 = 1.2, \mu_3 = 1.3, \mu_4 = 1.5$; the other parameters are unchanged. In both cases, the demand is higher under altruism at relatively low prices (complements) and lower at relatively high prices (substitutes). With high heterogeneity, the two demands cross once in the intermediate range while with low heterogeneity, the two demands coincide on some intermediate price range.

3.5. Welfare

Finally, we analyze the welfare properties of the insurance game. By Lemma 2, we know that every agent's expected utility is proportional to a common function, $\mathbb{E}(v_i(x)) = V_i v(\bar{\mu}, \bar{x})$. This implies that agents' interests are aligned. When one agent takes a decision which increases her utility, the utility of all other agents also increases. As a consequence, individual incentives are aligned with social welfare. Formally, say that $x \in \Pi_i[0, D_i]$ is a constrained Pareto optimum of the insurance game if there exists no other profile $x' \in \Pi_i[0, D_i]$ such that $\forall i, \mathbb{E}v_i(x') \geq \mathbb{E}v_i(x)$ and $\exists i, \mathbb{E}v_i(x') > \mathbb{E}v_i(x)$. The Pareto optimum is constrained since we are considering *ex-ante* insurance decisions only, while maintaining the assumption of a non-cooperative Nash equilibrium in *ex-post* altruistic transfers.

Proposition 3. The Nash equilibria of the insurance game coincide with its constrained Pareto optima.

In general, informal transfers generate externalities in decisions to take up formal insurance. When an agent adopts formal insurance, her income stream changes. Through informal transfers, this affects others' income streams and utilities. Under our assumptions, however, and quite remarkably, individual incentives are aligned with social welfare. This is due to the multiplicative separability identified in Lemma 2. Proposition 3 can thus be viewed as a form of *second-best* welfare theorem. Note, however, that the Nash equilibria of the insurance game are not first-best efficient. Since agents are risk-averse, first-best outcomes would involve full insurance over both risks at actuarial prices. However, Proposition 3 shows that, conditional on the fact that the individual risk is imperfectly insured by altruistic transfers, the Nash equilibria of the insurance game are constrained Pareto-efficient.

4. Discussion and conclusion

We provide the first analysis of the introduction of formal insurance into a community connected through altruistic ties. Agents face a common and an individual shock and can buy formal insurance to cover the common shock, up to a coverage cap. We assume that the altruism network is connected, that the individual shock is large, and that damages are heterogeneous. *Ex-post* altruistic transfers make *ex-ante* decisions to buy formal insurance interdependent, and we characterize the Nash equilibria of this insurance game. Under CARA utilities, agents act as if they are trying to maximize the expected utility of a representative agent with average demand and average damages. A main effect of altruism is therefore to homogenize the demands for formal insurance. It increases demand of low-damage agents and decreases

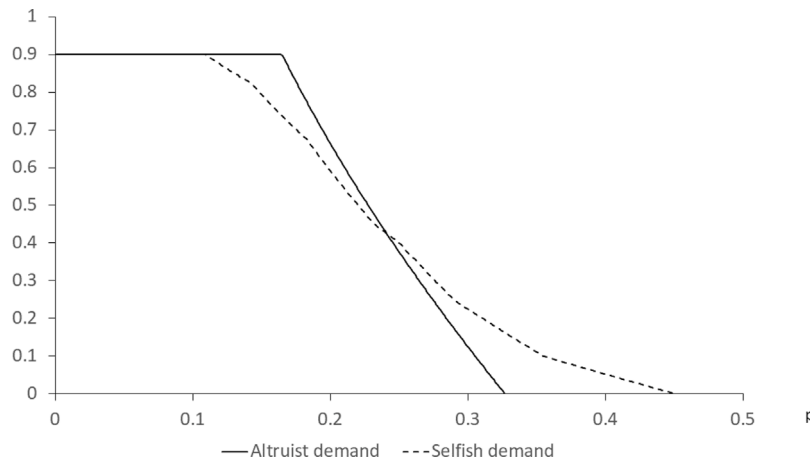


Fig. 3. Impact of altruism on aggregate demand for formal insurance under high damage heterogeneity.

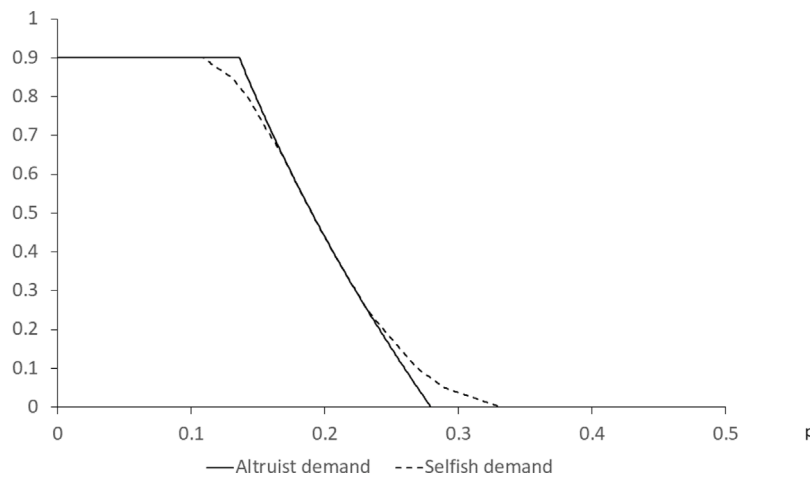


Fig. 4. Impact of altruism on aggregate demand for formal insurance under low damage heterogeneity.

demand of high-damage agents. The overall impact then depends on the interplay between damage heterogeneity and the two constraints on the amount of formal insurance agents can buy, the zero lower bound and the coverage cap. When all agents can be fully covered, the demand for formal insurance is unaffected by altruism at low prices and lower with altruism than without at high prices. When no agent can be fully covered, the demand for formal insurance is higher at low prices under altruism, and lower at high prices. Overall, we find that altruism networks have a first-order effect on the adoption of formal insurance.

Any analysis of formal insurance adoption in the presence of network-based informal transfers has to address, somehow, the combinatorial complexity inherent to networks. In our analysis, we address this complexity by assuming that agents face a large, uninsured individual shock affecting one, and only one, agent at a time. This shock then leads to indirect support from the full community. We argue that these assumptions are realistic, and provide a natural benchmark to analyze the impact of altruism networks on formal insurance. Relaxing these assumptions is an interesting, and challenging, direction for future research.

Assumption 1 and indirect support from the full community may not hold, for instance, if the individual shock is small or if it can be formally insured. In general, Lemma 1 extends as follows.¹⁸ Given equilibrium transfers \mathbf{t} , denote by \mathbf{g} the binary graph of transfers, $g_{ij} = 1 \Leftrightarrow t_{ij} > 0$

and $g_{ij} = 0$ otherwise, by C a weak component of \mathbf{g} , and by \mathbf{g}_C the subgraph of \mathbf{g} induced by C . Then, with CARA utilities,

$$c_i = \bar{y}_C + f(i, \mathbf{g}_C, \alpha) \tag{12}$$

where C is i 's component, $\bar{y}_C = \frac{\sum_{i \in C} y_i}{|C|}$ is the average income in this component and $f(i, \mathbf{g}_C, \alpha)$ depends on the subgraph \mathbf{g}_C and on i 's position in this subgraph. The difficulty is that components C and subgraphs \mathbf{g}_C are equilibrium objects which, in general, vary in complex ways with realized incomes.

This complexity already appears in our setup, for instance, if we assume that there is no individual shock, $\forall i, \lambda_i = 0$. Who gives to whom and how much then depends on baseline wealth w_i , damages μ_i , whether the common shock is realized, and insurance decisions x_i . To analyze the impact of altruism networks on formal insurance, we need to somehow discipline this dependence.

A natural way to go beyond Assumption 1 is to consider networks with specific structures. To illustrate, consider complete networks where every agent cares equally about every one else. We show in the Appendix that for complete networks, our analysis directly extends to situations with multiple individual shocks.¹⁹ Alternatively,

¹⁹ More precisely, Theorems 1–3 extend for complete networks to any stochastic structure of individual shocks under the following three conditions. (1) Individual and common shocks are independent, (2) individual shocks are high enough and not too different from each other in magnitude, and (3) at least one agent gets a shock and at least one agent does not get a shock.

¹⁸ See Theorem 1 in Bourlès et al. (2021).

we could consider separate communities with strong ties within and weak ties across. This could capture, for instance, subcastes in an Indian village, e.g., [Mazzocco and Saini \(2012\)](#). Informal transfers would then mostly happen within communities, and would flow between communities in specific circumstances only, such as large community-level shocks.

Another important direction for future research is to understand how interactions between formal insurance and informal transfers may depend on the motives underlying the informal transfers. We focused on altruism in our analysis. In reality, mutually beneficial informal insurance arrangements are another important source of informal transfers.²⁰ We know that different motives generally yield qualitatively different departures from efficient risk-sharing, see [Bourlès et al. \(2021\)](#). Presumably, different motives will also have different impacts on formal insurance. For instance, under altruism agents partially internalize the effect of their decisions to buy formal insurance on other people they care about. This should help align interests and reduce free-riding and miscoordination in formal insurance decisions, compared to informal insurance.

CRedit authorship contribution statement

Tizié Bene: Writing – review & editing, Writing – original draft, Investigation, Formal analysis, Conceptualization. **Yann Bramoullé:** Writing – review & editing, Writing – original draft, Investigation, Formal analysis, Conceptualization. **Frédéric Deroian:** Writing – review & editing, Writing – original draft, Investigation, Formal analysis, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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²⁰ Social pressure and social norms provide another documented motive. The empirical identification of these motives is the focus of a growing literature, e.g., [Ligon and Schechter \(2012\)](#) and [De Weerd et al. \(2019\)](#). This literature has ignored, so far, the interactions with formal insurance.

Appendix

Proofs of statements on Assumption 1. From Proposition 4 and its proof in [Bourlès et al. \(2021\)](#), we know that for any connected altruism network and any income realizations of others y_{-i} , agent i receives indirect support from the full community if

$$y_i \leq \min_{j \neq i} n(y_j + \frac{1}{A} \hat{\kappa}_{ji}) - \sum_{k \neq i} (y_k + \frac{1}{A} \hat{\kappa}_{ki})$$

When i is hit by the idiosyncratic shock, $y_i = w_i - px_i - (\mu_i - x_i)1_c - \lambda_i$ while $y_j = w_j - px_j - (\mu_j - x_j)1_c$ for $j \neq i$. Thus, i receives indirect support from the full community when

$$\lambda_i \geq w_i - px_i - (\mu_i - x_i)1_c - \min_{j \neq i} n(w_j - px_j - (\mu_j - x_j)1_c + \frac{1}{A} \hat{\kappa}_{ji}) + \sum_{k \neq i} (w_k - px_k - (\mu_k - x_k)1_c + \frac{1}{A} \hat{\kappa}_{ki})$$

Define $\bar{\lambda}$ as the maximum of the right hand side over situations where the common shock occurs or not ($1_c = 0$ or 1) and over choices of formal insurance ($x \in \Pi_i[0, D_i]$). This maximum exists because the right hand side is a continuous function of x and $\Pi_i[0, D_i]$ is a compact space. This shows that [Assumption 1](#) holds if $\forall i, \lambda_i \geq \bar{\lambda}$. \square

Proof of Lemma 1. From [Assumption 1](#) and [Bourlès et al. \(2017\)](#), we know that $u'(c_i) = \hat{\alpha}_{ij} u'(c_{i_0})$ for every $i \neq i_0$. This is equivalent to

$$c_i - c_{i_0} = \frac{\hat{\kappa}_{ii_0}}{A}$$

Conservation of income then implies that

$$\sum_i c_i = nc_{i_0} + \sum_{i \neq i_0} \frac{\hat{\kappa}_{ii_0}}{A} = \sum_i y_i$$

Therefore, $c_{i_0} = \bar{y} - \frac{1}{n} \sum_{i \neq i_0} \frac{\hat{\kappa}_{ii_0}}{A}$ while $c_i = c_{i_0} + \frac{\hat{\kappa}_{ii_0}}{A}$. \square

Proof of Lemma 2. When agent i_0 is hit by the individual shock, average income before transfers is equal to $\bar{y} = \bar{w} - p\bar{x} - (\bar{\mu} - \bar{x})1_c - \frac{\lambda_{i_0}}{n}$. By [Lemma 1](#), income after transfers of agent i is then equal to $c_i = \bar{y} + \frac{\hat{\kappa}_{ii_0}}{A} - \frac{1}{n} \sum_j \frac{\hat{\kappa}_{ji_0}}{A}$. Taking the expectation over realizations of the common shock and the individual shocks yields

$$\mathbb{E}u_i = -(1 - q_c) \left(\sum_{i_0} q_{i_0} e^{-A(\bar{w} - p\bar{x} - \frac{\lambda_{i_0}}{n} + \frac{\hat{\kappa}_{ii_0}}{A} - \frac{1}{n} \sum_j \frac{\hat{\kappa}_{ji_0}}{A})} \right) - q_c \left(\sum_{i_0} q_{i_0} e^{-A(\bar{w} - p\bar{x} - \frac{\lambda_{i_0}}{n} - (\bar{\mu} - \bar{x}) - \frac{\hat{\kappa}_{ii_0}}{A} - \frac{1}{n} \sum_j \frac{\hat{\kappa}_{ji_0}}{A})} \right)$$

which can be rewritten

$$\mathbb{E}u_i = [- \sum_{i_0} q_{i_0} e^{-A(\bar{w} - \frac{\lambda_{i_0}}{n} + \frac{\hat{\kappa}_{ii_0}}{A} - \frac{1}{n} \sum_j \frac{\hat{\kappa}_{ji_0}}{A})}] e^{Ap\bar{x}} [1 - q_c + q_c e^{A(\bar{\mu} - \bar{x})}]$$

And define $U_i = \sum_{i_0} q_{i_0} e^{-A(\bar{w} - \frac{\lambda_{i_0}}{n} + \frac{\hat{\kappa}_{ii_0}}{A} - \frac{1}{n} \sum_j \frac{\hat{\kappa}_{ji_0}}{A})} > 0$ such that $\mathbb{E}u_i = -U_i e^{Ap\bar{x}} [1 - q_c + q_c e^{A(\bar{\mu} - \bar{x})}]$. Next, we have

$$\mathbb{E}v_i = \mathbb{E}u_i + \sum_j \alpha_{ij} \mathbb{E}u_j$$

Define $V_i = U_i + \sum_j \alpha_{ij} U_j > 0$. This yields

$$\mathbb{E}v_i = -V_i e^{Ap\bar{x}} [1 - q_c + q_c e^{A(\bar{\mu} - \bar{x})}]. \quad \square$$

Proof of Theorem 1. The first order conditions of the problem $\max_{x_i \in [0, D_i]} \mathbb{E}v_i(x_i, \mathbf{x}_{-i})$ are: (1) $0 < x_i < D_i \Rightarrow \frac{\partial \mathbb{E}v_i}{\partial x_i} = 0$, (2) $x_i = 0 \Rightarrow \frac{\partial \mathbb{E}v_i}{\partial x_i} \leq 0$, and (3) $x_i = D_i \Rightarrow \frac{\partial \mathbb{E}v_i}{\partial x_i} \geq 0$. These conditions are necessary and sufficient by concavity. By [Lemma 2](#),

$$\frac{\partial \mathbb{E}v_i}{\partial x_i} = V_i \frac{1}{n} \frac{\partial v}{\partial x}(\bar{\mu}, \bar{x})$$

Nash conditions are then equivalent to the first order conditions of the problem $\max_{\mathbf{x} \in \Pi_i[0, D_i]} v(\bar{\mu}, \bar{x})$, which are also necessary and sufficient. Moreover, a solution \mathbf{x}^* is such that $\mathbf{x}^* \in \Pi_i[0, D_i]$ and \bar{x}^* is a solution to $\max_{x \in [0, \bar{D}]} v(\bar{\mu}, x)$. \square

Proof of Proposition 2. Payoffs in the perturbed game are equal to $\pi_i(x_i, \mathbf{x}_{-i}) = V_i v(\bar{\mu}, \bar{x}) e^{\frac{1}{2}\epsilon(x_i - \bar{x})^2}$. The first-order derivative is equal to

$$\frac{\partial \pi_i}{\partial x_i} = \left[\frac{1}{n} \frac{\partial v}{\partial x}(\bar{\mu}, \bar{x}) + \epsilon \left(1 - \frac{1}{n}\right) (x_i - \bar{x}) v(\bar{\mu}, \bar{x}) \right] V_i e^{\frac{1}{2}\epsilon(x_i - \bar{x})^2}$$

and, moreover, π_i is concave in x_i when ϵ is small enough. Introduce the functions $V(\mathbf{x}) = \frac{1}{2} \sum_i (x_i - \bar{x})^2$ and $\varphi(\mathbf{x}) = -\ln(-v(\bar{\mu}, \bar{x})) - \epsilon \left(1 - \frac{1}{n}\right) V(\mathbf{x})$. With some algebra, we see that: $\frac{\partial V}{\partial x_i} = x_i - \bar{x}$ and $\frac{\partial \varphi}{\partial x_i} = -\frac{1}{n} \frac{\partial v / \partial x}{v}(\bar{\mu}, \bar{x}) - \epsilon \left(1 - \frac{1}{n}\right) (x_i - \bar{x})$. Therefore, $\frac{\partial \pi_i}{\partial x_i} = (-v) V_i e^{\frac{1}{2}\epsilon(x_i - \bar{x})^2} \frac{\partial \varphi}{\partial x_i}$.

Since $(-v) V_i e^{\frac{1}{2}\epsilon(x_i - \bar{x})^2} > 0$, the function φ is an ordinal potential for the perturbed game. A profile \mathbf{x} is a Nash equilibrium iff it satisfies the first order conditions of the program $\max_{\mathbf{x}} \varphi(\mathbf{x})$ under the constraints $0 \leq x_i \leq D_i$.

Next, let us show that φ is strictly concave if $\epsilon > 0$ is small enough. Compute the second order derivatives of φ . Introduce the function f such that $f(\mu, x) = \frac{q_c(1-q_c)Ae^{A(\mu-x)}}{(1-q_c+q_c e^{A(\mu-x)})^2} > 0$. We have: $\frac{\partial^2 \varphi}{\partial x_i^2} = -\frac{1}{n^2} f(\bar{\mu}, \bar{x}) - \epsilon \left(1 - \frac{1}{n}\right)^2$ and, if $j \neq i$, $\frac{\partial^2 \varphi}{\partial x_i \partial x_j} = -\frac{1}{n^2} f(\bar{\mu}, \bar{x}) + \epsilon \frac{1}{n} \left(1 - \frac{1}{n}\right)$. Denote by \mathbf{J} a matrix of ones and by \mathbf{I} the identity matrix. The hessian of φ can then be written as

$$\nabla^2 \varphi = \left(-\frac{1}{n^2} f(\bar{\mu}, \bar{x}) + \epsilon \frac{1}{n} \left(1 - \frac{1}{n}\right)\right) \mathbf{J} - \epsilon \left(1 - \frac{1}{n}\right) \mathbf{I}$$

and this matrix is negative definite when ϵ is positive and small enough.²¹

This shows that the perturbed game has a unique Nash equilibrium. Next, consider a sequence of $\epsilon > 0$ converging to 0 and let \mathbf{x}_ϵ be the Nash equilibrium of the perturbed game. Consider a subsequence of \mathbf{x}_ϵ converging to some profile \mathbf{x} . Let us show that \mathbf{x} must be a Nash equilibrium of the original game with lowest variance.

Equilibria of the perturbed game satisfy the necessary and sufficient conditions: $0 < x_{ei} < D_i \Rightarrow \frac{\partial \varphi}{\partial x_i} = 0$; $x_{ei} = D_i \Rightarrow \frac{\partial \varphi}{\partial x_i} \geq 0$; and $x_{ei} = 0 \Rightarrow \frac{\partial \varphi}{\partial x_i} \leq 0$. Taking the limit of these conditions as ϵ tends to 0 yields: $0 < x_i < D_i \Rightarrow \frac{\partial v}{\partial x}(\bar{\mu}, \bar{x}) = 0$; $x_i = D_i \Rightarrow \frac{\partial v}{\partial x}(\bar{\mu}, \bar{x}) \geq 0$; and $x_i = 0 \Rightarrow \frac{\partial v}{\partial x}(\bar{\mu}, \bar{x}) \leq 0$. This shows that \mathbf{x} is a Nash equilibrium of the original game. Suppose that it does not have lowest variance, and let \mathbf{x}' be another Nash equilibrium such that $V(\mathbf{x}') < V(\mathbf{x})$. By continuity, $V(\mathbf{x}_\epsilon)$ converges to $V(\mathbf{x})$. Consider ϵ small enough such that $V(\mathbf{x}_\epsilon) > V(\mathbf{x}')$. Then,

$$\varphi(\mathbf{x}_\epsilon) = -\ln(-v(\bar{\mu}, \bar{x}_\epsilon)) - \epsilon \left(1 - \frac{1}{n}\right) V(\mathbf{x}_\epsilon) < -\ln(-v(\bar{\mu}, \bar{x}')) - \epsilon \left(1 - \frac{1}{n}\right) V(\mathbf{x}')$$

since $-\ln(-v(\bar{\mu}, \bar{x}_\epsilon)) \leq -\ln(-v(\bar{\mu}, \bar{x}'))$ because \mathbf{x}' is an equilibrium of the original game. This shows that \mathbf{x}_ϵ does not maximize φ , a contradiction. Therefore, any converging subsequence of \mathbf{x}_ϵ converges to an equilibrium of the original with lowest variance. We show in the final step that there is a unique equilibrium that minimizes variance over all equilibria. Since the strategy space is compact, \mathbf{x}_ϵ must then converge to this equilibrium.

An equilibrium of the original game with lowest variance solves $\min_{\mathbf{x}} V(\mathbf{x})$ under the constraints $0 \leq x_i \leq D_i$ and $\sum_i x_i = n\mathbf{x}^*$, where \mathbf{x}^* is the equilibrium average demand. Note that we can exclude situations where $x_i = 0$ for some i . In that case, there exists j such that $x_j > \mathbf{x}^*$ and a small Pigou-Dalton transfer from j to i decreases variance. Denote by ν the Lagrange multiplier of the constraint $\sum_i x_i = n\mathbf{x}^*$. The necessary and sufficient first order conditions are: $x_i < D_i \Rightarrow x_i = \mathbf{x}^* + \nu$ and $x_i = D_i \Rightarrow x_i \leq \mathbf{x}^* + \nu$. This implies that agents with relatively low

values of D_i 's demand the maximal amount of insurance, while agents with relatively high value of D_i 's all demand the same amount.

Order agents through increasing D_i and suppose that there are two different equilibria with lowest variance, $\mathbf{x} = (D_1, \dots, D_p, x, \dots, x)$ and $\mathbf{y} = (D_1, \dots, D_q, y, \dots, y)$ with $p < q$. We know that $D_p \leq x$ and $D_q \leq y$. Moreover, $x \leq D_q$ since \mathbf{x} is a feasible profile and $y < x$ since the average is constant and agents $p + 1$ to q have a higher action in the second profile. This means that $y < x$, a contradiction. Therefore, there is a unique equilibrium with lowest variance, and hence a unique equilibrium robust to conformism. Finally, note that if this equilibrium is symmetric, $x_i = x^S(\bar{\mu})$ and hence $\mu_i \leq \bar{\mu} \Rightarrow x_i \geq x_i^S$ while $\mu_i \geq \bar{\mu} \Rightarrow x_i \leq x_i^S$. Otherwise, $x_i = D_i \Rightarrow x_i \geq x_i^S$ and if $x_i = x < D_i$, demand decreases for agents with damage above a threshold and increases for agents with damage below this threshold. \square

Proof of Theorem 2. There are three cases. If $q_c \leq p \leq \bar{p}(\mu_{min})$, then $x^S(\mu_i)$ and $x^S(\bar{\mu})$ are all interior. By Proposition 1, $x^S(\mu_i) = \mu_i - \frac{1}{A} \ln\left(\frac{1-q_c}{q_c} \frac{p}{1-p}\right)$ and hence $x^S = \sum_i \mu_i - \frac{n}{A} \ln\left(\frac{1-q_c}{q_c} \frac{p}{1-p}\right) = n x^S(\bar{\mu}) = x^A$.

If $\bar{p}(\mu_{min}) < p < \bar{p}(\bar{\mu})$, then demand is zero for agents with lowest damages and interior for the representative agent. Then, $x^S(\mu_i) \geq \mu_i - \frac{1}{A} \ln\left(\frac{1-q_c}{q_c} \frac{p}{1-p}\right)$ with at least one strict inequality. This yields $x^S > \sum_i \mu_i - \frac{n}{A} \ln\left(\frac{1-q_c}{q_c} \frac{p}{1-p}\right) = n x^S(\bar{\mu}) = x^A$.

If $\bar{p}(\bar{\mu}) \leq p < \bar{p}(\mu_{max})$, then demand is equal to zero for the representative agents and is positive for agents with highest damages. This implies that $x^S > 0 = x^A$. \square

Proof of Theorem 3. Denote by $\hat{x}_i = \hat{x}(\mu_i) = \mu_i - \frac{1}{A} \ln\left(\frac{1-q_c}{q_c} \frac{p}{1-p}\right)$, such that $x_i^S = \min(\max(\hat{x}_i, 0), D_i)$. (1) Suppose first that $\mu_{max} - \mu_{min} < D$ and $\underline{p}(\mu_{max}) < \bar{p}(\mu_{min})$. (1.1) If $\underline{p}(\mu_{min}) < p < \underline{p}(\mu_{max})$, then $x^S(\mu_{min}) = \hat{x}(\mu_{min}) > 0$ and $x^S(\mu_{max}) = D < \hat{x}(\mu_{max})$. Thus, $x^S = \sum_i x_i^S < \min(nD, \sum_i \hat{x}_i) = x^A$. (1.2) If $\underline{p}(\mu_{max}) \leq p \leq \bar{p}(\mu_{min})$, then $x_i^S = \hat{x}_i$ and $x^S = \sum_i \hat{x}_i = x^A$. (1.3) If $\bar{p}(\mu_{min}) < p < \bar{p}(\mu_{max})$, then $x^S(\mu_{min}) = 0 > \hat{x}(\mu_{min})$ and $x^S(\mu_{max}) = \hat{x}(\mu_{max})$. Thus, $x^S = \sum_i x_i^S > \max(0, \sum_i \hat{x}_i) = x^A$. (2) Suppose, second, that $\mu_{max} - \mu_{min} \geq D$ and $\bar{p}(\mu_{min}) \leq \underline{p}(\mu_{max})$. (2.1) If $\underline{p}(\mu_{min}) < p \leq \underline{p}(\bar{\mu})$, then $x^S(\mu_{min}) < D$ and hence $x^S < nD = x^A$. (2.2) If $\bar{p}(\bar{\mu}) \leq p < \bar{p}(\mu_{max})$, then $x^S(\mu_{max}) > 0$ and hence $x^S > 0 = x^A$. (2.3) If $\underline{p}(\bar{\mu}) \leq p \leq \bar{p}(\bar{\mu})$, then $x^A = n\hat{x}(\bar{\mu}) = \sum_i \hat{x}(\mu_i)$. Therefore, $x^S - x^A = \sum_i x^S(\mu_i) - \hat{x}(\mu_i)$. For a given μ_i , the function $x^S(\mu_i) - \hat{x}(\mu_i)$ is equal to $D - \hat{x}(\mu_i)$ if $0 \leq p \leq \underline{p}(\mu_i)$, 0 if $\underline{p}(\mu_i) \leq p \leq \bar{p}(\mu_i)$, and $-\hat{x}(\mu_i)$ if $\bar{p}(\mu_i) \leq p \leq 1$. This function is thus weakly increasing overall, and strictly increasing over $[0, \underline{p}(\mu_i)] \cup [\bar{p}(\mu_i), 1]$. The sum of these functions is thus strictly increasing over the union of these sets, which is equal to the whole interval $[0, 1]$ since $\bar{p}(\mu_{min}) \leq \underline{p}(\mu_{max})$. Therefore, $x^S - x^A$ is a continuous and strictly increasing function of price, is strictly negative at $\underline{p}(\bar{\mu})$ and strictly positive at $\bar{p}(\bar{\mu})$. Therefore, it is equal to zero precisely once, at p^* , on the interval $[\underline{p}(\bar{\mu}), \bar{p}(\bar{\mu})]$.

Finally, if $\underline{p}(\mu_{min}) < p < \bar{p}(\mu_{min})$, then $x^S(\mu_{min}) = \hat{x}(\mu_{min}) > 0$ and $x^S(\mu_{max}) = D < \hat{x}(\mu_{max})$. Thus, $x^S = \sum_i x_i^S < \min(nD, \sum_i \hat{x}_i) = x^A$. And if $\underline{p}(\mu_{max}) < p < \bar{p}(\mu_{max})$, then $x^S(\mu_{min}) = 0 > \hat{x}(\mu_{min})$ and $x^S(\mu_{max}) = \hat{x}(\mu_{max})$. Thus, $x^S = \sum_i x_i^S > \max(0, \sum_i \hat{x}_i) = x^A$. This shows that $\bar{p}(\mu_{min}) \leq p^* \leq \underline{p}(\mu_{max})$. \square

Proof of Proposition 3. By Theorem 1, we know that the Nash equilibria are the maxima of the function $\varphi : \mathbf{x} \rightarrow v(\bar{\mu}, \bar{x})$. Let us show now that Pareto optima are also the maxima of this function. If \mathbf{x} is not a Pareto optimum, there exists \mathbf{x}' and i such that $\mathbb{E}v_i(\mathbf{x}') > \mathbb{E}v_i(\mathbf{x})$. This implies that $V_i \varphi(\mathbf{x}') > V_i \varphi(\mathbf{x})$ and hence \mathbf{x} is not a maximum of φ . Reciprocally, suppose that \mathbf{x} is not a maximum of φ and let \mathbf{x}' be such that $\varphi(\mathbf{x}') > \varphi(\mathbf{x})$. Then, for every i , $V_i \varphi(\mathbf{x}') > V_i \varphi(\mathbf{x})$ and hence $\mathbb{E}v_i(\mathbf{x}') > \mathbb{E}v_i(\mathbf{x})$ and \mathbf{x} is not a Pareto optimum. \square

Complete networks and multiple shocks

Consider complete networks, with $\alpha_{ij} = \alpha$, as in Arrow (1981). Relax Assumption 1, and assume that all agents in set S are affected by the idiosyncratic shock, with $|S| = s$ and $1 \leq s \leq n-1$. We first characterize

²¹ In this case, $\nabla^2 \varphi = -a\mathbf{J} - b\mathbf{I}$ with $a, b > 0$. For any vector $\mathbf{z} \neq \mathbf{0}$, $\mathbf{z}'(\nabla^2 \varphi)\mathbf{z} = -a(\sum_i z_i)^2 - b(\sum_i z_i^2) < 0$.

equilibrium consumption under the assumption that every agent with a shock receives informal support while every agent without a shock gives informal support. In a second stage, we will derive conditions under which this assumption holds. Under this assumption, every agent with a shock ends up with the same consumption level, denoted by c_L , while every agent without a shock ends up with consumption level c_H . These consumption levels must satisfy two equations: $sc_L + (n-s)c_H = n\bar{y}$ and $c_H - c_L = \kappa/A$ where $\kappa = -ln(\alpha)$. This yields $c_L = \bar{y} - (1 - \frac{s}{n})\frac{\kappa}{A}$ and $c_H = \bar{y} + \frac{s}{n}\frac{\kappa}{A}$.

Next, consider general stochastic structures of individual shocks. Assume that with probability q_S , all agents in set S are affected by the shock. Assume, also, that individual and common shocks are independent. Denote by $1_S(i)$ an indicator variable equal to 1 if $i \in S$ and 0 otherwise, and by $\lambda_S = \sum_{i \in S} \lambda_i$. Expected selfish utility is equal to

$$\mathbb{E}u_i = -(1 - q_c) \sum_S q_S e^{-A(\bar{w} - p\bar{x} - \frac{\lambda_S}{n} + \frac{s}{n}\frac{\kappa}{A} - 1_S(i)\frac{\kappa}{A})} - q_c \sum_S q_S e^{-A(\bar{w} - p\bar{x} - (\bar{\mu} - \bar{x}) - \frac{\lambda_S}{n} + \frac{s}{n}\frac{\kappa}{A} - 1_S(i)\frac{\kappa}{A})}$$

and hence $\mathbb{E}u_i = U_i v(\bar{\mu}, \bar{x})$ with $U_i = \sum_S q_S e^{-A(-\frac{\lambda_S}{n} + \frac{s}{n}\frac{\kappa}{A} - 1_S(i)\frac{\kappa}{A})} > 0$. This means that Lemma 2 extends, and hence that Theorems 1–3 extend.

This reasoning is valid when every agent with a shock receives support while every agent without a shock gives support. In turn, this holds when

$$i \notin S \Rightarrow w_i - px_i - (\mu_i - x_i)1_c \geq \bar{w} - p\bar{x} - (\bar{\mu} - \bar{x})1_c - \frac{\lambda_S}{n} + \frac{s}{n}\frac{\kappa}{A}$$

$$i \in S \Rightarrow w_i - px_i - (\mu_i - x_i)1_c - \lambda_i \leq \bar{w} - p\bar{x} - (\bar{\mu} - \bar{x})1_c - \frac{\lambda_S}{n} - (1 - \frac{s}{n})\frac{\kappa}{A}$$

These inequalities hold for any choice of formal insurance and any realization of the common shock when $\frac{\lambda_S}{n}$ and $\lambda_i - \frac{\lambda_S}{n}$ are large enough. This is guaranteed, for instance, for any δ such that $\lambda_i \in [\lambda, \lambda + \delta]$ for λ high enough since $\frac{\lambda_S}{n} \geq \frac{\lambda}{n}$ and $\lambda_i - \frac{\lambda_S}{n} \geq \frac{\lambda}{n} - \delta$. \square

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